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FERMI-TELLER THEORY OF LOW VELOCITY IONIZATION LOSSES APPLIED TO MONOPOLES*

D. M. Ritson
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Recently there has developed a renewed interest in the ionization losses of slow monopoles. An excellent summary of the literature is contained in a paper of Ahlen and Kinoshita[1] (hereafter referred to as A.K.). A.K. have derived lower bounds for the ionization losses of a slow monopole passing through matter. However, they neglected the effects of an energy gap, in insulating materials, on the excitation of atomic electrons. Using the Thomas-Fermi statistical model of atomic wave functions, we have calculated lower bounds for ionization losses in "insulators" including effects due to an energy gap.

The Fermi-Teller theory[2] (hereafter referred to as F.T.) was originally used to predict the stopping, by ionization losses, of slow charged particles in materials. The theory is based on a calculation of the energy lost to a uniform "Fermi sea". However, the authors note that the actual "Fermi velocity" cancels out in the derivation. Therefore their calculated results also apply to a "Thomas-Fermi" atom, in which each volume element is considered to be a Fermi sea filled to the top of the potential well with atomic electrons.

We sketch below an outline of the modifications required to make the F.T. theory valid for a slow monopole traversing an insulator using the Thomas-Fermi model of the atom.

Consider a small volume element dV at a distant r from a nucleus of

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atomic number Z. The Thomas-Fermi atomic model considers this volume element to be filled to a maximum kinetic energy of $-U(r)$ with a Fermi sea of electrons. The potential can be solved for by a self-consistent equation and is described in terms of the function $\phi(x)$. The variable x is defined by

$$U(x) = \frac{-Z\ \phi(x)\ e^2}{x\ b_o} \tag{1}$$

and $x = r/b_o$ and $b_o = 0.885\ a_o/Z^{1/3}$ where a_o is the Bohr hydrogen radius $\hbar/\alpha m_e c$.

The function $\phi(x)$ is a universal function for all atomic numbers and is tabulated.[3] The maximum electron velocity is $v_f(x)$ given by

$$\frac{1}{2}\, m_e\, v_f(x)^2 = \frac{Z\ \phi(x)\ e^2}{x\ b_o}$$

or

$$\frac{v_f(x)^2}{c^2} = \frac{2Z\ \phi(x)}{x\ b_o}\ r_o \tag{2}$$

where r_o is the classical electron radius $e^2/m_e c^2$.

A slow heavy particle with charge e proceeding through this medium with a velocity v_o or β_o can, in the absence of an energy gap, excite electrons into the continuum within $\sim v_o$ of the Fermi surface. The density of such particles available for excitation is

$$\rho \sim \frac{m_e^3\ v_f^2\ v_o}{\hbar^3} \quad . \tag{3}$$

The collision cross section σ_e is

$$\sigma_e \sim \left(\frac{e^2}{m\ v_f^2}\right)^2 \quad . \tag{4}$$

From kinematics the energy transfer is

$$\Delta E_o \sim m_e v_f v_o \tag{5}$$

and the total energy transferred in inelastic collisions is

$$\frac{\Delta E}{\Delta T} = v_f \rho \sigma_e \, \Delta E_o$$

or

$$\frac{\Delta E}{\Delta T} \sim v_f m_e v_f v_o \left(\frac{e^2}{m\ v_f^2}\right)^2 \frac{m_e^3\ v_f^2\ v_o}{\hbar^3}$$

or

$$\frac{dE}{dZ} \sim \frac{m_e c^2\ \alpha^2\ \beta_o}{(\hbar/mc)} \quad , \tag{6}$$

where Z is the particle path length.

If the slow heavy particle has magnetic charge g (a monopole), the preceding estimate is modified by the substitution of equation (4) by

$$\sigma_e \sim \left(\frac{e\ g\ v_f}{m\ v_f^2}\right)^2 \tag{7}$$

leading to the result

$$\left(\frac{dE}{dZ}\right)_{\text{monopole}} \sim \left(\frac{dE}{dZ}\right)_{\text{charged particle}} \left(\frac{g^2\ \overline{v_f^2}}{e^2}\right) \tag{8}$$

with

$$\overline{v_f^2} = \int_0^{x_{max}} v_f^2 \, x^2 \, dx \Bigg/ \int_0^{x_{max}} x^2 \, dx \tag{9}$$

where x_{max} = atomic radius/b_o.

