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EFFECTIVE POTENTIALS FOR SUPERSYMMETRIC THREE SCALE HIERARCHIES*

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ABSTRACT

We consider the effective potential in models in which supersymmetry breaks at a scale μ but the Goldstino couples only to fields of mass $M \gg \mu$. We show that all large perturbative logarithms are removed by taking the renormalization point to be $O(M)$. This makes it possible to calculate the effective potential at large X in those inverted hierarchy models where the Goldstino couples only to superheavy fields. A general formula for the one-loop logarithm in these models is given. We illustrate the results with an SU(n) example in which the direction as well as the magnitude of the gauge symmetry breaking is undetermined at tree level. For this example a large perturbative hierarchy does not form and the unbroken subgroup is always $SU(n-1)\times U(1)$. In an Appendix we show that O'Raifeartaigh models with just one undetermined scalar field always have a decoupled Goldstino when the undetermined field is large, but that this need not be true in more general inverted hierarchy models.

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1. INTRODUCTION

Several authors[1-7] have recently considered models in which supersymmetry is broken at a scale μ midway between the superheavy scale M and the weak scale $\sim\mu^2/M$. In these models the Goldstino may couple directly only to superheavy fields, so that supersymmetry breaking in the low energy theory is suppressed by powers of 1/M. In Ref. 6, L. Susskind and the author have analyzed the low energy physics of these models by integrating out the superheavy fields to obtain effective supersymmetric interactions involving the Goldstino and the other light fields. The three scale structure is found to be stable in most cases.

The method used in Ref. 6 can be applied to the calculation of the vacuum energy in these models. Models with broken sypersymmetry typically have many degenerate vacua at tree level. Perturbative corrections to the energy determine which is the true vacuum of the theory. In a model with multiple scales, perturbation theory can lead to large logarithms of the ratios of scales. In Section 2 we will show that the special structure of the theories considered here leads to a simple result: there are no large logarithms in the order μ^4 piece of the effective potential when all fields and couplings are renormalized at the scale M.

The original motivation for this work was to control the large logarithms in the effective potential for inverted hierarchy models.[8] These models are studied in Sec. 3. For those inverted hierarchy models in which the Goldstino couples only to heavy fields, the result of Section 2 makes it possible to obtain the effective potential at large X by using the renormalization group. The X dependence of the vacuum energy is governed by the β and γ functions of the theory above the scale X. From this a general formula for the coefficient of the one loop logarithm in the effective potential can be derived. Our general formalism is illustrated with an SU(n) model in which the direction of the symmetry breaking as well as the magnitude is undetermined at tree level. It is found that this particular model does not develop a large perturbative hierarchy for reasonable couplings, and that the one loop effective potential determines the unbroken symmetry to be SU(n-1) × U(1).

Section 4 briefly discusses supergravity, which can make a significant contribution to the effective potential, some recent papers [7,9-11,30] on the effective potential in inverted hierarchy models, and the extension of the results to hierarchy models in which the Goldstino does not decouple. An Appendix derives some general results about the scalar potential in O'Raifeartaigh models.[12] We show for simple O'Raifeartaigh models (those with only one undetermined scalar field) that when the scalar field is large the Goldstino couples at tree level only to superheavy fields. In more general inverted hierarchy models (those with multiple undetermined fields or D-term SS breaking) this need not be true.

2. THE EFFECTIVE POTENTIAL

Start with a general supersymmetric theory, with chiral superfields $\hat{A}_i$ having components (A_i, ψ_i, F_i) and gauge superfields $\hat{V}_a$ with components $(V_a^\mu, \lambda_a, D_a)$. The Lagrangian is[13]

$$\tfrac{1}{2}(\hat{W}^\alpha \hat{W}_\alpha)_F + (\hat{A}^+ e^{gV} \hat{A})_D + \left\{(W(\hat{A}))_F + \text{h.c.}\right\} + \xi_a(\hat{V}_a)_D \tag{2.1}$$

where $W(\hat{A})$ is the superpotential, and ξ_a, the coefficient of the Fayet-Iliopoulos term, may be nonzero only for U(1) components of the gauge group. The tree level scalar potential is

$$U^{(0)}(A) = \sum_i (W,_i(A))^* W,_i(A) + \sum_a \tfrac{1}{2}(D_a(A,A^*))^2 \tag{2.2}$$

$$D_a(A,A^*) = -\tfrac{1}{2} g_a A_i^* \tau_{ij}{}^a A_j - \xi_a \tag{2.3}$$

One is interested in the radiative corrections to $U(A)$ at values A_i^0 of the scalar fields such that the tree level potential (2.2) is minimized.[14] For convenience we restrict our attention to the case that[15]

$$D_a(A^0, A^{0*}) = 0 \tag{2.4}$$

The Lagrangian may be expanded in terms of the shifted superfields,

$$\hat{A}'_i = \hat{A}_i - A_i^0 \quad . \tag{2.5}$$

The Goldstino superfield is identified as

$$\hat{X} = \sum_i F_i^{0*} \hat{A}'_i / f \tag{2.6}$$

where

$$F_i^0 = -(W,_i(A^0))^* \tag{2.7a}$$

$$f = \sum_i |F_i^0|^2 \tag{2.7b}$$

It is shown in the Appendix that $\hat{X}$ is massless at tree level. For the models to be considered here the $\hat{V}^a$ and the remaining linear combinations of the $\hat{A}'_i$ divide into heavy gauge and matter superfields, $\hat{V}_H{}^a$ and $\hat{H}_i$, and light gauge and matter superfields, $\hat{V}_L{}^a$ and $\hat{L}_i$. From (2.4) and (2.6), the only nonvanishing auxiliary field at tree level is $F_X{}^0 = f$. Fields of mass $O(\mu)$, $O(\mu^2/M)$, and zero have been grouped together as "light". The models considered here and in Ref. 6 are required to have the property that at tree level $\hat{X}$ couples only to the heavy fields.

To find the scalar potential, first obtain the full effective potential $U(A^0,F_X,F_L{}^i,D_L{}^a)$ for all the light scalar fields, dynamical and auxiliary. This is defined by summing all graphs with external F_X, $F_L{}^i$, and $D_L{}^a$ fields (recall that A^0 has already been shifted away in (2.5)) and which are one particle irreducible (1PI) with respect to $\hat{X}$, $\hat{V}_L{}^a$, and $\hat{L}_i$. Including graphs which are one particle reducible (1PR) with respect to the heavy fields takes the place of explicitly minimizing the effective potential for these fields. Extremizing $U(A^0,F_X,F_L{}^i,D_L{}^a)$ with respect to the auxiliary fields then leads to the scalar potential $U(A^0)$. This gives the same result as would have been found by working with the dynamical component fields from the start. In the latter case one does not extremize the potential with respect to the auxiliary fields, but the set of 1PI diagrams is correspondingly larger, since a graph which can only be divided by cutting an auxiliary field propagator is 1PI when written in terms of the dynamical fields.

