SLAC-PUB-2945
June 1982
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TRANSVERSE QUADRUPOLE WAKE FIELD EFFECTS IN HIGH INTENSITY LINACS*

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ABSTRACT

Transverse quadrupole wake fields exist whenever the beam is not round, and therefore in an alternating gradient focusing system these fields cannot be eliminated. As a result, these fields represent a potential limitation on high intensity linac performance. In this note we calculate the magnitude of quadrupole wake field effects for the SLAC linac.

(Submitted to Particle Accelerators)

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*Work supported by the Department of Energy, contract DE-AC03-76SF00515.

INTRODUCTION

As an intense bunch of charged particles travels down a linear accelerator, it interacts with the surrounding accelerating structure and generates wake electromagnetic fields behind it. If the beam intensity is high and therefore the wake field is strong, the particle distribution within the bunch may be significantly affected and the effective beam emittance will grow. This effect may impose an important limit on the bunch intensity for the linear colliding beam devices now being proposed since the luminosity of these devices depends critically on maintaining a beam size of the order of 1 μm at the collision point. As we report in this note, the inclusion of the quadrupole wake effects in calculations for the SLAC Linear Collider (SLC)[1] indicates that these effects may be important for other high intensity linear accelerators as well.

The wake field generated by an intense particle bunch in a cylindrically symmetric accelerator pipe can most conveniently be described by decomposing it into several components, each giving rise to different beam dynamical effects. The first component is the longitudinal wake field; it consists mainly of a longitudinal electric field which is uniform across the transverse cross section of the accelerator pipe and varies longitudinally along the pipe. The longitudinal wake field gives rise to a variation in particle energy within the length of the bunch. The second component of the wake field is the transverse dipole wake generated by an off-axis motion (i.e., the dipole moment) of the bunch; it deflects the tail portions of the bunch.

The third component of the wake field is a transverse quadrupole wake caused by the bunch having a transverse spatial distribution that is not round (i.e., having a finite quadrupole moment). The effect of the quadrupole wake is to cause the transverse distribution in the tail portions of the bunch to change without affecting the beam center of a properly centered beam.

The longitudinal wake and the transverse dipole wake have been extensively studied.[1] In this paper we will study the beam dynamical effects caused by the quadrupole wake field component. We first present some analysis assuming a simplified accelerator lattice model. The results are then applied to the presently proposed SLAC linac collider. A numerical tracking program has been written to test these results. The properties of the quadrupole wake fields are taken from the results of Ref. 2.

The longitudinal wake field effects can be minimized by adjusting the phase of the radio-frequency accelerating field relative to the arrival time of the particle bunch. The effects of the transverse dipole fields can in principle be eliminated by proper alignment of the accelerator pipe, and by accurate bunch injection. In contrast to the dipole wake component, the quadrupole wake field exists even if the particle bunch is perfectly centered on the pipe axis. This observation follows from the fact that the quadrupole wake field is generated by the quadrupole moment rather than the off-axis dipole moment of the particle distribution. Thus the quadrupole wake field represents a potential fundamental limiting influence on the transverse beam dynamics in the linac and as such needs to be studied. This is especially the case if the linac lattice is of the alternating gradient type so that the transverse spatial distribution of the beam is in general elliptical.

QUADRUPOLE WAKE FIELD

A quadrupole wake is generated if the transverse beam distribution posesses a quadrupole moment relative to the accelerator pipe axis. In that case, two types of quadrupole moments may exist:

$$Q_1 = \langle x^2 \rangle - \langle y^2 \rangle \qquad (1)$$
$$Q_2 = 2\langle xy \rangle$$

where x and y refer to the transverse coordinates defined by the focusing scheme of the accelerator, and $\langle\ \rangle$ means averaging over the transverse beam dimensions.

Consider a slice of charge travelling down the linac. Let the charge slice have quadrupole moments Q_1 and Q_2. The wake field generated by this charge slice at a point z following the slice gives rise to a transverse Lorentz force given by

$$e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) = e^2 W(z) \left[Q_1(x\hat{x} - y\hat{y}) + Q_2(y\hat{x} + x\hat{y})\right] \qquad (2)$$

where W(z) is a certain quadrupole wake function characteristic of the accelerator pipe structure, $\hat{x}$ and $\hat{y}$ are the unit vectors in the x and y directions. The electromagnetic force generated by Q_1 resembles that of a quadrupole focusing magnet, while the force generated by Q_2 resembles that of a skew quadrupole magnet. For simplicity, in the following we assume the transverse beam shape is an upright ellipse for which $Q_2 = 0$ and therefore there are no skew quadrupolar fields.

