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SPONTANEOUSLY BROKEN SUPERSYMMETRIC SYSTEMS OF THE NONLINEAR FIELDS AND GAUGE FIELDS*

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ABSTRACT

We consider $N = 1$ supersymmetric systems of nonlinear fields and gauge fields in $3 + 1$ dimensional space-time. The nonlinear fields take values on Kählerian complex manifolds. In a Lagrangian formulation of the systems based on Grassmann manifolds, which is a class of Kähler manifolds, we show explicitly that both gauge symmetry and supersymmetry are spontaneously broken. A general argument, in terms of counting of degrees of freedom, further shows that spontaneous gauge symmetry breakdown is also necessarily accompanied by supersymmetry breakdown in systems based on other classes of Kähler manifolds. The resulting particle spectrums of the systems have remarkable massless sectors consisting of gauge fields and fermions only.

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I. INTRODUCTION

Nonlinear realizations of an internal continuous symmetry group (G) which become linear representations when restricted to a given continuous subgroup (H) had been applied, and extensively studied, in connection with pion dynamics and its generalizations. Fields forming nonlinear realizations are self-interacting scalar/pseudoscalar fields which take values on the coset space G/H. There exist[1] standard form of nonlinear realizations, and a systematic procedure for constructing Lagrangian densities which are invariant under the nonlinear field transformations. The method applies to global as well as local internal symmetry groups, and puts little constraint[2] on the choice of G and H.

It is plausible that dynamical symmetry breakdowns may happen in supersymmetric theory, leading to the formation of composite Goldstone particles analogous to pions of low energy hadronic physics, which may be described as supersymmetric nonlinear fields. Supersymmetric generalization of the nonlinear fields has been considered by several authors.[3] Indeed, idea of nonlinear realization plays an important role in a recent formulation[4] of the SO(8) supergravity.

Not all the nonlinear fields can be supersymmetrized. In $3+1$ dimensional space-time, and for $N=1$ supersymmetry, the supersymmetric generalization is restricted[5] to chiral supermultiplets whose scalar components take values on a special class of complex manifolds termed Kähler manifolds.[6] This necessary condition greatly reduces the choice of G and H. For example, for G = SO(2m), H must be U(m) so that G/H is a Kähler manifold. Thus supersymmetry has an effect of circumscribing the possible patterns of dynamical symmetry breakdowns.

Some work has been done to study systems in which the supersymmetric nonlinear fields are coupled to supergravity.[7] In this paper we study supersymmetric systems containing both nonlinear fields and gauge fields. The specific class of Kähler manifolds involved directly in our Langrangian formulation and analysis are called Grassmann manifolds ($G_{p,q}$). $G_{p,q}$ can be represented as a coset space $SU(q+p)/SU(q) \times SU(p) \times U(1)$. The gauge fields are those associated with $SU(q+p)$.

In Section II, after defining Grassmann manifold $G_{p,q}$ as a certain class of complex matrices, and the supersymmetric generalization of these matrices, we construct an action in $N=1$ superspace. The action is supersymmetric as well as invariant with respect to the $SU(p+q)$ gauge transformations and auxiliary local $U(p)$ transformations.[8] Demanding that the action be stationary with respect to variations in the auxiliary $U(p)$ gauge fields leads to a constraint equation. The explicit expression for the Lagrangian density, after integration over super-coordinates θ and $\bar{\theta}$, is obtained in the Wess-Zumino gauge.

