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FLAVOR-CHANGING Z-DECAYS: A WINDOW TO ULTRAHEAVY QUARKS?*

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ABSTRACT

We study flavor-changing Z decays into quarks

$$Z \to Q + \bar{q}$$

in the standard $SU(2) \times U(1)$ theory with sequential generations. Such decays occur in higher order electroweak interactions, with a probability growing as the fourth power of the mass of the heaviest (virtual) quark mediating the transition. With the possible exception of $Z \to b\bar{s}$, these decay modes are generally very rare in the three-generation scheme. However with four generations $Z \to b'\bar{b}$ is observable if the t' mass is a few hundred GeV. Such decay modes could thus provide a glimpse of the ultraheavy quark spectrum.

1. INTRODUCTION

A most important problem of particle physics, the investigation of the number and the properties of quarks as the basic constituents of hadronic matter, will be very hard to tackle in the mass range above ~100 GeV. While operating or planned (anti)proton-proton and e^+e^- colliders can pair-produce quarks with masses up to ~50 GeV (and later ~100 GeV), ultraheavy quarks (mass $\gtrsim$ 100 GeV) don't appear to be directly accessible in the foreseeable future.

However, the possible existence of yet heavier quarks than b (and t?) does affect higher order electroweak corrections to Born amplitudes and determines the strength of processes forbidden in lowest order. The most celebrated examples of this latter sort are the deduction of the charmed quark's existence from the absence of lowest order strangeness-changing neutral-current processes[1] and the deduction of the charmed quark mass from the strength of higher order processes.[2] The key fact enabling ultraheavy virtual quarks to influence low energy measurements is the absence of a decoupling theorem for spontaneously broken gauge theories. Because the couplings of the Higgs particles (longitudinal modes of the massive vectors) to fermions are proportional to fermion masses, amplitudes with internal fermions need not be suppressed as the internal fermion masses m_i increase [below the limit set by the validity of the perturbation expansion]. Indeed, they may even grow as powers of m_i. An analysis of the nondecoupling one-loop amplitudes is presented in Appendix A.

It is therefore tempting to investigate processes sensitive to the possible existence of ultraheavy quarks. A search for an ultraheavy influence on K_L - K_S transitions or neutral K decay is hampered by the weak mixing of light u,d,s quarks to very heavy (4th generation) quarks, suppressed presumably by powers of the Cabibbo angle. A process where the ultraheavy influence is not multiply Cabibbo suppressed is flavor-changing neutral-current (FCNC) production of a heavy quark(s). The simplest example is Z decay into two different quarks, at least one of which is heavy,

$$Z \to Q + \bar{q} \quad . \tag{1}$$

In the standard electroweak model with all left-handed fermions in identical SU(2) doublets and all right-handed fermions in singlets, such decays are suppressed by the GIM mechanism[1] to order αG_F. But such decays do occur in electroweak interactions at the one-loop level. Internal W bosons can mediate the conversion of very heavy internal fermions into the kinematically allowed final state, $Q\bar{q}$. All amplitudes induced at one-loop level [in 't Hooft-Feynman gauge] are shown in Fig. 1.

Before proceeding with the exact calculation let us estimate the branching ratio B of $Z \to Q\bar{q}$. Compared to the lowest order flavor-neutral Z decays, there are two additional vertices in the vertex loop with characteristic electroweak strength $\alpha_W/\pi = e^2/4\pi^2\sin^2\theta_W$. In addition, the longitudinal modes of the W in unitary gauge, or equivalently the unphysical Higgs scalars in 't Hooft-Feynman gauge, couple proportional to fermion mass divided by W mass. Denote the entries in the charged current mixing matrix by λ_{ij}. Then one expects, up to possible logarithms coming from the internal loop momentum integral, a branching ratio of

$$B(Z \rightarrow Q\bar{q}) \sim \left(\frac{\alpha_W}{\pi}\right)^2 \left|\sum_i v_i m_i^2/m_W^2\right|^2/N_F \tag{2a}$$

to leading order in m_i; $v_i = \lambda_{iq}\lambda^*_{iQ}$, and N_F is the number of final quark states accessible to diagonal Z decay [lepton states have been neglected since their relative color weighting is 1/3.] Taking $N_F \sim 6$ and retaining only the heaviest quark mass in the sum, one gets

$$B(Z \rightarrow Q\bar{q}) \sim 10^{-5}|v_i|^2 \ (m_i/m_W)^4 \quad . \tag{2b}$$

If the Cabibbo angle is a typical mixing angle, $|v_i|^2 \lesssim 10^{-1}$, a branching ratio of order 10^{-4} can only be reached if one or more internal fermion masses m_i greatly exceeds the W mass. From this it is clear that measuring FCNC's in e^+e^- collisions will need the huge Z resonance yield to be measurable. Planned accelerators anticipate millions of Z's per year, and the contrast of a heavy $\bar{Q}$ jet ($m_Q \gtrsim 50$ GeV) with a recoiling light jet ($m_q \lesssim 5$ GeV) may provide a reasonable signature, although identification may be difficult. Semileptonic decay patterns might also be useful. A further benefit of this process is that one may hope to discover in the final state new quarks with masses nearly as large as the Z itself.

Section 2 describes some details of our calculation. Formulae for the exact results are too lengthy to be presented in this paper. Instead we give a simple phenomenological approximation. A summary of theoretical upper bounds on fermion masses in the standard electroweak model and a brief discussion of mixing angles is presented in Section 3. Section 4 contains results for 3 and 4 generation scenarios, and Section 5 contains a summary and some concluding remarks.

2. $Z \to Q\bar{q}$ DECAY WIDTH

We assume the pattern of the standard SU(2) × U(1) model: left-handed fermion doublets and right-handed singlets occur in repeating generations, and there is a single Higgs doublet. We use $\sin^2\theta_W = 0.22$, $m_Z = 90$ GeV and $m_W = m_Z \cos\theta_W = 80$ GeV. The general fermion-antifermion current to which an on-shell Z couples can be written

$$J_\mu = \bar{Q}[\gamma_\mu(LF_L + RF_R) + i\,\sigma_{\mu\nu}\frac{k^\nu}{m_Z}(LF_{TL} + RF_{TR})]q \tag{3}$$

where $L/R = 1/2\,(1 \mp \gamma_5)$ are the usual chirality projectors, and k_μ is the Z four-momentum vector. Neglecting the q mass [this assumption can easily be dropped at the expense of rather lengthy expressions] we find for the partial width

$$\Gamma(Z \to Q\bar{q}) = \frac{m_Z}{16\pi}(1-x^2)\left\{(2-x^2-x^4)|F_L|^2 + (1+x^2-2x^4)|F_{TL}|^2 \right.$$
$$\left. + 6x(1-x^2)\,\mathrm{Re}\,(F_L F^*_{TL}) + (L \to R)\right\} \tag{4}$$

with $x = m_Q/m_Z$.

