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THE NATURE OF NONPERTURBATIVE EFFECTS IN LEPTON-NUCLEON SCATTERING*

R. Michael Barnett
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

There is evidence for substantial nonperturbative effects in the structure functions extracted from deep-inelastic lepton-nucleon scattering. These effects have a major impact on α_s and Λ determinations. Results which I reported previously for μN and νN scattering (F_2 and xF_3) are extended to include ed data which are at lower W^2 and higher x and have extremely high statistics. Possible analytical forms for higher-twist terms and for α_s are considered in detail here using μN, νN and ed data.

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1. INTRODUCTION

There has been a strong conviction in recent years that the data for deep-inelastic lepton-nucleon scattering give clear evidence for the validity of Quantum Chromodynamics (QCD), calculated in perturbation theory. Furthermore it has been assumed that nonperturbative effects are small so that one may extract the magnitude of the strong coupling constant α_s [or equivalently the parameter Λ where $\alpha_s \propto 1/\ell n(Q^2/\Lambda^2)$]. The results from deep-inelastic scattering have been considered a primary window into perturbative QCD. However, in recent work[1] I have shown that there may be a major problem with these results, and I will expand upon this problem in this paper.

It is certainly true that the data are in rough qualitative agreement with the predictions of perturbative QCD.[2] In fact I believe it is a non-trivial success of the QCD theory that its zeroth-order predictions reproduce the parton model's predictions (scaling) for deep-inelastic scattering. The numbers found for α_s extracted from these data are small enough that one has every right to hope that perturbation theory might be an adequate means to calculate the predictions of Quantum Chromodynamics.

However, if these data are to be taken as evidence for QCD and if they are to be used to extract the magnitude of α_s, then we must examine the data for evidence of nonperturbative corrections.[3] These nonperturbative corrections could radically alter the magnitude of α_s extracted and then weaken the evidence for the validity of the theory.

So I have undertaken the examination of three major data sets to find the nature of nonperturbative effects. They are the data of the European Muon Collaboration[4] (EMC), the CERN-Dortmund-Heidelberg-Saclay (CDHS) collaboration[5] and the Stanford Linear Accelerator Center-Massachusetts Institute of Technology (SLAC-MIT) collaboration.[6] These data are for μN, νN and ed scattering respectively. Each data set was examined separately rather than in combination. For each, I examined and report here on the structure function F_2 in the singlet case. For CDHS I also studied xF_3 which gave similar results, but with poorer statistical significance. For SLAC-MIT I found similar results for F_2(e + proton) but the additional free parameters of the mixed singlet-nonsinglet case reduce the significance. I therefore do not report on these latter two cases.

The structure functions for deep-inelastic lepton-nucleon scattering can be extracted from the cross-sections. In the case of μN or eN scattering

$$\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^2}\left[yF_1(x,Q^2) + \frac{1}{x}\left(1 - y - \frac{Mxy}{2E} \right) F_2(x,Q^2) \right] , \tag{1.1}$$

where

$$2xF_1(x,Q^2) = F_2(x,Q^2) \left(1 + \frac{4M^2x^2}{Q^2} \right) \Big/ (1 + R) , \tag{1.2}$$

$$y = (E - E')/E = Q^2/(2MxE) , \tag{1.3}$$

$$R \equiv \sigma_L/\sigma_T . \tag{1.4}$$

As discussed elsewhere,[7] the impact of assumptions of R in extracting F_2 is of limited importance for these results. In each case here I report on results using the R values favored by each of the respective experimental collaborations. With the EMC data I eliminated a few endpoints which apparently are not at the center of their energy bins.[8] Since I wanted singlet data I chose the EMC data on iron.[4] These data were at E = 120, 250 and 280 GeV. These latter two energy sets overlapped in Q^2 and x so I normalized them. I tested the E = 120 GeV normalization by allowing it to be a free parameter, but there was little change. For the SLAC-MIT[6] data the statistical significance is so great I felt it essential to include a 1.5% systematic error. Systematic errors were not included for the other data sets. I found that my results were not sensitive to endpoints where systematics were expected to be large.

Since I wished to exclude the gross nonperturbative effects (such as simple mass effects), I chose to exclude all data with low Q^2 and low W^2. W^2 is the invariant hadronic mass ($W^2 = Q^2(1-x)/x + m^2$). Since W^2 is the variable of greatest concern I used the most stringent cut there. For EMC and CDHS data I required $W^2 > 10$ GeV^2. Since the SLAC-MIT data occur at much lower E, it is not possible to use such a severe cut. Since I was able to use the SLAC-MIT data to confirm and strengthen results established using the EMC and CDHS data, I used the cut $W^2 > 4$ GeV^2 for the SLAC-MIT data. The Q^2 cuts were $Q^2 \geq 4$ GeV^2 (EMC), $Q^2 \geq 2$ GeV^2 (CDHS) and $Q^2 \geq 3.5$ GeV^2 (SLAC-MIT). These effects were established in Ref. 1 using $Q^2 \geq 10$ GeV^2; use of these lower cuts increases the statistical significance. We expect perturbation theory to break down at small x, so in all three cases, I required $x \geq 0.15$.