Supergraphs can be used to best advantage by writing $U(A^0,F_X,F_L{}^i,D_L{}^a)$ as $U(A^0,\hat{X},\hat{L}_i,\hat{V}_L{}^a)$, where

$$\hat{X} = \theta^2 F_X$$

$$\hat{L}_i = \theta^2 F_L{}^i$$

$$\hat{V}_L{}^a = \tfrac{1}{2}\theta^2\bar{\theta}^2 D_L{}^a \qquad (2.8)$$

The effective action then consists in the usual fashion of superspace integrals of products of superfields.[16] For example the tree level SS effective potential is

$$U(A^0,\hat{X},\hat{L}_i,\hat{V}_L{}^a) = -(\hat{X}^+\hat{X})_D - (\hat{L}_i{}^+\hat{L}_i)_D - \tfrac{1}{2}(\hat{W}_L{}^{a\alpha}\hat{W}_{L\alpha}{}^a)_F + [(\hat{X})_F + (\hat{X}^+)_{F*}]f + \ldots. \qquad (2.9)$$

where the ellipsis represents terms such as $[L_iL_j]_F$ which vanish for the values (2.8). The form of the linear term in (2.9) follows from (2.6) and (2.7). From (2.9),

$$U(A^0,F_X,F_L{}^i,D_L{}^a) = -F_X{}^*F_X - F_L{}^{i*}F_L{}^i - \tfrac{1}{2}D_L{}^aD_L{}^a + (F_X + F_X{}^*)f \qquad (2.10)$$

which extremizes to

$$U(A^0) = f^2 \qquad (2.11)$$

reproducing (2.2). It will be shown below that the only significant radiative contribution is to the coefficient of $F_X{}^*F_X$ in (2.10) (except when $\hat{X}$ can mix with other light fields). This would follow immediately, by dimensional analysis, if we had only graphs with internal heavy lines, but it is necessary to give some attention to graphs with internal light lines.

Radiative corrections to the effective potential are restricted by the GRS theorem[16] to be D-terms (the result of Ref. 16 was for 1PI graphs, but it can be readily extended to any graph that contains a loop). This means that any correction to (2.9) must be at least quadratic in F or linear in D. For gauge symmetries which remain unbroken below μ, the linear D term cannot be induced.[17,18] It may be induced at $O(\mu)$ for gauge symmetries broken at that scale, but it can always be removed by a small ($O(\mu)$) shift in the scalar fields, leaving the vacuum energy unchanged.[19] Thus, radiative corrections to Eq. (2.10) are at least quadratic in the auxiliary fields.

Let us first consider radiative corrections involving only external $\hat{X}$ fields. Figure 1 shows a one loop correction to the coefficient of $(\hat{X}^+\hat{X})_D$, Γ_{X^+X}, in the supersymmetric effective potential. This graph gives rise to $\log(\Lambda^2/m^2)$, where Λ is the renormalization point and m is the mass of the field circulating in the loop. Since $\hat{X}$ couples only to superheavy fields, there will be no large logarithm when $\Lambda \sim M$. It is clear that if $\hat{X}$ coupled to lighter fields as well, no single choice of Λ would remove all large logarithms.

Higher order contributions to Γ_{X^+X} will not contain large logarithms as long as all internal lines are superheavy and $\Lambda \sim M$. The potentially dangerous graphs are those such as Fig. 2 with internal light lines. Since all external momenta vanish, this graph could have a singular dependence on the light internal masses from the region where the internal momentum, q, is much less than M. In fact, this does not happen. In the small q region the heavy blobs can be replaced with

$$\frac{1}{M}(\hat{X}\hat{L}\hat{L}^+)_D \tag{2.12}$$

plus operators of higher dimension suppressed by further powers of 1/M.

With this effective vertex the contribution of the graph to $\Gamma_{X^\dagger X}$ is of the form

$$\frac{1}{M^2}\int\frac{d^4q}{q^2} \tag{2.13}$$

for $q \ll M$. This is quadratically convergent in the infrared and gives an $O(1)$ contribution only for $q \sim M$. (Recall that we are studying $O(\mu^4)$ in the effective potential, so we need keep only $O(1)$ in $\Gamma_{X^\dagger X}$.) This is true as well for all other $\Gamma_{X^\dagger X}$ graphs with internal light lines. Absorbing all heavy lines into effective vertices, there will be at least two dimension 5 vertices or one dimension 6 vertex coupling the external $\hat{X}$ to the light internal lines. This makes the infrared behavior of the graph under uniform scaling of the light line momenta at least two powers better than the canonical logarithmic divergence for $\Gamma_{X^\dagger X}$, as seen in Eq. (2.13) for the example of Fig. 2.[20] Thus, the dominant contribution comes when the light line momenta are scaled up until at least one is $O(M)$; the line may then be absorbed into a hard vertex and the argument repeated until all light line momenta are $O(M)$. The conclusion is that in graphs contributing to $\Gamma_{X^\dagger X}$, all lines are either heavy or at large momentum. Other regions are suppressed by powers of q/M. From this it follows that:

(a) There can be no large logarithm in the $O(1)$ part of $\Gamma_{X^\dagger X}$ when all fields and couplings are renormalized at $\Lambda \sim M$.

(b) The dependence of $\Gamma_{X^\dagger X}$ on the $O(\mu)$ dimensional couplings must bring in a power of μ/M. $\Gamma_{X^\dagger X}$ may be evaluated with these couplings set to zero. These properties will be referred to as (a) and (b) throughout the paper.

The next correction to the effective potential is $(\hat{X}^+\hat{X}D^2\hat{X})_D \sim F_X{}^*F_X{}^2$. This is of dimension 6 and any graph with all internal lines superheavy must give a coefficient $\sim 1/M^2$. The whole term is then of order μ^6/M^2 and can be neglected. Analysis parallel to that used for Γ_{X^+X} shows that this continues to hold true when there are internal light lines.

All higher terms with only $\hat{X}$ fields externally are suppressed by powers of M as well. There are some infrared divergences, but they do not affect this conclusion. For example, Fig. 3 generates $(\hat{X}^+\hat{X}\bar{D}^2\hat{X}^+D^2\hat{X})_D \sim (F_X{}^*F_X)^2$ with a coefficient of $1/M^4$ times a large logarithm. The whole term is then of order

$$\frac{\mu^8}{M^4}\,\ell n\,(M^2/\mu^2) \qquad (2.14)$$

which is negligible. Other terms have greater infrared divergences, but these are spurious, resulting from multiple insertions of operators such as $(\hat{X}\hat{L}^+\hat{L})_D$ into a light line. Summing these insertions into the propagator just gives a field dependent logarithm, $\ell n\,(M^2/F_X{}^*F_X)$, in (2.14) and leaves the second derivative of the effective potential, Γ_{X^+X}, essentially unchanged (by the same logic as (b)). To summarize, the only significant radiative correction found thus far is to the coefficient of $F_X{}^*F_X$ in (2.10), and this correction satisfies (a) and (b).