In Section III we perform a point transformation on the constrained chiral supermultiplet to get rid of superfluous degrees of freedom. By also consistently redefining other fields in the system, we eventually arrive at a unitary picture. It is then clearly visible that the $SU(q+p)$ gauge group is broken into $SU(q) \times SU(p) \times U(1)$ subgroup. The 2pq gauge fields, which correspond to the 2pq broken group generators, have eaten all the nonlinear scalar fields, just the right number, and become massive vector bosons. Of the 2pq two-component gauginos corresponding to the broken group generators, half of them absorb the fermionic super-partners of the scalar fields and consequently become pq four-component massive fermions, and the other half remain massless, forming a (q,p) representation of the $SU(q) \times SU(p)$ gauge group. The other massless particles are the gauge fields and gauginos of the unbroken $SU(q) \times SU(p) \times U(1)$ gauge group. Thus the mass spectrum reveals that supersymmetry is also spontaneously broken. Indeed we calculate the energy of the ground state; it turns out to be greater than zero.

What would happen if the nonlinear fields take values on Kähler manifolds other than the Grassmannian? We argue in Section IV that spontaneous breaking of supersymmetry will still happen. We then discuss the unbroken subgroups (H), and the representation contents ($\underline{\Gamma}$ plus one adjoint) of the massless fermions that would result in our systems, when the nonlinear fields take values on different classes of irreducible, compact, symmetric Kähler manifolds. Two remarkable cases are (i) $H = Spin(10) \times U(1)$ and $\underline{\Gamma} = \underline{16}$; and (ii) $H = E_6 \times U(1)$ and $\underline{\Gamma} = \underline{27}$. We also discuss a number of other topics related to the findings reported above in Section V.

II. LAGRANGIAN FORMULATION OF THE SYSTEMS

We shall construct in this Section Lagrangian densities for the systems in which the nonlinear fields take values on Grassmannian complex manifolds $G_{p,q}$ where p and q are two positive integer indices. The manifold $G_{p,q}$ has pq complex dimensions. It can be represented by $p\times(q+p)$-dimensional complex matrices A with the identification that, for any A, A and vA, where v is any nonsingular $p\times p$-dimensional unitary matrix, are to be taken as equivalent.

For the purpose of constructing a supersymmetric theory, one replaces the complex matrices A by $p\times(q+p)$-dimensional matrices (Φ) whose elements are chiral superfields. We shall retain A to denote the scalar components of the superfields, and use ψ and F to denote the fermionic components and the auxiliary fields.[9] The equivalence relation is incorporated into the theory in a form of an auxiliary local U(p) symmetry. The auxiliary symmetry reduces the number of actual, independent chiral superfields in the theory to $p(q+p) - p^2 = pq$. The action for a system of purely Grassmannian nonlinear superfields is[10]

$$I_0 = \int d^4x\ d^2\theta\ d^2\bar{\theta}\ \mathrm{Tr}\left(-\mu^2 V + \Phi^\dagger e^V \Phi\right) \tag{1}$$

where μ is a mass characterizing the system, and V are the U(p) gauge superfields in a form of $p\times p$-dimensional matrix. The action is supersymmetric as well as invariant with respect to the local U(p) transformation:

$$\left.\begin{array}{lcl} \Phi & \to & e^{-i\Lambda\Phi} \\ \Phi^\dagger & \to & \Phi^\dagger\, e^{i\Lambda^\dagger} \\ & & \\ e^V & \to & e^{-i\Lambda^\dagger}\, e^V\, e^{i\Lambda} \end{array}\right\} \qquad (2)$$

and

where Λ are chiral superfields parametrizing the group U(p). Besides these expected symmetries, we observe that the action is also invariant under a global SU(q + p) transformation:

$$\left.\begin{array}{lcl} \Phi & \to & \Phi\, e^{-i\Omega} \\ \Phi^\dagger & \to & e^{i\Omega}\, \Phi^\dagger \end{array}\right\} \qquad (3)$$

with Ω taking values on SU(q + p) algebra. The action required for a supersymmetric system of Grassmannian nonlinear fields interacting with SU(q + p) gauge fields then naturally suggests itself to be the following:

$$I = \int d^4x\, d^2\theta\, d^2\bar{\theta} \left\{ \mathrm{Tr}\left(-\mu^2 V + \Phi^\dagger e^V \Phi e^U\right) + \frac{1}{2}\, \mathrm{Tr}\left[WW\delta(\bar{\theta}) + \bar{W}\bar{W}\delta(\theta)\right]\right\} \qquad (4)$$

