Y SCALING IN d, ^{3}He AND ^{4}He[*]

P. Bosted, R. G. Arnold, S. Rock and Z. Szalata
Department of Physics
The American University, Washington, D.C. 20016

and

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

Data for electron scattering from d, ^{3}He and ^{4}He in the region between the elastic and quasielastic peaks are found to be unified by a nuclear scaling function. A simple model is used to understand the general features of the data, to understand the approach to scaling with increasing momentum transfer, and to extract the deuteron wave function from the deuteron data.

Submitted to Physical Review Letters

In recent years it has been recognized[1] that cross sections for quasielastic electron scattering from nuclei at a variety of kinematical conditions can be unifed by a nuclear scaling function F(y). In the simplest picture, and using the impulse approximation, the variable y is the smallest possible Fermi momentum that can give a final state electron of the observed energy and angle. The function F(y) describes the shape of the quasielastic peak and is related to an integral over the single particle wave function.

The scaling prediction has been verified for $|y| < 0.3$ GeV/c for calcium[2] with incident energies of 0.2 to 0.5 GeV and angles of 60° to 160°, and for a much larger range of y using ^{3}He data at 8° and much higher energies.[3]

In this letter we use new data for deuterium which exhibit remarkable y scaling, and compare the results to those from ^{3}He and ^{4}He. We use a simple model to understand the approach to scaling and extract the deuteron wave function.

The relation between experimental inclusive electron cross sections $d^{2}\sigma / d\Omega / dE'$ and the corresponding scaling function value that we use is

$$F(y) = \left(d^{2}\sigma / d\Omega / dE'\right) (dE' / dy) \left[Z\sigma_{p}(E,\theta) + (A-Z)\sigma_{n}(E,\theta)\right] \qquad (1)$$

where y is the solution of

$$E + M_{A} = E' + \left(M + q^{2} + 2qy + y^{2}\right)^{\frac{1}{2}} + \left(M_{A-1}^{2} + y^{2}\right)^{\frac{1}{2}} \quad , \qquad (2)$$

where $\sigma_{p}(E,\theta)$ and $\sigma_{n}(E,\theta)$ are the on-shell proton and neutron elastic cross sections evaluated for stationary nucleons, E and E' are the incident and final electron energies, and q is the absolute value of the three-

momentum transfer. The variables A and Z are the nucleon and atomic numbers, and M_A, M and M_{A-1} are the masses of the target, ejected nucleon and spectator nucleons.

There now exists data for electron scattering from deuterium from SLAC experiments E101[4] and E133,[5] at scattering angles 8° and 10° respectively, with incident energies ranging from 6 to 21 GeV. There are ten data sets in E101, with average four-momentum transfer Q^2 values from 0.8 to 6.0 $(GeV/c)^2$, all of which span a final electron energy range very close to the deuteron elastic peak (the threshold inelastic region). These data probe the region of the largest possible negative y values for a given Q^2 and angle. The E133 data come in five sets with average Q^2 values from 2.5 to 10 $(GeV/c)^2$ and concentrate on the region around the quasielastic peak.

In Fig. 1. we show F(y) for deuterium obtained using Eq. 1 with on-shell elastic cross sections σ_p and σ_n taken from a fit to the world's data.[6] There was a substantial contribution to the high Q^2 data from pion production (up to 60% at the highest Q^2 for y < 0). This was subtracted by using smeared inelastic electron-proton data normalized to the deuteron data in the large positive y region. In order to minimize effects from final state interactions, a few data points very close to the deuteron elastic peak for which p* < 0.2 GeV/c were not used, where p* is the momentum of the final state nucleons in their center of mass. It can be seen that the remaining data cluster in a very narrow band.