Since the GIM mechanism prevents flavor-changing neutral currents in the bare Lagrangian, the form factors are zero to lowest order. Renormalizability of spontaneously broken SU(2) × U(1) then ensures that loop contributions to $Z \to Q\bar{q}$ are finite and dependent on no parameters beyond those already present in the bare Lagrangian. [To the order we are interested in, the parameters are sufficiently defined by processes in Born approximation.] We work in the 't Hooft-Feynman gauge and employ dimensional regularization with a completely anticommuting γ_5. The ten diagrams contributing in this gauge at the one-loop level are shown in

Fig. 1. A sum over the internal quarks of the appropriate charge is implied.

Our detailed calculation confirms all prior discussion. Divergent terms which don't have a dependence on the internal masses m_i are canceled by the unitarity sum $\sum v_i = 0$. Divergent terms with an m_i dependence cancel when the diagrams are summed. [As a check we have carried out a formal renormalization program, generating counterterms via the substitutions $\psi_{j\,Bare}{}^{L,R} = \sum \sqrt{Z_{ji}}{}^{L,R}\ \psi_{i\,Ren}{}^{L,R}$; the counterterms cancel among themselves for q and Q on-shell. Details are described in Appendix B.] Following the methods of 't Hooft and Veltman,[3] we have reduced the integrals of all two- and three-point functions to Spence functions and logarithms. The four graphs with the unphysical Higgs coupling twice to fermions give rise to a leading $(m_i/m_H)^2$ term in the amplitude.

The leading mass term is already evident in the approximate result of Ma and Pramudita[4] who calculated $Z \to q_1\bar{q}_2$ in the zero-external mass limit $m_Z = m_{q_1} = m_{q_2} = 0$. We have checked that our calculation reproduces their result in this limit. We also checked our results in the limit of zero external quark masses, but keeping m_Z physical, by an independent dispersive calculation. The dispersion integral over the (finite) imaginary parts of the diagrams has an m_i-independent leading singularity which is canceled by the unitarity sum, and a next-to-leading singularity which requires one subtraction. Choosing the subtraction at $s = 0$ fixes the substraction constant to be the $m_Z = 0$ result of Ma and Pramudita.

Denote each form factor for fixed internal quark flavor i by $F_k(m_i)$ ($k = L,R,TL,TR$). Approximating all but the highest internal mass by $m_i = 0$ ($i \neq I$ with $m_I = \max m_i$) and using $v_I = -\sum_{i \neq I} v_i$ one gets for the form factors summed over all internal fermion species

$$F_k = v_I \, \Delta F_k(m_I) \tag{5a}$$

$$\Delta F_k(m_I) = F_k(m_I) - F_k(0) \quad . \tag{5b}$$

[The infinite, i-independent part of the form factors cancel in $\Delta F_k(m_I)$.] The real and imaginary parts of $\Delta F_L(m_I)$ and the corresponding width are shown in Fig. 2 for the zero external quark mass limit. In this limit the other three form factors vanish. The leading $(m_i/m_W)^2$ behavior is evident. The infinite derivatives of ΔF_L as m_I approaches $1/2\, m_Z$ are threshold effects. The two graphs with two internal quark propagators have imaginary parts proportional to $(m_Z{}^2 - 4m_i{}^2)^{1/2}$ for $m_Z > 2m_i$; the dispersive integral then gives a real part proportional to $(4m_i{}^2 - m_Z{}^2)^{1/2}$ for $m_Z < 2m_i$. As a consequence the partial width of Z will show a cusp at $m_I \to 1/2 m_Z$.

Even in the zero external mass limit the analytic expressions for ΔF_L are lengthy. However, a useful approximation is

$$|\Delta F_L|^2 \simeq K^2[3/2 + (m_I/m_W)^4] \tag{6}$$

for $m_I \geq 1/2\, m_Z$ and negligible otherwise, where $K = g^3/64\pi^2 \cos\theta_W \simeq 5 \times 10^{-4}$. The asymptotic m_I dependence in (6) is exact. The constant 3/2 is fitted to the width in Fig. 2 [The cusp is of course not reproduced by the approximate formula; this is irrelevant as long range binding effects will distort the width in the threshold regime anyway.] In the exact calculation F_R, F_{TL} and F_{TR} are non-zero but less than F_L. Neglecting these form factors and adopting (6) for F_L gives reasonably good results even for non-zero external masses. The reader is invited to compare this single form factor approximation to the exact results presented in Section 4.

3. MASS BOUNDS AND QUARK MIXING ANGLES

Since the FCNC amplitude grows with the internal fermion mass, its magnitude is bounded only if m_i is bounded [the quark mixing angle assumed to vanish more slowly than m_i^{-1}, see next paragraph]. There are theoretical arguments for fermion mass upper bounds. Partial wave unitarity sets a limit of several hundred GeV to quark masses beyond which couplings in the theory become so strong that the perturbation expansions fails.[5] The empirical success of $m_W = m_Z \cos\theta_W$ as measured in the ratio of neutral to charge current neutrino scattering implies a mass splitting of $\lesssim 700$ GeV within heavy quark doublets.[6] Stability of the Higgs minimum at one loop in the effective potential together with the postulate of perturbatively small Higgs self-couplings imply $m_i \lesssim 800$ GeV.[7] Finally, postulating the validity of perturbation theory and stability of the one-loop Higgs vacuum up to grand unifying energies gives $m_i \lesssim 250$ GeV.[8] Using these results we will entertain the possibility that ultraheavy quarks are bounded by $\lesssim 500$ GeV.