where U and W are respectively the supersymmetric generalizations of the gauge fields and field strengths of the SU(q + p). Note that in fundamental representation, we normalize the generators (T^a) such that $\mathrm{Tr}(T^a\, T^b) = \frac{1}{2}\delta_{ab}$. The action I is obviously supersymmetric and invariant with respect to the U(p) × SU(q + p) gauge transformations. Applying variational principle on I with respect to variations in V leads to a constraint equation, namely

$$\int d\beta\; e^{(1-\beta)V}\, \Phi\, e^U\, \Phi^\dagger\, e^{\beta V} = \mu^2\, \mathbb{1}_p \qquad (5)$$

where $\mathbb{1}_p$ denotes a $p \times p$-dimensional unit matrix. Consequently the action I can be expressed as

$$I = \int d^4x \; d^2\theta \; d^2\bar{\theta} \left\{ \mathrm{Tr}\left(-\mu^2 V\right) + \frac{1}{2} \mathrm{Tr}\left[W W \delta(\bar{\theta}) + \bar{W}\bar{W}\delta(\theta) \right] \right\} \tag{6}$$

with the V to be determined by the constraint Eq. (5).

The constraint equation takes its simplest form in Wess-Zumino (WZ) gauge. The validity of WZ gauge is insured by the $U(p) \times SU(p+q)$ gauge invariance. In WZ gauge we write

$$U = -\theta\sigma^\mu\bar{\theta}\, U_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\, D \tag{7a}$$

$$V = -\theta\sigma^\mu\bar{\theta}\, V_\mu + i\theta\theta\bar{\theta}\bar{\lambda}_v - i\bar{\theta}\bar{\theta}\theta\lambda_v + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\, D_v \quad . \tag{7b}$$

Thus we have $V^m = 0 = U^m$ for $m \geq 3$, and the exponentials in Eq. (5) become polynomials. Equating coefficients of various powers of θ and $\bar{\theta}$ at both sides of Eq. (5) yields the following:

$$AA^\dagger = \mu^2 \, \mathbb{1}_p \tag{8a}$$

$$\psi A^\dagger = 0 = A\bar{\psi} \tag{8b}$$

$$FA^\dagger = 0 = AF^\dagger \tag{8c}$$

$$\mu^2 V_\mu + \psi\sigma^\mu\bar{\psi} + iA\overleftrightarrow{\partial}_\mu A^\dagger + AU_\mu A^\dagger = 0 \tag{8d}$$

$$i\mu^2\lambda_v + \frac{i}{\sqrt{2}}\,\partial_\mu A\sigma^\mu\psi - \frac{i}{\sqrt{2}}\,A\sigma^\mu\partial_\mu\bar{\psi} - \sqrt{2}\,\psi F^\dagger - \frac{1}{\sqrt{2}}\,AU_\mu\sigma^\mu\psi^\dagger + iA\lambda A^\dagger = 0 \tag{8e}$$

$$\frac{1}{2}\mu^2 D_v - \left(\partial_\mu A + \frac{i}{2} AU_\mu\right)\left(\partial^\mu A^\dagger - \frac{i}{2} U^\mu A^\dagger\right) + \frac{i}{2}\left(\partial_\mu\psi + \frac{i}{2}\psi U_\mu\right)\sigma^\mu\bar{\psi}$$
$$- \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu\bar{\psi} - \frac{i}{2} U_\mu\bar{\psi}\right) + \frac{i}{\sqrt{2}}\left(\psi\lambda A^\dagger - A\bar{\lambda}\bar{\psi}\right) + \frac{1}{4}\mu^2 V_\mu V^\mu + \frac{1}{2} ADA^\dagger + FF^\dagger = 0 \tag{8f}$$

and, of course, the adjoint of (8e). The first three constraints are necessary for defining Grassmannian nonlinear superfields; the others determine V_μ, λ_v, $\bar{\lambda}_v$, and D_v.