Also shown in Fig. 1 are data from SLAC experiment E121[7] for ^{3}He and ^{4}He at 8°. There are threshold inelastic data sets with average Q^2 values from 0.8 to 5.0 $(GeV/c)^2$, with fourteen sets for ^{3}He and seven sets

for ^{4}He. There are six quasielastic sets for ^{3}He, with Q^2 from 0.15 to 4.0 $(GeV/c)^2$. A cut on p* was made, and the subtraction of real pion production was performed, in the same manner as for deuterium. It can be seen that again F(y) for ^{3}He and ^{4}He groups into narrow bands. Notice that the ^{3}He peak is wider than the d peak, reflecting the larger average Fermi momentum and stronger two-body correlations. At large negative y, the F(y) values for all three elements are about the same order of magnitude, but the slope increases slightly with A.

In order to understand the general features of y scaling, we derive a simple formula for F(y) for deuterium, starting with the nonrelativistic model of McGee as modified by Durand.[8] We have kept only the terms proportional to u^2 and w^2, where u and w are the s-state and d-state parts of the deuteron wave function. The inclusion of other terms, such as those proportional to uw or u' make less than a 6% difference over the kinematic range of our data, but may be more important at other energies and angles. We then obtain

$$d^2\sigma \,/\, d\Omega \,/\, dE' \;=\; \left[M^2 p^* E (\sigma_p + \sigma_n) \,/\, 8\pi E^* \right] \int \psi^2(k) d\Omega^* \qquad (3)$$

where $\psi^2 = u^2 + w^2$, k is the lab momentum of the spectator nucleon in the impulse approximation, and E* and dΩ* are the center-of-mass energy and solid angle of the final state nucleons. Using Eq. 1,

$$F(y) \;=\; (M^2 E \,/\, 2qE')(dE' \,/\, dy) \int_{|y|}^{|y+q|} \psi^2(k) k\, dk \Big/ \left(k^2 + M^2 \right)^{\frac{1}{2}} \,. \qquad (4)$$

It should be noted that the range of integration

$$|q+y| - |y| \;=\; (E + 2M - E')\, p^*/E^*$$

is proportional to p*. Since p* grows, at fixed y, with Q^2, and ψ^2 decreases quickly as a function of k, the upper limit of the integral can be replaced with infinity at large Q^2. Taking the limit $Q^2 \to \infty$,

$$F(y) \longrightarrow (M^2/2)\ (1 + y/E_y) \Big/ (2m - y - E_y)\ \int_{|y|}^{\infty} \psi^2(k)k\,dk \Big/ \left(k^2 + M^2\right)^{\frac{1}{2}} \qquad (5)$$

where $E_y = (y^2 + M^2)^{\frac{1}{2}}$. This equation now depends only on y.

Similar expressions have been derived for elements other than deuterium,[9] where the high Q^2 limits take the same form as Eq. 5. At finite Q^2 the wave function must be replaced by a spectral function $S(k, E_{rel})$, where E_{rel} is the relative energy of the spectator nucleons, and both k and E_{rel} must be integrated over.

Shown in Fig. 1 are the predictions for the asymptotic F(y) for deuterium using Eq. 5 with three different commonly used nonrelativistic deuteron wave functions.[10-12] The predictions agree very well with the data near the quasielastic peak, but are somewhat low around y = -0.3 GeV/c. This could be due to a lack of high momentum components in the wave functions used (see below), or to corrections to our simple model from relativistic effects or off-shell form factors. Other processes that could account for the discrepencies, such as final state interactions or meson exchange currents, would not in general be expected to y scale, but could still be of some importance. It is interesting to note that a calculation[3] for ^{3}He using the best available Faddeev wave functions falls below the data in a manner similar to our deuteron calculation. This discrepancy is attributed to a lack of high momentum components in the ^{3}He wave functions.

Although on a log scale the grouping of the experimental F(y) data is impressive, systematic differences between different data sets in a

given y region exist. This could be expected since much of the data was taken at Q^2 values for which Eq. 4 has not yet reached the asymptotic limit of Eq. 5. The gradual increase of the ratio of Eq. 4 to Eq. 5 towards 1.0 is depicted by the curves in Fig. 2, where it can be seen that the onset of scaling occurs at higher Q^2 as $|y|$ is increased. When evaluating Eq. 4 for ^{3}He and ^{4}He, the kinematics were changed, but no integral over the relative energy was performed. In addition, the same wave function[10] was used as for deuterium. Since the effects of the wave function largely cancel out in taking the ratio of Eq. 4 to Eq. 5, this approximation should be reasonably valid. The solid points in Fig. 2 represent the average of the experimental data points in the y intervals indicated. Each group has been normalized with a single multiplicitive constant to each of the theoretical curves. It can be seen that the shape of the experimentally observed approach to scaling is consistent with that expected from Eqs. 4 and 5.