The best means of analyzing deep-inelastic scattering data is to use the evolution equations of Altarelli and Parisi.[9] By contrast, moment analyses are once removed from the measured quantities and require extrapolation into unmeasured regions. Use of the evolution equations allows point-by-point comparisons using any cuts on the data. The results reported are always with the leading-order equations. Based on my work[10] with xF_3 and on the very small values of Λ found, it is safe to say that the magnitude of Λ and of nonperturbative effects reported here would be changed only in small amounts in going to next-to-leading order, but that the basic conclusions would remain unchanged.

The leading order evolution equations for eN and μN scattering in the singlet case are:

$$Q^2 \frac{\partial}{\partial Q^2} F_2{}^S(x,Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \left\{ [3 + 4 \ln (1-x)]\, F_2{}^S(x,Q^2) \right.$$

$$+ \int_x^1 dw \left[\frac{2}{(1-w)} \left[(1+w^2)\, F_2{}^S\left(\frac{x}{w}, Q^2\right) - 2F_2{}^S(x,Q^2) \right] \right.$$

$$\left.\left. + \frac{3}{2} N_f \left[w^2 + (1-w)^2\right] G\left(\frac{x}{w}, Q^2\right) \right] \right\} \qquad (1.5)$$

and

$$Q^2 \frac{\partial}{\partial Q^2} G(x,Q^2) = \frac{3\alpha_s(Q^2)}{\pi} \left\{ \left(\frac{11}{12} - \frac{N_f}{18} + \ln(1-x) \right) G(x,Q^2) \right.$$

$$+ \int_x^1 dw \left[\frac{wG\left(\frac{x}{w},Q^2\right) - G(x,Q^2)}{1-w} + \left(w(1-w) + \frac{1-w}{w} \right) G\left(\frac{x}{w},Q^2\right) \right.$$

$$\left. \left. + \frac{2}{9} \left(\frac{1+(1-w)^2}{w} \right) F_2{}^S\left(\frac{x}{w},Q^2\right) \right] \right\} \tag{1.6}$$

where N_f is the number of quark flavors. Note that my definition of the gluon distribution $G(x,Q^2)$ differs from some others by a factor of x. The momentum sum rule relates $F_2(x,Q^2)$ to $G(x,Q^2)$:

$$\int_0^1 dx\ (F_2{}^S + G) = 1 \tag{1.7}$$

Since the higher-twist corrections for the sum rule and for G are unknown, I have neglected <u>all</u> higher-twist effects in Eq. (1.7). Equations (1.5)-(1.6) require boundary conditions at $Q^2 = Q_0{}^2$ which I chose to be:

$$F_2{}^S(x,Q_0{}^2) = C(1-x)^\alpha\ (1 + ax) \tag{1.8}$$

$$G(x,Q_0{}^2) = A(1-x)^\beta\ (1 + gx) \tag{1.9}$$

Use of Eq. (1.7) fixes the value of A. Unfortunately there is no good way to determine the values of the glue parameters (ß and g). As reported in Ref. 7 the magnitude of A is sensitive to the choice of ß and g. The data prefer the approximate values $\beta \approx 5$ and $g \approx 0$ and for most work, I have fixed these parameters at values close to these.

It is essential for the (lower-energy) SLAC-MIT data to incorporate the target-mass corrections of ξ-scaling[11] into the QCD predictions for F_2 (they were also included for EMC and CDHS data). These are obtained by using

$$\tilde{F}_2(x,Q^2) = \frac{x^2}{\xi^2} v^3 F_2(\xi,Q^2) + \frac{6M^2}{Q^2} x^3 v^4 \int_\xi^1 dz \frac{F_2(z,Q^2)}{z^2}$$

$$+ \frac{12M^4}{Q^4} x^4 v^5 \int_\xi^1 dz \int_z^1 dy \frac{F_2(y,Q^2)}{y^2} \tag{1.10}$$

where

$$\xi = \frac{2x}{1 + (1 + 4M^2x^2/Q^2)^{1/2}} \tag{1.11}$$

and

$$v \equiv \left[1 + \frac{4M^2x^2}{Q^2} \right]^{-1/2} \tag{1.12}$$

F_2 is the output of the evolution equation (plus any higher-twist term), and $\tilde{F}_2$ is compared with data.

2. RESULTS

If there are substantial nonperturbative effects[3], there is one variable in deep-inelastic scattering which seems most appropriate for describing such effects. This is the invariant hadronic mass which is

$$W^2 = \frac{Q^2(1-x)}{x} + M^2 \tag{2.1}$$

It is the variable expected (from power counting rules) to occur in higher-twist terms. And it is the only large mass which is likely to affect the perturbative QCD predictions. Gupta and Quinn[12] have argued that there may be substantial nonperturbative effects and that they may not even fall as powers of W^{-2} (or Q^{-2}). Perturbative QCD predicts little W^2 dependence in the structure functions. One way to search for anomalous W^2 dependence is to look for such dependence in the strong coupling parameter Λ.

