SLAC-PUB-2961
August 1982
(T)

CONSTANT FIELDS IN QUANTUM CHROMODYNAMICS*

Curt A. Flory
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

A space-time constant solution to the equations of motion for Quantum Chromodynamics with massive quarks is shown to exist. This field configuration satisfies several properties that may be of phenomenological interest.

(Submitted to Physical Review D)

*Work supported by the Department of Energy, contract DE-AC03-76SF00515.

Many solutions to the non-Abelian Yang-Mills classical equations of motion which generate a non-vanishing field strength have been shown to exist. Much of the interest has been focused upon Abelian solutions to the non-Abelian equations, where a space-time constant field strength is generated by a linear Abelian gauge potential as exists in classical electrodynamics.[1] In non-Abelian theories, an alternative gauge potential exists which also generates a space-time constant field strength. These configurations are space-time constant gauge fields which give rise to a non-vanishing field strength through the commutator term of the field strength tensor.[2] These different gauge potentials which are associated with identical field strengths can give rise to very different physics,[3] which is a simple manifestation of the Wu-Yang ambiguity.[4] In this note, a space-time constant solution to the equations of motion for QCD with a flavor doublet of color triplet massive quarks will be shown to exist, and some interesting properties exhibited.

The SU(3) invariant action of QCD with massive quarks is given by

$$S = -\frac{1}{4}\int d^4x\ F^a_{\mu\nu}\ F^{\mu\nu}_a + \int d^4x\ \bar{\psi}(i\not{\partial} + g\not{A} - m)\psi \quad , \tag{1}$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu \quad , \tag{2}$$

$$\not{A} = A^a_\mu \gamma^\mu T^a \quad , \tag{3}$$

and T^a the fundamental representation matrices of SU(3). The equations of motion are obtained by extremizing the action with respect to each independent variable, yielding

$$(i\not{\partial} + g\not{A} - m)\psi = 0 \tag{4a}$$

$$\bar{\psi}(-i\overleftarrow{\partial\!\!\!/} + g A\!\!\!/ - m) = 0 \qquad (4b)$$

$$D_\nu^{ab} F_b^{\mu\nu} = g\,\bar{\psi}\,\gamma^\mu\,T^a\,\psi \quad , \qquad (4c)$$

where

$$D_\nu^{ab} = \delta^{ab}\partial_\nu + g f^{acb} A_\nu^c \quad . \qquad (5)$$

When the fields satisfying (4) are required to be space-time constants, the partial differential equations become simply algebraic,

$$(g A\!\!\!/ - m)\psi = 0 \qquad (4a')$$

$$\bar{\psi}(g A\!\!\!/ - m) = 0 \qquad (4b')$$

$$g^2 f^{acb} f^{bde}\, A_\nu^c A_d^\mu A_e^\nu = g\,\bar{\psi}\,\gamma^\mu\,T^a\,\psi \quad . \qquad (4c')$$

Before solving (4'), the following additional constraint will be imposed upon the solution. We will search for a set of fields that is an extremum of the QCD action which generates the vacuum fields necessary to motivate the remarkably successful work of Shifman, Vainshtein, and Zakharov (SVZ).[5] They identify the vacuum correlation of the gluon field

$$\langle 0|F_{\mu\alpha}^a F_{\beta\nu}^a|0\rangle = \frac{A}{3}\,(g_{\mu\beta}g_{\alpha\nu} - g_{\mu\nu}g_{\alpha\beta}) \qquad (6)$$

as the origin of the leading power corrections to perturbative QCD, with A a positive quantity. The ansatz for our solution will first be required to satisfy the restrictions of (6).