This being the case, it is plausible to use Eq. 4 to explore the discrepency described earlier between the experimental and calculated F(y) values for deuterium, the only case for which we calculate absolute cross sections. If we make the asumption that the discrepency is not due to relativistic corrections, off-shell form factors, or other reaction mechanisms, but solely to a deficiency in high momentum components, we can use Eq. 4 to find a wave function that would better describe the experimental F(y) values. This is of particular interest since there seems to be a similar lack of high momentum components in ^{3}He. Using Eq. 4 and neglecting some small terms proportional to F(y), we find

$$\psi^2(k) = \left(2E_yE'q \,/\, EM^2y\right) (dy \,/\, dE') \left(dF(y) \,/\, dy\right) \qquad (6)$$

where $k = |y|$. The wave function values obtained from Eq. 6 are shown in Fig. 3. They were obtained by differentiating all pairs of adjacent F(y) points in each data set. Values at different Q^2 in a given k region were found to be consistent with each other, and were therefore averaged together to obtain smaller error bars. No systematic errors are included. Also shown in Fig. 3 are three commonly used potential models and data from two (e, e'p) experiments.[13,14] It can be seen that the results of our analysis contain more high momentum components in the 0.2 to 0.5 GeV/c region than any of the models, and disagree with one (e,e'p) experiment, but agree with the other.

As with the (e,e'p) experiments, a full study of all the processes that could complicate the interpretation of the inclusive electron measurements in terms of the plane wave impulse approximation and nonrelativistic wave functions must be undertaken before any definite conclusions can be drawn. The result remains, however, that any theory must be able to explain the impressive experimental observation that nucleon scaling unifies inclusive electron scattering data for the three lightest nuclei over a large kinematic region and for y values that imply very short distances within the nucleus.

REFERENCES

1. G. B. West, Phys. Rep. 18C, 263 (1975); and Proceedings of Int. School on Electron and Pion Interactions with Nuclei at Intermediate Energies, eds., W. Bertozzi, S. Costa, C. Schaerf, Ariccia (Rome), Italy (1979), p. 417.

2. P. D. Zimmerman et al., Phys. Rev. C19, 279 (1979).

3. I. Sick, D. Day, J. S. McCarthy, Phys. Rev. Lett. 45, 871 (1980).

4. W. Schütz et al., Phys. Rev. Lett. 38, 259 (1977).

5. S. Rock et al., SLAC-PUB-2949, submitted to Phys. Rev. Lett., with emphasis on the extraction of the neutron cross section.

6. For the proton we used the fit Dipole (Γ_ρ = 112 MeV) from F. Iachello, A. Jackson and A. Lande, Phys. Lett. 43B, 191 (1973). For the neutron, we assumed G_{En} = 0, and G_{Mn} follows the empirical dipole law, reduced slightly at high Q^2 to agree with Ref. 5.

7. D. Day et al., Phys. Rev. Lett. 43, 1143 (1979); S. Rock et al., SLAC-PUB-2838 (to be published in Phys. Rev. C). We have used a revised analysis of the data originally published by D. Day et al., to be submitted for publication.

8. A summary of, and references to, the work of McGee and Durand are given in W. Bartel et al., Nucl. Phys. B58, 429 (1973).