A.K. perform an exact calculation, assuming that the value of $\overline{v_f^2}$ is $\alpha^2 c^2$. This is close to the value obtained by numerically integrating equation (9). The major numerical uncertainty in equating monopole losses to charged particle losses is the screening effect of the atomic electrons, small for a monopole, but finite for a charged particle. A.K. set a lower bound on the ionization produced by a monopole by assuming similar shielding effects.

In one respect, however, the A.K. estimates are inadequate. Implicitly they assume that there is no energy gap. Both Argon gas (proportional chambers) and scintillators are insulators. In the case of scintillating plastic the material is relatively transparent to transmitted radiation down to 2,900 Å[4] implying an energy gap of 4 eV, and proportional chambers require ionization of the atoms with an energy gap therefore of the order of 13 eV.[5] While it is possible that the distortion of the atomic wave functions associated with the passage of a slow particle through an atom is sufficient to substantially decrease the energy gap, the absence of detailed calculations makes this an unconservative assumption.

To modify the above estimate, we note that the inelastic energy loss ΔE_o, given by equation (5), must exceed the energy gap G_o, therefore

$$\Delta E_o \sim m_e v_f v_o > G_o$$

or

$$v_f \gtrsim \frac{G_o}{m_e v_o} \quad . \tag{10}$$

This restricts the volume of the material which can participate to x_o by the condition from equation (2) and equation (10)

$$\left(\frac{2Z\ \phi(x_o)}{x_o}\ r_o \right)^{1/2} = \frac{G_o}{m_e v_o} \quad . \tag{11}$$

Therefore to obtain the correct result, equation (9) must be changed to

$$\overline{v_f^{\,2}} = \int_o^{x_o} v_f^{\,2}\, x^2\, dx \Big/ \int_o^{x_{max}} x^2\, dx \tag{12}$$

where the first limit of integration has been changed from x_{max} corresponding to the atomic radius, to x_o set by equation (11).

Table I tabulates parameters of interest for Carbon (scintillator) for a monopole of charge $e/2\alpha$. Values are numerically calculated using the above equations and the functional form of the Thomas-Fermi atom. Listed are the maximum radii from which collisions with energy losses in excess of 4 eV can occur (column 1), the corresponding values of beta for the slow particle (column 2), $\overline{v_f^{\,2}}$ in units of αc (column 3), and the energy loss in units of "minimum" ionization loss (column 4). The assumed atomic radius for Carbon is 1.1 Å. Table II tabulates the same values for Argon assuming an energy gap of 13 eV and an atomic radius of 1.8 Å.

The quoted ionization values are in units of minimum ionization, and are calculated using the following formula for monopoles of charge $e/2\alpha$,

$$\left(\frac{dE}{dZ}\right)_{\text{monopole}} \geq \frac{1}{4}\left(\frac{\beta_o}{5\text{x}10^{-3}}\right)\left(\frac{\overline{v_f^2}}{\alpha^2 c^2}\right)\left|\frac{dE}{dZ}\right|_{\substack{\text{proton}\\ \text{at } \beta=5\text{x}10^{-3}}}$$

Z is in units of $\text{gm}\cdot\text{cm}^{-2}$ and $(dE/dZ)_{\substack{\text{proton}\\ \text{at } \beta=5\text{x}10^{-3}}}$ is evaluated from experimental results[6] at a β of $5\text{x}10^{-3}$ and is $240\cdot I_{min}$ for Carbon and $75\cdot I_{min}$ for Argon. Values for β_o's > $1\text{x}10^{-3}$ are similar to those given by A.K. As discussed by A.K., equation (13) should give a conservative lower bound to the ionization losses.

For scintillator (Carbon) the modification to the A.K. estimates becomes important for β's below $5\text{x}10^{-4}$. For Argon the result of taking into account the energy gap becomes important at β's below $2\text{x}10^{-3}$. Losses calculated using equation (13) are plotted for Carbon (scintillator) and Argon in Figs. 1 and 2.

Using the above results, we estimate the limits on detection on monopoles to be $\beta \gtrsim 1\text{x}10^{-4}$ for scintillator and $\beta \gtrsim 3\text{x}10^{-4}$ for proportional counters.

