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PAULI-CORRECTED BREIT-WIGNER FORMULAE FOR COMPOUND RESONANCES IN TWO- AND THREE-CLUSTER SYSTEMS*

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ABSTRACT

Breit-Wigner formulae for compound resonances in two- and three-cluster systems are derived. Analytic corrections due to the Pauli Principle with regard to corresponding formulae for partial resonance widths and resonance pole shifts are given. In the case of a system of three composite particles a model is formulated, in which sudden decay into three clusters is determined by the knowledge of the microscopical compound state of the metastable decaying nucleus. The derivation of the Breit-Wigner formulae in the case of a three-cluster system is based on the study of the asymptotic behavior of the full three-body Green's function. In the case of the two-body channel situation the formalism presented here is an alternative to the Wildermuth-Benöhr reaction theory.

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1. INTRODUCTION

The purpose of this paper is the study of the influence of the Pauli Principle on the compound resonance behavior in two- and three-cluster systems. The method proposed here is based on the introduction of a Pauli-corrected microscopic Hamiltonian. This deviates from the usual procedure as for example in the resonating group method [1,2,3], in which the Pauli Principle is introduced by a fully antisymmetrized ansatz for the wave function. Consequently our approach differs from the reaction theory by Wildermuth and Benöhr [4] already in the two-body channel situation, which is based on the resonating group method (RGM). Therefore, in the two-cluster situation our approach may be considered as an alternative to the reaction theory by Wildermuth and Benöhr. In the three-cluster case we formulate a model for a three-cluster compound resonance, and derive the resulting three-body Breit-Wigner formulae.

In Ref. [4] it has been shown that compound states give rise to resonances of Breit-Wigner type in the single channel case. Also the coupled channel case has been investigated and the influence of a second open channel on the resonance behavior in the first channel has been studied. However a compound resonance for three clusters with internal structure found no treatment hitherto.

On the other side, resonances in three-particle systems have been a topic of great interest during the last decade, in particular in elementary particle physics. Most investigations, however, are based on the assumption that the three interacting particles are point particles without internal structure. In the present paper we want to consider resonances which occur in systems of three composite particles, or

clusters. The new feature will be the appearance of compound resonances and the difficulty will be that they will interfere with subsystem resonances or resonance-like structures such as the final state peak.

In terms of mathematics, the difficulty will be that, instead of the two-body Green's function, we will need the three-body Green's function. This means that we have to assume that a Faddeev equation has already been solved for the motion of the three clusters without the presence of a compound state (and in absence of the Pauli Principle). It will be seen that in our model the inclusion of the Pauli Principle leads to a strong three-body force. This three-cluster force is represented by a three-cluster separable potential (Pauli potential). The first difficulty will be the inclusion of the Pauli Principle by an appropriate treatment of the Pauli potential. Another difficulty arises from the complicated dynamics of the three-cluster system.

In terms of physics, the latter difficulty is this. Since there is a spectator cluster which can carry an arbitrary amount of energy, a subsystem resonance or a final state peak will not only appear in a narrow energy range of the three-body system. It will appear at all energies above a certain minimum which corresponds to zero spectator energy. The compound resonance will thus never be an isolated resonance but will always find something to interfere with. This will be seen more clearly from the relevant mathematical expressions.

In Section II we propose a method to include the Pauli Principle in the microscopic Hamiltonian and how to derive the Pauli-corrected effective intercluster interaction in the case of two- and three-cluster system. In Section III we derive the Breit-Wigner formula for the two-

cluster compound resonance and discuss how it is affected by the Pauli Principle. In Section IV a model for the three-cluster resonance - based on the method presented in Section II - is formulated. A technique for solving this model is proposed in Section V. The technique is based on the asymptotic formulae of the full three-cluster resolvent in the presence of two-body forces only. The corresponding formulae are discussed in the Appendix.

By applying this technique we succeed in derivation of generalized Breit-Wigner resonance formulae for the three-cluster system. In Section VI the Breit-Wigner formulae and the influence of the Pauli Principle on them are discussed in more detail.

2. MICROSCOPIC TREATMENT OF THE PAULI PRINCIPLE

We consider a microscopic Hamiltonian $H = H_0 + V$, of an A-nucleon system, where the potential V is a sum of two-body interactions, and the corresponding A-particle Schroedinger equation is

$$(H_0 + V)\psi = E\psi \quad . \tag{2.1}$$

We rewrite the time-independent Schroedinger equation (2.1) in the form of a projection equation

$$\langle \delta\psi \mid H-E \mid \psi \rangle = 0 \quad . \tag{2.2}$$

If $\delta\psi$ represents a completely arbitrary variation in the space of all A-nucleon functions, then Eqs. (2.1) and (2.2) are entirely equivalent.

In the case of two-cluster single channel the RGM was very successful in solving Eq. (2.2) for the ansatz

$$\psi = \mathcal{A}\{\phi(1)\ \phi(2)\ \chi\} \quad . \tag{2.3}$$

Here the functions ϕ describe the internal behaviour of the cluster, while the function χ is relative-motion function in two-cluster configuration. $\mathcal{A}$ denotes the antisymmetrization operator of the A-nucleon system. The variation $\delta\psi$ is now restricted to the arbitrary variations of the linear function χ.

In contrast to the RGM our method introduces the influence of the Pauli Principle not by a fully antisymmetrized ansatz for the wave function ψ but by a modification of the microscopic Hamiltonian [5].

On the microscopic level we are able to express the Pauli Principle by mutual orthogonal Pauli forbidden states $|n\rangle$. Note that on the cluster level there are in addition to Pauli forbidden states also partly Pauli forbidden states, which in many cases carry the main part of the Pauli principle.[1] On the microscopic level this separation in Pauli forbidden and Pauli partly forbidden states can be avoided by explicit construction for example in the harmonic oscillator shell model space. In principle there is an infinite number of the microscopic Pauli forbidden states. However, in any realistic (variational) bound state calculation the number of microscopic Pauli forbidden states is limited by the dimension of the space used in the calculation; and the dimension of the space is determined by a reasonably chosen highest energy quantum excitation. The same holds also for the scattering problem. Here, at any fixed scattering energy, but still consistent with the cluster ansatz under consideration, we have to choose high enough quantum excitation and to include the corresponding microscopic Pauli forbidden states. It is true that at high energies the number of microscopic Pauli forbidden states increases rapidly. However, this problem is not too serious for two reasons. First, at high energies the exchange effects due to the Pauli principle become small. Second, the validity of the cluster model is restricted to the low energy range. Thus we can restrict the number of Pauli forbidden states to some reasonable and manageable number.

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[1] In the RGM the Pauli forbidden states correspond to the eigenstates of the norm kernel with eigenvalues equal to one. The eigenvalues equal to one appear only in so-called equal width limits of the RGM.

It should be noted that the states $|n\rangle$ carry all information on the spin and angular momentum configuration of the system under consideration and therefore when introducing cluster internal functions for a system of two or more clusters one has to take the appropriate projection of the spin and angular momentum variables onto the channel under consideration.

$$|n\rangle = \sum_i c_i^n \{\zeta_i^n \prod_\nu u(i_\nu^n; \xi_\nu)\} \quad .$$

Here ζ_i^n are spin-isospin functions, u is a simple particle function labeled by i_ν, and ξ_ν are the ν-th member of set of Jacobi coordinates which describe the configuration of the system of nucleons. It holds $\mathcal{A}|n\rangle = 0$. We also assume that the coefficients c_i^n are chosen such that $\langle n|m\rangle = \delta_{nm}$. We wish now to solve this Schroedinger equation not in the complete space H, but in the subspace $H^{P^a} \subset H$ which is the orthogonal complement of a subspace H^P specified constructively using the orthoprojector p given by

$$p = \sum_n |n\rangle \langle n| \quad ; \tag{2.4}$$

i.e. H^{P^a} denotes the Pauli allowed microscopic subspace. In the cluster problems, it is not only required to project the solution of the Schroedinger equation for the Hamiltonian H on the allowed subspace H^P but also to eliminate all virtual transitions to the forbidden states. This can be achieved by applying the orthogonalizing pseudopotential method (OPP) [6] on the microscopic level by going over to a modified microscopic hermitian Hamiltonian $\tilde{H}$:

$$\tilde{H} = H + \lambda p = H + \lambda \sum_n |n\rangle \langle n| \quad . \tag{2.5}$$

Since the states $|n\rangle$ are not eigenstates of the Hamiltonian H the real constant λ should be brought to infinity in the final solution. One can easily show that in the limit $\lambda \to \infty$ the solution $\tilde{\Psi}$ corresponding to the Hamiltonian $\tilde{H}$ is orthogonal to all microscopic Pauli forbidden states $|n\rangle$. In this approach we extend the orthogonality scattering [7] to an "orthogonality reaction method" by introducing the orthogonalizing microscopic pseudopotential.