Substituting Eq. (8) in Eq. (6) we finally obtain the expression for the Lagrangian density

$$\mathscr{L}(x) = \mathrm{Tr}\Big\{-\mathscr{D}_\mu A \mathscr{D}^\mu A^\dagger + \frac{i}{2}\left(\mathscr{D}_\mu \psi\sigma^\mu\bar{\psi} - \psi\sigma^\mu\mathscr{D}_\mu\bar{\psi}\right)$$

$$+ \frac{1}{4}\mu^2 V_\mu V^\mu + FF^\dagger + \frac{1}{2} ADA^\dagger + \frac{i}{\sqrt{2}}\left(\psi\lambda A^\dagger - A\bar{\lambda}\bar{\psi}\right)$$

$$+ i\left(\mathscr{D}_\mu\lambda\sigma^\mu\bar{\lambda} - \lambda\sigma^\mu\mathscr{D}_\mu\bar{\lambda}\right) - \frac{1}{2} F_{\mu\nu}F^{\mu\nu} + DD\Big\} \tag{9}$$

where

$$\mathscr{D}_\mu A = \partial_\mu A + \frac{i}{2} AU_\mu \tag{10a}$$

$$\mathscr{D}_\mu \psi = \partial_\mu \psi + \frac{i}{2} \psi U_\mu \tag{10b}$$

and

$$\mathscr{D}_\mu \lambda = \partial_\mu \lambda + \frac{i}{2} \lambda U_\mu - \frac{i}{2} U_\mu \lambda \quad . \tag{10c}$$

$F_{\mu\nu}$ is simply the field strength of SU(q + p) gauge fields and V_μ is given by Eq. (8d).

III. PHYSICAL CONTENT OF THE SYSTEMS

Let us now install the SU(q + p) gauge coupling constant (g) in the theory. This is achieved by first scaling each field in the Lagrangian density of Eq. (9) by a factor 2g and then discarding an overall factor of $(2g)^2$ from the resulting Lagrangian density. The new expression, after some rearrangements, is

$$\begin{aligned}\mathscr{L}(x) = \mathrm{Tr}\Big\{ & -\tilde{\mathscr{D}}_\mu A \tilde{\mathscr{D}}^\mu A^\dagger + \frac{i}{2}\left(\tilde{\mathscr{D}}_\mu \psi \sigma^\mu \bar{\psi} - \psi \sigma^\mu \tilde{\mathscr{D}}_\mu \bar{\psi}\right) \\ & + FF^\dagger + gADA^\dagger + i\sqrt{2}\, g\left(\psi\lambda A^\dagger - A\bar{\lambda}\bar{\psi}\right) \\ & + i\left(\mathscr{D}_\mu \lambda \sigma^\mu \bar{\lambda} - \lambda\sigma^\mu \mathscr{D}_\mu \bar{\lambda}\right) - \frac{1}{2} F_{\mu\nu}F^{\mu\nu} + DD \Big\} \end{aligned} \tag{11}$$

with

$$\tilde{\mathscr{D}}_\mu A = \partial_\mu A + igAU_\mu + igV_\mu A \tag{12a}$$

$$\tilde{\mathscr{D}}_\mu \psi = \partial_\mu \psi + ig\psi U_\mu + igV_\mu \psi \tag{12b}$$

$$\mathscr{D}_\mu \lambda = \partial_\mu \lambda + ig\lambda U_\mu - igU_\nu \lambda \tag{12c}$$

$$F_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu + ig\left[U_\mu, U_\nu\right] \tag{12d}$$

and

$$V_\mu = -\frac{2g}{\mu^2}\left(\psi\sigma^\mu\bar{\psi} + iA\overleftrightarrow{\partial}_\mu A^\dagger + 2gAU_\mu A^\dagger\right) \quad . \tag{12e}$$