References

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TABLE I

Monopole Ionization in Carbon (Scintillator)

Assumed energy gap 4 eV. Atomic radius 1.1×10^{-8} cm. Monopole magnetic charge 137 e/2.

Radius in cms	β_o	$\overline{v_f^2}$ in units of α^2c^2	Ionization in Units of I_{min}
$2.5757\ 10^{-12}$	$6.845\ 10^{-6}$	0	0
$5.1514\ 10^{-11}$	$3.1082\ 10^{-5}$	$1.8514\ 10^{-4}$	$6.9053\ 10^{-5}$
$1.0303\ 10^{-10}$	$4.4416\ 10^{-5}$	$7.3085\ 10^{-4}$	$3.8954\ 10^{-4}$
$1.5454\ 10^{-10}$	$5.5279\ 10^{-5}$	.0016168	.0010725
$2.5757\ 10^{-10}$	$7.2968\ 10^{-5}$	.0043425	.0038024
$5.1514\ 10^{-10}$	$1.0891\ 10^{-4}$	.016123	.021072
$1.0303\ 10^{-9}$	$1.6851\ 10^{-4}$	.056648	.11455
$1.5454\ 10^{-9}$	$2.2406\ 10^{-4}$	.11407	.30669
$2.0606\ 10^{-9}$	$2.7945\ 10^{-4}$	.18278	.61292
$2.5757\ 10^{-9}$	$3.3203\ 10^{-4}$	.2596	1.0344
$3.606\ 10^{-9}$	$4.4383\ 10^{-4}$	.4296	2.288
$4.6362\ 10^{-9}$	$5.5889\ 10^{-4}$	.61108	4.0983
$5.6665\ 10^{-9}$	$6.8295\ 10^{-4}$	.79597	6.5234
$6.6968\ 10^{-9}$	$8.1148\ 10^{-4}$	.97979	9.5409
$7.7271\ 10^{-9}$	$9.3729\ 10^{-4}$	1.1613	13.062
$1.0303\ 10^{-8}$	.0013173	1.5936	25.192

TABLE II

Monopole Ionization in Argon

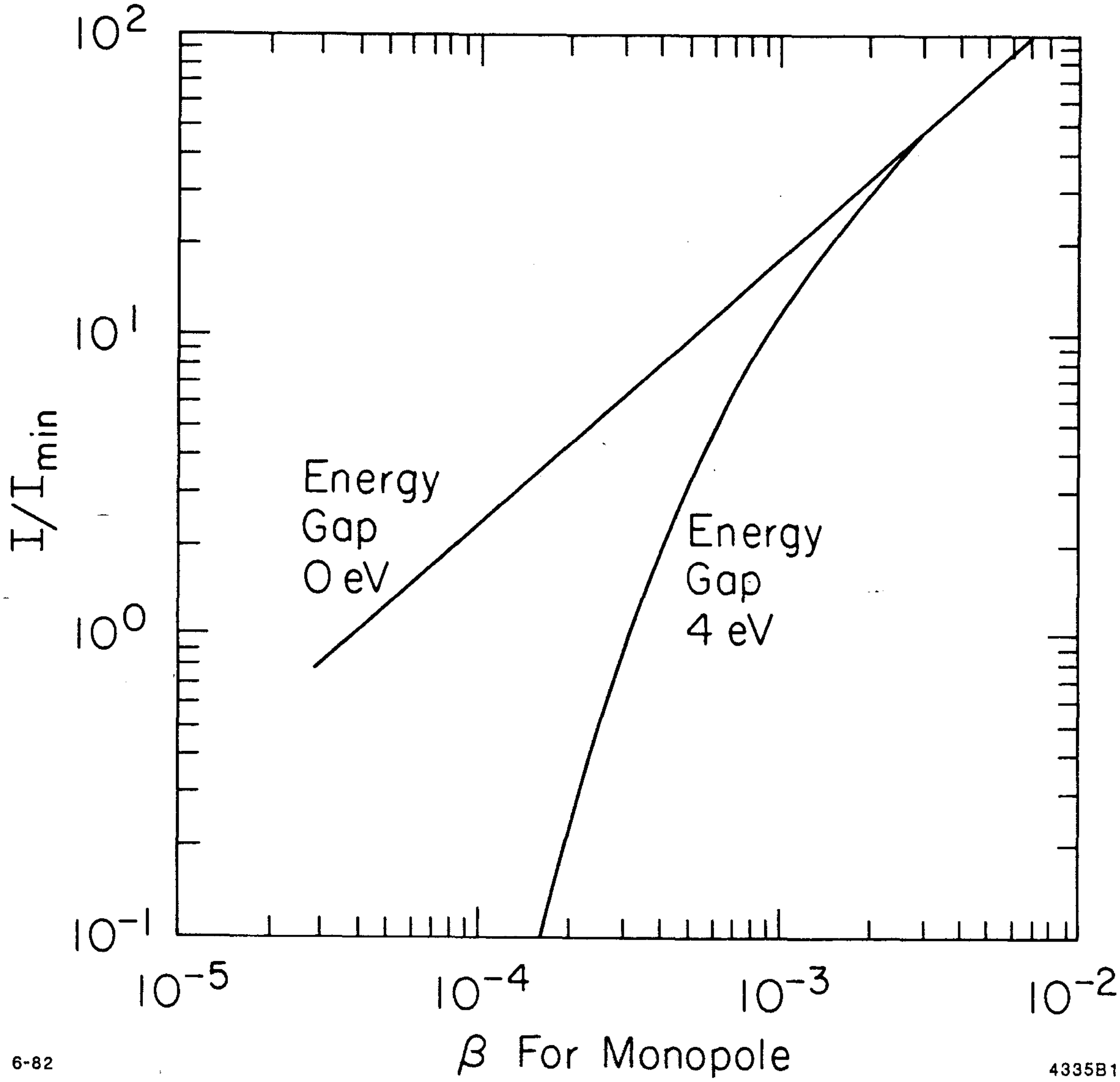
Assumed energy gap 13 eV. Atomic radius 1.8×10^{-8} cm. Monopole magnetic charge 137 e/2.