$$V(\lambda) = \lambda p \qquad (\lambda \to \infty) \quad , \tag{2.6}$$

and deriving the cluster equations by suitably chosen ansatzes for the wave function $\tilde{\Psi}$. Thus, in contrast, to the OPP method, we are not interested in the direct solutions of the Hamiltonian $\tilde{H}$. $\tilde{H}$ serves only as a Pauli-corrected basic microscopic Hamiltonian for a subsequent RGM-like treatment. We use the ansatz

$$\tilde{\Psi} = \phi(1)\,\phi(2)\,\chi \quad . \tag{2.7}$$

Note that since the Pauli Principle is considered in the Hamiltonian (2.5), we can drop in (2.7) the antisymmetrization operator. Using the Hamiltonian (2.5) and the projection equation (2.2) we obtain the following Schroedinger equation for clusters:

$$(T_r + V_D + \lambda \sum_n |N_n\rangle \langle N_n|)\chi = E_r\chi \quad . \tag{2.8}$$

Here V_D is the double folding two-cluster potential; for antisymmetrized cluster internal functions V_D is the direct potential of the RGM equation. E_r is the relative energy of the two clusters in the c.m. system, given by $E_r = E-E_1-E_2$. The cluster internal energies E_1 and E_2 are obtained by computing the expectation values (in realistic calculation ground state energies) of their internal Hamiltonian, T_r is the corresponding kinetic energy operators, and

$$N_n(R) = \langle n \mid \phi(1)\phi(2)R\rangle_{int} \quad . \tag{2.9}$$

The bracket $\langle \ \rangle_{int}$ means that the integration is performed only over the internal degrees of the clusters and includes the appropriate evaluation of the spin and angular momentum variables. In evaluating the separable term in Eq. (2.8) we have assumed that according to the no-distortion ansatz made in (2.7) the unit operator in the microscopic space can be represented by

$$\mathbb{1} = \int |\phi(1)\phi(2)R\rangle \langle R\phi(2)\phi(1)| \, dR \quad . \tag{2.10}$$

The transition from the microscopic equation (2.5) to the cluster-Schroedinger equation (2.8) as for the Pauli exclusion principle is concerned will be discussed now in more detail. It is true that on the microscopic level in an arbitrary Hilbert space we can express the

antisymmetrization operator $\mathbb{A}$ by $\mathbb{A} = \mathbb{1} - \sum_n |n\rangle\langle n|$. It is also valid that in the limit $\lambda \to 0$ the Hamiltonian $\tilde{H}$ leads to a fully antisymmetrized solution $\tilde{\Psi}$, for which it holds $\langle n|\tilde{\Psi}\rangle = 0$ and therefore $\mathbb{A}\tilde{\Psi} = 0$. However the interpretation of the separable Pauli-correction term in Eq. (2.8) must be treated with more caution than in the case of Eq. (2.5). As one knows from the resonating group theory the Pauli Principle on the cluster level leads to fully and partly Pauli forbidden states. The difference between them is expressed quantitatively by the corresponding eigenvalues of the RGM kernel. Thus in a chosen normalization the difference between fully and partly Pauli forbidden states consists in their different weights. This difference disappears in the limit $\lambda \to 0$ [see Eq. (2.8)]. Therefore the Pauli forbidden (or inhibited) states can be treated in Eq. (2.8) only as fully Pauli forbidden intercluster states. Therefore the treatment of the Pauli principle in Eq. (2.8) is essentially the same as in the orthogonality condition model [9]. The treatment proposed by Eq. (2.8) can be extended to almost Pauli forbidden states [3] and also in some approximation to partly Pauli forbidden states. The latter aproximation is the better the more the state is Pauli suppressed. With this understanding the intercluster Pauli-corrected interaction can be written as:

$$V(\lambda \to \infty) = V_D + (\lambda \to \infty) \sum_n |N_n\rangle \langle N_n| \quad . \tag{2.11}$$

Using potential V we are now able to derive all the helpful properties owned by the OPP method, as for example the nice feature of improved convergence of Born series especially in the critical low energy region.

Since these properties are discussed and derived in Ref. 8 and are easily adapted for the inter-cluster potential $V(\lambda)$, we dispense with their discussion here. We rather like to discuss the extension of the microscopic method to a three-cluster situation. Let us consider the most simple no-distortion ansatz for a three-cluster system:

$$\Psi = \mathcal{A}(1)\,\mathcal{A}(2)\,\mathcal{A}(3)\,\chi \quad . \tag{2.12}$$

Using the projection equation (2.2) and the microscopic Hamiltonian with the potential V we obtain with ansatz (2.12):

$$(T_\xi + T_\eta + V_{1,D} + V_{2,D} + V_{3,D} + \lambda \sum |N_i\rangle\langle N_i|)\chi = E_r\chi \quad . \tag{2.13}$$

Here the functions N_i depend on two Jacobi coordinates ξ and η and are square integrable in respect to them. $V_{i,D}$ are the local double-folding potentials of the cluster subsystems. $T_\xi+T_\eta$ denotes the kinetic energy operator in the c.m. system. From Eq. (2.13) we observe that there in only one non-local potential, namely $\lambda \sum |N_i\rangle\langle N_i|$ which is separable three-cluster potential with finite Hilbert-Schmidt norm, and which carries the influence of the Pauli Principle as far the fully or highly suppressed Pauli intercluster states are considered.

The nice feature of the Hamiltonians (2.8) and (2.13) is that they clearly separate the symmetry properties (Pauli Principle) of the system, described by operator p, from the details (specific nucleon-nucleon interaction) of the dynamics of the composites.

To conclude this section we give a useful formula for the resolvent

$$GP(E) = (E - h)^{-1} \quad , \tag{2.14}$$

defining

$$G(E) = (E - h + \lambda p)^{-1} \quad , \tag{2.15}$$

we obtain the following expression for G(E) in the limit $\lambda \to \infty$

$$GP = G - Gp\,(pGp)^{-1}\,pG \quad , \tag{2.16}$$

Here h represents two-cluster or three-cluster Pauli-corrected Hamiltonian given in (2.8) and (2.13). In resolvent G the Pauli Principle is neglected. The importance of this formula is that the limes $\lambda \to \infty$ was carried out analytically and thereby the parameter λ is no longer present. For numerical application this is a convenient feature, since the equations are free from the strong coupling constant which λ represents.

Finally we consider the two-cluster Schroedinger equations $(E-h)\chi P = 0$ and $(E-h+\lambda p)\chi = 0$. Then it holds

$$\chi P = \chi - \frac{Gp\chi}{pGp} \quad . \tag{2.17}$$

The derivation of Eq. (2.16) and Eq. (2.17) can be found in Refs. [3,8,10]. In Section III and VI we will see that resolvent (2.16) and expression (2.17) allow us to give analytical corrections of the resonance width and resonance pole shift due to the inclusion of Pauli Principle.

3. PAULI-CORRECTED BREIT-WIGNER FORMULA FOR AN ISOLATED TWO-CLUSTER COMPOUND RESONANCE

In this section we discuss the influence of the Pauli Principle on the characteristic quantities of the two-cluster Breit-Wigner formula. Since the formal method of deriving the Breit-Wigner formula can be found in Ref. 4 we treat it only briefly. Note however that in our case the Pauli Principle is treated differently.

In order to allow excitations of one single compound state we enlarge the ansatz for the wave function Ψ as follows:

$$\Psi = \phi(1)\,\phi(2)\,\chi + af = \Psi_D + af \quad , \tag{3.1}$$

where f describes the compound state. Here f is antisymmetric and square integrable function by construction. The technique of introducing such square integrable compound (or distortion) state embedded in continuum is described in detail in Refs. [1,4]. Then from the projection equation and after some simple algebra we obtain the following Lippman-Schwinger equation for χ^P:

$$\chi^P = \tilde{\chi}^P + \tilde{G}^P|F\rangle \frac{\langle F|\tilde{\chi}^P\rangle}{E_c - E - \langle F|\tilde{G}^P|F\rangle} \quad , \tag{3.2}$$

where

$$E_c = \langle f\,|H|\,f\rangle \quad , \tag{3.3a}$$

$$F(\underline{r}') = \int dr^{3A}\, f(\underline{r}_1 \ldots \underline{r}_A)(H-E)\,\phi(1)\,\phi(2)\,\delta(\underline{r}-\underline{r}') \quad , \tag{3.3b}$$

$$\tilde{G}^P(E_r) = (E_r - T_r - V_D - \lambda \sum |N_i\rangle\langle N_i|)^{-1} \quad , \tag{3.3c}$$

and $\tilde{\chi}P$ is the solution of Eq. (2.8), i.e. in absence of the compound resonance.