As described in Ref. 1, I first observed a problem with the predictions of perturbative QCD by extracting the magnitude of Λ separately at small invariant hadronic mass W^2 and at large W^2. In all cases I excluded the region $W^2 < 10$ GeV^2 to avoid unimportant mass effects. I found that the magnitude of Λ was radically different in the two W^2 regions: Λ(large W^2) $\approx$ 0.05 GeV whereas Λ(small W^2) $\approx$ 0.3-0.5 GeV.

It was not reported in Ref. 1 (since the $W^2 > 10$ GeV^2 cut cannot be maintained), but the SLAC-MIT data[6] divided into two W^2 bins give very similar results. I cannot rule out the possibility that these three different experiments[4-6] all have systematic errors of the same form. However, I believe that this is quite unlikely since the three experiments are quite different. In either case, these results clearly mean that α_s or Λ determinations are not possible from these data until the source of this anomaly is understood. And they cast a cloud over the "evidence" for perturbative QCD.

Before investigating nonperturbative effects, one should note that the source of this anomaly is not in thresholds for hadrons (including intrinsic charm)[13] or for leptons. Hadronic thresholds would enhance the discrepancy not explain it. The variety of leptonic beams make that hypothesis untenable. Furthermore, while ξ scaling is important for lower W^2 values, it is not the source of the anomaly. Neither are Q^2 dependence in the β function or the value of R. The use of the V^2 evolution approach[14] has little impact.

The primary focus of this paper is to see if the anomalous W^2 dependence implies nonperturbative corrections which can be parameterized as higher-twist corrections (later I will discuss modifications of the strong coupling constant α_s). The procedure I have followed is to evolve the leading-twist piece of $F_2(x,Q^2)$ according to Eqs. (1.5)-(1.6); then multiply by a higher-twist term (say $[1 + W_0{}^2/W^2]$); then calculate the ξ-scaling version of F_2 according to Eq. (1.10) and then determine the parameters (Λ, $W_0{}^2$, etc.) by fitting to the data. This procedure has several faults. We have assumed that the higher-twist term evolves exactly as the leading-twist term, and that there is only one higher-twist term. This may not be entirely reasonable since we find large higher-twist terms. Furthermore, since F_2 appears to have such corrections, it is possible that $G(x,Q^2)$ also has such corrections. And I have neglected all such effects in the momentum sum rule. All of this means that the exact values and parameterizations of the correction terms that I find might have to be modified. However, present data (and probably future data) certainly lack the precision to allow us to add parameters to account for these additional effects.

Before proceeding, let me comment on the impact of the ξ variable of Eqs. (1.10)-(1.12) (i.e., the impact of target mass effects). With the higher energy EMC and CDHS data, the impact is minimal. However, with the SLAC-MIT data it is crucial to include ξ-scaling. In Fig. 1 we see this effect for a sample of one-third of the data. By including the ξ variable in analysis of SLAC-MIT data, χ^2 improves from 240 to 105 for 79 degrees of freedom for the case with no higher-twist. When higher-twist is included, the impact on χ^2 is very small, which indicates that the higher-twist terms can also compensate for target-mass effects.

As described above, higher-twist effects were included by multiplying the leading-twist results by terms of the form $[1 + f(x,1/W^2)]$ or $[1 + f(x,1/Q^2)]$. Most early analyses of these effects in deep-inelastic scattering assumed that these terms were

$$\left(1 + \frac{W_0^2}{W^2}\right) \tag{2.2}$$

or equivalently

$$\left(1 + \frac{Q_0^2}{Q^2(1-x)}\right) \quad . \tag{2.3}$$

With these forms one finds that there is little improvement to fits and that W_0^2 and Q_0^2 are small and consistent with zero.

In Ref. 1, I described use of a modified higher-twist term:

$$\left[1 + \left(\frac{x}{0.4} - 1 \right) \left(\frac{9}{W^2} \right)^4 \right] \quad . \tag{2.4}$$

While this term gives an excellent fit to the EMC data, it has several faults. It is a peculiar, ad hoc form which changes sign at $x = 0.4$. This form is not the best parameterization of the anomalous W^2 dependence for the CDHS data. It is not necessary for higher-twist effects to be the same in neutrino and muon scattering, but it would be more believable if they were similar. It is obvious that this term cannot be extrapolated down to the W^2 region of the SLAC-MIT data (it would blow up). (The SLAC-MIT data were not considered in Ref. 1.) The effect of the high power of $1/W^2$ in term (2.4) is to damp out the low x region. The $(x/0.4 - 1)$ piece causes a sharp rise at large x.