There are three possible sources of the beam quadrupole moment:

(1) The transverse beam sizes $\langle x^2 \rangle$ and $\langle y^2 \rangle$ scale with the envelope functions β_x and β_y in the focusing system of the linac. A quadrupole moment then arises from the beating of the two beta-functions. This quadrupole moment will be present even if there are no misalignments of the linac structure.

(2) A quadrupole moment may result from injecting a beam whose phase ellipses are not matched to those prescribed by the linac lattice and as a result envelope oscillations occur.

(3) A quadrupole moment results from an off-axis motion of the beam center.

The first two quadrupole moments occur as a result of the beam not being round; the third exists even if the beam distribution is round.

The first type of quadrupole moment has an oscillation period equal to that of the lattice cells. The focusing force associated with the quadrupole moment oscillates with the cell periodicity. When this focusing force is applied to the particle betatron motion, the motion will be driven resonantly, leading to an instability, if the betatron phase advance is close to 180 degrees per cell. In fact, even if the phase advance per cell is sufficiently far away from 180 degrees so that particle motions are stable, there will still be a finite increase of the effective beam emittance.

The second type of quadrupole moment oscillates with a frequency equal to twice the betatron oscillation frequency ω_β. A focusing force that oscillates with $2\omega_\beta$ necessarily drives the single particle betatron motions on resonance, leading always to an instability. A similar resonance driving mechanism is the cause of the dipole wake instability studied in Ref. 3. Obviously if the beam is injected with poor phase ellipse matching, there will be an emittance growth of the beam.

If the beam has an off-axis motion, the associated dipole moment will generate a dipole wake whose effects dominate the beam behavior if the pipe radius is much greater than the transverse beam dimension. Thus if the dipole wake effects have been controlled properly, this component of the quadrupole wake becomes negligible. In the following we will assume that the beam center does not execute off-axis motion.

ROUGH ESTIMATES

Consider a particle located at a longitudinal position z relative to the bunch center (z>0 for bunch head). This particle is acted upon by a quadrupole field arising from all the charges at larger z values. To first order in the wake field strength, the quadrupole field is given by, assuming the entire bunch has the same transverse distribution and thus the same quadrupole moment Q_1,

$$e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) = e^2 Q_1 (x\hat{x} - y\hat{y}) \int_z^\infty dz' \rho(z') \, W(z' - z) \qquad (3)$$

where $\rho(z)$ is the density distribution of the bunch with $\int \rho(z)dz = N$ with N the total number of particles in the bunch.

The wake function W(z) has been extensively studied for a periodic corrugated pipe structure in Ref. 2. This function vanishes for $z < 0$ because of causality, i.e., the charge slice does not produce any wake field in front of it. For positive z, the wake function first increases linearly with z and then starts to oscillate at higher values of z. The result of Ref. 2 for the SLAC linac, for instance, is replotted in Fig. 1 (in units consistent with Eq. (3)).

To get a rough quantitative estimate of the quadrupole wake on particle motion, let us consider a bunch with uniform longitudinal distribution of total length ℓ. We assume that ℓ is short enough that the relevant portion of the wake function is approximately linear in z. Then for a particle at the tail of the bunch, the integral in Eq. (3) is roughly equal to NW_o, where W_o is the value of the wake function evaluated at a distance $\ell/2$ behind the charge slice. From Fig. 1, we estimate that $W_o = 0.4\ \mathrm{cm}^{-5}$ if we take $\ell = 3$ mm.

We will now do two rough estimates on the quadrupole wake effects.

A. First Estimate

Consider one FODO cell. Let s be the distance along the linac. Let the horizontal and the vertical emittances of the beam be equal, $\epsilon_x = \epsilon_y = \epsilon$. The quadrupole moment at position s is given by

$$Q_1(s) = \epsilon(s)\ (\beta_x(s) - \beta_y(s)) \tag{4}$$

The corresponding change in betatron phase advance, accumulated from injection (s = 0) to L, due to the focusing effect of the quadrupole wake field is

$$\Delta\Psi_x = -\int_o^L ds\ \frac{1}{2}\ \beta_x(s)\ \frac{N\ r_e W_o}{\gamma(s)}\ Q_1(s) \tag{5}$$

$$= -\frac{N\ r_e W_o \epsilon_o}{2\gamma_o} \int_o^L ds\ \frac{\beta_x(s)[\beta_x(s) - \beta_y(s)]}{(1 + Gs)^2}$$

$\gamma(s)$ = (beam energy)/$mc^2 = \gamma_o(1 + Gs)$

$r_e = e^2/mc^2$

G = acceleration gradient

$\epsilon(s) = \epsilon_o\gamma_o/\gamma(s)$

Assuming the beam energy does not change appreciably within one FODO cell, we can approximate the quantity $\beta_x(\beta_x - \beta_y)$ in the integrand of Eq. (5) by its value averaged over the FODO cell:

$$\langle\beta_x(\beta_x - \beta_y)\rangle = 1/6\ (\beta_{max} - \beta_{min})^2 \tag{6}$$

where β_{max} and β_{min} are the maximum and minimum values of the beta-function in the FODO cell. After making this approximation, Eq. (5) becomes

$$\Delta\Psi_x = -\frac{N\ r_e W_o \epsilon_o L}{12\ \gamma_f}\ (\beta_{max} - \beta_{min})^2 \tag{7}$$

where $\gamma_f = \gamma_o(1 + GL)$ is the final beam energy in units of mc^2. The negative sign occurring in this expression is the result of the averaged quadrupole wake force yielding a net defocusing action in both planes.