Consider now terms in the supersymmetric effective potential with external light fields but no external $\hat{X}$. One knows that (a) and (b) need not hold for these terms. However, since they must be quadratic in the light auxiliary fields, they can never drive the extremum away from it tree level value

$$F_L{}^i = D_L{}^a = 0 \qquad (2.15)$$

Thus, by themselves they do not contribute to the effective scalar potential.

Finally consider terms involving both $\hat{X}$ and the other external light fields. Contributing graphs must involve at least one effective hard vertex coupling $\hat{X}$ to the other fields. Suppose first that there is no light field which is allowed, by the symmetries unbroken at A_i^0, to mix with $\hat{X}$. Then the effective vertex of lowest dimension is $(\hat{X}\hat{L}^+\hat{L})_D$, of dimension 5: this has coefficient 1/M. All terms involving $\hat{X}$ plus other light fields are thus suppressed by a power of 1/M and do not contribute at order μ^4. Then (2.15) holds to order μ^2 and the relevant part of the SS effective potential is just

$$U(A^0,F_X,F_L{}^i,D_L{}^a) = -\Gamma_{X^+X}(A^0)F_X{}^*F_X + (F_X + F_X{}^*)f \qquad (2.16)$$

so that

$$U(A^0) = f^2/\Gamma_{X^+X}(A^0) \qquad (2.17)$$

and the absence of large logarithms in the scalar potential follows from (a).

If $\hat{X}$ can mix with a light field, say $\hat{Y}$, the term

$$(\hat{X}^+\hat{Y})_D = F_X{}^*F_Y \qquad (2.18)$$

appears in the effective potential unsuppressed by M and drives a nonzero value for F_Y. (There is no dimension 4 gauge invariant SS operator which contains $F_XD_L{}^a$ and so can mix $\hat{X}$ with a light gauge field.) If it happens that $\hat{Y}$ is also decoupled from the other light fields, the argument applied before can be extended. For example, Eq. (2.18) becomes

$$U(A^0) = f^2(\Gamma^{-1}(A^0))_{X^+X} \qquad (2.19)$$

as Γ is now a matrix. This often happens in inverted hierarchy models, as will be seen in the next section. If $\hat{Y}$ couples to light fields, the effective potential will contain large logarithms at some order (though not before three loops).[21] These logarithms do not make perturbation theory invalid, but one must work harder. The heavy fields are integrated out with $\Lambda \sim M$, and then the effective couplings are run down to the appropriate scale to evaluate other terms in the SS effective potential. Incidentally, such Goldstino mixing was also the one case found in Ref. 6 for which radiative corrections could induce large SS breaking for the light fields.

3. INVERTED HIERARCHY MODELS

In Witten's inverted hierarchy models[8], SS breaks at tree level and a scalar field X (or perhaps several scalar fields) is undetermined. The one loop correction to the effective potential[22]

$$V^{(1\ \mathrm{loop})}(X) = \sum_i \frac{(-1)^F}{64\pi^2} m_i(X)^4 \ln (m_i(X)^2/\Lambda^2) \tag{3.1}$$

breaks the tree level vacuum degeneracy. For some values of the parameters, the one loop effective potential decreases as X grows, so the stable minimum, if it exists, lies at $X \gg \mu$, μ being the typical scale in the Lagrangian. Thus, one would like to determine the behavior of the effective potential at large X. Factors of $\ln (X^2/\Lambda^2)$ make simple perturbation theory invalid in this region. On the other hand, if one knew that factors such as $\ln (\mu^2/\Lambda^2)$ did not appear in the effective potential, choosing $\Lambda^2 \sim X^2$ would make perturbation theory valid as long as couplings were small.

Banks[2,11] observed that in simple examples of the inverted hierarchy, the Goldstino coupled only to heavy fields. In the Appendix, this is shown to be true in all O'Raifeartaigh models with a single undetermined scalar field. For models in which the Goldstino decouples from light fields, the analysis of the preceding section makes it possible to apply the renormalization group to the effective potential. Take first the standard renormalization group equation[22]

$$\left[\Lambda \frac{\partial}{\partial\Lambda} + \sum_\alpha \beta_\alpha \frac{\partial}{\partial g_\alpha} - X_i\gamma_{ij} \frac{\partial}{\partial X_j} - Y_k(X)\gamma_{kj} \frac{\partial}{\partial X_j}\right] U(X_i,\Lambda,g) = 0 \tag{3.2}$$

where g_α includes both dimensionless and dimensional couplings. Here $U(X_i,\Lambda,g)$ is a "reduced" effective potential – a function only of X_i, which are the undetermined scalar fields and those superlight (mass $\sim\mu^2/M$) fields with which they can mix. The effective potential has already been minimized with respect to the heavy fields and those mass $\sim\mu$ fields which can mix with the $\hat{X}_i$ (see Footnote 21). These (dependent) fields have been designated $Y_k(X)$. The Y_k are all $O(\mu)$ or less, while the $\partial/\partial X_j$ each bring in a factor of $1/M$ (by reasoning parallel to Section 2) so the term proportional to $Y_k(X)$ in (3.2) can be neglected. When the conditions of Section 2 are met, Eqs. (2.17) and (2.19) show that $U(X_i,\Lambda,g)$ depends on f, which is a function of the couplings but not of X_i or Λ[23], and on Γ, which by (b) is a function of X_i and Λ but not of the dimensional couplings. Thus,

$$U(X_i,\Lambda,g) = U(X_i/\Lambda, X_i/X_j, g) + O(\mu^5/M) \tag{3.3}$$

Combined with the standard equation (3.2), this gives to order μ^4,

$$X_i(\delta_{ij} + \gamma_{ij}) \frac{\partial}{\partial X_j} U(X_i,\Lambda,g) = \sum_\alpha \beta_\alpha \frac{\partial}{\partial g_\alpha} U(X_i,\Lambda,g) \tag{3.4}$$

Thus, the variation of U along certain curves in configuration space, defined by the LHS of (3.4), is given simply by running the couplings. For inverted hierarchy models without a decoupled Goldstino, this is not the case. For these models the potential depends on ratios of the dimensional couplings with X_i or Λ, and Eq. (3.3) does not hold.

The β functions which appear in (3.2) are always those for the unbroken theory above X, not those which apply between X and μ. This would seem to conflict with the idea that μ is the "fundamental" scale of the theory, but it must be so.[24] One way to see this is to note that $g(X)$ would be obtained from $g(\Lambda_0)$, with $\Lambda_0 \gg X$, by using the β-functions of the unbroken theory. As X varies, $g(\Lambda_0)$ is essentially constant. (It is defined, for example, in terms of a Green's function at Euclidean momenta $\sim\Lambda_0$, and depends on an external field X only as powers of X/Λ_0). Thus, the change in $g(X)$ is given entirely by the β-function. On the other hand, $g(X)$ would be obtained from $g(\mu)$ with the low energy β function, but as X varies so does the initial value $g(\mu)$. The point is that μ is not really the "fundamental" scale, as we continue to apply local field theory down to much shorter distances.