This suggests use of a higher-twist term of the form (multiplied times the leading-twist QCD results):

$$\left(1 + x^3 \frac{W_0^2}{W^2} \right) \quad . \tag{2.5}$$

The precise power of x might be slightly different. For the EMC data the fit with term (2.5) is almost the same as with (2.4). For the CDHS data, term (2.5) gives an improved fit. For the SLAC-MIT data this term gives a greatly improved fit. (The χ^2 relative to the χ^2 with $W_0^2 \equiv 0$ are EMC 133/143 for 118 dof, CDHS 83.4/87.4 for 76 dof, and SLAC-MIT 68/105 for 78 dof.) The values of W_0^2 obtained are

$$W_0^2 = 12.5 \pm 4.3 \ \text{GeV}^2 \ (\Lambda \approx 0.075 \ \text{GeV}) \ \text{EMC}$$

$$W_0^2 = 8.3 \pm 5.3 \ \text{GeV}^2 \ (\Lambda \approx 0.130 \ \text{GeV}) \ \text{CDHS}$$

$$W_0^2 = 4.4 \pm 0.47 \ \text{GeV}^2 \ (\Lambda \approx 0.048 \ \text{GeV}) \ \text{SLAC-MIT}$$

These Λ values should not be taken as quantitative since they are quite sensitive to the precise parameterization of higher-twist effects and to the precise value of W_0^2 (and because of the deficiencies of this approach described in Section I). The difference between the EMC and SLAC-MIT values for W_0^2 may reflect differing systematic errors, or the neglect of other higher-twist terms (which could be relevant in the energy region of one data set) or the inadequacy of term (2.5) for describing the data. Recall that the SLAC-MIT data have a different W^2 cut and are at lower energies.

The implication of these results from three different types of experiments is that W_0^2 is large and clearly non-zero. Such a higher-twist term is strongly peaked at higher x values. Note that the SLAC-MIT data have x values as high as 0.90 whereas other data have $x \lesssim 0.65$.

Further understanding of the nature of nonperturbative corrections may be obtained by considering a higher inverse-power of W^2 in the higher-twist term. Specifically using the term

$$\left[1 + x^2 \frac{W_0^4}{W^4} \right] \qquad (2.6)$$

in QCD gives a fit to EMC data which is slightly inferior to that using term (2.5) but still superior to that using leading-twist QCD. For CDHS data, fits with terms (2.5) and (2.6) are very similar. For the SLAC-MIT data term (2.6) gives a noticeably superior fit to the data. (The χ^2 relative to the χ^2 for $W_0^2 \equiv 0$ are EMC 136/143 for 118 dof, CDHS 83.2/87.4 for 76 dof and SLAC-MIT 57/105 for 78 dof.) The values of W_0^2 obtained are

$$W_0^2 = 7.1 \pm 1.7 \ \text{GeV}^2 \ (\Lambda \approx 0.11 \ \text{GeV}) \ \text{EMC}$$

$$W_0^2 = 6.3 \pm 2.4 \ \text{GeV}^2 \ (\Lambda \approx 0.15 \ \text{GeV}) \ \text{CDHS}$$

$$W_0^2 = 3.6 \pm 0.35 \ \text{GeV}^2 \ (\Lambda \approx 0.006 \ \text{GeV}) \ \text{SLAC-MIT}$$

Again, the Λ values should not be taken as quantitative.

As for term (2.5) three different types of experiments give large values of W_0^2 which are clearly non-zero. These are substantial nonperturbative effects which cannot be neglected and which can have a major impact on the value of Λ. For the SLAC-MIT data, I show the impact of term (2.6) in Fig. 2. Note that only one-third of the data appears in Fig. 2 due to the large number of x-bins; as a result one doesn't see the full statistical significance of the difference between the two fits.

I believe that the comparison of the two terms (2.5) and (2.6) using the three data sets suggests that both the W^{-2} and W^{-4} terms may be substantial. It would not be useful to attempt to use both terms in a fit since the number of parameters increases beyond the signifcance of the data and since one already has obtained adequate fits to the data.

Also of interest is a recent calculation by Gunion, Nason and Blankenbecler[15] of leading power-law correction to F_2 at large x near 1 and large Q^2. Their analysis is based on the extension of the Brodsky-Lepage formalism[16] first employed by Berger and Brodsky[17] in their calculation of higher-twist contributions for pion beams. They find a term

$$\left[1 - 7x^{\alpha} \left(\frac{m^2}{Q^2(1-x)^2} \right) + 600x^{\beta} \left(\frac{m^2}{Q^2(1-x)^2} \right)^2 \right] \quad . \tag{2.7}$$

The powers (α and β) of x are not calculated. Since this term contributes at large x only, there is very little sensitivity in the data to these powers. For convenience, let us take $\alpha = 1$ and $\beta = 2$. The SLAC-MIT data are very well fit by these data [the fit is comparble to that from term (2.6)]. One finds $m = 0.132$ and $\Lambda = 0.128$ GeV. The EMC and CDHS data are consistent with term (2.7) but show no improvement in fit over leading-twist for $m = 0$-0.13 GeV. It is possible that the EMC and CDHS data would require additional nonperturbative corrections.