Taking the present SLAC linac with the linac collider parameters as an illustrative example, we let $N = 5 \times 10^{10}$, $\epsilon_o = 1.4 \times 10^{-6}$ m, $\gamma_o = 100$ MeV/mc^2, $G = 0.11\ m^{-1}$ (10 MeV/m acceleration), $\beta_{max} = 42$ m and $\beta_{min} = 7.2$ m. We find then that $\Delta\Psi_x = -3.6$ rad. Since most of this phase advance is acquired in the first FODO cell or so and since this phase advance occurs at the tail of the bunch while the head is unperturbed, this means that the quadrupole wake can in fact do some damage to the effective beam emittance.

B. Second Estimate

We now consider the quadrupole wake caused by injecting the beam with an error in the matching of the phase space ellipse. The equation of motion for a tail particle can be modeled by

$$(\gamma(s)x'(s))' + k_o^2\,\gamma(s)\,x(s) = N\,W_o r_e Q_1(s)x(s) \tag{8}$$

where k_o is the reciprocal of the average beta-function in the first FODO cell and $Q_1(s)$ is the quadrupole moment caused by an injection mismatch.

The homogeneous solution of Eq. (8) is approximately

$$x^{(1)} \approx \frac{x_o}{\sqrt{1 + Gs}} \cos k_o s \tag{9}$$

Since the quadrupole moment is quadratic in displacement, we will model the quadrupole moment Q_1 as

$$Q \approx (Q_1)_{mismatch} \cdot \frac{1}{1 + Gs} \cos 2k_o s \tag{10}$$

That is, the magnitude of Q_1 at $s = 0$ is given by the mismatch at injection and then Q_1 oscillates with twice the natural betatron frequency. The first order perturbation of x, $x^{(1)}$, is then obtained from Eq. (8) by replacing the x on the right right-hand side by $x^{(o)}$. The result for $x^{(1)}$ at the end of acceleration, $s = L$, is

$$x^{(1)} \approx \xi \frac{x_o}{\sqrt{1 + Gs}} \sin k_o s \tag{11}$$

where we have defined

$$\xi = \frac{N\,W_o r_e}{4\,\gamma_o G k_o} (Q_1)_{mismatch} \tag{12}$$

It follows from comparing (9) and (11) that the beam emittance grows appreciably if the parameter ξ is not too small compared with unity.

For the SLAC linac collider, if we somewhat arbitrarily take $(Q_1)_{mismatch} = \epsilon_o\beta_o/10$ with $\beta_o = 20$ m, $\epsilon_o = 1.4 \times 10^{-6}$ m, $\gamma_o = 100$ MeV/mc^2, $G = 0.11$ m^{-1}, we find $\xi \approx 0.3$. This means the mismatching problem would also show up if we use the present linac lattice to handle the SLC beam parameters.

NUMERICAL RESULTS

A computer code has been written to simulate the beam behavior in the presence of quadrupole wake fields for the first sector of the SLAC linac. The code is structured much like a standard TRANSPORT-type code.[4] That is, transfer matrices are calculated for such elements as drifts, quadrupole magnets, solenoids and accelerating units. In addition, in the accelerating units the wake field forces are computed at intervals that are frequent enough that their influence on particle motion is adequately taken into account. We assume that there are wake fields only in the accelerating units.

The beam bunch is divided into a number of slices and each slice affects the motion of all slices behind it. All slices are assumed to move with the speed of light so that they do not overtake one another. All particles in a given slice are assumed to have the same energy. Each slice is characterized by the following parameters:

- number of particles in the slice,
- its longitudinal position relative to the bunch center,
- its energy,

- parameters describing the location and motion of its centroid (x, x', y, y'),

- parameters describing the phase space distribution of particles in the slice (the 4×4 Σ-matrix).[4]

A slice centroid is then tracked through an element with transfer matrix T by

$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_{out} = T \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_{in} \tag{13}$$

While the slice distribution is transformed as

$$\Sigma_{out} = T\ \Sigma_{in}\ \tilde{T} \tag{14}$$

The transfer matrix T depends on the energy (rigidity) of the slice being tracked. In the case of an accelerating unit, the unit is divided up (external to the code) into subunits. In each subunit, in addition to the usual acceleration transformation, the longitudinal wake field is calculated and its effect on slice energy integrated over the length of the subunit.* Moreover, the dipole and quadrupole transverse wake fields are calculated and applied to the beam motion as impulsive "kicks". The transfer matrices used in the code are given in detail in the Appendix.