If we neglect the influence of the Pauli Principle we have to omit the potential $\lambda \sum |N_n\rangle\langle N_n|$ and we obtain for the solution $\tilde{\chi}$ the same formula (3.2), where $\tilde{G}P$ has to be replaced by

$$\tilde{G}(E_r) = (E_r - T_r - V_D)^{-1} \quad , \tag{3.4}$$

and correspondingly $\tilde{\chi}P$ by $\tilde{\chi}$. For the latter quantities we obtain the following Breit-Wigner formula for resonant part of the amplitude in the partial wave of total spin j:

$$A_j^{res} \pm \frac{\Gamma_j(E_r)}{E_c - E - \Delta_j - \Gamma_j(E_r)/2} \quad . \tag{3.5}$$

Here

$$\Gamma_j(E_r) = 2\pi \left|\langle F_j|\tilde{\chi}_j(E_r)\rangle\right|^2 \quad , \tag{3.6}$$

and

$$\Delta_j = \langle F_j|\mathrm{Re}\ (\tilde{G}_j)|F_j\rangle \quad . \tag{3.7}$$

Note that

$$\mathrm{Im}\ [\tilde{G}_j(E_r)] = \pi\ |\tilde{\chi}_j(E_r)\rangle\ \langle\tilde{\chi}_j(E_r)| \quad .$$

Now we like to study the influence of the Pauli Principle on the quantities Γ_j and Δ_j. For this purpose we reconsider formula (3.2). Then, using the resolvent (2.17) as well as expression (2.18) we arrive formally at the same Breit-Wigner formula as given in Eq. (3.5) however now with Pauli-corrected quantites Γ_jP and Δ_jP.

In order to make the structure of this modification more transparent, we present the corresponding formulae assuming that the separable potential p is only of rank one. The extension to higher rank is straightforward. For $p = |N\rangle\langle N|$ we obtain

$$\Gamma_j P = 2\pi \left| \langle F_j|\tilde{\chi}_j\rangle - \frac{\langle F_j|\tilde{G}_j|N_j\rangle \langle N_j|\tilde{\chi}_j\rangle}{\langle N_j|\tilde{G}_j|N_j\rangle} \right|^2 , \tag{3.8}$$

and

$$\Delta_j P = \Delta_j - 2\mathrm{Re}\left(\frac{\langle F_j|\tilde{G}_j|N_j\rangle \langle N_j|\tilde{\chi}_j\rangle \langle\tilde{\chi}_j|F_j\rangle}{\langle N_j|\tilde{G}_j|N_j\rangle} \right) + \left| \frac{\langle F_j|\tilde{G}_j|N_j\rangle \langle N_j|\tilde{\chi}_j\rangle}{\langle N_j|\tilde{G}_j|N_j\rangle} \right|^2 . \tag{3.9}$$

In order to see how the Pauli Principle influences the lifetime of the resonance we reexpress the important term in Eq. (3.8) in the following way

$$\langle F_j|\tilde{\chi}_j\rangle - \frac{\langle F_j|\tilde{G}_j|N_j\rangle \langle N_j|\tilde{\chi}_j\rangle}{\langle N_j|\tilde{G}_j|N_j\rangle}$$

$$= \langle F_j \mid 1 - \tilde{G}_j|N_j\rangle\langle N_j|/\langle N_j|\tilde{G}_j|N_j\rangle \mid \tilde{\chi}_j\rangle , \tag{3.10}$$

in which the operator $\tilde{G}_j/\langle N_j|\tilde{G}_j|N_j\rangle$ is a dynamical weight-operator determining the weight with which the Pauli forbidden state N_j is projected out of the wave function $\tilde{\chi}_j$. It is straightforward to prove that the operator

$$D = 1 - \frac{\tilde{G}_j|N_j\rangle\langle N_j|}{\langle N_j|\tilde{G}_j|N_j\rangle} \qquad (3.11)$$

is a projector by verifying the relation $D^2 = D$. Because of this property we conclude that

$$|\langle F_j|\tilde{\chi}_j\rangle|^2 \;>\; |\langle F_j|D|\tilde{\chi}_j\rangle|^2 \quad .$$

This means [see formula (3.8)] that the Pauli Principle increases the lifetime of resonances. This result is in agreement with the numerical finding of Arima et al. [11] and Fliessbach [12] that the inclusion of Pauli Principle reduces the resonance widths. It supports also the interpretation of the Pauli Principle given by Schmid et al. [13] as an additional (Pauli) barrier which inhibits transitions between the inner and asymptotic regions.

The resonance shift formula (3.10) shows that $\Delta_j P$ depends sensitively on the inclusion of the Pauli Principle. This confirms the recent findings reported in Ref. [14] that the negligence of the Pauli Principle in some cases can even reverse the order of rotational bands.

4. THE MODEL FOR A THREE-CLUSTER COMPOUND RESONANCE

We consider a single three-body channel and allow the presence of one simple compound state. This means that we are solving the microscopic Schroedinger equation in a restricted function space given by

$$\Psi = \mathcal{A}(1)\,\mathcal{A}(2)\,\mathcal{A}(3)\,\chi + af = \Psi_D + af \quad , \tag{4.1}$$

using the Pauli-corrected micorscopic Hamiltonian HP(λ)

$$HP(\lambda) = H + \lambda\,|n\rangle\langle n| \quad .$$

We get from

$$\langle\delta\Psi|(HP(\lambda)-E)|\Psi\rangle = 0 \quad , \tag{4.2}$$

the following set of equations

$$\begin{aligned} \langle\delta\Psi_D|HP(\lambda)-E|\Psi_D\rangle + a\langle\delta\Psi_D|HP(\lambda)-E|f\rangle &= 0 \\ \langle f|HP(\lambda)-E|\Psi_D\rangle + a\langle f|HP(\lambda)-E|f\rangle &= 0 \quad . \end{aligned} \tag{4.3}$$

Formal elimination of the second equation leads to

$$\langle\delta\Psi_D|HP(\lambda)-E|\Psi_D\rangle - \frac{\langle\delta\Psi_D|H-E|f\rangle\,\langle f|H-E|\Psi_D\rangle}{\langle f|H-E|f\rangle} = 0 \quad . \tag{4.4}$$

The technique of evaluating the terms appearing in this equation is similar to that known from the resonating group method [4]. It is even simpler since in our ansatz for Ψ_D there is no antisymmetrization operator. Without the second term we would just get the integro-differential equation for $\chi(\underline{\xi},\underline{\eta})$ given already in (2.14).

The interaction $V'_{eff}(1,2,3)$ appearing in this part of the equation decomposes into three subsystem interactions and into a three-body separable interaction,

$$V'_{eff}(1,2,3) = V_{1,D} + V_{2,D} + V_{3,D} + \lambda \sum |N_i\rangle \langle N_i| \quad . \tag{4.5}$$

The mathematical structure of the second term of (4.4) is a separable potential of rank one with a resonance denominator

$$V_c = |F\rangle \frac{1}{\langle f|H|f\rangle - E} \langle F| \quad , \tag{4.6}$$

with

$$F(\underline{\xi}',\underline{\eta}') = \int dr^{3A}\, f(\underline{r}_1,\ldots,\underline{r}_A)(H-E) \times \phi(1)\phi(2)\phi(3)\, \delta(\underline{\xi}-\underline{\xi}')\, \delta(\underline{\eta}-\underline{\eta}') \tag{4.7}$$

Note that in the second term of (4.4) and in the formula (4.7) we can use the Hamiltonian H instead of HP(λ), since f is already antisymmetric. The full potential reads now

$$V_{eff}(1,2,3) = V'_{eff}(1,2,3) + V_c \quad . \tag{4.8}$$

The interaction $V'_{eff}(1,2,3)$ is energy independent. For the derivation given in the following section we assume that the Faddeev equations with the two-body potentials $V_{1,D}$, $V_{2,D}$ and $V_{3,D}$ has been solved, and that the spectrum of wave function solutions is known. We can include the influence of the Pauli Principle by considering the potential $\lambda \sum |N_i\rangle\langle N_i|$. Then, by virtue of formula (2.17) and (2.18), we can construct the complete spectrum of Pauli-corrected wave function solutions.