In order to explore the physical content of the systems we shall replace A and ψ (F = 0 obviously) by unconstrained fields. We find it useful to make the following polar decomposition:

$$A \equiv \frac{\mu}{2g}\, e^{i\phi}\left(0 \mid \mathbb{1}_p\right) e^{i\xi}$$

and

$$\psi \equiv e^{i\phi}\left(\chi \mid 0\right) e^{i\xi} \tag{13}$$

where $\exp(i\phi)$ and $\exp(i\xi)$ take values respectively on $U(p)$ and $SU(q+p)/SU(q) \times SU(p) \times U(1)$ and χ is a $p \times q$-dimensional matrix of two-component fermions. They are uncostrained. It is easily seen that the constraints $AA^{\dagger} = \mu^2/4g^2$, and $A\bar{\psi} = 0 = \psi A^{\dagger}$ are well respected by the polar decomposition.

The apparent degrees of freedom associated with ϕ, though not constrained, are superfluous; their contributions to the Lagrangian density eventually cancel completely because of a local $U(p)$ invariance associated with the composite gauge fields V_{μ}. On the other hand, ξ associate with genuine degrees of freedom, their contributions do not cancel but are summarized and absorbed by a new definition of the $SU(q+p)$ gauge fields and gauginos. The new gauge fields U'_{μ} are given by the following,

$$igU'_{\mu} \equiv ige^{i\xi} U_{\mu} e^{-i\xi} - e^{i\xi} \partial_{\mu} e^{-i\xi} \qquad (14)$$

and the new gauginos λ' by

$$\lambda' \equiv ie^{i\xi} \lambda e^{-i\xi} \quad . \qquad (15)$$

We will also use subscripts N, E, W, S to denote the four submatrices, being of dimensions $q \times q$, $q \times p$, $p \times q$, and $p \times p$, respectively, of $(q+p) \times (q+p)$-dimensional matrix.

In terms of unconstrained fields, the Lagrangian density, after elimination of the auxiliary fields F and D, takes the following form

$$\mathscr{L}(x) = -\frac{1}{2}\,\mathrm{Tr}\,F'_{\mu\nu}F'^{\mu\nu} + 2i\,\mathrm{Tr}\left[\left(\partial_\mu\lambda' + ig\lambda' U'_\mu - igU'_\mu\lambda'\right)\sigma^\mu\bar{\lambda}'\right]$$
$$+ i\,\mathrm{Tr}\left[\left(\partial_\mu\chi + ig\chi U'_{\mu N} - igU'_{\mu S}\chi\right)\sigma^\mu\bar{\chi}\right]$$
$$+ \frac{\mu}{\sqrt{2}}\,\mathrm{Tr}\left(\chi\lambda'_E + \bar{\lambda}'_E\bar{\chi}\right) - \frac{\mu^2}{4}\,\mathrm{Tr}\left(U'_{\mu W}\,U'_{\mu E}\right)$$
$$+ \frac{g^2}{\mu^2}\,\mathrm{Tr}\left(\chi\sigma^\mu\bar{\chi}\right)\left(\chi\sigma^\mu\bar{\chi}\right) - \frac{\mu^4\,pq}{64g^2(q+p)} \quad . \tag{16}$$