9. E. Pace and G. Salme, Inst. Naz. Fisica Nucl. IPFN-ISS 82/1, submitted to Phys. Lett.

10. M. Lacombe et al., Phys. Rev. C21, 861 (1980).

11. E. Loman and H. Feshbach, Ann. Phys. (N.Y.), 48, 94 (1968).

12. K. Holinde and R. Machleidt, Nucl. Phys. A256, 497 (1976).

13. M. Berheim et al., Nucl. Phys. A365, 349 (1981).

14. U. L. Agranovich et al., Yad. Phy. 25, 1123 (1977).

FIGURE CAPTIONS

Fig. 1. Experimental values for F(y) obtained from recent SLAC data for

d, ^{3}He and ^{4}He (Refs. 4, 5, 7). The data have been radiatively

corrected and pion production contributions have been subtracted.

The solid lines are calculations of F(y) for dueterium using

Eq. 5 with three different deuteron wave functions: Paris

(Ref. 10), Loman-Feshbach with 5.2 % d-state (Ref. 11), and

Holinde-Machleidt (Ref. 12).

Fig. 2. Comparison of experimental and theoretical approach to scaling.

The solid lines are the ratio of Eq. 4 to Eq. 5, using the Paris

wave function (Ref. 10). The points are the average experimental

data in the indicated y bins, normalized to the solid lines with

a single multiplicative constant for each set.

Fig. 3. Experimental deuteron wave functions obtained from Eq. 6

compared to three wave function models labeled as in Fig. 1.