From the general expressions (4) it is clear that the largest value of $v_I \Delta F_k(m_I)$ will dominate the amplitude, viz. $\Delta F_k = F_k(m_I) - F_k(0)$. If $m_I > m_W$, we have $v_I \Delta F_L(m_I) \propto v_I (m_I/m_W)^2$ from (6). A similar behavior obtains in the kaon system where it has been suggested[9] that in order to maintain the suppression of FCNC amplitudes to order αG_F, $v_i(ds) \lesssim m_W^2/m_i^2$ for heavy flavors. On the other hand, motivated by the empirical relation $v_u(ds) \sim \sqrt{m_d/m_s}$, many authors proceeding from a wide variety of model postulates have found ($i > Q,q$)

$$v_i(Qq) = \sqrt{m_Q/m_i} \quad \sqrt{m_q/m_i} \tag{7}$$

This behavior apparently guarantees the suppression as long as $m_i < m_W^2/\sqrt{m_d m_s} \sim 10$ TeV. This bound is much less restrictive than the theoretical heavy mass bounds quoted before. A behavior of the mixing angles as suggested by (7) implies a linear growth of the FCNC amplitude with m_i. We may also deduce from (7) a crude rule of thumb. $Q = -1/3$ quark masses increase by an approximate factor of 25 per generation. Together with (7) this then suggests $v_i(Qq) \sim (0.2)^n$ where n is the number of generation jumps from Q to i to q.

Convenient parametrizations of the mixing matrix λ_{Qi} can be found for the case of three generations[10] and also for four generations.[11] Unfortunately, the matrix entries involving heavy quarks are poorly known. Our approach will be to obviate the question of mixing angles whenever possible. If the amplitude is dominated by just one internal quark flavor as in (5), uncertainties of unknown mixing angles are lumped into a single factorizing v_I. In this approximation universal curves of $\Gamma(Z \to Q\bar{q})/|v_I(Qq)|^2$ are obtained. In the following section we present results utilizing the single internal mass dominance formulae of Eq. (5). We have checked that results in this approximation differ little from the exact results obtained with a realistic unitary mixing matrix and spectrum of quark masses.

4. RESULTS

It follows from our general discussion that flavor-changing Z decays are expected to be very rare in the standard three-generation scenario as long as the t quark mass does not greatly exceed the W mass. $Z \to t\bar{c}$ decays mediated by b quark exchange will be negligible as $(m_b/m_W)^4 \sim 10^{-5}$, resulting in branching ratios of order 10^{-10}. The same holds true for the e^+e^- continuum, $\gamma^* \to t\bar{c}$. Since the t quark mass is considerably larger than the b quark mass, the branching ratio of $Z \to b\bar{s}$ will be many orders of magnitude larger. The partial width from this decay channel is shown in Fig. 3, the quark mixing matrix element $v_t^2(bs)$ being factored out. By the CPT theorem $\Gamma(Z \to \bar{b}s) = \Gamma(Z \to b\bar{s})$.

Presumably the most realistic possibility for observing flavor-changing Z decays is the decay into the third-generation b and the fourth-generation b′,

$$Z \to b' + \bar{b} \tag{8}$$

mediated by heavy t′ quark exchange. In Fig. 4 we display $\Gamma(Z \to b\bar{b}' + \bar{b}b')/v_{t'}^2(b'b)$ as a function of the b′ mass varying the t′ mass between 200 and 500 GeV. A $v_{t'}^2(b'b) \sim 10^{-1}$ value would possibly allow a branching ratio as large as 10^{-4}. The dotted line indicates the expectation for models with an approximate constant progression of quark masses between generations, resulting in $m_{b'} \sim 70$ GeV and $m_{t'} \sim$ 350 GeV.[12] With these mass values, FCNC Z decays may afford us a glimpse at the fourth generation. For the sake of completeness we have also drawn $\Gamma(Z \to t\bar{c} + \bar{t}c)/v_{b'}^2(tc)$ for four generations in Fig. 5, assuming a t quark mass of 30 GeV and varying the b′ mass.

5. DISCUSSION

Assuming the standard model, it appears that if flavor-changing Z decays are observed, it will most likely be in the reaction $Z \to b'\bar{b}$, producing the b and b′ quark of the third and fourth generation. With $m_b \sim 5$ GeV, $m_{b'}$ up to ~80 GeV could be accessible. The large internal mass driving $b'\bar{b}$ production is the fourth-generation t′ mass, and Cabibbo-valued mixing angles suggest a branching ratio up to 10^{-4}. $t\bar{c}$ will probably not be seen; the relative lightness of charge -1/3 (internal) quarks and multiple mixing angles yield too small an amplitude for the 3 or 4 generation schemes. Unless the t-quark mass is ultraheavy, the prognosis for $b\bar{s}$ final states is similarly pessimistic.

Of course, the standard model may be wrong. Embellishing it with multi Z′s or more Higgs doublets would undoubtedly increase FCNC rates. In general, more than one doublet of Higgs particles leads directly to flavor-changing neutral Higgs couplings.[13] These must be suppressed in K decay by choosing the masses of such Higgs′ to be heavy. If the masses are chosen very heavy, one is effectively returned to the neutral single Higgs sector[14] assumed in this paper. However, richer Higgs structures are possible. Let $g_i^{a,Q}$ be the Yukawa coupling of the a-th Higgs doublet to the i-th generation of charge Q. Then two doublets are allowed to couple to quarks without inducing flavor-changing neutral Higgs couplings provided $\sum g_i^{1,2/3} \; g_i^{2,-1/3} = 0$.[13] If this condition is introduced by fiat or by new symmetries, there results 5 potentially light physical Higgs and potentially larger induced FCNC rates in heavy quark systems.

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APPENDIX A: DECOUPLING THE DECOUPLING THEOREM

There is no decoupling theorem for spontaneously broken gauge theories. Because the coupling of the Higgs particles, or equivalently in unitary gauge, the longitudinal modes of the massive vectors, to fermions are proportional to the fermion masses, amplitudes with internal fermions need not be suppressed as the internal mass m_i increases [up to the limit where the perturbation expansion fails]. Indeed, amlitudes may even grow as powers of m_i/m_W. This behavior seems less surprising when it is realized that this mass ratio is just $\sqrt{2}$ times the ratio of Yukawa to gauge coupling constants.