The initial value of $U(X_i/\Lambda, X_i/X_j, g)$ may safely be calculated for $\Lambda^2 = \sum X_i^2$, by (a). Again, this would not be true if the Goldstino coupled to light fields. To leading order it is just the tree level energy

$$\epsilon^{(0)}(g) = \min_A U^{(0)}(A,g) = \min_A \sum_k \left|W_{,k}(A,g)\right|^2 \tag{3.5}$$

To this same order we may use the one loop β-function and neglect the γ_{ij} in (3.4). This does not change the qualitative behavior of the effective potential or the $O(1/g^2)$ part of $\ln(X_{min}/\mu)$ if there is a minimum; to get the $O(1)$ part of $\ln(X_{min}/\mu)$ would require going to higher order. The solution to (3.4) is now

$$U(X_i,\Lambda,g) = \epsilon^{(0)}(g(\tfrac{1}{2}\ln(\sum_i X_i^2/\Lambda^2)) \tag{3.6}$$

where

$$\frac{d}{dt} g_\alpha(t) = \beta_\alpha(g(t))$$

There is a general one loop formula for the RHS of (3.4). We have

$$\sum_\alpha \beta_\alpha \frac{\partial}{\partial g_\alpha} \epsilon^{(0)}(g) = \sum_\alpha \left(\beta_\alpha \frac{\partial}{\partial g_\alpha} + \beta_\alpha \frac{\partial A_i{}^{min}}{\partial g_\alpha} \frac{\partial}{\partial A_i} \right) \Big|\sum_k W,_k(A,g)\Big|^2_{A=A^{min}(g)} \tag{3.7}$$

where $A^{min}(g)$ is the point where (3.5) is minimized. The relation between coupling constant and wavefunction renormalization[16,25] implies

$$\sum_\alpha \beta_\alpha \frac{\partial}{\partial g_\alpha} W(A,g) = A_i \gamma_{ij} \frac{\partial}{\partial A_j} W(A,g) \tag{3.8}$$

or

$$\sum_\alpha \beta_\alpha \frac{\partial}{\partial g_\alpha} W,_k(A,g) = A_i \gamma_{ij} \frac{\partial}{\partial A_j} W,_k(A,g) + \gamma_{kj} W,_j(A,g) \tag{3.9}$$

Combining (3.7) with (3.9) and using the fact that A_{min} is an extremum,

$$\sum_\alpha \beta_\alpha \frac{\partial}{\partial g_\alpha} \epsilon^{(0)}(g) = F_k{}^0 (\gamma_{jk}{}^* + \gamma_{kj}) F_j{}^{0*} \tag{3.10}$$

With the cubic term in the superpotential normalized to be $1/3\ g_{ijk} \hat{A}_i \hat{A}_j \hat{A}_k$,

$$\gamma_{kj} = \frac{1}{8\pi^2} \left(g_{k\ell m} g_{j\ell m} - e^2 (C_2)_{kj} \right) \tag{3.11}$$

Thus,

$$U(X_i,\Lambda,g) = F_k{}^0 F_j{}^{0*} \left(\delta_{jk} + \frac{1}{8\pi^2} [g_{k\ell m} g_{j\ell m}{}^* - e^2 (C_2)_{kj}] \ln \left(\sum_i \frac{X_i{}^2}{\Lambda^2} \right) \right) \tag{3.12}$$

This reproduces the one loop results found in Refs. 9 and 26 from Eq. (3.1).

We now consider an SU(n) example with adjoint fields $\hat{A}$ and $\hat{Y}$, singlet $\hat{Z}$, and superpotential

$$W(\hat{Z},\hat{Y},\hat{A}) = \lambda\hat{Z} - g\hat{Z}\mathrm{tr}(\hat{A}^2) + m\mathrm{tr}(\hat{A}\hat{Y}) \tag{3.13}$$

with phases chosen to make all couplings real. For SU(2) this example was considered in Refs. 6 and 26; for SU(5) it was considered in Ref. 17. The tree level scalar potential is

$$U^{(0)}(Z,Y,A) = m^2\mathrm{tr}(A^*A) + \mathrm{tr}(mY - 2gAZ)(mY^* - 2gA^*Z^*)$$
$$+ |\lambda - g\mathrm{tr}(A^2)|^2 + \frac{e^2}{4}\mathrm{tr}([A,A^*] + [Y,Y^*])^2 \tag{3.14}$$

For all values of the couplings the minimum satisfies

$$[Y,Y^*] = 0 \tag{3.15a}$$

$$A = mY/2gZ \tag{3.15b}$$

and Z is arbitrary (it can be made real by an R-rotation). For $m^2 > 2g\lambda$ there is the further condition

$$Y = 0 \tag{3.16}$$

which fixes the minimum except for the arbitrary value of Z. For $m^2 < 2g\lambda$, the condition is

$$Y = Y^* \tag{3.17a}$$

$$\mathrm{tr}(Y^2) = \left(\frac{4g\lambda}{m^2} - 2\right)Z^2 \tag{3.17b}$$

In this case, up to an SU(n) rotation, Y is an arbitrary real traceless diagonal matrix, with magnitude given by (3.17b). There are many possible unbroken subgroups. The vacuum energy is

$$\epsilon^{(0)}(\lambda,m,g) = \begin{cases} \lambda^2 & , \quad m^2 > 2g\lambda \\ \dfrac{m^2\lambda}{g} - \dfrac{m^4}{4g^2} & , \quad m^2 < 2g\lambda \end{cases} \tag{3.18}$$

The potential in directions which violate (3.15) by significant amounts is large ($O(Z^4)$ or $O(Z^2\mu^2)$) while that in directions which violate (3.16) or (3.17) is small ($O(\mu^4)$).[27] In fact, radiative corrections make the effective values of the couplings in (3.17b) a function of Z, so the trough in the potential, straight at tree level, actually bends in the Y-Z plane due to radiative corrections. It can even change from the form (3.16) to (3.17) as $m^2(Z)-2g(Z)\lambda(Z)$ changes sign. We should therefore verify decoupling for all configurations which satisfy (3.15). Then Z^0 is real and arbitrary, Y^0 is an arbitrary complex traceless diagonal matrix, and A^0 is fixed by (3.15b); Z^0 and Y^0 are taken to be $\gg \mu$. The shifted superpotential is

$$\begin{aligned} W(\hat{Z}',\hat{Y}',\hat{A}') = &-g\hat{Z}\mathrm{tr}(\hat{A}')^2 + m\mathrm{tr}(\hat{A}'\hat{Y}') \\ &+ m\hat{Z}'\mathrm{tr}(\hat{A}'w^0) + gZ^0\mathrm{tr}(\hat{A}')^2 \\ &+ \hat{Z}'(\lambda - m^2/2g\ \mathrm{tr}(w^0)^2) - m^2/2g\ \mathrm{tr}(\hat{Y}'w^0) \end{aligned} \tag{3.19}$$

where $w^0 = Y^0/Z^0$. The Goldstino superfield $\hat{X}$ is identified as that linear combination of $\hat{Z}'$ and $\hat{Y}'$ which appears linearly in (3.19). It has cubic interactions and small ($O(\mu)$) mixing with the heavy $\hat{A}$ field, consistent with decoupling.[6] It also has interactions with gauge fields from the $\hat{Y}$ kinetic term. $\hat{X}$ is neutral under the gauge symmetries unbroken by Y^0, so the argument of (A.24) shows that the gauge couplings of $\hat{X}$ all involve heavy gauge fields. $\hat{X}$ can mix with the neutral components of $\hat{Y}$, which also decouple by (A.24). The conditions of Section 2 are thus met and Eq. (3.3) and (3.4) apply.