The linac lattice starts with a weak triplet of quadrupole magnets. At this position, the beam energy is 42.6 MeV, the beam has an upright round distribution with $\sigma_x = \sigma_y = 2.1$ mm, $\sigma_{x'} = \sigma_{y'} = 0.87$ mrad and $\sigma_z = 1$ mm. The invariant x- and y-emittances are equal to $0.015\ \pi m_o c$ - cm. In this lattice, the beam is kept round in the first few accelerator

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*We have included in the longitudinal wake field only the "m = 0" component. This means all particles in a given slice see the same longitudinal wake.

sections by using triplets for focussing. The beam, of course, becomes elliptical as soon as FODO cells are used.

For the numerical calculations reported on here, the bunch is divided into 11 slices evenly spaced in their longitudinal positions with a total span of $4\sigma_z$. The particle population in the slices is such that it describes a gaussian bunch with 5×10^{10} particles in total. The slices are then tracked using the computer code. Accelerator sections are divided into subsections; wake field effects are accounted for at the end of each subsection. Earlier accelerator sections are divided more frequently. At the end of tracking, the total distance travelled is 112 meters and the energy of the central slice is 1.21 GeV.

In this simulation we assume no off-axis motion of slices. Transverse dipole effects are therefore not studied (although the code is written to handle them). The longitudinal wake is included, modifying the slice energies. Most of the energy variation caused by the longitudinal wake, however, has been removed by phasing the bunch center 15° ahead of the accelerating voltage. The quadrupole wake effects are then studied by comparing the results with and without turning on the quadrupole wake in the tracking.

Figure 2 gives the x- and y-beam sigma sizes of the 11 slices without taking into account the quadrupole wake. The slight variation of the transverse beam size throughout the bunch is due to the different slice energies. Figure 3 gives the same quantities when the quadrupole wake is taken into account. The perturbation of the distribution in the tail of the beam shows clearly.

Figure 4 gives the result when the total number of particles is increased to 7×10^{10}. The influence of the quadrupole wake becomes much more pronounced. This example illustrates the fact that the wake effects depend sensitively on the beam intensity, as well as the accuracy of wake field calculation, in the region of interest.

We have also studied the quadrupole wake effect caused by an injection mismatch. In Fig. 5, the phase space distribution of the beam at the injection point is 20% mismatched so that σ_x = 2.52 mm (20% more than the nominal value prescribed by the lattice), $\sigma_{x'}$ = 0.726 mrad (20% less than the nominal value), σ_y and $\sigma_{y'}$ are kept to be the nominal values and the total number of particles is 5×10^{10}. Compared with Fig. 3, we see that the injection mismatch greatly enhances the quadrupole wake effect on the beam. One reason for this behavior is that the mismatched beam is not round at the low energy end of the linac where the wake effect plays an important role.

ACKNOWLEDGEMENTS

We would like to thank Dick Helm, Martin Lee, Roger Miller, Phil Morton, John Sheppard and Perry Wilson for several illuminating discussions. We would also like to thank Mike Sullenberger who transferred the TRANSPORT lattice into the input of our program, and John Sheppard who provided us the SLAC linac lattice.

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1. SLAC Linear Collider Conceptual Design Report, SLAC-229 (1980).

2. K. Bane, P. Wilson, XIth Int. Conf. on High Energy Accelerators, Geneva, 1980, page 592.

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4. K. L. Brown et al., SLAC-91/UC-28 (1977).

APPENDIX

In this appendix we give the transfer matrices used in the numerical tracking code for the various elements.

Drift

For a drift space of length L,

$$T_{drift} = \begin{bmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Quadrupole

If the quadrupole length is L and its gradient is G, then $k_Q \equiv (G/33.356 * E_{Gev})^{1/2}$, where the magnetic rigidity of a 1 GeV electron is 33.356 kG m, and E_{Gev} is the energy in GeV of the slice being tracked. We then define:

$$c \equiv \cos k_Q L,$$

$$s \equiv \sin k_Q L,$$

$$ch \equiv \cosh k_Q L,$$

$$sh \equiv \sinh k_Q L.$$

For a focusing quadrupole the transfer matrix is

$$T_{focus} = \begin{bmatrix} c & s/k_Q & 0 & 0 \\ -k_Q s & c & 0 & 0 \\ 0 & 0 & ch & sh/k_Q \\ 0 & 0 & k_Q sh & ch \end{bmatrix}$$

while for a defocusing quadrupole,

$$T_{defocus} = \begin{bmatrix} ch & sh/k_Q & 0 & 0 \\ k_Q sh & ch & 0 & 0 \\ 0 & 0 & c & s/k_Q \\ 0 & 0 & -k_Q s & c \end{bmatrix}$$