An alternative approach to consideration of nonperturbative effects is the possibility that there are power-law terms in the strong coupling constant α_s which are important at small Q^2 or W^2. This would result in α_s being much larger at small Q^2 than expected from higher Q^2 data. In Ref. 1, I discussed the form for α_s:

$$\alpha_s = \frac{W_0{}^2}{W^2} + \frac{12\pi}{25 \ln Q^2/\Lambda^2} \quad . \tag{2.8}$$

One can only make the approximation that the evolution equations (1.5)-(1.6) are unaffected except for the substitution of Eq. (2.8) for α_s.

With term (2.8) one finds only modest improvements in fits to EMC, CDHS and SLAC-MIT data. [The χ^2 for the fits relative to the χ^2 for fits with $W_0{}^2 \equiv 0$ are EMC 137/143 (118 dof), CDHS 85.5/87.4 (76 dof) and SLAC-MIT 89/105 (78 dof).] The $W_0{}^2$ values are

$$W_0^2 = 2.3 \pm 0.8 \quad \text{GeV}^2 \quad \text{EMC}$$

$$W_0^2 = 0.25^{+2.0}_{-0.25} \quad \text{GeV}^2 \quad \text{CDHS}$$

$$W_0^2 = 1.6 \pm 0.5 \quad \text{GeV}^2 \quad \text{SLAC-MIT}$$

The resulting Λ values are very small. These numbers at least raise the possibility that α_s is quite large even while Λ is small. The values of α_s in the SLAC-MIT and EMC data can be as large as 0.5, and this clearly makes use of perturbation theory very unwise (including for this extraction of α_s).

Equation (2.8) may not be the best parameterization for α_s. An improved fit to the EMC data ($x^2 = 132$) is obtained using

$$\alpha_s = \left(\frac{x}{0.42} - 1\right) \theta(x-0.42) \frac{8}{Q^2} + \frac{12\pi}{25 \ln Q^2/\Lambda^2} \quad . \tag{2.9}$$

An improved fit to the SLAC-MIT data ($x^2 = 82$) is obtained using

$$\alpha_s = \left(\frac{1.5}{W^2}\right)^2 + \frac{12\pi}{25 \ln Q^2/\Lambda^2} \quad . \tag{2.10}$$

Given the large values of α_s found, I feel that it is not fruitful to search for the "perfect" parameterization of α_s.

3. CONCLUSIONS

It is clear that there are substantial nonperturbative effects present in the data for deep-inelastic lepton-nucleon scattering. The evidence appears in ed, μN and νN data for $F_2(x,Q^2)$ and $xF_3(x,Q^2)$. The evidence has the most statistical significance in the ed data but the size of such effects appear to be somewhat smaller there (in terms of the parameter W_0^2) than in the μN or νN data. These nonperturbative effects may be parameterized as $x^3W_0^2/W^2$ or $x^2W_0^4/W^4$ relative to the leading-twist QCD results. Multiplying (or dividing) these terms by powers of $(1-x)$ offers no improvement. The magnitude of W_0^2 may be in the range 4-12 GeV^2. Alternatively one may consider that the strong couping constant α_s is modified at small Q^2 or W^2 to include an inverse power term [$\alpha_s \approx W_0^2/W^2 + 12\pi/(25 \log Q^2/\Lambda^2)$]. In this case $W_0^2 \approx 1.5$-2.0 GeV^2, and one finds that in some regions α_s is extremely large, $\alpha_s \approx 0.5$, even though Λ is quite small.

From either viewpoint these are large effects which cast doubt on the appropriateness of perturbation theory calculations of Quantum Chromodynamics in these kinematic regions. The magnitude of these effects also means that any simple attempt to parameterize nonperturbative effects in these regions (such as reported here) can at best give very qualitative understandings of the nature of the phenomena. These effects clearly are confined to the large x region and involve powers of the invariant hadronic mass. It also appears that Λ might be quite small (perhaps Λ = 0-150 MeV). But it is not clear that it is appropriate to use perturbation theory to study such substantial nonperturbative effects.

It is tempting to suggest that the situation might improve if we only had data in a higher energh region. This was said at an earlier time and has now proven to be wrong. There is nothing in the results reported here to suggest that the situation will improve in a still higher energy region. Given the magnitude of these effects it is not appropriate to extrapolate up in energy. We can, however, hope that at higher energies if we do not enter a perturbative regime, we will at least learn considerably more about the nonperturbative nature of QCD.

There is no question that α_s and Λ extractions from the present data for deep-inelastic lepton-nucleon scattering are not valid (it is conceivable that Λ is zero).[18] There may well be no reliable means of determining α_s or Λ from any present experimental data. Our ignorance is greater than we would like.

Finally there will undoubtedly be those who feel that these results cast doubt on all of QCD by removing much of the evidential foundation of QCD. I don't share that view, but hope that further theoretical and experimental work can clarify the situation.