It is clear that massive gauge fields are $\sqrt{2}\,U'_{\mu E}$ and $\sqrt{2}\,U'_{\mu W}$ ($U'_{\mu E}$ and $U'_{\mu W}$ are adjoint of each other), with mass squared $\mu^2/8$. Meanwhile fermionic partners of the nonlinear scalar fields, namely χ, combine with gauginos $\sqrt{2}\,\lambda'_E$ to form pq massive four-component fermions of mass $\mu/2$. The other degrees of freedom, including gauge fields $U'_{\mu N}$ and $U'_{\mu S}$, and gauginos λ'_N, λ'_S and λ'_W, remain as massless particles. The gauge group $SU(q+p)$ is spontaneously broken to $SU(q) \times SU(p) \times U(1)$, as manifested on the mass spectrum of the particles. The most remarkable phenomenon is, however, that supersymmetry is also spontaneously broken in the systems. It is shown both by the mass spectrum and by the presence of a nonzero positive ground state energy, namely the constant $\mu^4 pq/64g^2(q+p)$ in $\mathscr{L}(x)$ of Eq. (16). The constant results from $\mathrm{Tr}(gADA^\dagger + DD)$ term of Lagangian density of Eq. (11).

IV. GENERALIZATION TO SYSTEMS BASED ON OTHER KÄHLER MANIFOLDS

Algebraic manipulations in the last section demonstrate explicitly spontaneous supersymmetry breakdowns happening in the systems of Grassmannian nonlinear fields interacting with gauge fields. The breaking of supersymmetry is intimately tied to that of gauge symmetry. Let us show the necessity of this connection by an argument based on counting of degrees of freedom.

Consider a supersymmetric system of nonlinear fields taking values on a coset space G/H, assuming to be a Kähler manifold, and gauge fields of gauge group G. The number of degrees of freedom is $N_i = (4 \times \dim G + 2 \times \dim G/H)$. If the gauge group G is broken to H, assuming that gauge fields corresponding to generators of G/H eat the nonlinear scalar fields, without triggering a spontaneous supersymmetry breakdown. Then we would expect at least the number of massless vector supermultiplet to be dim H; each vector supermultiplet contains one gauge field. The total number of degrees of freedom would then be at least $N_f = 4 \times \dim H + 8 \times \dim G/H$ because each massless vector supermultiplet has four degrees of freedom but the massive one has eight. Since $N_i - N_f = -2 \times \dim G/H$, the assumption of having gauge symmetry breaking $G \to H$ without supersymmetry breakdown is therefore false in the systems we are interested.

Let us continue the above line of reasoning but admit that supersymmetry breaks while the gauge group G breaks to H. Thus $N_f = N_i = 4 \times \dim H + 6 \times \dim G/H$. After symmetry breakings, a half of the degrees of freedom form the massive and massless gauge fields, the remaining half are fermionic: $(2 \times \dim H + 2 \times \dim G/H)$ for gauginos, and $1 \times \dim G/H$

for fermionic partners of nonlinear scalar fields. They form definite representations of the unbroken gauge group H: an adjoint plus representations $\underline{\Gamma}$ and $\underline{\Gamma}^*$ for gauginos, and a $\underline{\Gamma}$ for the other fermions. [$\underline{\Gamma}$ is representation content of the nonlinear scalar fields.] Assuming that the mechanism for generating masses for gauginos is Yukawa coupling, which is supersymmetric counterpart of minimal gauge coupling of nonlinear fields, then gauginos in adjoint representation and $\underline{\Gamma}$ representation would remain as massless particles while the other fermionic degrees of freedom become massive.

What are the available subgroups H and representations $\underline{\Gamma}$? Kähler manifolds expressible as G/H with G being a compact, connected, simple Lie group are classified into six classes.[11] They are (i) $G/H = SU(q+p)/SU(q) \times SU(p) \times U(1)$ with number of complex dimensions $\dim_c G/H = qp$, so $\underline{\Gamma}$ is a (q,p) representation. (ii) $G/H = SO(2q)/U(q)$, $\dim_c G/H = q(q-1)/2$, so $\underline{\Gamma}$ is a second rank antisymmetric tensor representation. (iii) $G/H = Sp(2q)/U(q)$, $\dim_c G/H = q(q+1)/2$, so $\underline{\Gamma}$ is a second rank symmetric tensor representation. (iv) $G/H = SO(q+2)/SO(q) \times U(1)$, $\dim_c G/H = q$, so $\underline{\Gamma}$ is a vector representation. (v) $G/H = E_6/Spin(10) \times U(1)$, $\dim_c G/H = 16$, so $\underline{\Gamma}$ is a spinor representation $\underline{16}$. (vi) $G/H = E_7/E_6 \times U(1)$, $\dim_c G/H = 27$, so $\underline{\Gamma}$ is a minimal representation $\underline{27}$. Except in case (iv), $\underline{\Gamma}$ are always complex representations of non-Abelian part of H. H carries a U(1) factor, which is a necessary condition for G/H to be Kählerian.