Solenoid

If the solenoid length is L and its field strength is B, k ≡ B/(2* 33.356 * E_{GeV}). Let c ≡ cos kL and s ≡ sin kL, then the transfer matrix for a solenoid is

$$T_{solenoid} = \begin{bmatrix} c & sc/k & sc & s^2/k \\ -ksc & c & -ks^2 & sc \\ -sc & -s^2/k & c & sc/k \\ ks^2 & -sc & -ksc & c \end{bmatrix}.$$

Accelerating Unit

Given an accelerating unit of length L, an accelerating peak voltage (including any transit time factor) V in GV, the rf phase of the bunch center ϕ, the free space wavelength of the rf frequency λ, define

$$\delta = eV \cos(\phi + z_k 2\pi/\lambda)/E_{GeV} \quad ,$$

where z_k is the position of the k^{th} slice relative to the center of the bunch, and E_{GeV} is the energy of the k^{th} slice in GeV. Then the transfer matrix for the accelerating unit is

$$T_{acc} = \begin{bmatrix} 1 & L/\delta \ \ln(1+\delta) & 0 & 0 \\ 0 & 1/(1+\delta) & 0 & 0 \\ 0 & 0 & 1 & L/\delta \ \ln(1+\delta) \\ 0 & 0 & 0 & 1/(1+\delta) \end{bmatrix}$$

with the energy of the slice being changed by $E_{GeV} \rightarrow E_{GeV}(1+\delta)$.

Wake Field Transformation

In an accelerating unit of length L, assumed to be short enough that the wake field forces can be treated as impulses, the energy change of the k^{th} slice is of the form

$$\Delta E_k = - L \sum_{i=1}^{k-1} N_i W_o(z_i - z_k)$$

where N_i is the number of particles in the i^{th} slice, z_i and z_k are the longitudinal coordinates of the i^{th} and k^{th} slices, and z_1 is the slice at the head of the bunch. For the SLAC linac we take the following analytical fit to the wake function[1,2]

$$W_o(\Delta z) = [0.115 \times 10^{-9}\ \mathrm{GeV}/3000\ \mathrm{m}]\ \exp(- \sqrt{\Delta z/1.62\ \mathrm{mm}})$$

The dipole wake fields deflect all particles in a given slice by the same amount, so the Σ-matrix is unchanged, but the centroid coordinates are modified. In the impulse approximation only x' and y' are changed, with the functional dependence being of the form

$$\Delta x'_k = \frac{r_o L}{\gamma_k} \sum_{i=1}^{k-1} N_i W_1(z_i - z_k)\ x_i$$

where r_o is the classical electron radius and γ_k is the energy of the k^{th} slice in units of $m_o c^2$. For the SLAC linac the wake function W_1 is taken to be[2]

$$W_1(\Delta z) = 2.7 \times 10^5 \mathrm{m}^{-3}\ (\Delta z/1.5\ \mathrm{mm})^{0.81 - 0.127\ (\Delta z/1\ \mathrm{mm})}$$

If a given slice of the beam is not round in cross section, or if it is off axis, it will generate quadrupolar wake fields. These fields will act to deflect particles in following slices in the same way as a quadrupole magnet does. There are both regular and skew quadrupole moments of a beam slice, designated by Q_{1i} and Q_{2i} in Eq. (1), respectively, for the i^{th} slice. In the impulse approximation we take the transfer matrix to be, for the k^{th} slice,

$$\bar{}_{\text{quad}\cdot\text{wake}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ q_1 & 1 & q_2 & 0 \\ 0 & 0 & 1 & 0 \\ q_2 & 0 & -q_1 & 1 \end{bmatrix}$$

where

$$q_1 = \frac{r_o L}{\gamma_k} \sum_{i=1}^{k-1} N_i \, W_2(z_i - z_k) \, Q_{1i}$$

and

$$q_2 = \frac{r_o L}{\gamma_k} \sum_{i=1}^{k-1} N_i \, W_2(z_i - z_k) \, Q_{2i} \quad .$$

The quadupole moments of the i^{th} slice are given by

$$Q_{1i} = \Sigma_{11} - \Sigma_{33} + x^2{}_i - y^2{}_i$$

and

$$Q_{2i} = 2\,\Sigma_{13} + 2\,x_i y_i$$

where the Σ's are elements of the Σ-matrix for the i^{th} slice.