5. SOLUTION OF THE MODEL

The aim is to obtain the solution of

$$(h_0 + \bar{V}_{eff}(1,2,3))\chi = E_r\chi \quad , \tag{5.1}$$

with the boundary condition for bound state scattering, i.e. for the scattering of a free particle 1 by the pair (1) = (2,3) initially in a bound state of energy E_{in}. Here $\bar{V}_{eff}(1,2,3)$ is given by

$$\bar{V}_{eff}(1,2,3) = V_{eff}(1,2,3) - \lambda \sum |N_i\rangle \langle N_i| \quad .$$

Thus at this stage we are neglecting the Pauli Principle.

h_o represents the kinetic energy operator: $h_o = \hbar^2(\Delta_\xi + \Delta_\eta)$, and E_r is the relative energy of the three-cluster system. (We work in the center-of-mass system of the three clusters and use Newton's notation of coordinates and momenta [15].) We define the following operators:

$$\begin{aligned} h &= h_0 + \bar{V}_{eff} \quad , \qquad \tilde{h} = h - V_c \\ g(z) &= (z - h)^{-1} \quad , \qquad \tilde{g}(z) = (z - \tilde{h})^{-1} \\ g_0(z) &= (z - h_0)^{-1} \quad . \end{aligned} \tag{5.2}$$

with $z = E_r + i\epsilon$. It holds

$$g(z) = \tilde{g}(z) + \tilde{g}(z)\, V_c\, g(z) \quad . \tag{5.3}$$

It is assumed that the bound state scattering problem, i.e. the equation

$$\lim_{\epsilon\to 0} i\epsilon\, \tilde{g}(E_r + i\epsilon)\, \Phi_1 = \tilde{\chi}_1^{(+)} \tag{5.4}$$

is already solved. Here Φ_1 denotes the product of the plane wave $\vec{F}_1$ and the bound state function of clusters 2 and 3, ϕ_{1n}, where n denotes the n-th bound state of clusters 2 and 3. This problem can be solved using some of the many techniques for solving the Faddeev equations. Using the operator identity (5.3) one obtains the Lippmann-Schwinger equation

$$\lim_{\epsilon\to 0} i\epsilon\, g(E_r+i\epsilon)\Phi_1 = \chi_1^{+} = \tilde{\chi}_1^{+} + \tilde{g}(E_r+i0^+)\, V_c\, \chi_1^{(+)} \quad . \tag{5.5}$$

The last equation has a unique solution, since V_c has a finite Hilbert-Schmidt norm. Due to the separability of V_c the solution $\chi_1^{(+)}$ can be obtained immediately after a simple algebraic calculations. The result is

$$\chi_1^{(+)} = \tilde{\chi}_1^{(+)} + \tilde{g}(E_r+i0^+)\, \frac{|F\rangle \langle F|\tilde{\chi}_1^{(+)}\rangle}{\langle f|H|f\rangle - E - \langle F|\tilde{g}^+(E_r)|F\rangle} \quad . \tag{5.6}$$

In order to evaluate Eq. (5.6) one needs the asymptotic formulae of the resolvent $\tilde{g}^+(E_r) = \tilde{g}(E_r+i0^+)$ in all channels. The derivation of the formulae for the three-body Green's function assuming two-body interactions only is given in the Appendix [19]. The results are:

$$\tilde{g}^{+}(E_r;\underline{R},\underline{R}') \xrightarrow[\substack{|\underline{\eta}_i| \to \infty \\ |\underline{R}'|,|\underline{\xi}_i| < \infty}]{} -\frac{1}{4\pi(2\pi_i)^{3/2}}$$

$$\times \sum_n^e \frac{i|\underline{q}_{in}||\underline{\eta}_i|}{|\underline{\eta}_i|} \phi_{in}(\underline{\xi}_i)\, \tilde{\chi}_{in}^{(-)*}(\underline{q}_{in},E_n;\underline{R}') \quad , \tag{5.7}$$

where $\underline{q}_{in} = \underline{\eta}_i^0|\underline{q}_{in}|$ $[|\underline{q}_{in}| = (E_r-E_{in})^{1/2}]$, i=1,2,3 for the two-body channels and

$$\tilde{g}^{+}(E_r;\underline{R},\underline{R}') \xrightarrow[\substack{|\underline{R}| \to \infty \\ |\underline{R}'| < \infty}]{} \frac{e^{i\pi/4}\, E_r^{3/4}\, e^{i|\underline{K}||\underline{R}|}}{2(2\pi)^{5/2}\, |\underline{R}|^{5/2}}\, \tilde{\chi}_0^{(-)*}(|\underline{K}|\underline{R}^0,\underline{R}') \, , \tag{5.8}$$

for the breakup-channel, where $\tilde{\chi}_0^{(-)*}$ is the three-particle scattering function for three free incident particles. Here, the six-dimensional coordinates $(\underline{\xi}_i,\underline{\eta}_i)$ i=1,2,3 are denoted by a vector $\underline{R}$, and the corresponding six-dimensional conjugate momentum $(\underline{p}_i,\underline{q}_i)$ i=1,2,3 by a vector $\underline{K}$. Then the following formulae are valid [15]:

$$|\underline{R}|^2 = |\underline{\xi}_i|^2+|\underline{\eta}_i|^2 \; ; \; E_r = |\underline{K}|^2 = |\underline{p}_i|^2+|\underline{q}_i|^2 \quad ,$$

$$\underline{KR} = \underline{\xi}_i\underline{p}_i + \underline{\eta}_i\underline{q}_i \; ; \; d\underline{R} = d\underline{\xi}_i\, d\underline{\eta}_i \; , \; i = 1,2,3 \; .$$

Using the formulae (5.7) and (5.8) and the known structure of the asymptotic form of the three-particle scattering function $\tilde{\chi}_1^{(+)}$ [15,16], one obtains all scattering amplitudes of the solution $\chi_1^{(+)}$ of the Eq. (5.6). One obtains

a) for the breakup channel.

$$\chi_1^{(+)}(\underline{q}_1,E_1,\underline{R}) \xrightarrow[|\underline{R}|\to\infty]{} \frac{e^{i|\underline{K}||\underline{R}|}}{|\underline{R}|^{5/2}}\left[R_{01}(\underline{K}';\underline{q}_1,E_1) + R_{01}^{res}(\underline{K}',\underline{q}_1,E_1)\right] , \tag{5.9}$$

b) for the elastic channel

$$\chi_1^{(+)}(\underline{q}_1,E_1;\underline{R}) \xrightarrow[\substack{|\underline{\eta}_1|\to\infty \\ |\underline{\xi}_1|<\infty}]{} e^{i\underline{q}_1\underline{\eta}_1}\phi_{1n}(E_{1n},\underline{\xi}_1)$$

$$- \frac{m_1(2/\bar{\mu}_1)^{1/2}}{|\underline{\eta}_1|}\sum_n e^{i|\underline{q}'_{1n}||\underline{\eta}_1|}\phi_{1n}(\underline{\xi}_1)$$

$$\times \left[R_{11}(\underline{q}'_{1n},E_{1n};\underline{q}_1,E_1) + R_{11}^{res}(\underline{q}'_{1n},E_{1n};\underline{q}_1,E_1)\right] , \tag{5.10}$$

c) for the rearrangement channel $j\neq 1$

$$\chi_1^{(+)}(\underline{q}_1,E_1;\underline{R}) \xrightarrow[\substack{|\eta_j|\to\infty \\ |\underline{\xi}_j|<\infty}]{} - \frac{m_j(2/\bar{\mu}_j)^{1/2}}{|\underline{\eta}_j|}\sum_n e^{i|\underline{q}'_{jn}||\underline{\eta}_j|}$$

$$\times \phi_{jn}(\underline{\xi}_j)\left[R_{j1}(\underline{q}'_{1n},E'_{1n};\underline{q}_j,E_j) + R_{j1}^{res}(\underline{q}'_{1n},E'_{1n};\underline{q}_j,E_j)\right] . \tag{5.11}$$