For amplitudes with closed fermion loops, simple operator analysis establishes the leading m_i behavior. Let n be the number of external Higgs particles in the amlitude. Since the Higgs couples proportional to the fermion mass, the leading power in m_i for an amplitude is $4+n-d$ where d is the dimension of the effective operator. We list all operators having positive definite power dependence on m_i in Table 1. Amplitude (b), with $n = 0$, renormalizes the relation $m_W = m_Z \cos\theta_W$ and has been used to establish upper bounds on heavy fermions in non-degenerate doublets.[6] Amplitudes (b) with $n = 0$ and (a) with $n = 2$ have been used to relate the renormalized Higgs couplings to heavy fermion masses.[5] Amplitude (c) was calculated for $Z \to \phi\gamma$.[15] Amplitude (d) occurs in $\phi \to \gamma\gamma$,[16,17] $\phi \to gg$,[18] $\gamma^* \to \gamma\phi$[19] and $gg \to \phi$.[20] Except for (d) with two external gluons, all amplitudes also receive contributions from closed Z and/or W loops. The interference between the graphs suppresses the leading behavior in m_i until $m_i \gg m_{W,Z}$.

We next turn to an investigation of amplitudes with (light) fermions as external particles. Representative graphs of all such amplitudes induced at one loop which increase with increasing internal fermion mass are shown in the lower half of Table 1. [To the heavy fermions in these graphs may be attached an arbitrary number of external Higgs fields. The power lost by the increase in loop convergence is compensated by the m_i in the Higgs coupling.] Amplitude (e) has been calculated in the standard model with approximation[17] and in a two Higgs doublet model exactly.[21] Amplitude (g) has been extensively treated with respect to rare decays and CP mixing in strange, charmed and bottom systems.[2,9,22] Amplitude (f) with an external real photon has been used to estimate light quark radiative decay,[23] and has been exactly analyzed with respect to $\mu \to e\gamma$ decay.[24] For off-shell photons the result, without quadrature, has also been given.[25] Amplitude (f) with a space-like gluon is just the penguin process of weak decay. Amplitude (f) has also been considered with the external gluon on-shell[26] and time-like.[27]

APPENDIX B: RENORMALIZATION OF NON-DIAGONAL Z DECAY

Direct calculation of $Z \to q_i\bar{q}_j$, $i \neq j$ shows that the ultraviolet divergences cancel. This is due to the following fact: generating the counterterms as usual by expressing the bare Lagrangian by renormalized quantities, $\mathcal{L}_{Count} = \mathcal{L}_{Bare}(\psi,g,m) - \mathcal{L}_{Bare}(\psi_{Ren},g_{Ren},m_{Ren})$, one finds that in lowest non-vanishing order the counterterms relevant for non-diagonal Z decay arise only from field renormalization

$$u_j{}^R = \sum_i \sqrt{Z_{ji}{}^{u,R}}\; u_{i,Ren}{}^R$$

$$(\lambda d)_j{}^R = \sum_i \sqrt{Z_{ji}{}^{d,R}}\; (\lambda d)_{i,Ren}{}^R$$

$$\begin{pmatrix} u_j{}^L \\ (\lambda d)_j{}^L \end{pmatrix} = \sum_i \sqrt{Z_{ji}{}^L} \begin{pmatrix} u_{i,Ren}{}^L \\ (\lambda d)_{i,Ren}{}^L \end{pmatrix}$$

where $(\lambda d)_j = \sum \lambda_{jm} d_m$ and R/L label the $(1 \pm \gamma_5)/2$ chirality projected fields. [For the sake of simplicity we omit CP violation, i.e., λ is taken orthogonal.]

In the one loop approximation we get the following 2 and 3-point counterterms for Z decay into different up quarks (cf. Fig. 6a),

$$\delta\mathcal{L}_2 = \bar{u}_i\; A_{ij}\; u_j$$

$$\delta\mathcal{L}_3 = \bar{u}_i\; i\gamma_\mu\; B_{ij}\; u_j\; Z^\mu$$

where

$$A_{ij} = \delta Z_{ij}{}^{u,R}\left[i\not{\partial}R - \frac{1}{2}(m_{u_j}L + m_{u_i}R)\right] + \delta Z_{ij}{}^L\left[i\not{\partial}L - \frac{1}{2}(m_{u_i}L + m_{u_j}R)\right] ,$$

$$B_{ij} = \frac{ig}{\cos\theta_W}\left\{\frac{2}{3}\sin^2\theta_W\; R\; \delta Z_{ij}{}^{u,R} + \left(\frac{2}{3}\sin^2\theta_W - \frac{1}{2}\right) L\; \delta Z_{ij}{}^L\right\} .$$

The counterterm renormalizing the $u_i \to u_j$ propagator and the counterterm renormalizing the proper vertex $Zu_i\bar{u}_j$ are given by the same Z factors $\delta Z_{ij}{}^{u,R}$ and $\delta Z_{ij}{}^{L}$. Direct calculation then shows that the contribution of the counterterms to non-diagonal Z decay vanishes so that $Z \to u_i\bar{u}_j$ is not renormalized (cf. Fig. 6b).

The argument for Z decay into down quarks is similar. The propagator $d_i \to d_j$ and the proper vertex are both renormalized by $\delta Z_{ij}{}^{d,R}$ and $\delta Z_{ij}{}^{L}$ and the sum of the counterterm contributions relevant for $Z \to d_i\bar{d}_j$ vanishes. $\delta Z_{ij}{}^{L}$ is the same for up and down current quarks though the left-handed parts of the counterterms are different for up and down mass eigenstates. In fact if we defined $\delta Z_{ij}{}^{L}$ such that $\delta\mathcal{L}_2{}^u = \bar{u}_i{}^L\, i\not{\partial}\, u_j{}^L\, \delta Z_{ij}{}^L + \ldots.$ renders the $u_i \to u_j$ propagator finite, then the corresponding counterterm for down quarks is given by $\delta\mathcal{L}_2{}^d = \bar{d}_i{}^L\, i\not{\partial}\, d_j{}^L\, \delta\tilde{Z}_{ij}{}^L + \ldots.$ where

$$\delta\tilde{Z}_{ij}{}^L = \lambda^\dagger{}_{i\ell}\, \delta Z_{\ell m}{}^L\, \lambda_{mj} \quad . \tag{B1}$$

Explicit calculation for the infinite parts gives

$$\delta Z_{ij}{}^L = \frac{-g^2}{16\pi^2}\,\frac{2}{\epsilon}\left\{\left(\frac{-5}{18} + \frac{1}{36\cos^2\theta_W} + \frac{m_{ui}{}^2}{4m_W{}^2}\right)\delta_{ij} + \frac{m_{dk}{}^2}{4m_W{}^2}\,\lambda_{ik}\lambda_{kj}{}^\dagger\right\},$$

$$\delta\tilde{Z}_{ij}{}^L = \frac{-g^2}{16\pi^2}\,\frac{2}{\epsilon}\left\{\left(\frac{-5}{18} + \frac{1}{36\cos^2\theta_W} + \frac{m_{di}{}^2}{4m_W{}^2}\right)\delta_{ij} + \frac{m_{uk}{}^2}{4m_W{}^2}\,\lambda_{ik}{}^\dagger\lambda_{kj}\right\},$$

which fulfill relation (B.1). Note that $\delta Z_{ij} = \delta Z_{ji}$ for orthogonal λ.