Radius in cms	β_o	$\overline{v_f^2}$ in units of $\alpha^2 c^2$	Ionization in Units of I_{min}
$1.7859\ 10^{-12}$	$1.0695\ 10^{-5}$	0	0
$3.5719\ 10^{-11}$	$4.8564\ 10^{-5}$	$6.0944\ 10^{-5}$	$1.1247\ 10^{-5}$
$7.1438\ 10^{-11}$	$6.9399\ 10^{-5}$	$2.4058\ 10^{-4}$	$6.3445\ 10^{-5}$
$1.0716\ 10^{-10}$	$8.6371\ 10^{-5}$	$5.3222\ 10^{-4}$	$1.7468\ 10^{-4}$
$1.7859\ 10^{-10}$	$1.1401\ 10^{-4}$	.0014295	$6.193\ 10^{-4}$
$3.5719\ 10^{-10}$	$1.7017\ 10^{-4}$	.0053073	.003432
$7.1438\ 10^{-10}$	$2.633\ 10^{-4}$	.018647	.018657
$1.0716\ 10^{-9}$	$3.5008\ 10^{-4}$	.037548	.04995
$1.4288\ 10^{-9}$	$4.3663\ 10^{-4}$	.060167	.099827
$1.7859\ 10^{-9}$	$5.1879\ 10^{-4}$	.085456	.16847
$2.5003\ 10^{-9}$	$6.9347\ 10^{-4}$	.14142	.37266
$3.2147\ 10^{-9}$	$8.7325\ 10^{-4}$	.20115	.6675
$3.9291\ 10^{-9}$	.0010671	.26202	1.0625
$4.6435\ 10^{-9}$	.0012679	.32253	1.5539
$5.3578\ 10^{-9}$	.0014645	.38229	2.1275
$7.1438\ 10^{-9}$	.0020583	.52459	4.1031
$1.3395\ 10^{-8}$	.0045743	.93084	16.18

Figure Captions

Fig. 1. This shows a plot of ionization in units of "minimum ionization" versus β for a monopole of charge $e/2$. The plot is calculated for Carbon with an energy gap of 4 eV and without an energy gap for comparison.

Fig. 2. This shows a similar plot to Fig. 1 for ionization loss versus β for a monopole traversing Argon gas with an energy gap of 13 eV and without an energy gap for comparison.