Here R_{ij} are the usual Faddeev scattering amplitudes in absence of the three-cluster potential V_c and are given by

$$R_{j1}(\underline{q}'_{1n},E'_{1n};\underline{q}_j,E_j) = -\frac{\bar{\mu}_j^{1/2}}{16\pi m_j\,\mu_j^{3/2}}$$

$$\times \int d\underline{R}\; e^{-i\underline{q}'_{1n}\underline{\mathcal{R}}_1}\, \phi_{1n}^*(\underline{\xi}_1) \sum_{i\neq j} V_{D,i}(\underline{\xi}_i)\, \tilde{\chi}_j^{(+)}(\underline{q}_j,E_j;\underline{R}) \;, \tag{5.12a}$$

j=1,2,3 and

$$R_{01}(\underline{K}';\underline{q}_1,E_1) = \frac{e^{i\pi/4}\,E_r^{3/4}}{2(2\pi)^{5/2}}$$

$$\times \int d\underline{R}\; \Phi_1^{(-)*}(\underline{K}',\underline{R}) \sum_{i=1} V_{D,i}(\underline{\xi}_i)\, \tilde{\chi}_1^{(+)}(\underline{q}_1,E_1;\underline{R}) \;, \tag{5.12b}$$

in which $\Phi_1^{(-)}(\underline{K},\underline{R}) = e^{i\underline{q}_1\underline{\mathcal{R}}_1}\phi_1^{(-)}(\underline{p}_1,\underline{\xi}_1)$ and $\phi_1^{(-)}(\underline{\xi}_1)$ is a scattering state in the subsystem (1). The potentials $V_{D,i}$ are defined by $V_{D,i} = V_D(j,k)$, (i,j,k) cyclic. The three-cluster resonance scattering amplitudes R_{i1}, i=1,2,3 which arise from the potential V_c can be found as

$$R_{i1}^{res} = -\frac{\bar{\mu}_i^{3/2}}{16\pi m_i\,\mu_i^{3/2}}\;\frac{\langle\tilde{\chi}_i^{(-)}|F\rangle\,\langle F|\tilde{\chi}_1^{(+)}\rangle}{\langle f|H|f\rangle - E - \langle F|\tilde{g}^+|F\rangle} \;, \tag{5.13a}$$

i=1,2,3 and

$$R_{01}^{res} = \frac{e^{i\pi/4}\,E_r^{3/4}}{2(2\pi)^{5/2}}\;\frac{\langle\tilde{\chi}_0^{(-)}|F\rangle\,\langle F|\tilde{\chi}_1^{(+)}\rangle}{\langle f|H|f\rangle - E - \langle F|\tilde{g}^+|F\rangle} \;. \tag{5.13b}$$

Then the corresponding cross sections are given by:

a) for the two-body channels i=1,2,3:

$$\frac{d\sigma}{d\Omega} = \frac{|\underline{q}'_{in}|}{|\underline{q}_1|} \frac{m_1 \bar{\mu}_i^{1/2}}{m_i \bar{\mu}_1^{1/2}} |R_{i1} + R_{i1}^{res}|^2 \quad , \tag{5.14a}$$

b) for the breakup channel

$$\frac{d\sigma}{d\Omega} = \frac{\bar{\mu}_1 |\underline{q}'_1|^3}{2^{5/2} m_1^2 \mu_1^{3/2} |\underline{q}_1||\underline{K}|^5} |R_{o1} + R_{o1}^{res}|^2 \, d\underline{q}'_1 \quad . \tag{5.14b}$$

It can be observed already here [formulae (5.14)] that the interference effects between the structures contained in R_{ij} such as final state interaction and quasi-free scattering and R_{ij} are present in all channels. On the other side, the decay of the three-cluster resonance itself is influenced by the existing structures produced by the two-body potentials [formulae (5.13)]. Thus formulae (5.12)-(5.14) give a qualitative insight into the variety of energy correlations in the case of a sudden decay of a three-cluster system. In the next section the scattering amplitudes R_{ij}, j=0,1,2,3 will be studied in more detail.

6. BREIT-WIGNER RESONANCE FORMULAE FOR A THREE CLUSTER COMPOUND RESONANCE

In this section, first the structure of the scattering amplitudes R_{i1}, i=0,1,2,3 will be discussed. Then we discuss the corrections to these quantities due to the Pauli Principle. The terms of interest are:

$$\frac{\langle \tilde{\chi}_0^{(-)}|F\rangle \langle F|\tilde{\chi}_{1n}^{(+)}\rangle}{\langle f|H|f\rangle - E - \langle F|\tilde{g}^+|F\rangle} , \tag{6.1a}$$

and

$$\frac{\langle \tilde{\chi}_{im}^{(-)}|F\rangle \langle F|\tilde{\chi}_{1n}^{(+)}\rangle}{\langle f|H|f\rangle - E - \langle F|\tilde{g}^+|F\rangle} . \tag{6.1b}$$

The resolvent $\tilde{g}^+(E_r)$ can be represented in the following spectral decomposition:

$$\tilde{g}^+(E_r) = \sum_v \frac{|\tilde{\chi}_v\rangle\langle\tilde{\chi}_v|}{E_r - E_v} + \sum_{jn} \int d\underline{q}_j \frac{|\tilde{\chi}_{jn}^{(+)}\rangle\langle\tilde{\chi}_{jn}^{(+)}|}{E_r - E_{jn} - |\underline{q}_j|^2 + i0^+}$$

$$+ \iint d\underline{q}_k \, d\underline{p}_k \frac{|\tilde{\chi}_0^{(+)}\rangle\langle\tilde{\chi}_0^{(+)}|}{E_r - |\underline{q}_k|^2 - |\underline{p}_k|^2 + i0^+} \equiv \sum_H \frac{|\tilde{\chi}_H\rangle\langle\tilde{\chi}_H|}{E_r - E_H + i0^+} , \tag{6.2}$$

where k=1,2,3 are arbitrary. The above formula follows from the principle of asymptotic completeness for a three-body system derived independently by Ginibre and Moulin, and Thomas [17]. Here $\tilde{\chi}_v$ is the three-body bound state, $\tilde{\chi}_{in}^{(+)}$ is a bound state scattering function with initially bound state in subsystem (i), and $\tilde{\chi}_0^{(+)}$ is a scattering function for three free incident particles. We define a complex resonance shift by

$$\Delta^{+}(E_r) = \sum_{H} \frac{\langle F|\tilde{\chi}_H\rangle\langle\tilde{\chi}_H|F\rangle}{E_r - E_H + i0^{+}} \quad . \tag{6.3}$$

Further, by using the identity

$$\lim_{\epsilon\to 0} \frac{1}{E_H - E_r - i\epsilon} = \mathrm{CPV}\,\frac{1}{E_H - E_r} + i\pi\delta(E_H - E_r) \quad , \tag{6.4}$$

one can separate the non-hermitian operator $\tilde{g}^{+}(E_r)$ into a hermitian and antihermitian part $\tilde{g}^{+} = \tilde{g}^{h} + i\pi\tilde{g}^{ah}$, with

$$\begin{aligned}\tilde{g}^{h}(E_r) = &\sum_{v} \frac{|\tilde{\chi}_v\rangle\langle\tilde{\chi}_v|}{E_r - E_v} + \sum_{in}\int \mathrm{CPV}\,\frac{|\tilde{\chi}_{in}^{(+)}\rangle\langle\tilde{\chi}_{in}^{(+)}|}{E_r - |\underline{q}_i|^2 - E_{in}} \\ &+ \int\int \mathrm{CPV}\,\frac{|\tilde{\chi}_0^{(+)}\rangle\langle\tilde{\chi}_0^{(+)}|}{E_r - |\underline{q}_i|^2 - |\underline{p}_i|^2} \quad ,\end{aligned} \tag{6.5}$$

$$\begin{aligned}\tilde{g}^{ah}(E_r) = &\sum_{in}\int |\tilde{\chi}_{in}^{(+)}\rangle\langle\tilde{\chi}_{in}^{(+)}|\,\delta(E_r - E_{in} - |\underline{q}|^2)\,d\underline{q}_i \\ &+ \int\int |\tilde{\chi}_0^{(+)}\rangle\langle\tilde{\chi}_0^{(+)}\,\delta(E_r - |\underline{q}_k|^2 - |\underline{p}_k|^2)\,d\underline{p}_k\,d\underline{q}_k \quad .\end{aligned}$$

Then defining the following quantities

$$\begin{aligned}&\gamma_{in}^{(\pm)}(E_r) = \langle F|\tilde{\chi}_{in}^{(\pm)}\rangle \qquad , i = 0,1,2,3 \\ &\Gamma_{in}^{(\pm)}(E_r) = 2\pi\,|\gamma_{in}^{(\pm)}(E_r)|^2 \quad ,\end{aligned} \tag{6.6}$$

we can rewrite the formula (6.3) for the complex resonance shift

$$\Delta^{+}(E_r) = \Delta_c(E_r) + \frac{i}{2}\left[\sum_{in}\Gamma_{in}^{(+)}(E_r-E_{in}) + \int \Gamma_0^{(+)}(|\underline{q}_k|^2)\, d\underline{q}_k\right], \qquad (6.7)$$

where $\Delta_c(E_r) = \langle F|\tilde{g}^h(E_r)|F\rangle$.

The magnitudes $\gamma^{(\pm)}(E_r)$ describe the probability amplitude of the resonance cluster-state F in the different scattering states at the same energy. The variety of the scattering states results from different initial conditions.