For the SLAC linac the wake function W_2 is taken to be[2]

$$W_2(\Delta z) = 0.38 \times 10^{10}\ \text{m}^{-5}\ (\Delta z/1.5\ \text{mm})^{0.693\ -\ 0.16(\Delta z/1\ \text{mm})} \quad .$$

FIGURE CAPTIONS

Fig. 1. The quadrupole wake function obtained from Ref. 2.

Fig. 2. The horizontal and vertical rms beam sizes at the end of tracking. These results assume no quadrupole wake effects.

Fig. 3. The horizontal and vertical rms beam sizes at end of tracking. Quadrupole wake is included. The intensity is 5×10^{10} particles in the beam pulse.

Fig. 4. The horizontal and vertical rms beam sizes at end of tracking. The intensity is 7×10^{10} particles per pulse. The very last slice will scrape against the accelerator pipe.

Fig. 5. The horizontal and vertical rms beam sizes at the end of tracking. The beam is assumed to be mismatched by 20% at injection. The last three slices scrape against the pipe.

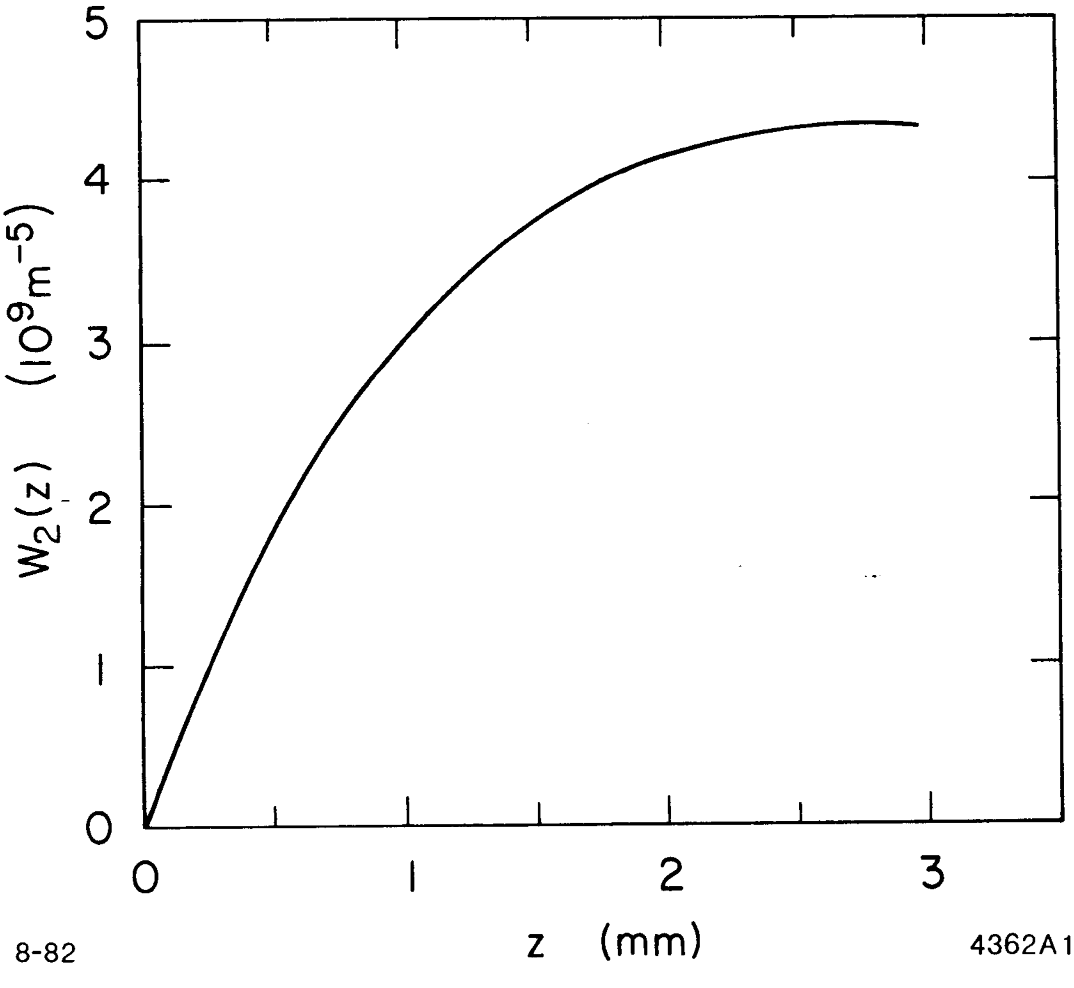
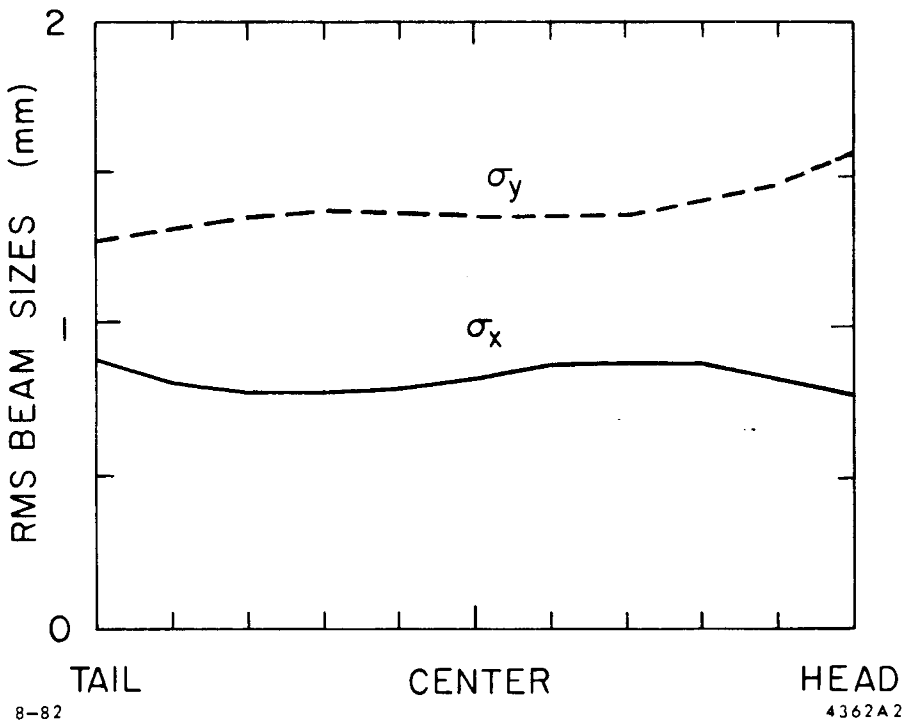