By performing contractions of the free Lorentz indices in (6), and introducing the standard definitions

$$E_i^a \equiv F_{0i}^a \quad , \quad B_i^a \equiv \frac{1}{2}\,\varepsilon_{ijk} F_{jk}^a \qquad (7)$$

it is straightforward to show that Eq. (6) implies

$$-\vec{E}^a \cdot \vec{E}^a = \vec{B}^a \cdot \vec{B}^a = A$$
$$\vec{E}^a \times \vec{B}^a = 0 \quad . \qquad (6')$$

The fields of (6') are necessarily complex, with the possible solution $\vec{E}^a = \pm i\vec{B}^a$. (In distinction with electrodynamics, it is thought that complex fields may have physical relevance in non-Abelian gauge theories, provided the action and energy remain real.[6]) Translating this condition onto the space-time constant vector potentials, and using $F^a_{\mu\nu} = gf^{abc}A^b_\mu A^c_\nu$ gives

$$f^{abc}A^b_0 A^c_i = \pm \frac{i}{2} f^{ade} \varepsilon_{ijk} A^d_j A^e_k \quad . \tag{8}$$

Gauge invariance allows space-time constant gauge rotations of the A^a_μ-fields. A global SU(3) rotation can diagonalize the matrix $A^a_\mu A^\mu_b$, which chooses an orthogonal set of at most four 4-vector A^a_μ-fields. It is then easy to see that all the conditions necessary to satisfy the SVZ requirements as written in Eq. (8) are incorporated by the ansatz

$$\begin{aligned} A^4_\mu &= (\lambda,0,0,0) \\ A^5_\mu &= (0,i\lambda,0,0) \\ A^6_\mu &= (0,0,i\lambda,0) \\ A^7_\mu &= (0,0,0,i\lambda) \end{aligned} \tag{9}$$

with all other A^a_μ set equal to zero, and using the standard form for the structure constants. It remains to show that this ansatz yields a solution to the equations of motion for the appropriate choice of λ.

The equations of motion given by (4') must be self-consistently solved for the fermionic fields and the gluonic fields as given by the ansatz of (9). This is done in a straightforward fashion by making explicit the individual color components of the fields. Using the Gell-Mann form for the SU(3) generators[7] and denoting color components by subscripts, Eq. (4a') becomes

$$g A_4 \psi_3 - ig A_5 \psi_3 - 2m\psi_1 = 0$$

$$g A_6 \psi_3 - ig A_7 \psi_3 - 2m\psi_2 = 0 \qquad (10)$$

$$g A_4 \psi_1 + ig A_5 \psi_1 + g A_6 \psi_2 + ig A_7 \psi_2 - 2m\psi_3 = 0 \quad ,$$

or, more simply

$$\psi_1 = \frac{g}{2m} (A_4 - iA_5)\psi_3$$

$$\psi_2 = \frac{g}{2m} (A_6 - iA_7)\psi_3$$

$$\left[(A_4 + iA_5)(A_4 - iA_5) + (A_6 + iA_7)(A_6 - iA_7) - \frac{4m^2}{g^2}\right]\psi_3 = 0 \qquad (11)$$

Implementing the ansatz of Eq. (9), we quickly find that $\lambda = m/g$, and two independent fermionic solutions to (11) exist which differ only by their spin eigenvalue. Assigning the two different solutions to the two different fermion flavors (denoted by superscript u,d), yields the explicit solution

$$\psi_1^u = 0 \qquad \psi_1^d = N \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\psi_2^u = N \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \qquad \psi_2^d = 0 \qquad (12)$$

$$\psi_3^u = N \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \qquad \psi_3^d = N \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

with N an overall normalization. Similar analysis of (4b') gives $\bar{\psi}_a^{u,d} = (\psi_a^{u,d})^t$. Note that $\bar{\psi} \neq \psi^+\gamma^0$ for classical anti-commuting Fermi fields viewed as integration variables in the functional integral formulation of QCD.[8] $\bar{\psi}$ and ψ are totally independent variables, which is necessary to have a sensible integration theory.

The last equation of motion that must be satisfied is (4c'), which reduces to

$$\frac{3m^2}{2g} A^a_\mu = \bar{\psi} \gamma_\mu T_a \psi \tag{13}$$

for the gluon fields given by (9). It is simple algebra to verify that the fermionic solutions of (12) satisfy this equation for $N^2 = 3m^3/8g^2$.