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REFERENCES

1. R. M. Barnett, Phys. Rev. Lett. 48, 1657 (1982).

2. A. J. Buras, in Proceedings 1981 International Symposium on Lepton and Photon Interactions at High Energies, edited by W. Pfeil (Physikalisches Institut, Universitat Bonn, 1981), p. 636; S. D. Drell, ibid., p. 1003; A. H. Mueller, Phys. Rep. 73, 237 (1981).

3. L. F. Abbott and R. M. Barnett, Ann. Phys. 125, 276 (1980); L. F. Abbott, W. B. Atwood and R. M. Barnett, Phys. Rev. D 22, 582 (1980); R. Blankenbecler, in Proceedings of the 15th Rencontre de Moriond, Les Arcs, France, 1980, edited by J. Tran Thanh Van (Editions Frontieres, Dreux, France 1980), Vol. 1, p. 463; H. D. Politzer, Physica Scripta 23, 934 (1981); S. J. Brodsky and G. P. Lepage, in High Energy Physics - 1980 (XX International Conference, Madison, Wisconsin), edited by L. Durand and L. G. Pondrom (American Institute of Physics, New York, 1981), p. 805; R. L. Jaffe and M. Soldate, report no. MIT-CTP-954 and Phys. Lett. 105B, 467 (1981); F. Eisele et al., report no. DO-TH81/11; A. Donnachie and P. V. Landshoff, Phys. Lett. 95B, 437 (1980); E. Berger, Z. Phys. C4, 289 (1980); I. A. Schmidt and R. Blankenbecler, Phys. Rev. D 16, 1318 (1977); I. A. Schmidt, Ph.D. Thesis, SLAC Report No. 203; C. A. Dominguez, Phys. Rev. D 25, 3084 (1982); E. L. Berger, Phys. Lett. 89B, 241 (1980); L. Baulieu et al., Phys. Lett. 81B, 41 (1979); R. K. Ellis, W. Furmanski and R. Petronzio, report no. CERN-TH-3254; E. V. Shuryak and A. I. Vainshtein, Nucl. Phys. B199, 451 (1982) and Phys. Lett. 105B, 65, (1981).

4. J. J. Aubert et al., Phys. Lett. 105B, 322 (1981); see also Phys. Lett. 105B, 315 (1981).

5. F. Eisele, in Proceedings of the International Conference on Neutrino Physics and Astrophysics: Neutrino 81, Maui, Hawaii, 1981, edited by R. J. Cence, E. Ma, and A. Roberts (University of Hawaii Press, Honolulu, 1982); see also H. Abramowicz et al., report nos. CERN-EP/82-23 and CERN-EP/81-168.

6. J. S. Poucher et al., Phys. Rev. Lett. 32, 118 (1974); J. S. Poucher, Ph.D. Thesis, MIT, 1971 (unpublished); A. Bodek et al., in Particles and Fields - 1973 (Proceedings of the Berkeley APS Meeting, August 13-17, 1973), edited by H. H. Bingham, M. Davier and G. R. Lynch (American Institute of Physics, New York, 1973), p. 98; A. Bodek, Ph.D. Thesis, MIT LNS Report No. COO-3069-116, 1972 (unpublished); E. M. Riordan, Ph.D. Thesis, MIT LNS Report No. COO-3069-176, 1972 (unpublished); W. B. Atwood, Ph.D. Thesis, SLAC Report No. 185 (unpublished); M. D. Mestayer, Ph.D. Thesis, SLAC Report No. 214 (unpublished); E. M. Riordan et al., Report No. SLAC-PUB-1634 (unpublished); A. Bodek et al., Phys. Rev. D 20, 1471 (1979); W. B. Atwood et al., Phys. Lett. 64B, 479 (1976); S. Stein et al., Phys. Rev. D 12, 1884 (1975).

7. R. M. Barnett and D. Schlatter, Phys. Rev. 112B, 475 (1982).

8. Private communication from John Carr.

9. G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977); Yu. L. Dokshitser, D. I. Dyakonov, and S. I. Troyan, Report No. SLAC-TRANS-0183, translated from Proceedings of the 13th Leningrad Winter School on Elementary Particle Physics (1978), pp. 1-89 (unpublished); J. Kogut and L. Susskind, Phys. Rev. D 9, 697 (1974); 9, 3391 (1974); G. Parisi, Phys. Lett. 43B, 207 (1973); D. J. Gross, Phys. Rev. Lett. 32, 1071 (1974).

10. An analysis of CDHS data for xF_3 using leading-order evolution and next-to-leading order evolution was done assuming a higher-twist term of the form $(1 + x^3W_0{}^2/W^2)$. I found in both cases that a very small value of Λ resulted and $W_0{}^2 \approx 40 \pm 15$ GeV^2. These results unlike those for F_2 were found to be sensitive to end points where systematics are expected to be large. It is important to note, however, that the value of $W_0{}^2$ is quite insensitive to whether leading or next-to-leading order evolution equations were used (largely because Λ is small).