Also shown are results from two d(e,e'p) experiments:
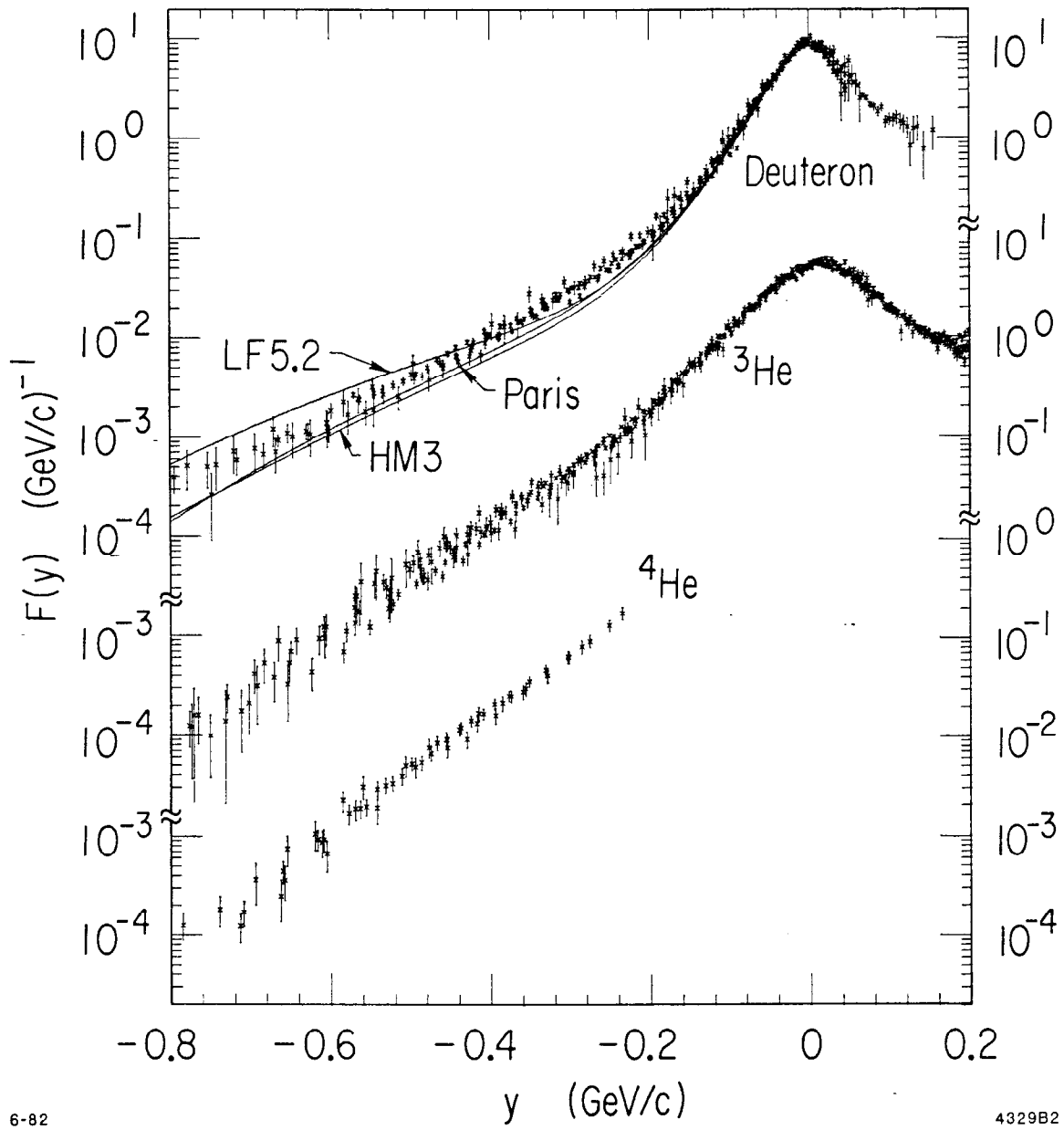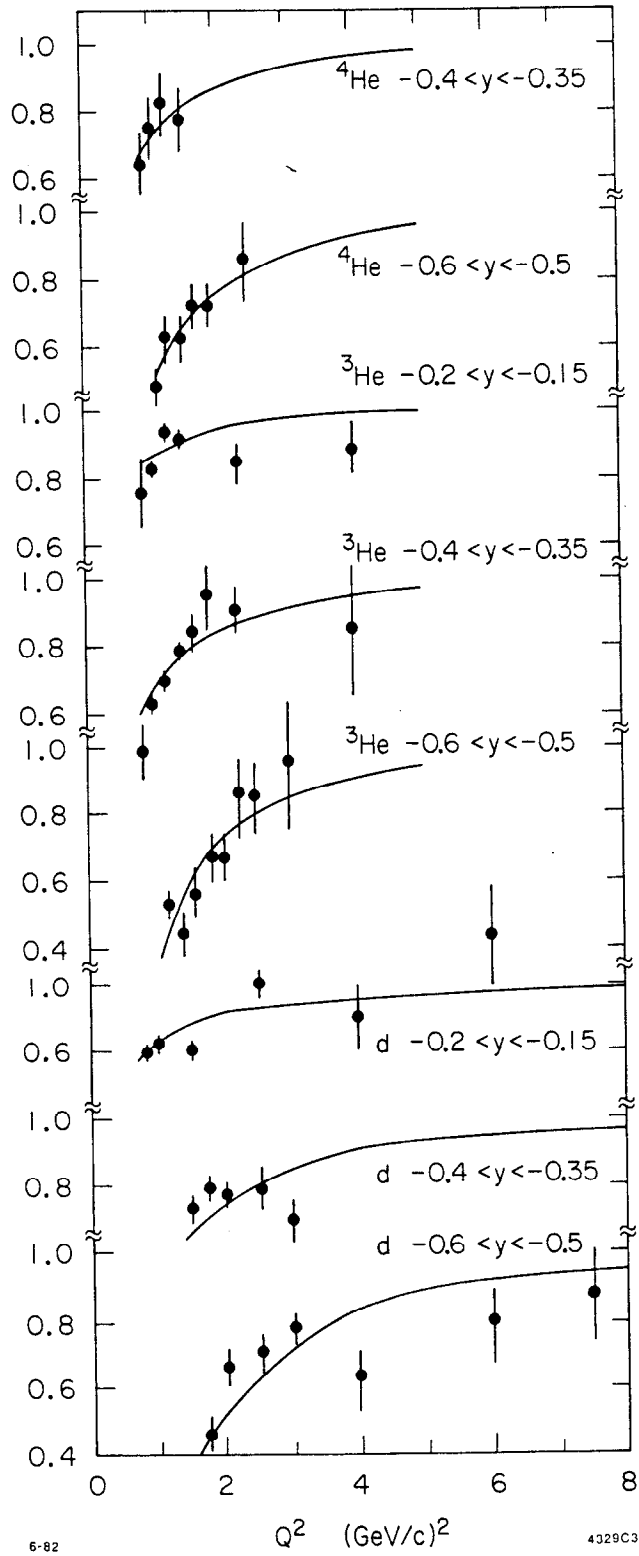