The RNG improved one loop energy is

$$\epsilon^{(1)}(X) = \epsilon^{(0)}(\lambda(X), m(X), g(X)) \tag{3.20}$$

The β functions are

$$\frac{X}{e}\frac{de}{dX} = -\frac{e^2}{16\pi^2} C_A \tag{3.21a}$$

$$\frac{X}{g}\frac{dg}{dX} = \frac{g^2}{8\pi^2}(D_A + 2) - \frac{e^2}{4\pi^2} C_A \tag{3.21b}$$

$$\frac{X}{m}\frac{dm}{dX} = \frac{g^2}{8\pi^2} - \frac{e^2}{4\pi^2} C_A \tag{3.21c}$$

$$\frac{X}{\lambda}\frac{d\lambda}{dX} = \frac{g^2}{8\pi^2} D_A \tag{3.21d}$$

where D_A and C_A are the dimension and Casimir of the adjoint representation. The ratio $m^2(X)/2g(X)\lambda(X)$ decreases monotonically, so that (3.16) holds at sufficiently small X and (3.17) at sufficiently large X. The energy satisfies

$$X\frac{d\epsilon^{(1)}}{dX} = \frac{g^2\lambda^2 D_A}{4\pi^2} \qquad \text{for } m^2(X) > 2g(X)\Lambda(X)$$

$$= -\frac{m^2\lambda e^2}{4\pi^2 g} C_A + \frac{m^4 e^2}{8\pi^2 g^2} C_A + \frac{m^4}{16\pi^2} D_A \quad \text{for } m^2(X) < 2g(X)\Lambda(X) \tag{3.22}$$

The energy and its first derivative, as well as the scalar v.e.v.s., are continuous at $m^2(X) = 2g(X)\lambda(X)$. By (3.22), an extremum can only occur for $m^2(X) < 2g(X)\lambda(X)$. The second derivative at an extremum is

$$X^2 \left.\frac{d^2\epsilon^{(1)}}{dX^2}\right|_{extr} = \frac{m^4}{32\pi^4 g^2}\left(g^4 D_A - \frac{9}{4} e^2 g^2 D_A C_A - e^4 C_A^2\right) \tag{3.23}$$

Equations (3.21) and (3.22) are readily integrated. We shall give the qualitative results. There are two general behaviors for the dimensionless couplings e and g. If initially

$$\frac{g^2}{e^2} < \frac{3C_A}{2D_A + 4} \qquad (3.24)$$

then g^2/e^2 decreases monotonically and both couplings are asympotically free. If the inequality (3.22) is reversed, g^2/e^2 grows monotonically and at some scale the positive term in (3.21b) dominates and g diverges. For the asympotically free case, the energy always has the behavior shown in Fig. 4, rising to a maximum and then falling asymptotically. One may check that for (3.24), the second derivative at the extremum, Eq. (3.23), is always negative. Actually, Fig. 4 applies only for $Z \gg \mu$. The scale μ may lie anywhere along the curve of Fig. 4a, depending on the initial values of the couplings. Only the part of Fig. 4a to the right of μ then applies. Our analysis does not apply to $X \lesssim \mu$; $\epsilon(X)$ should be approximately constant in this region.

For the non-asymptotically free case, the potential resembles Fig. 4a until g(X) begins to grow, then turns up as shown in Fig. 4b. This may occur anywhere along the curve, depending on the parameters. When it occurs to the right of the maximum, a minimum forms, but in most cases this occurs when g(X) is large and perturbation theory is not valid. The only time the minimum is perturbative is when it is quite close to the maximum, as in Fig. 4c, and then one sees that for a large hierarchy the minimum will not be absolute. (Only for α_e quite small, $O(10^{-3})$, do the couplings run sufficiently slowly to put many decades between the maximum and minimum in Fig. 4c.) The superpotential (3.13) does not, then, lead to large fixed perturbative hierarchies. Rather, depending on the parameters, the large X potential has one of three general behaviors:

a) it falls indefinitely, with both couplings asymptotically free, leading to a time dependent hierarchy;

b) it rises until $g(X)$ diverges, so the minimum must lie at $X \sim \mu$ (this is when $g(X)$ diverges to the left of the minimum in Fig. 4); or

c) it falls until $g(X)$ diverges (as in Fig. 4b), so a large hierarchy may form but only at a scale where the theory is strongly coupled.

As a final exercise with this model we may determine which subgroup is unbroken when the minimum is perturbative, ignoring the fact that it is only a local minimum. Equation (3.1) may be used directly, but we will do it in a way which illustrates the formalism fo Section 2. We have $Y^0 = \mathrm{diag}(Y_1{}^0, \ldots, Y_n{}^0)$ with $Y_i{}^0$ complex and $\sum Y_i{}^0 = 0$. We shall start by assuming the $Y_i{}^0$ are all different, so that the unbroken subgroup is $[U(1)]_{n-1}$. This is the most general case, as the maximum number of fields (Z' and all diagonal components, $Y_i{}'$, of Y') can mix. After the one loop corrections of Fig. 5, the SS effective potential for these fields is

$$U(\hat{Z}',\hat{Y}'_i) = (\hat{Z}^+\hat{Z})_D \left[1 - \frac{g^2 D_A}{8\pi^2} \ln\left[\frac{2g^2(Z^0)^2}{\Lambda^2}\right]\right]$$
$$+ \sum_{i,j} [(\hat{Y}'_i{}^+ - \hat{Y}'_j{}^+)(\hat{Y}'_i - \hat{Y}'_j)]_D \left[1 + \frac{e^2}{16\pi^2} \ln\left[\frac{|Y_i{}^0 - Y_j{}^0|^2}{\Lambda^2}\right]\right]$$
$$+ (\hat{Z}')_F \left[\lambda - \frac{m^2}{2g(Z^0)^2} \sum_i (Y_i{}^0)^2\right] - \frac{m^2}{2gZ^0} \sum_i (\hat{Y}'_i)_F Y_i{}^0 \qquad (3.25)$$

which is accurate as long as $\Lambda \sim Z^0 \sim Y_i{}^0$. Extremizing (3.25) with respect to F_Z and F_{Y_i},

$$U(Z^0,Y_i{}^0) = \left|\lambda - \frac{m^2}{2g(Z^0)^2} \sum_i (Y_i{}^0)^2\right|^2 \left[1 + \frac{g^2 D_A}{8\pi^2} \ln\left[\frac{2g(Z^0)^2}{\Lambda^2}\right]\right]$$

$$+ \frac{m^4}{2g^2(Z^0)^2} \sum_i |Y_i{}^0|^2$$

$$- \frac{m^4 e^2}{64\pi^2 g^2 (Z^0)^2} \sum_{i,j} |Y_i{}^0 - Y_j{}^0|^2 \ln\left[\frac{|Y_i{}^0 - Y_j{}^0|^2}{\Lambda^2}\right] \quad (3.26)$$