10^2
10^1
10^0
10^{-1}
I/I_{min}
Energy Gap 0 eV
Energy Gap 4 eV
10^{-5}
10^{-4}
10^{-3}
10^{-2}
β For Monopole
6-82
4335B1

Fig. 1

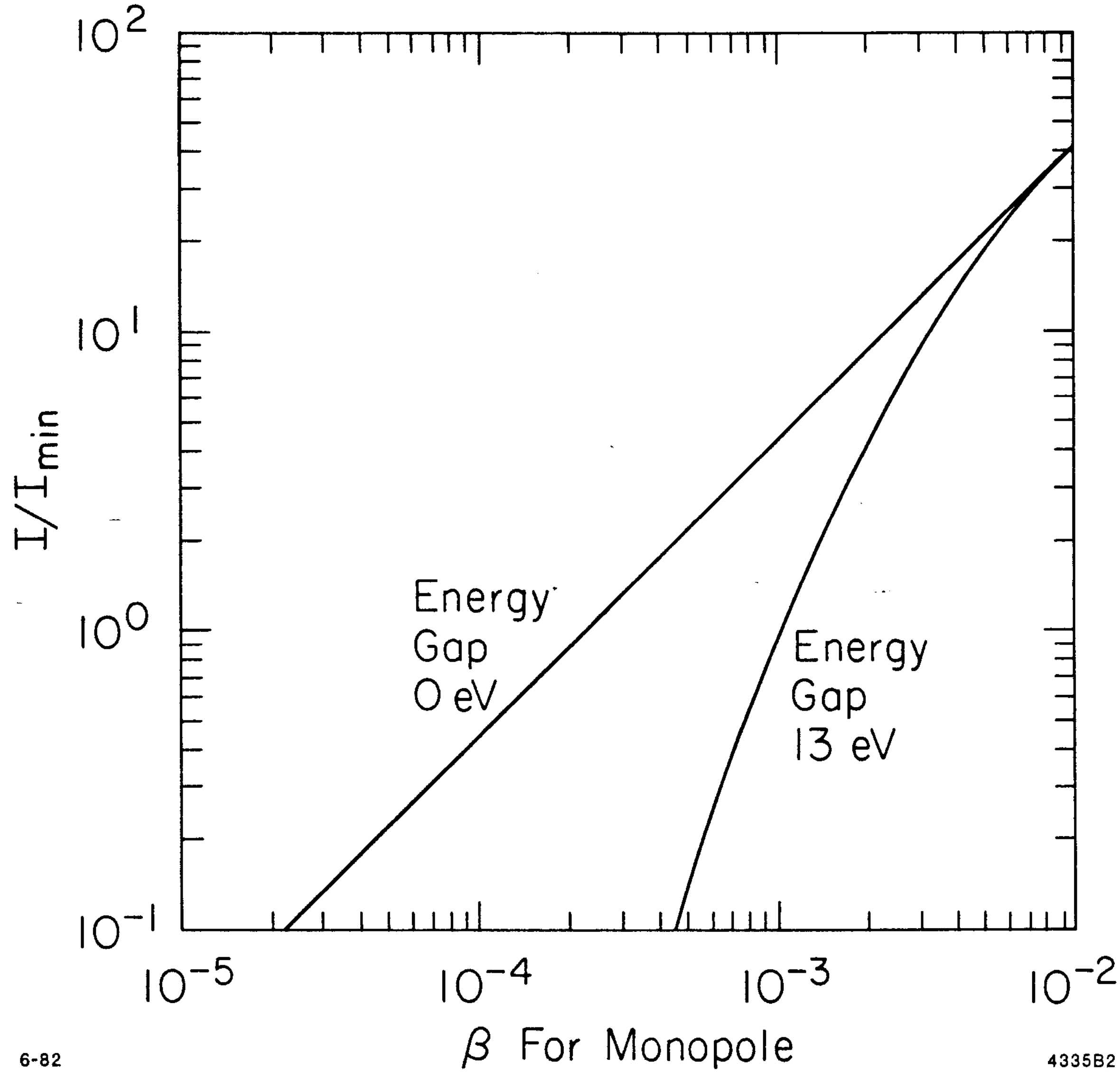

10^2
10^1
10^0
10^{-1}
I/I_{min}
Energy Gap 0 eV
Energy Gap 13 eV
10^{-5}
10^{-4}
10^{-3}
10^{-2}
β For Monopole
6-82
4335B2

Fig. 2