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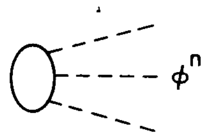
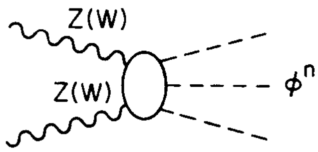
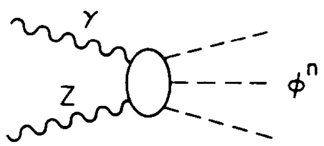
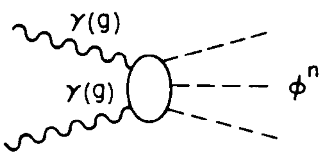
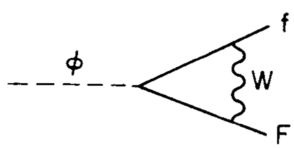
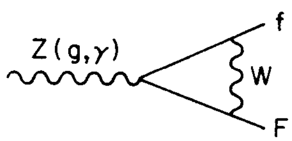
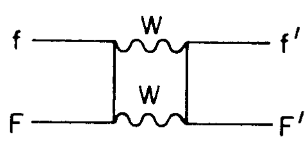
TABLE CAPTION

Amplitudes having positive definite dependence on the internal fermion mass m_i.

FIGURE CAPTIONS

1. Graphs contributing to $Z \to Q\bar{q}$ in the 't Hooft-Feynman gauge.
2. Real and imaginary parts of $\Delta F_L(m_I)$ in the zero external fermion mass limit, along with the corresponding width $\Gamma(Z \to Q\bar{q} + \bar{Q}q)/|v_I|^2 = m_Z/4\pi|\Delta F_L|^2$ in GeV.
3. The width for bs decay in MeV, divided by the generation mixing factor, as a function of the top quark mass in the three generation scheme.
4. The width for third generation b + fourth generation b′ decay, divided by the generation mixing factor, as a function of the b′ mass for three different t′ mass values. The box indicates the width for the predicted quark mass values of Ref. 12.
5. The width for tc decay in the four generation scheme, divided by the generation mixing factor, as a function of the b′ mass.
6. Non-diagonal counterterms (a) and their cancellation in $Z \to q_i\bar{q}_j$ (b).

Process		Operator	Operator Dimension	Leading Power of m_i
(a)		ϕ^n	n	4
(b)		$Z_\mu Z^\mu \phi^n$	n+2	2
(c)		$F^{\mu\nu} \partial_\mu Z_\nu \phi^n$	n+4	0
(d)		$F^{\mu\nu} F_{\mu\nu} \phi^n$	n+4	0
(e)				2
(f)				2
(g)				2