Due to the square integrability of F the resonance widths Γ are also very sensitive to subsystem resonances, since in that case the amplitude of this subsystem is large at small distances. Therefore the partial widths Γ are good candidates for probing the off-shell behavior of the three-cluster wave functions. With the definitions (6.6) and (6.7) we can transform (6.1a) and (6.1b) into

$$\frac{\langle\tilde{\chi}_0^{(-)}|F\rangle\,\langle F|\tilde{\chi}_{1n}^{+}\rangle}{\langle f|H|f\rangle - E - \langle F|\tilde{g}^{+}(E_r)|F\rangle}$$

$$= \frac{\gamma_0^{(-)*}(E_r)\,\gamma_{1n}^{(+)}(E_r)}{E_c - E - \Delta_c(E_r) + \frac{i}{2}\left[\sum_{in}\Gamma_{in}^{(+)}(E_r-E_{in}) + \int \Gamma_0^{(+)}(E_r-|\underline{q}|^2)\, d\underline{q}\right]}, \qquad (6.8a)$$

and

$$\frac{\langle \tilde{\chi}_{jm}^{(-)}|F\rangle\langle F|\tilde{\chi}_{1n}^{(+)}\rangle}{\langle f|H|f\rangle - E - \langle F|\tilde{g}^{+}(E_r)|F\rangle}$$

$$= -i \frac{i\gamma_{jm}^{(-)*}(E_r)\,\gamma_{1n}^{(+)}(E_r)}{E_c - E - \Delta_c(E_r) + \frac{i}{2}\left[\sum_{in}\Gamma_{in}^{(+)}(E_r - E_{in}) + \int \Gamma_0^{(+)}(E_r - |\underline{g}|^2)\,d\underline{g}\right]} \quad , \tag{6.8b}$$

respectively, where $E_c = \langle f|H|f\rangle$.

Obviously, these formulae reveal the Breit-Wigner structure. The resonance energy pole E_{res} - the pole at which the real part of the denominator in (6.8) becomes zero - is defined by

$$E_c - E_{res} - \Delta_c(E_{res}) = 0 \quad .$$

The formulae (6.8) exhibit that even in the case of the three-body breakup channel the Breit-Wigner resonance formula is still valid. Moreover it allows one to study the rich energy correlations of the decay products. The only difference to this formula is that $\Delta_c(E_r)$ and $\Gamma(E_r)$ are energy dependent. To eliminate this difference it is necessary to make certain approximations [19]. We have to use that fact that we are considering an isolated, relatively sharp resonance. For such a resonance level all other resonances lie energetically so far away that $\Gamma(E_r)$ can be considered to be approximately energy independent over the energy width of this level. Then we make the approximation that, in the neighborhood of E_r,

$$\Delta_c(E_r) \approx \Delta_c(E_r) + (E_r - E_{res}) \left[\frac{d\Delta_c(E_r)}{dE_r} \right]_{E_r=E_{res}} ,$$

$$\Gamma(E_r) \approx \Gamma(E_{res}) . \qquad (6.9)$$

Then one arrives to the same formulae as (6.8) where $\Delta_c(E_r)$ is replaced by $\Delta_c(E_{res})$ and the partial widths by:

$$\Gamma_{res}(E_{res}) = \left. \frac{\Gamma(E_r)}{1 + \frac{d}{dE_r} \Delta_c(E_r)} \right|_{E_r=E_{res}} ,$$

$$\gamma_{jn}^{res}(E_{res}) = \left. \frac{\gamma_{jn}(E_r)}{1 + \frac{d}{dE_r} \Delta_c(E_r)} \right|_{E_r=E_{res}} . \qquad (6.10)$$

The main difference in the structure of formulae (6.8) compared with the resonance formulae for the two-body channel situation [18] is the appearance of an integral over resonance widths $\Gamma_0^{(+)}(E_r - |q|^2)$. This reflects the fact that the three-body breakup can be considered as a continuum of infinite many two-body channels. Each distribution of the energy over the three bodies corresponds to one two-body channel. However the interpretation of the resonance widths Γ according to the picture of two-body channel situation is no longer possible.

This can be seen in the following way: though $\tilde{\chi}_0^{(+)}$ is a scattering function for three free incident particles, it contains apparently also contributions of all two-body channels. This aspect is lacking in the

Breit-Wigner formula in the two-body channel situation and is obviously characteristic for (N ≥ 3)-cluster systems. The analogy to the two-channel case is not possible because $\Gamma_0^{(+)}$ cannot be interpreted as the transition amplitude from the resonance state F whether to the breakup channel nor to some of the two-body channels; all these transition amplitudes are contained in $\Gamma_0^{(+)}$, but they altogether determine nevertheless effectively the partial resonance width of the breakup channel.

Therefore even in the case of a two-body channel, no straightforward parametrization by phase shifts of the resonance amplitude can be given. Analogous considerations can be made for Γ^{+}_{jm}. They show that vice versa the transition amplitudes from the compound state F to the breakup channel influence the resonance width of a two-body channel. It should be pointed out that this many-cluster effect is qualitatively different from the mutual influence of the partial widths of the two-body coupled channels. The latter effect depends on the ratio of the partial width to the total resonance width. The origin of the new effect is a dynamical one. It leads to some interesting and important consequences. Let us assume that in addition to the three-cluster resonance there is at a lower energy a subsystem resonance in the subsystem (i) consisting of clusters j and k. One considers now the three-cluster resonance behavior in the two-body channel, where clusters j and k form just a bound state. In this case the three-cluster resonance width of this channel is strongly influenced by the resonance of the same subsystem (i), though in this channel just this subsystem is bound at each energy.

Another difference is that the formulae given by Schranner [18] for two-body channels were derived under the strong approximation; namely that the direct coupling between the open two-body channels can be neglected and that the transition occur through the formation of the resonance state. Such approximation is valid only in specific situations, if for instance the resonance occurs in an energy region where all or all but one channel are shielded from the compound region by high potential barriers. The derivations presented here restricted to the two-body channel reactions are exact and cover all cases.

The influence of the three-cluster breakup structures such as quasi-free scattering, final state interactions, or two-body channel resonances on the three-cluster resonance is twofold. First, such structures strongly influence the three-cluster resonance widths $\Gamma(E)$; second, they interfere with the three-cluster resonance amplitudes R_{01}. The kinematic aspects of such interferences are extensively studied in Ref. [19].

Due to the analysis done in Section III and V and in virtue of formulae (2.17) and (2.18) (which hold also in the three-cluster case), it is easy now to answer the question how the symmetry generated by the Pauli Principle influences the derived Breit-Wigner formulae for the three-cluster compound resonance. Again, in order to make the Pauli-corrected formulae more clear, we assume for simplicity that the rank of the separable potential p is just one. Then after some algebra one finds the Pauli-corrected formulae for the characteristic quantities introduced in (6.6).

$$\gamma_{in}{}^{P(\pm)} = \gamma_{in}{}^{(\pm)} - \frac{\langle F|\tilde{g}^{+}|N\rangle\langle N|F\rangle}{\langle N|\tilde{g}^{+}|N\rangle} \quad , \tag{6.11}$$

for i=0,1,2,3

$$\Gamma_{in}{}^{P(\pm)} = 2\pi \left|\gamma_{in}{}^{P(\pm)}\right|^2 \quad , \tag{6.12}$$

and

$$\Delta_c{}^P = \Delta_c - \Delta_{cor}{}^P \quad . \tag{6.13}$$

Here the correction term $\Delta_{cor}{}^P$ due to the Pauli Principle can be written as:

$$\begin{aligned}
\Delta_{cor}{}^P = 2\mathrm{Re} \Bigg[& \frac{1}{\langle N|\tilde{g}^+|N\rangle} \Bigg(\sum_v \frac{\langle F|\tilde{g}^+|N\rangle\langle N|\tilde{\chi}_v\rangle}{E_r - E_v} \\
& + \sum_{in} \int \mathrm{CPV} \frac{\langle F|\tilde{g}^+|N\rangle\langle N|\tilde{\chi}_{in}{}^{(+)}\rangle}{E_r - |\underline{q}_i|^2 - E_{in}} + \int\int \mathrm{CPV} \frac{\langle F|\tilde{g}^+|N\rangle\langle N|\tilde{\chi}_0{}^+\rangle}{E_r - |\underline{q}_i|^2 - |\underline{p}_i|^2} \Bigg) \Bigg] \\
& - \left|\frac{\langle F|\tilde{g}^+|N\rangle}{\langle N|\tilde{g}^+|N\rangle}\right|^2 \Bigg(\sum_v \frac{|\langle N|\tilde{\chi}_v\rangle|^2}{E_r - E_v} + \sum_{in} \int \mathrm{CPV} \frac{|\langle N|\tilde{\chi}_{in}{}^{(+)}\rangle|^2}{E_r - |\underline{q}_i|^2 - E_{in}} \\
& + \int\int \mathrm{CPV} \frac{|\langle N|\tilde{\chi}_0{}^{(+)}\rangle|^2}{E_r - |\underline{q}_i|^2 - |\underline{p}_i|^2} \Bigg) \quad .
\end{aligned} \tag{6.14}$$

The formulae (6.11)-(6.14) are easily generalized to any finite rank of the separable potential p.