11. H. Georgi and H. D. Politzer, Phys. Rev. Lett. 36, 1281 (1976).

12. S. Gupta and H. R. Quinn, Phys. Rev. D 25, 838 (1982).

13. D. P. Roy, Phys. Rev. Lett. 47, 213 (1981); S. J. Brodsky *et al.*, Phys. Lett. 93B, 451 (1980); R. M. Godbole and D. P. Roy, Tata Institute report no. TIFR/TH-82-5.

14. R. M. Barnett, D. Schlatter and L. Trentadue, Phys. Rev. Lett. 46, 1659 (1981).

15. J. Gunion, P. Nason and R. Blankenbecler, SLAC Report No. SLAC-PUB-2937 and forthcoming publication.

16. S. J. Brodsky and G. P. Lepage, in *High Energy Physics - 1980 (XX International Conference, Madison, Wisconsin)*, edited by L. Durand and L. G. Pondrom (American Institute of Physics, New York, 1981), p. 805, and Phys. Rev. D 22, 2157 (1980).

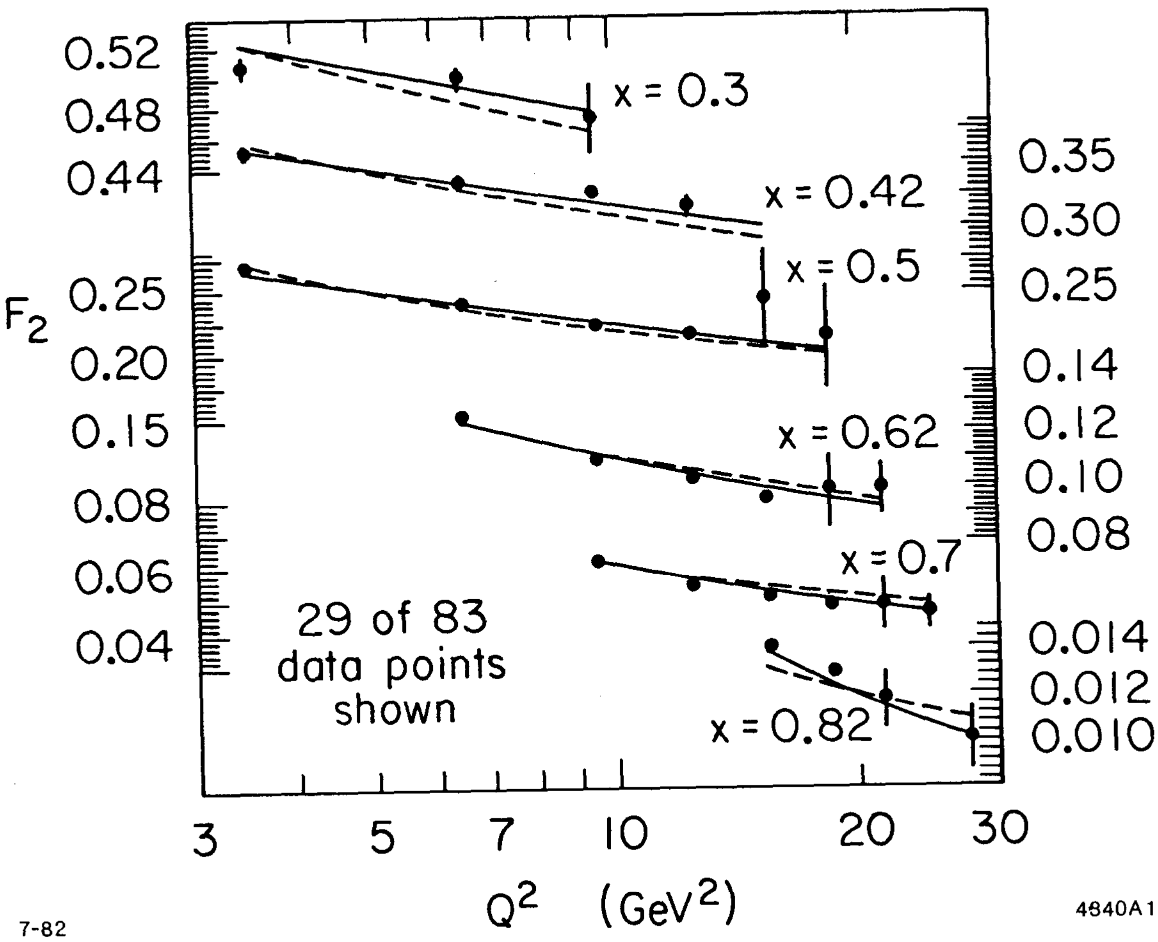
17. E. L. Berger and S. J. Brodsky, Phys. Rev. Lett. 42, 940 (1979).

18. With a term such as $[1 + (c+x^2)W_0{}^2/W^2]$ and $\alpha_s = \Lambda \equiv 0$, one can get fits which are almost as good as those from leading-twist QCD. There are undoubtedly other similar terms which can do even better.