Using Eq. (7), the non-zero components of the field strength generated by the above solution are

$$\begin{aligned} E^1_z &= \frac{im^2}{2g} & B^1_z &= \frac{m^2}{2g} \\ E^2_y &= \frac{im^2}{2g} & B^2_y &= \frac{m^2}{2g} \\ E^3_x &= \frac{im^2}{2g} & B^3_x &= \frac{m^2}{2g} \\ E^8_x &= \frac{i\sqrt{3}\,m^2}{2g} & B^8_x &= -\frac{\sqrt{3}\,m^2}{2g} \end{aligned} \tag{14}$$

The existence of these non-zero fields at an extremum of the QCD action perhaps implies a complicated vacuum structure. At the tree level, operators are observed to have non-zero vacuum expectation values (VEV). In particular, for the above solution,

$$\begin{aligned} \langle g^2 F^a_{\mu\nu} F^{\mu\nu}_a \rangle &= 6m^4 \\ \varepsilon_{\mu\nu\alpha\beta} \langle F^{\mu\nu}_a F^{\alpha\beta}_a \rangle &= 0 \\ \langle (\vec{E}^a \cdot \vec{E}^a + \vec{B}^a \cdot \vec{B}^a) \rangle &= 0 \\ \langle \bar{\psi}\psi \rangle &= 6\, m^3/g^2 \end{aligned} \tag{15}$$

It is straightforward but tedious to also verify that the solution exhibited has a symmetry where, for any of the $A^a_\mu \to -A^a_\mu$, another solution exists which yields the same VEV's as (15). The interesting implication of this symmetry is that $\langle F^a_{\mu\nu} \rangle = 0$ as all extrema of the action are summed.

The above field configurations may be of physical relevance for several reasons. It is possible that the solution may lead to a more fundamental formulation of the work of Shifman, Vainshtein, and Zakharov by illuminating details of the field potentials leading to the field strength vacuum expectation values. Also, this solution is an inherently non-Abelian field configuration, which satisfies the equations of motion by having a fermion background field as the source term. The fermion background field also satisfies the equations of motion in a self-consistent fashion, thus allowing for our gauge invariant result. This is in marked distinction to other work with inherently non-Abelian fields where an external source is necessary to support the configuration, and thus manifest gauge and Lorentz invariance lost.

It is not expected that the true physical vacuum would be frozen in the appropriately averaged configuration of (14). Entropy effects would be expected to dominate at large distance scales, and perhaps localized domains of different orientations could coexist on this larger scale. While these interpretations are pursued, it is also important to analyze quantum fluctuations about these configurations to determine the vacuum stability properties. These topics are currently being investigated.

ACKNOWLEDGEMENTS

I would like to thank P. Sikivie for a useful discussion.

REFERENCES

[1]G. K. Savvidy, Phys. Lett. 71B, 133 (1977); S. G. Matinyan and G. K. Savvidy, Nucl. Phys. B134, 539 (1978); N. K. Nielsen and P. Olesen, ibed. B144, 376 (1978); P. Sikivie, Phys. Rev. D 20, 877 (1979); H. Leutwyler, Nucl. Phys. B179, 129 (1981).

[2]R. Akhoury and W. I. Weisberger, Nucl. Phys. B174, 225 (1980); P. P. Martin and W. Mecklenburg, Phys. Rev. D 22, 3088 (1980); T. N. Tudron, Phys. Rev. D 22, 2566 (1980).

[3]L. S. Brown and W. I. Weisberger, Nucl. Phys. B157, 285 (1979).

[4]T. T. Wr and C. N. Yang, Phys. Rev. D 12, 3843 (1975).

[5]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147, 385 (1979).

[6]P. P. Martin and W. Mecklenburg, Phys. Rev. D 22, 3088 (1980).

[7]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[8]S. Coleman, in The Whys of Subnuclear Physics, International School of Subnuclear Physics, Erice, Italy, 1977, edited by I. Zichichi (Plenum Press, New York, 1979).