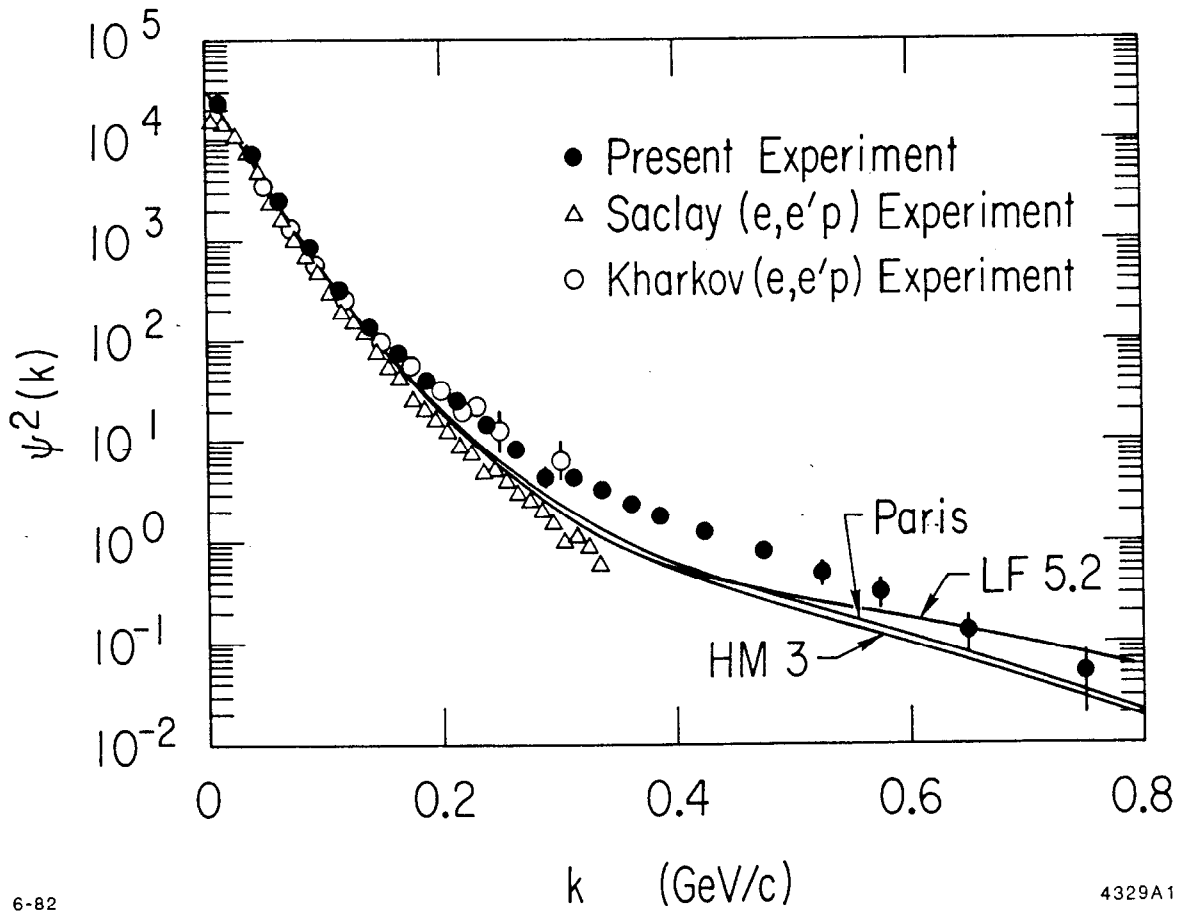
Saclay (Ref. 13) and Kharkov (Ref. 14).

Fig. 1

Fig. 2

Fig. 3