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4348A6

Table I

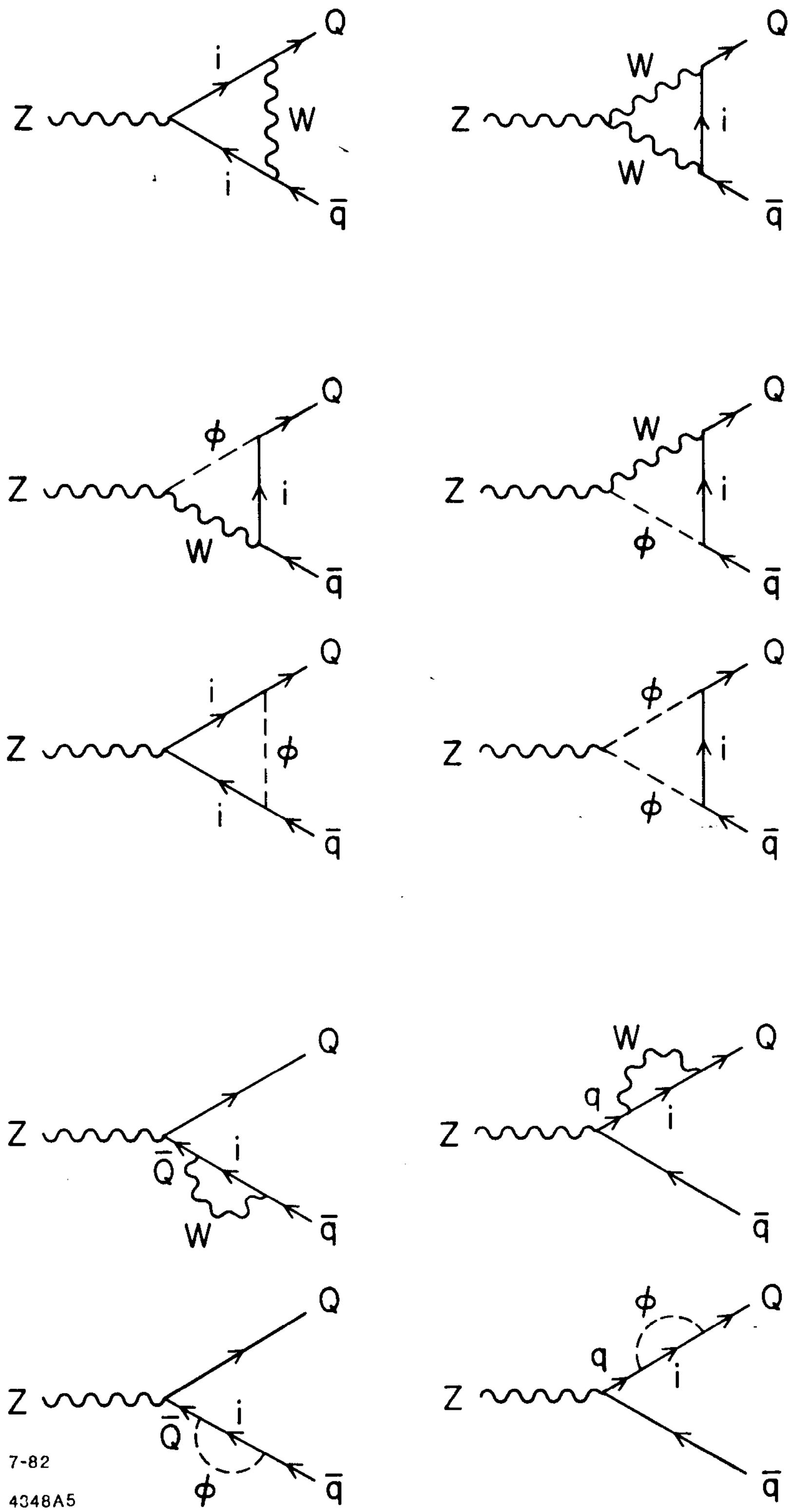
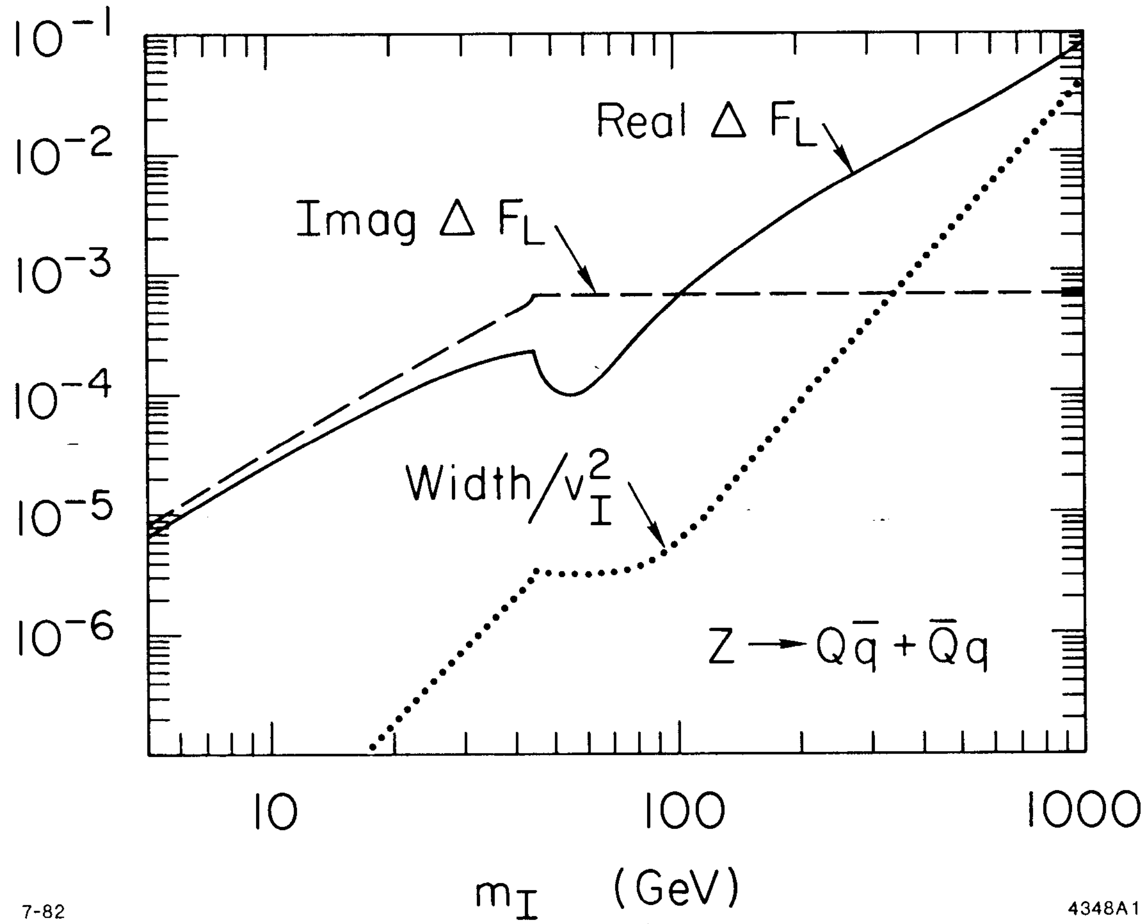


Fig. 1



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Real ΔF_L
Imag ΔF_L
Width/v_I^2
$Z \rightarrow Q\bar{q} + \bar{Q}q$
10
100
1000
m_I (GeV)
7-82
4348A1