One observes that the Pauli Principle shifts the resonance pole in all channels uniformly. However the partial resonance widths are influenced by the Pauli Principle in a different way. The formula (6.11) shows that the Pauli Principle has much stronger impact on the partial resonance widths Γ_{in} in the two-body channels than on the partial resonance width

Γ_0 in the breakup channel. This is so, since $|\langle N|\tilde{\chi}_0^{(+)}| \ll |\langle N|\tilde{\chi}_{in}^{(+)}\rangle|$. This shows that the major influence of Pauli Principle on the resonance behavior in the breakup channel consists only in shifting the resonance pole. However the Pauli Principle influences strongly the subsystem structures by making them more pronounced. Thus it makes the observation of the sudden three-cluster decay even more difficult.

7. CONCLUSION

A new method of considering Pauli Principle on a microscopic level has been presented. Based on this method a reaction theory for the decay of a nuclear compound state into two and three cluster has been developed. It allows a detailed study of the influence of the Pauli Principle on such quantites as resonance width and shift of the resonance pole. Explicit formulae for Pauli-corrected quantities have been given and discussed. The formalism for the three-cluster case shows clearly the interference of the sequential decay modes with the sudden decay mode. The differential cross section has a complicated structure, even when the compound resonance is sharp and isolated. As compared with the standard Breit-Wigner formalism, the three-cluster decay of a compound state corresponds to a case in infinitely many two-body channels. Correspondingly the total decay width is a sum plus an integrable over the partial widths. New effects of the mutual influence of the three-cluster resonance with subsystem resonances and other resonance-like structures have been discussed. It has been shown that the influence of the Pauli Principle on the partial resonance width in the breakup channel is small, whereas it is appreciable in the two-cluster channel of the three-cluster system. For a practical application of the theory one has to solve the three-cluster Faddeev equation in absence of the three-cluster compound resonance potential and one has to calculate matrix elements of this potential with the obtained solutions. The influence of the Pauli Principle enters by a simple separable potential which can be numerically evaluated in terms of transition amplitudes.

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APPENDIX

THE ASYMPTOTIC FORMULAE OF THE FULL THREE-BODY GREEN'S FUNCTION IN COORDINATE REPRESENTATION

The decomposition into channels of the three-body wave function was derived by Sasakawa [20] and the asymptotic form of the three-body wave function was studied by Glöckle [21], Newton [15,22], Merkuriew [16], Gignoux [16,23], Laverne [16,23], and Nutall [24,25].

As one knows from Refs. [21]-[25], the asymptotic form of the three-body wave function depends on the direction in the six-dimensional coordinate space. The same, of course, must be true of the Green's function. One can, therefore, expect that one should be able to use the various forms of the resolvent equation for the full resolvent $\tilde{g}$ in terms of the channel resolvent $\tilde{g}_i$, i=0,1,2,3 in order to find its asymptotic form in various directions. One has

$$\tilde{g} = \tilde{g}_i + \tilde{g}_i \, \bar{V}(i) \, \tilde{g} \quad , \tag{A.1}$$

where $\bar{V}(i) = [V(j) + V(k)]$, i,j,k cyclic, and $\bar{V}(0) = \sum V(i)$. We are considering a situation with pairwise interactions only. In Eq. (A.1) a term of the following form appears:

$$\int d\underline{R}'' \; \tilde{g}_1^{+}(\underline{R},\underline{R}'') \; V(2) \; (\underline{\xi}''_2) \; \tilde{g}^{+}(\underline{R}'',\underline{R}') \quad . \tag{A.2}$$

There is a convergence problem with $\underline{\eta}_2''$-integration. For this reason (A.1) is not the proper starting point for going to large $|\underline{R}|$. One can expect that one has to start with Faddeev-like equations for the Green's function $\tilde{g}$. In order to obtain a Faddeev-like splitting of a resolvent

identity, one considers Eq. (A.1) for i=0,1,2,3 and obtains after an easy calculation

$$\tilde{g} = \sum_{i=1} \tilde{g}_i[M_i + V(i)\,\tilde{g}_0\,\bar{V}(i)\,\tilde{g}] \quad , \tag{A.3}$$

in which

$$M_i = \begin{cases} V(i)\tilde{g}_0 & \text{if} \quad i \neq \lambda \\ \mathbb{1}_\lambda & \text{if} \quad i = \lambda \end{cases}$$

Here λ can be chosen arbitrarily $\lambda = 1,2,3$. One can show that the formula (A.3) corresponds to the Weinberg's decomposition of the three-particle wave function [26], and the choice of λ corresponds to the determination of the entrance channel. The arbitrariness of λ reflects the fact, that in the space of eigenstates of $\tilde{g}_i^{-1}$ the operator $V(i)\tilde{g}_0$ is identical with identity $\mathbb{1}_i$ acting in this subspace. An equivalent formulation of (A.3) is

$$\tilde{g} = \tilde{g}_0 + \tilde{g}_0 \sum_{i=1} [V(i)\tilde{g}_i + V(i)\tilde{g}_i\,\bar{V}(i)\tilde{g}] \quad . \tag{A.4}$$

We will show later that in Eqs. (A.3) and (A.4) the disconnected parts in the kernel are avoided.

In order to derive the asymptotic formulae for the full three-body Green's function one needs corresponding asymptotic formulae for the channel Green's functions $\tilde{g}_i$. Such formulae were already derived by Newton [15], who pointed out that $\tilde{g}_i$ can be split into two parts:

$$\tilde{g}_i^{+}(E) = \tilde{g}_i^{+c}(E) + \tilde{g}_i^{+b}(E) \quad ,$$

where $\tilde{g}_i^{+c}(E)$ is responsible only for the asymptotic formulae of $\tilde{g}_i^{+}(E)$ in the breakup channel, and $\tilde{g}_i^{+b}(E)$ determines uniquely the asymptotic behavior of $\tilde{g}_i^{+}(E)$ in the two-body channel i:

$$\lim_{\substack{|\underline{R}| \to \infty \\ |\underline{R}'| < \infty}} \tilde{g}_i^{+}(E;\underline{R},\underline{R}') = \lim_{\substack{|\underline{R}| \to \infty \\ |\underline{R}'| < \infty}} \tilde{g}_i^{+c}(E;\underline{R},\underline{R}')$$

$$= \frac{e^{i\pi/4}\, E^{3/4}\, e^{i|\underline{K}||\underline{R}|}}{2(2\pi)^{5/2}\, |\underline{R}|^{5/2}} \Phi_i^{(-)*}(\underline{K}',\underline{R}') \quad , \tag{A.5}$$

$$\lim_{\substack{|\underline{\eta}_i| \to \infty \\ |\underline{R}'|,|\underline{\xi}_i| < \infty}} \tilde{g}_i^{+}(E;\underline{R},\underline{R}') = \lim_{\substack{|\underline{\eta}_i| \to \infty \\ |\underline{R}'|,|\underline{\xi}_i| < \infty}} \tilde{g}_i^{+b}(E;\underline{R},\underline{R}')$$

$$= \frac{1}{4\pi(2\mu_i)^{3/2}} \sum_n \frac{e^{i|\underline{q}'_{in}||\underline{\eta}_i|}}{|\underline{\eta}_i|} \phi_{in}(\underline{\xi}_i)\, \Phi_i^{*}(\underline{q}_{in},E_{in};\underline{R}') \quad , \tag{A.6}$$

where

$$q'_{in} = \underline{\eta}_i^0 |q_{in}| \quad , \quad \Phi_i(\underline{q}_{in},E_{in};\underline{R}') = e^{i\underline{q}_{in}\underline{\eta}_i}\, \phi_{in}(\underline{\xi}_i) \quad .$$