Fig. 1



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RMS BEAM SIZES (mm)
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Fig. 2

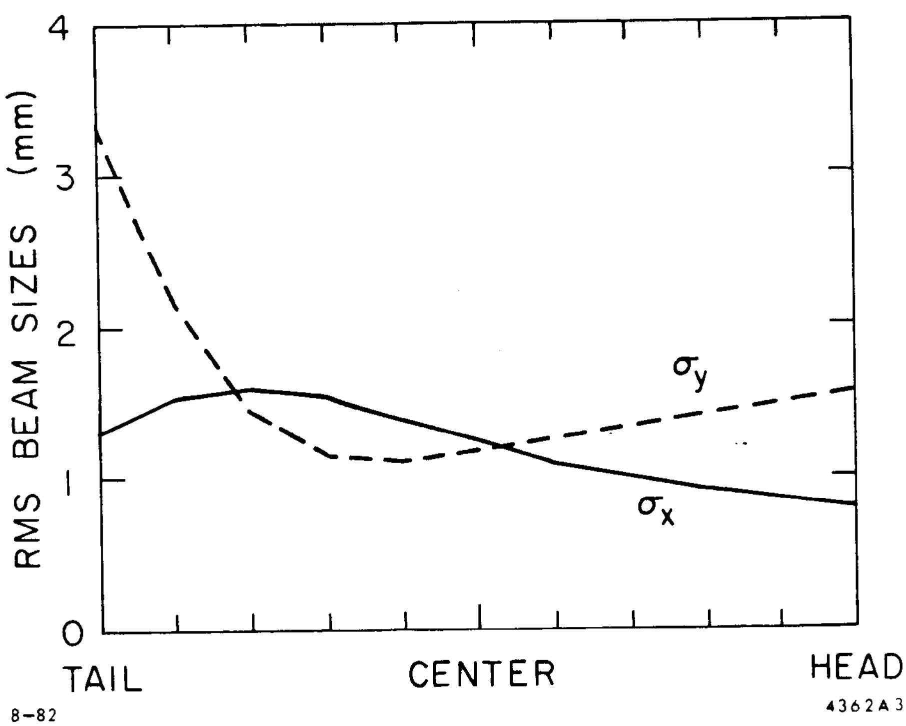
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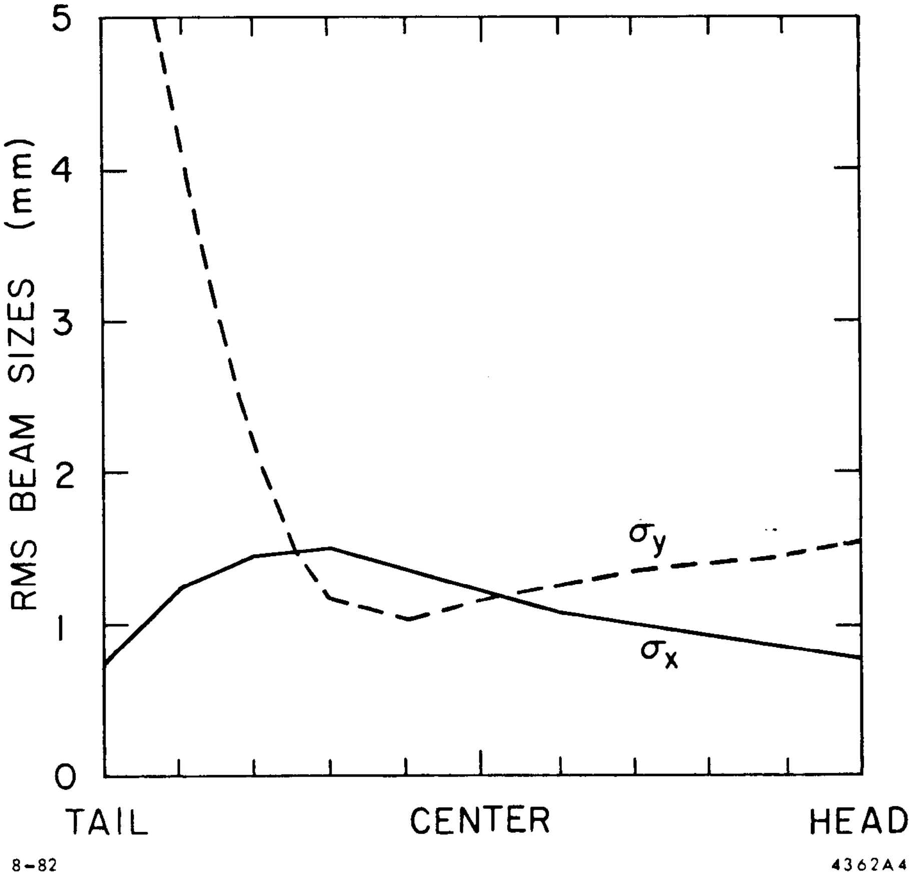
Fig. 3