One can now see that as two or more $Y_i{}^0$ become equal, although some terms in the SS effective potential (3.25) diverge, the scalar potential (3.26) is well behaved. This is an illustration of decoupling at the one loop level. The same holds in the limit that all $Y_i{}^0$ vanish.[27]

The perturbative corrections to (3.26) are even in $\mathrm{Im}(Y_i{}^0)$, so the tree level result, $\mathrm{Im}(Y_i{}^0) = 0$, remains true at the minimum. The direction of SU(n) breaking is determined by the last term of (3.26). Extensive experimentation (we have no general proof) indicates that this is always minimized, at fixed $\sum |Y_i{}^0|^2$, for the unbroken subgroup $SU(n-1) \times U(1)$.

4. CONCLUSIONS

We have shown that the radiative corrections to the leading, $O(\mu^4)$, part of the effective potential are dominated by the scale M. This will not be true, in general, for the nonleading terms, which are important in determining the realization of the low energy symmetries. In all cases, though, the effective potential should be calculable. It is useful to keep the auxiliary fields explicit for as long as possible, as in (2.10), so as to take advantage of supersymmetry and nonrenormalization theorems. One must then carefully determine the scales which dominate the radiative corrections to the various terms in the SS effective potential.

Supergravity fits into the formalism developed here and in Ref. 6. Supergravity has two effects of comparable magnitude to those in the pure matter theory.[28] It gives rise to additional terms (M-terms) in the tree level scalar potential, and it gives additional interactions between the Goldstino and other light fields, of the same form as those from the matter theory, both at tree level and in loops. The analysis of Section 2 should then continue to hold, while detailed conclusions such as those for the model of Section 3 may be changed with the inclusion of the contributions of supergravity.

While this paper was in preparation we received several papers dealing with the effective potential in inverted hierarchy models. Yamagishi[7], Einhorn and Jones[9] and Frampton, Georgi, and Kim[10] have worked out the one loop term (3.1) from the mass matrices for general models, in agreement with Eq. (3.12) from the renormalization group. References 7 and 9 study also the renormalization group for the effective potential and investigate the SU(5) example of Ref. 8, finding it to be more favorable for developing large hierarchies than the example studied in Section 3. Reference 9 makes interesting observations about the nature of the scales in inverted hierarchy models. It should be noted that all of these general analyses contain the assumption that large logarithms are removed by choosing $\Lambda \sim X$. We have found that this is true in inverted hierarchy models in which the Goldstino decouples from the light fields, and that this decoupling is the case in simple inverted hierarchy models (see the Appendix). However, it fails already at one loop in more general models, as can be seen from the discussion of Fig. 1, and then even (3.12) no longer holds. In the derivation of (3.12) from (3.1), decoupling is needed in order to make the replacement $\ell n\, X^2$ for $\ell n\, m_i{}^2$: it is necessary that only fields of mass O(X) contribute to the sum. For inverted hierarchy models without decoupling, a generalization of (3.12) is obtained by keeping only those one loop graphs for Γ_{X^*X} which contain a heavy line. In (3.12) one replaces $g_{k\ell m}$ with $\tilde{g}_{k\ell m}$, and $(C_2)_{k\ell}$ with $(\tilde{C}_2)_{k\ell}$, where

$$\tilde{g}_{k\ell m} = \begin{cases} g_{k\ell m} & \text{if any of } k,\ \ell,\ \text{or } m \text{ have mass } O(X) \\ 0 & \text{if all of } k,\ \ell,\ \text{or } m \text{ are light} \end{cases} \tag{4.1}$$

and

$$(\tilde{C}_2)_{k\ell} = \frac{1}{4} \sum_a{}' \, \tau_{km}{}^a \, \tau_{m\ell}{}^a \tag{4.2}$$

where the sum in (4.2) runs only over superheavy gauge fields. Again, in these models it should be possible to control the large logarithms to arbitrary order by renormalizing the various terms in the SS effective potential at appropriate scales.

Very recently we have also received the paper of Banks and Kaplunovsky,[11] which also discusses the SU(5) model of Ref. 7, 8, and 9 and touches upon many of the same questions as the present paper, and the paper of Hall and Hinchliffe,[30] which argues that this SU(5) model in fact develops a large hierarchy only for finely adjusted values of the coupling constants.

ACKNOWLEDGEMENTS

I would like to thank L. Susskind and B. Zumino for helpful discussions.

APPENDIX A

A THEOREM CONCERNING O'RAIFEARTAIGH MODELS

In many examples of the inverted hierarchy, the supersymmetry breaking is decoupled from the light fields. This decoupling occurs because the trough in the scalar potential is straight and parallel to the auxiliary field v.e.v. It is possible to show that these features are true in all models of O'Raifeartaigh (F-term) supersymmetry breaking. Some of these results have also been obtained by Banks and Kaplunovsky,[11] and Zumino.[29]

Consider a supersymmetic Lagrangian with superfields $\hat{A}_i, i = 1,\ldots,n$, superpotential $W(\hat{A})$, and gauge group defined by

$$\delta_a \hat{A}_i = i g_a \tau_{ij}{}^a \hat{A}_j \tag{A.1}$$

(repeated indices are summed, except for gauge group indices). Here the possibility of a semisimple group, with several couplings, is included. The gauge group may include U(1) factors, with Fayet-Iliopoulos terms $\xi_a D_a$ in the Lagrangian. The scalar potential is

$$U(A) = W_{,i}(A)(W_{,i}(A))^* + \tfrac{1}{2} \sum_a D_a{}^2(A,A^*) \tag{A.2}$$

$$D_a(A,A^*) = -\tfrac{1}{2}\, g_a A_i{}^* \tau_{ij}{}^a A_j - \xi_a \tag{A.3}$$

The result to be shown is the following:

Suppose the potential (A.2) has a minimum (it need only be local) for a certain value $A_i{}^0$ of the scalar fields, such that

$$D_a(A^0,A^{0*}) = 0 \tag{A.4}$$

$$W_{,i}(A^0) = -(F_i{}^0)^* \tag{A.5}$$

with $\sum F_i{}^0 F_i{}^{0*} \equiv f^2 > 0$. Define A^x by

$$A_j{}^x = A_j{}^0 + xF_j{}^0/f \tag{A.6}$$

Then for arbitrary complex x,

(i) $D_a(A^x, A^{x*}) = D_a(A^0, A^{0*}) = 0$

(ii) $W_{,i}(A^x) = W_{,i}(A^0) = -F_i^{0*}$

Thus, the scalar potential is constant along the line defined by (A.6).