Fig. 2

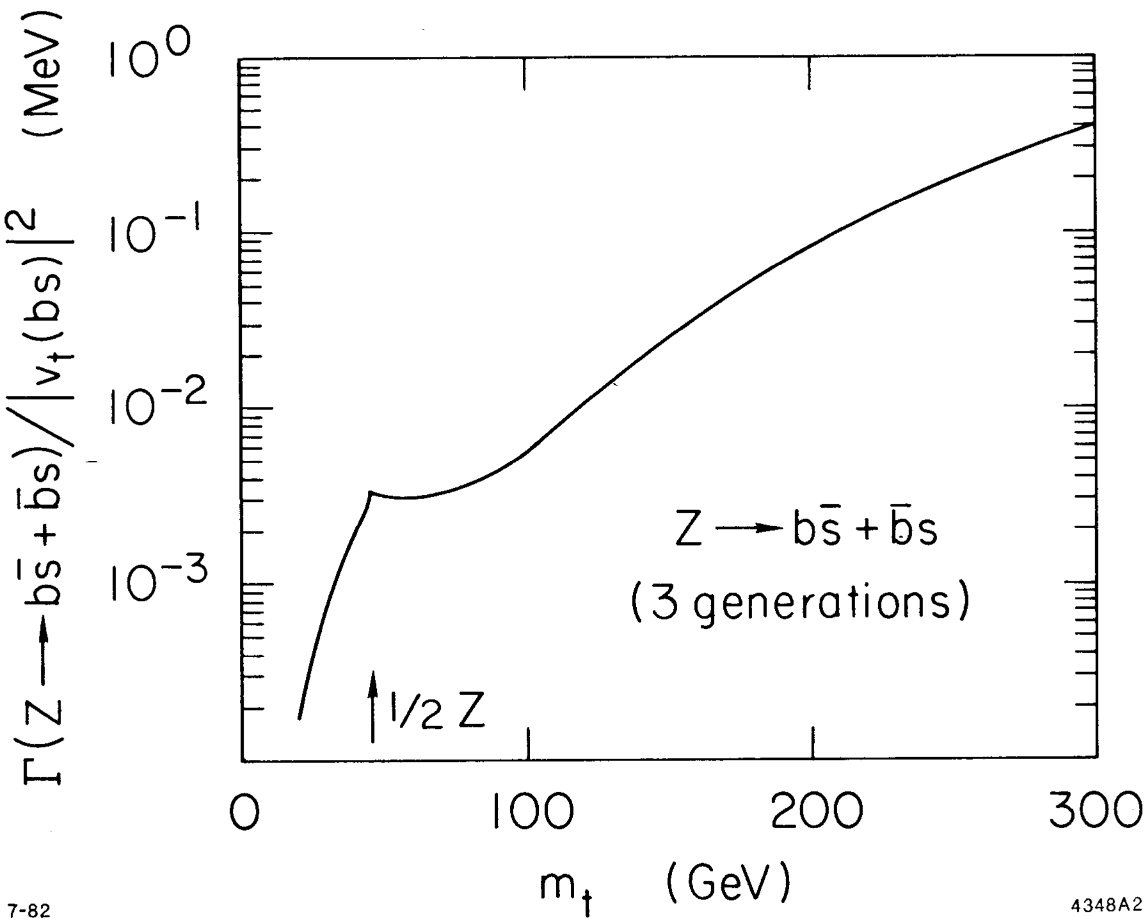
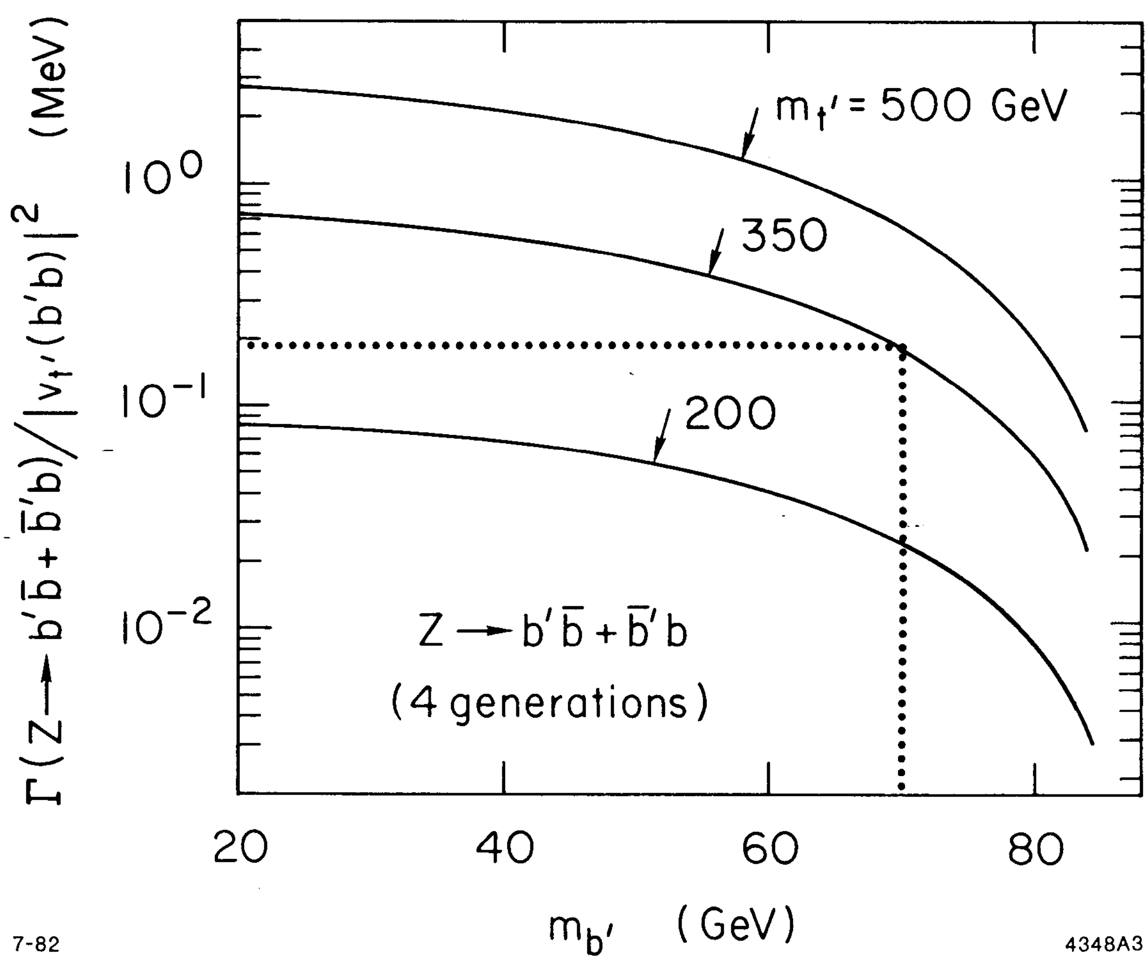
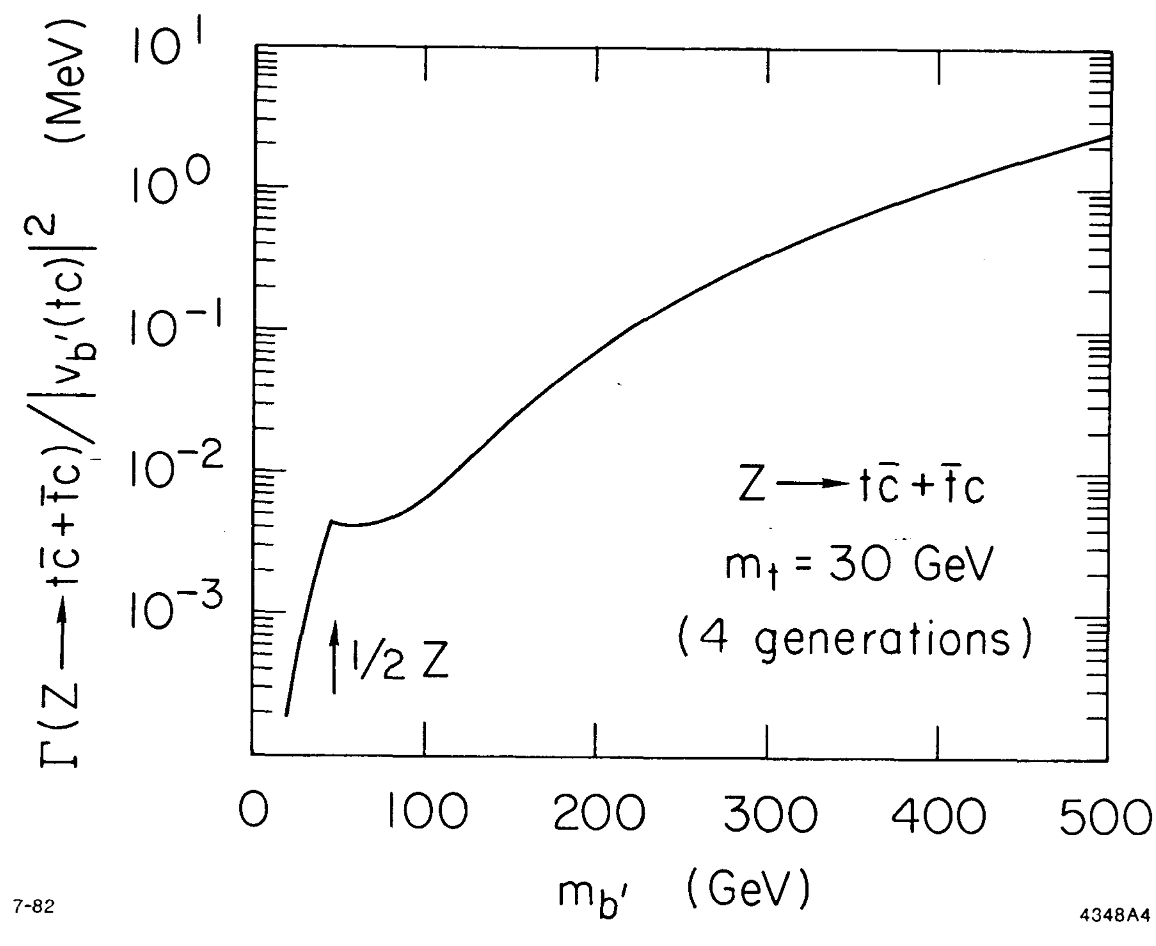