RMS BEAM SIZES (mm)
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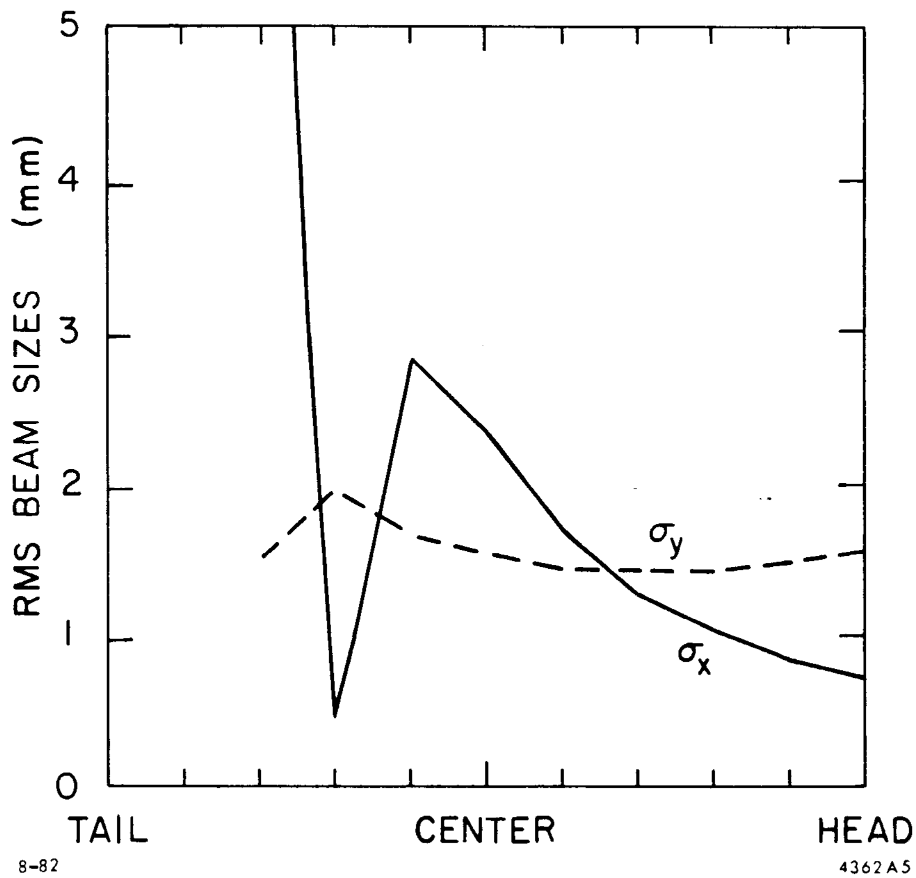
Fig. 4



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RMS BEAM SIZES (mm)
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Fig. 5