This has a simple corollary: suppose that x is much larger than any mass scale occuring in the Lagrangian or any other scalar field v.e.v. Then any vertex involving the supersymmetry breaking auxiliary field also involves at least one field of mass O(x).

Proof of (i): Gauge invariance of the superpotential implies that

$$W_{,i}(A)\tau_{ij}{}^a A_j = 0 \tag{A.7}$$

Expanding in powers of $A' = A - A^0$, the zeroth order term is

$$F_i^{0*}\tau_{ij}{}^a A_j^0 = 0 = A_i^{0*}\tau_{ij}{}^a F_j^0 \tag{A.8}$$

The first order term is

$$W_{,ij}(A^0)\tau_{ik}{}^a A_k^0 - F_i^{0*}\tau_{ij}{}^a = 0 \tag{A.9}$$

The condition that A^0 be an extremum of the potential (A.2) is

$$F_i^0 W_{,ij}(A^0) = 0 \tag{A.10}$$

Contracting (A.9) with F_j^0 and using (A.10) gives

$$F_i^{0*}\tau_{ij}{}^a F_j^0 = 0 \tag{A.11}$$

From (A.8) and (A.11) it follows that the x-dependent terms in $D_a(A^x, A^{x*})$ vanish.

Proof of (ii): For this it is convenient to redefine the superfields. First shift away the scalar v.e.v.:

$$\hat{A}'_i = \hat{A}_i - A_i^0 \tag{A.12}$$

Now choose new linear combinations $\hat{X}, \hat{B}$ of the $\hat{A}_i'$ such that

$$\hat{X} = \hat{A}'_i F_i^{0*}/f \tag{A.13}$$

and the $\hat{B}_\ell$, $\ell = 1,\ldots,n-1$, are orthogonal linear combinations. The transformed superpotential is

$$W'(\hat{X},\hat{B}) = W(\hat{A}) \tag{A.14}$$

Equation (A.5) is now

$$W'_{,X}(0,0) = -F_X^{0*} = -f \tag{A.15a}$$

$$W'_{,\ell}(0,0) = -F_\ell^{0*} = 0 \tag{A.15b}$$

Thus, $\hat{X}$ is the Goldstino superfield. Statement (ii) becomes

$$W'_{,X}(X,0) = -f \tag{A.16a}$$

$$W'_{,\ell}(X,0) = 0 \tag{A.16b}$$

The parameter x of (A.6) is seen to be the scalar field X.

The form of W' is quite restricted. From (A.15), the linear part is just

$$W'_{\rm linear} = -f\hat{X} \tag{A.17}$$

In terms of the new fields, (A.10) is

$$W'_{,XX}(0,0) = W'_{,X\ell}(0,0) = 0 \tag{A.18}$$

Thus, X does not appear at all in the quadratic part of W'. Consider now the quadratic part of the scalar potential (A.2), which by assumption is non-negative. From (A.17), (A.18), and the proof of (i), it has the form

$$\tfrac{1}{2}fW'_{,XXX}\,X^2 + fW'_{,XX\ell}\,XB_\ell + O(B^2) + \text{h.c.} \tag{A.19}$$

$W'_{,XXX}$ must vanish, or else (A.19) could be made negative by taking $B_\ell = 0$ and varying the phase of X. But then $W'_{,XX\ell}$ must also vanish, for if it did not (A.19) could again be made negative, by taking B sufficiently small that the O(B) term dominates the $O(B^2)$, and again varying the phase of X. In all, then, W' must have the form

$$W'(\hat{X},\hat{B}) = -f\hat{X} + O(\hat{B}^2) + O(\hat{X}\hat{B}^2) + O(\hat{B}^3) \quad . \tag{A.20}$$

From (A.20), (A.16) immediately follows. We have assumed a renormalizeable, cubic, superpotential, but this argument may be extended to general polynomial superpotentials. Zumino[29] has proven (ii) for general polynomial superpotentials by more elegant means.

To see the corollary, note that $\hat{X}$ couples to the chiral fields $\hat{B}$ through the vertex

$$\frac{1}{2}\, W'_{,X\ell m}(\hat{X}\hat{B}_\ell\hat{B}_m)_F \qquad (A.21)$$

while the large, O(X), part of the mass matrix is

$$\frac{1}{2}\, XW'_{,X\ell m}(\hat{B}_\ell\hat{B}_m)_F \qquad (A.22)$$

Going to a basis in which the mass matrix is diagonal, it is clear that in (A.21) $\hat{X}$ couples only to fields which have nonzero mass at O(X). $\hat{X}$ also couples to the other fields through

$$[\hat{A}_i{}^*(e^{gV})_{ik}(F_k{}^0/f)\hat{X}]_D \qquad (A.23)$$

Expanding the exponential, every interaction term has the structure

$$(\ldots.)_j g_a \hat{V}^a \tau_{jk}{}^a F_k{}^0 \qquad (A.24)$$

Since F_k (being parallel to the large scalar v.e.v.) is neutral under the subgroup unbroken at O(X), (A.24) vanishes for any gauge field which is not superheavy. Thus, all vertices involving X involve fields of mass O(X) as well.

This decoupling can be avoided either by having non-zero D fields or by having additional flat directions in the scalar potential beyond that required by the theorem and choosing the large v.e.v. in a different direction. This simplest way to arrange either of these is to have one O'Raifeartaigh model to provide the large v.e.v., plus a separate sector of either O'Raifeartaigh or Fayet-Iliopoulos type giving additional SS breaking but no large scalar v.e.v.

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21. In fact the mixing is harmless if $\hat{Y}$ is sufficiently massive. Adding a mass term mA_YF_Y to the SS effective potential, one finds that this is the leading linear term in A_Y for $m \gtrsim \mu$. The whole effect of a small shift in A_Y is then to produce a linear F_Y term, which can cancel that driven by the mixing; writing down a general effective potential, one finds that the energy is minimized when A_Y has just this magnitude. Thus the mixing shifts A_i slightly (by $O(\mu)$) from its vacuum value A_i^0, but leaves the energy unchanged. This is relevant to the SU(5) model of Ref. 8, in which such mixing occurs.

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FIGURE CAPTIONS

1. One loop contribution to Γ_{X^+X}.

2. Typical contribution to Γ_{X^+X} with light internal fields. The blobs are general superheavy subgraphs.

3. Infrared divergent contribution to $(\hat{X}^+\hat{X}\bar{D}^2\hat{X}^+D^2\hat{X})_D$.

4. a) U(X) in the asymptotically free case.

   b) U(X) with g(X) diverging at X_c.

   c) U(X) when the minimum is perturbative.