$\Gamma(Z \to b\bar{s} + \bar{b}s) / |v_t(bs)|^2$ (MeV)
10^0
10^{-1}
10^{-2}
10^{-3}
$Z \to b\bar{s} + \bar{b}s$
(3 generations)
1/2 Z
0
100
200
300
m_t (GeV)
7-82
4348A2

Fig. 3



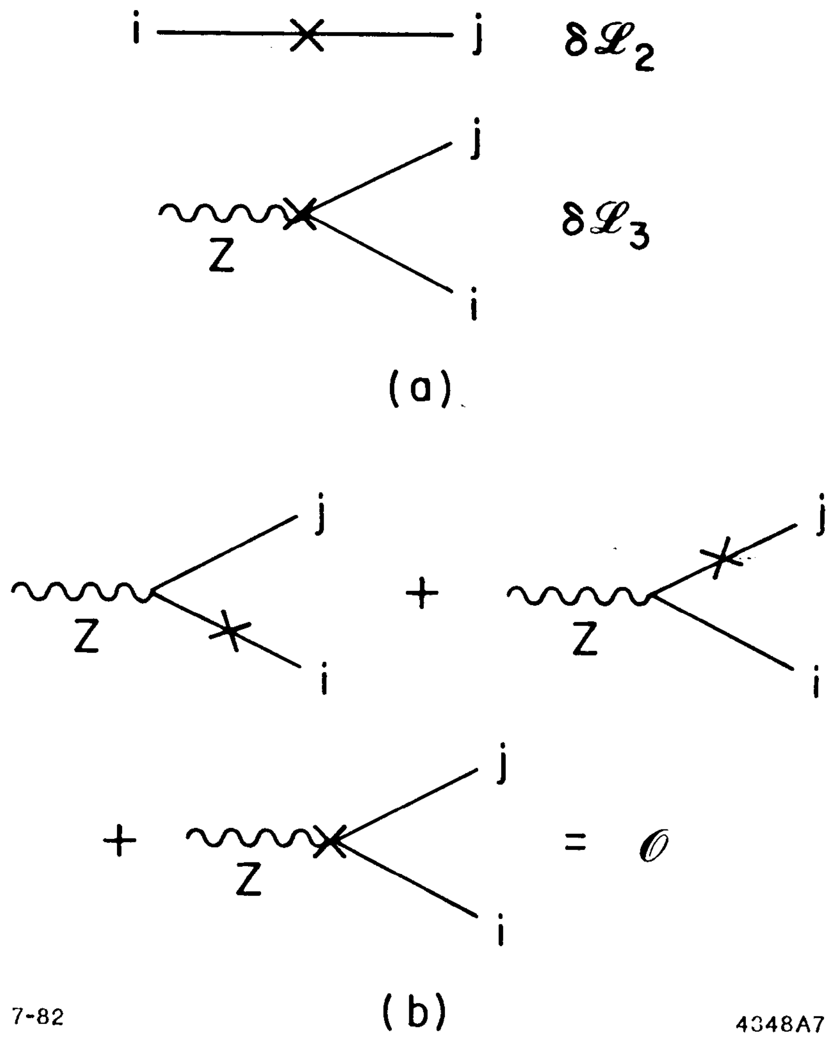
$\Gamma(Z \rightarrow b'\bar{b} + \bar{b}'b) / |v_{t'}(b'b)|^2$ (MeV)
10^0
10^{-1}
10^{-2}
$m_{t'} = 500$ GeV
350
200
$Z \rightarrow b'\bar{b} + \bar{b}'b$
(4 generations)
20
40
60
80
$m_{b'}$ (GeV)
7-82
4348A3

Fig. 4



$\Gamma(Z \longrightarrow t\bar{c}+\bar{t}c)/|v_{b'}(tc)|^2$ (MeV)
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$Z \longrightarrow t\bar{c}+\bar{t}c$
$m_t = 30$ GeV
(4 generations)
1/2 Z
0
100
200
300
400
500
$m_{b'}$ (GeV)
7-82
4348A4

Fig. 5



i
j
$\delta\mathscr{L}_2$
j
Z
i
$\delta\mathscr{L}_3$
(a)
j
Z
i
+
j
Z
i
+
j
Z
i
= $\mathcal{O}$
7-82
(b)
4348A7

Fig. 6