Figure Captions

1. The structure function $F_2(x,Q^2)$ versus Q^2 for selected values of x. The solid (dashed) curves are the predictions of leading-twist QCD with (without) ξ-scaling incorporated. The data are from the SLAC-MIT collaboration (Ref. 6). In some cases the error bars are smaller than the plotted points. There are 25 x bins. The two-thirds of the data which lie in other x-bins are not plotted here, but of course they contribute to the statistical significance of the difference of the two sets of curves.

2. The structure function $F_2(x,Q^2)$ versus Q^2 for selected values of x. The solid curves are the predictions of leading-twist QCD with ξ-scaling incorporated. The dotted curve is the leading-twist QCD result multiplied by $(1 + 13\ x^2/W^4)$. The data are from the SLAC-MIT collaboration (Ref. 6). In some cases the error bars are smaller than the plotted points. There are 25 x bins. The two-thirds of the data which lie in other x bins are not plotted here, but of course they contribute to the statistical significance of the difference of the two sets of curves.



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F_2
x = 0.3
x = 0.42
x = 0.5
x = 0.62
x = 0.7
x = 0.82
0.35
0.30
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29 of 83
data points
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Q^2 (GeV^2)
7-82
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Fig. 1

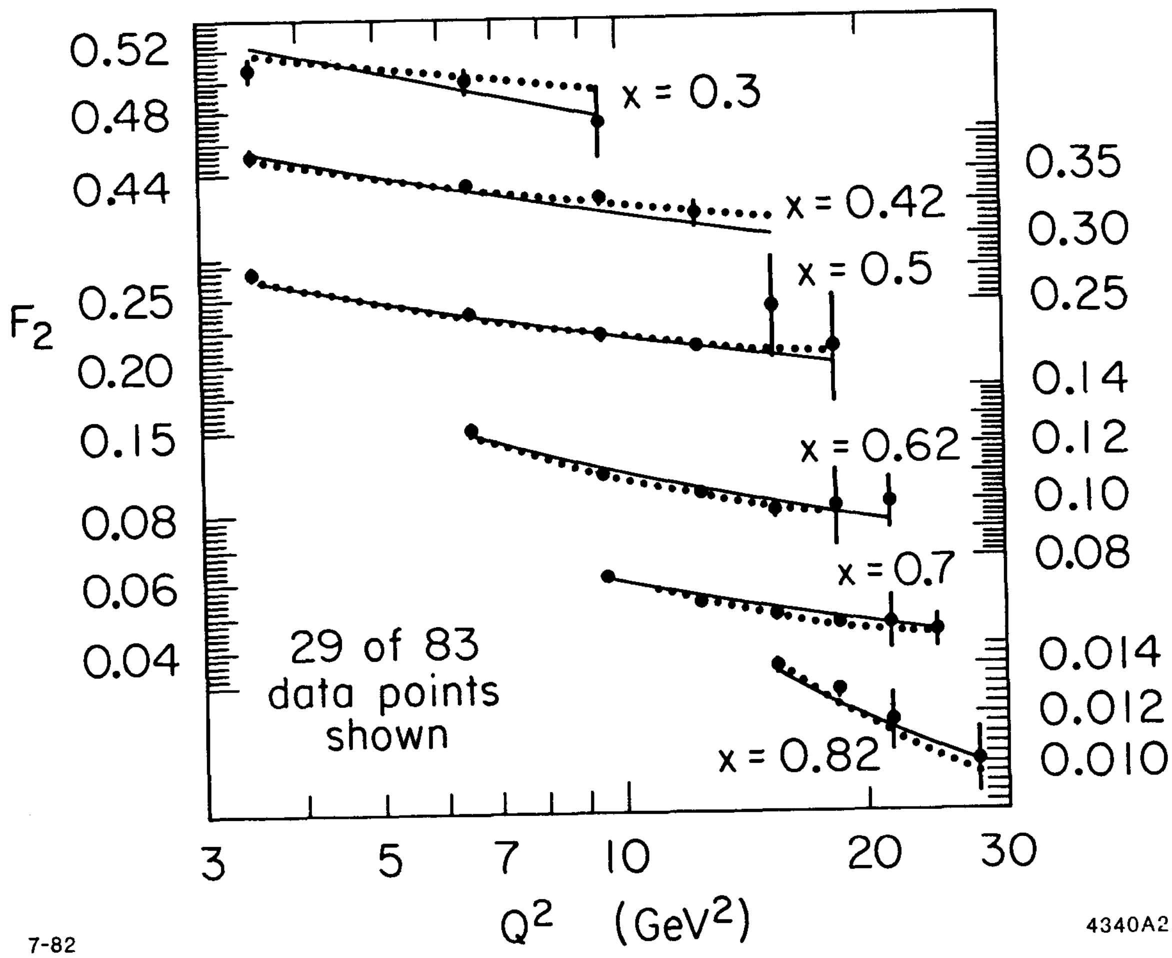

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Fig. 2