Generalization of our work to cases where G is noncompact will not be a simple task, if it is possible at all. This is so even in cases without supersymmetry.[2] We simply want to make a remark here that mathematically noncompact, irreducible, symmetric Kähler manifolds can also be classified[11] into six classes parallel to the compact cases, and have a list of H and Γ identical to the above one.

V. CONCLUSION AND DISCUSSION

We have shown in this article that there exists a class of models in which spontaneous breakdown of gauge symmetry is necessarily accompanied by spontaneous breakdown of supersymmetry. What we consider are $N = 1$ supersymmetric systems of the nonlinear fields interacting with gauge fields in $(3 + 1)$ dimensional space-time. The systems have a rich geometrical structure in the sense that the nonlinear fields take values on Kähler manifolds G/H, G is also the gauge group to be broken. Explicit Lagrangian formulation of the systems based on the first class of Kähler manifolds, namely, the Grassmannians, is achieved. It may be useful to do so for systems based on other types of Kähler manifolds,[11] and we see no obstacle of principle against it.

The above mentioned connection between breakings of gauge symmetry and supersymmetry is in strong contrast to what happens in renormalizable supersymmetric Yang-Mills theories. In such theories, if the chiral part of scalar potential, which is polynomial of scalar components of chiral superfields only, does not break supersymmetry then the presence of gauge interactions does not change this situation whether or not there is gauge symmetry breaking, if any. The reason is that the chiral part of the potential is invariant with respect to complex extension of the gauge group.[12] Interpreted along the line of reasoning used in Section IV, the complexification doubles the number of would-be Goldstone particles so that there are enough degrees of freedom for forming massive vector supermultiplets and thus not conflicting with supersymmetry.

Our result should not be taken as implying that supersymmetry will break down whenever there is an interaction between gauge fields and the nonlinear fields. For example, if we modified the systems such that the gauge group were H, or smaller subgroups, instead of G, there would be no breakdown of supersymmetry nor that of gauge symmetry. However, for modified systems with gauge group X such that $G \supset X \supset H$, one obtains breakdowns of both gauge symmetry and supersymmetry. Lagrangian densities for modified systems can be easily obtained by modifying that of the original systems. For example, in the Grassmannian cases, the modifications are achieved by taking U of Eq. (4) to be that of the desired gauge groups.

The resulting particle spectrums of our systems have some remarkable properties. First one notices that a well known sum rule[13] for masses is violated, at least in the Grassmannian cases. Secondly one notices that there is no scalar particle, the spectrums consist of only gauge bosons and fermions. In particular, the massless sector consists of gauge fields of H, their corresponding gauginos, and a $\underline{\Gamma}$ representation of other gauginos. In most cases $\underline{\Gamma}$ are complex representations (see Section IV for a complete listing) of non-Abelian part of H. Since $\underline{\Gamma}$ runs through representations $\underline{16}$ and $\underline{27}$ respectively for the cases where the non-Abelian part of H is Spin(10) and E_6, one cannot resist speculating on a possible connection between the $\underline{\Gamma}$ gauginos and quarks and leptons.[14] But in this respect our systems appear to be incomplete; they cannot accomodate the phenomenon of repetition of the family-structure of quarks and leptons.

We also observe that the systems presented are nonrenormalizable, as manifested by, for instance, the presence of a quartic fermion interaction term in the Lagrangian density, Eq. (16).

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