The derivation of the asymptotic formula for the two-body channel is trivial. In this case one can start directly from the identity (A.1) i≠0. There is no trouble in the integration in the coordinates $\underline{\xi}'_i$ and $\underline{\eta}'_i$, since in the two-body channel the leading contribution in the Green's function comes only from $\tilde{g}_i^{+b}(E)$. Thus the range of $\underline{\xi}'_i$ is limited by the bound state function ϕ_{in} and the range of $\underline{\xi}'_j$ is limited by the potential V(j), j≠i. If the ranges of $\underline{\xi}'_i$ and of $\underline{\xi}_j$, j≠i are limited, then the same holds true for $\underline{R}' = (\underline{\xi}'_i,\underline{\eta}'_i)$. Starting from (A.1) and using (A.6) one obtains

$$\lim_{\substack{|\underline{\eta}_i| \to \infty \\ |\underline{R}'|, |\underline{\xi}_i| < \infty}} \tilde{g}^{+}(E;\underline{R},\underline{R}') = -\frac{1}{4\pi(2\mu_i)^{3/2}} \sum_n \frac{e^{i|\underline{q}'_{in}||\underline{\eta}_{in}|}}{|\underline{\eta}_i|} \tilde{\chi}_{in}^{(-)*}(\underline{R}') \,, \tag{A.7}$$

where

$$\lim_{\epsilon \to 0} i\epsilon\, \tilde{g}(E-i\epsilon)\Phi_{in} = \Phi_{in} + \tilde{g}^{(-)}\, \tilde{V}(i)\, \Phi_{in} \equiv \tilde{\chi}_{in}^{(-)} \,.$$

In the case of the breakup channel the use of Eq. (A.1) is no longer justified, since the corresponding kernels do not fulfill the requirement of the connectivity. In that case, it is suitable to start from the identity (A.3) or (A.4). Studying Eq. (A.3) one finds that the terms $V(i)g_0$ cause no trouble because they represent compact two-body kernels embedded in the three-body space and the corresponding function can be factored out. But we have to ensure that $\langle \underline{R}|V(i)\tilde{g}_0 V(j)$, $j \neq i$ decreases asymptotically sufficiently rapidly. It is known that the compactness of the operator $V(i)\tilde{g}_0 V(j)$ or its cyclic iteratives is a sufficient condition for the required asymptotical decrease. Faddeev [27] has shown that such kernels are compact in a suitable Banach space for real energies greater than E_{min} (where E_{min} is the lowest energy level of the subsystem). One can show directly that the kernel

$$k(\underline{\xi}_i,\underline{\eta}_i,\underline{\xi}'_i,\underline{\eta}'_i) = V(i)\ (\underline{\xi}_i)\ \tilde{g}_0(\underline{\xi}_i,\underline{\eta}_i;\underline{\xi}'_i,\underline{\eta}'_i)$$

$$\times\ V(j)\ (a_{ij}\underline{\xi}'_i + b_{ij}\underline{\eta}'_i)$$

fulfills the above mentioned requirement: the behavior of the kernel k for large $|\underline{\xi}_i|$ is evident because of the potential V(i), for the

coordinate $\underline{\eta}_i$ one can show evaluating a representation of $\tilde{g}_0$ in the form of the convolution

$$\tilde{g}_0(E;\underline{\xi}_i,\underline{\eta}_i;\underline{\xi}'_i,\underline{\eta}'_i) = \frac{i}{2\pi}\int_{-\infty}^{\infty} dz$$

$$\times \langle \underline{\xi}_i|\tilde{g}_0(E-z)|\underline{\xi}'_i\rangle \langle \underline{\eta}_i|\tilde{g}_0(z)|\underline{\eta}'_i\rangle$$

by the stationary phase method [28] that k decreases in $|\underline{\eta}_i|$ stronger than $1/(|\underline{\eta}_i|)^2$.

Thus we are allowed to use the formula (A.4) in order to derive the asymptotic formula of the resolvent $\tilde{g}$ in the breakup channel. To this purpose we need the corresponding formula for the free resolvent $\tilde{g}_0$. It holds

$$\lim_{\substack{|\underline{R}|\to\infty \\ |\underline{R}'|<\infty}} \tilde{g}_0(E;\underline{R},\underline{R}') = \frac{e^{i\pi/4}\, E^{3/4}\, e^{i|\underline{K}|\underline{R}^0\underline{R}'}}{2(2\pi)^{5/2}\,|\underline{R}|^{5/2}}\, e^{-i|\underline{K}|\underline{R}^0\underline{R}'} . \qquad (A.8)$$

This formula is obtained using methods already applied by Newton [15]. Starting from (A.3) and using (A.8) one gets

$$\lim_{\substack{|\underline{R}|\to\infty \\ |\underline{R}'|<\infty}} \tilde{g}(E,\underline{R},\underline{R}') = \frac{e^{i\pi/4}\, E^{3/4}\, e^{i|\underline{K}|\underline{R}^0\underline{R}'}}{2(2\pi)^{5/2}\,|\underline{R}|^{5/2}}$$

$$\times \left[\Phi_0^*(\underline{K}',\underline{R}') + \sum_i (\Phi_0 V(i)\tilde{g}_i + \Phi_0 V(i)\tilde{g}_i\bar{V}(i)\tilde{g})^*|\underline{R}'\rangle\right] , \qquad (A.9)$$

where

$$\Phi_0(\underline{K}',\underline{R}') = e^{i|\underline{K}|\underline{R}^0\underline{R}'} \quad , \quad \underline{K}' = |\underline{K}|\underline{R}^0 \quad .$$

Considering that $\lim_{\epsilon\to 0} i\epsilon\, g_i(E-i\epsilon)\Phi_0 = \Phi_i^{(-)}$, one obtains

$$\sum_{i=1}^{3} \left(\Phi_0\, V(i)\, \tilde{g}_i + \Phi_0\, V(i)\, \tilde{g}_i\, \bar{V}(i)\tilde{g}\right)$$

$$= \sum_{i=1}^{3} \left(\Phi_i^{(-)} + \Phi_i^{(-)}\, \bar{V}(i)\tilde{g}\right) - \sum_{i=1}^{3} \left(\Phi_0 + \Phi_0\, \bar{V}(i)\tilde{g}\right) ,$$

and with this result finally

$$\lim_{\substack{|\underline{R}|\to\infty \\ |\underline{R}'|<\infty}} \tilde{g}(E,\underline{R},\underline{R}') = \frac{e^{i\pi/4}\, E^{3/4}\, e^{i|\underline{K}||\underline{R}|}}{2(2\pi)^{5/2}\, |\underline{R}|^{5/2}}$$

$$\times \left[\sum_{i=1}^{3} \tilde{\chi}_i^{(-)*}(\underline{K}',\underline{R}') - 2\tilde{\chi}_0^{(-)*}(\underline{K}',\underline{R}')\right] , \qquad \text{(A.10)}$$

in which

$$\tilde{\chi}_i^{(-)} = \lim_{\epsilon\to 0} i\epsilon\, \tilde{g}(E-i\epsilon)\, \Phi_i^{(-)} \quad , \qquad \text{(A.11)}$$

and

$$\tilde{\chi}_0^{(-)} = \lim_{\epsilon\to 0} i\epsilon\, \tilde{g}(E-i\epsilon)\, \Phi_0 \quad . \qquad \text{(A.12)}$$

One can easily show that $\chi_i^{(-)}$ as defined in (A.11) and (A.12) are identical for i=0,1,2,3. Therefore one obtains

$$\lim_{\substack{|\underline{R}| \to \infty \\ |\underline{R}'| < \infty}} \tilde{g}(E;\underline{R},\underline{R}') = \frac{e^{i\pi/4}\, E^{3/4}\, e^{i|\underline{K}||\underline{R}|}}{2(2\pi)^{5/2}\, |\underline{R}|^{5/2}}\, \tilde{\chi}_0^{(-)*}(\underline{K}',\underline{R}') \quad . \tag{A.13}$$

Finally, we mention another useful asymptotic formula for the breakup channel formulated not in terms of scattering function but in terms of the scattering amplitude. This shows that the scattering amplitude R_{00} without the compound state F is already contained in the resonance scattering amplitude R_{10}^{res}. It is known that the scattering amplitude R^{00} is obtained by quadrature from the scattering amplitude R_{0i}, i=1,2,3. The result is

$$\lim_{\substack{|\underline{R}| \to \infty \\ |\underline{R}'| < \infty}} \tilde{g}(E;\underline{R},\underline{R}') = \frac{e^{i\pi/4}\, E^{3/4}\, e^{i|\underline{K}||\underline{R}|}}{2(2\pi)^{5/2}\, |\underline{R}|^{5/2}}$$

$$\times \left[e^{-i\underline{K}'\underline{R}'} + (2\pi)^3 \int R_{00}^{(-)}(\underline{K}',\underline{K}'') \langle \underline{K}''|\tilde{g}_0|\underline{R}'\rangle \, d\underline{K}'' \right] , \tag{A.14}$$

in which $R_{00}^{(-)}$ denotes the free-free transition amplitude in a three-body system.

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