5. a) Radiative correction to $(\hat{Z}'^{\dagger}\hat{Z}')_D$.

   b) Radiative correction to $(\hat{Y}'_i{}^{+}\hat{Y}'_j)_D$.

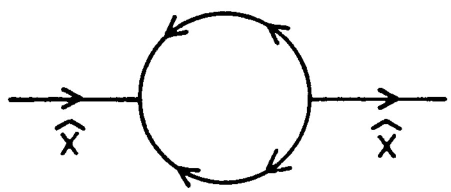

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Fig. 1

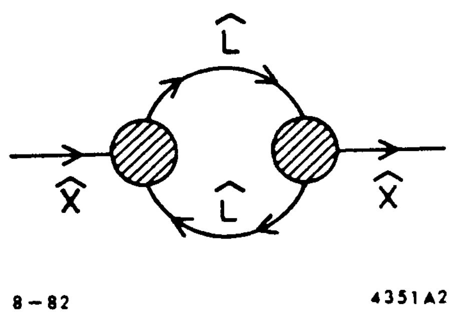

Fig. 2

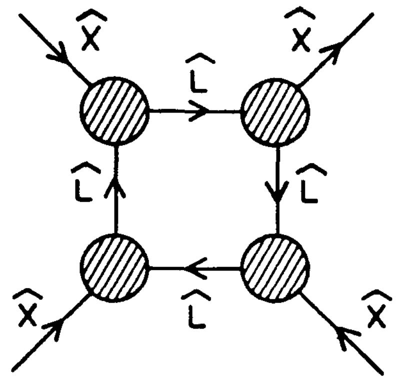

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Fig. 3

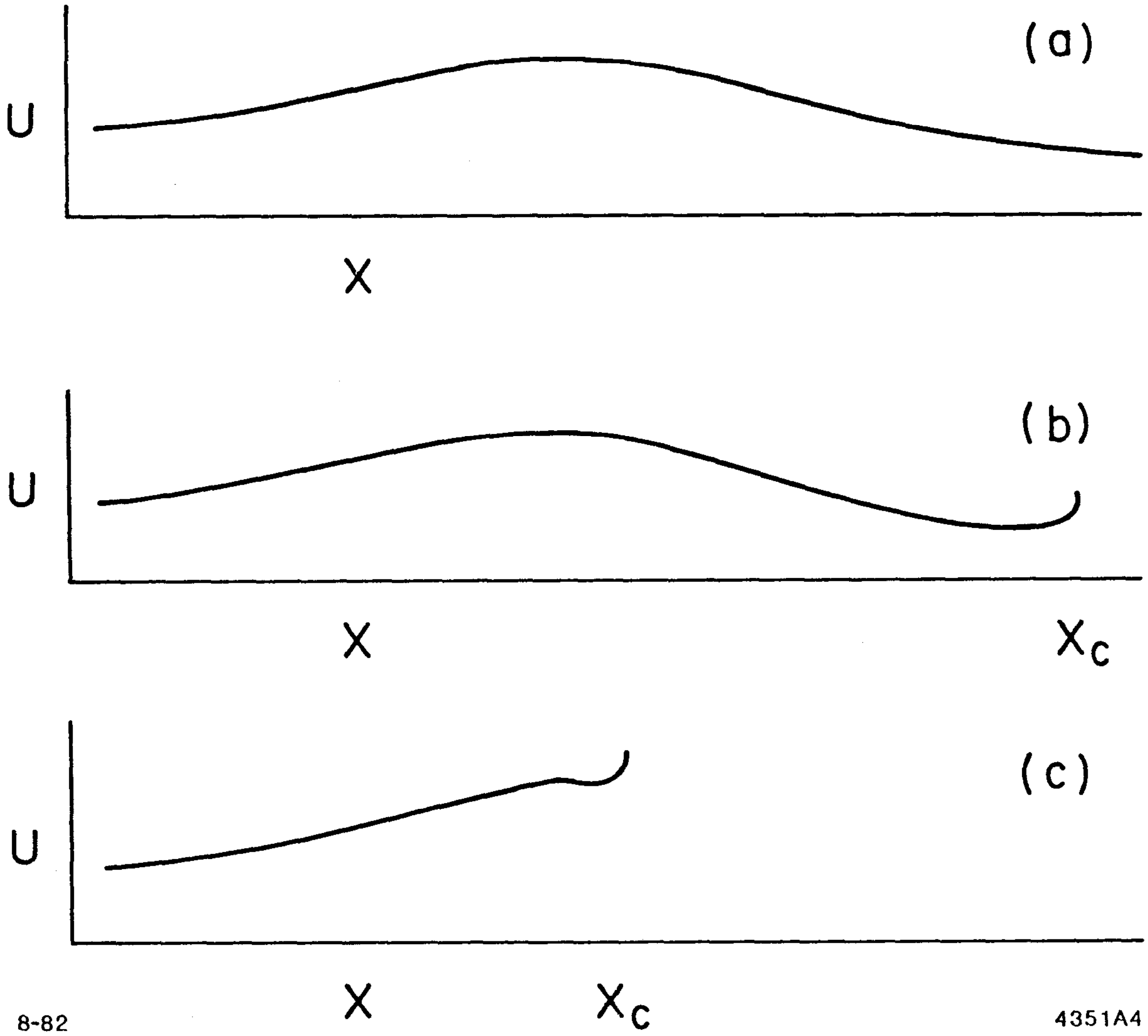

Fig. 4

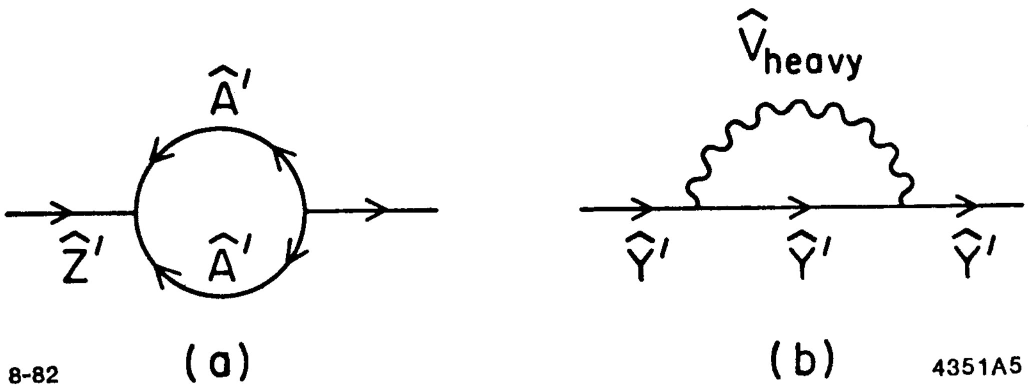

Fig. 5