POWER SUPPRESSED CORRECTIONS TO DEEP INELASTIC SCATTERING*

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ABSTRACT

A direct calculation of the leading power law corrections to the
structure functions at large x is described. Three unexpected results
are that these terms have a form different from that usually assumed, the
leading correction to W_2 is negative, and R is large and slowly varying.

While QCD is widely accepted as the theory of the strong interactions, detailed comparison with experiment is far from perfect. The deep inelastic structure functions are a case in point. It now seems clear that the leading asymptotic terms predicted by QCD do not explain the low to moderate Q^2 sructure function data nor the ratio $R = \sigma_L/\sigma_T$ [1]. Higher twist power law corrections to asymptotic QCD predictions could be important for both; several sucessful phenomenological descriptions based on higher twist have been given [2]. In this letter we present a direct calculation of the leading power law corrections to W_2 and W_L at large x near 1 and large Q^2 [3]. We do not present a detailed fit to data but, rather, a qualitative picture of the results. Two surprises emerge: the leading higher twist contribution to νW_2 is negative and of a form different than that assumed in the literature; and $\nu W_L/\nu W_2$ is remarkably large.

Our analysis is based on the extension of the Brodsky-Lepage formalism [4] first employed by Berger and Brodsky [5] in their calculation of higher twist contributions for pion beams. In the $x \rightarrow 1$ limit the bound state quark struck by the virtual photon is required to carry most of the "+" - component of longitudinal momentum. In the case of the proton target, there are two spectator quarks which must transfer most of their longitudinal momentum to the struck quark. The simplest diagrams allowing this transfer are illustrated in Fig. 1(a), where we imagine attaching the virtual photon to the upper quark line as in Fig. 1(b). Simple kinematics forces the interior propagators off-shell by an amount roughly proportional to $(\ell_t^2+m^2)/(1-x)$ where ℓ_t is a typical transverse momentum and m the mass of the spectator quarks. The diagrams of Fig. 1

determine the average value of ℓ_t in terms of m. As $x \to 1$ the off-shell-ness becomes large and perturbative techniques become reliable. In this domain, proton Fock states with more than the three basic constituents have more suppressed (1-x) power law behavior and will not be considered here.

The method used is to first directly compute the matrix elements of the perturbative born graphs as a function of the longitudinal fractions of the initial quarks and of the final state spectator quark longitudinal and transverse momenta. The transverse momenta of the initial quarks do not enter into the leading large x behavior of the born graphs; they are integrated over in defining the evolved wave function for the initial state. The resulting amplitudes are then summed and squared and the final state integrals and helicity sums performed. The relevant born graphs are obtained from Fig. 1(a) by attaching the virtual photon to all places on the upper quark line [as for example in Fig. 1(b)], and by summing over the roles of the various quarks including different helicity states. Since the quark mass plays an important role it is not possible to ignore helicity flip terms in computing a given born graph.

The structure functions for deep inelastic scattering are defined by

$$
W_{\mu\nu} = - \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] W_1 + \left[p_\mu - q_\mu \frac{p \cdot q}{q^2} \right] \left[p_\nu - q_\nu \frac{p \cdot q}{q^2} \right] W_2 \quad (1)
$$

Using light-cone $(+,-,\perp)$ notation and a frame with $q^+ = 0, q^- = \nu/p^+$, we have

$$W_2 = \frac{W^{++}}{p^{+2}} \quad ; \quad W_L = Q^2 \, p^{+2} \frac{W^{--}}{4\nu^2}$$

$$R = \frac{\sigma_L}{\sigma_T} = \frac{r}{1-r} \quad ; \quad r = \frac{4x^2}{Q^2} \left[\frac{\nu W_L}{\nu W_2} \right] \qquad (2)$$

The born graphs were computed in both Feynman and axial gauges. The axial gauge forms, with a two loop momentum subtracted $\alpha(k^2)$ included for every gluon propagator of momentum k, were used in the actual integrations. This procedure may reduce the magnitude of higher order corrections [6]. The leading terms at large x and Q^2 are of the form

$$\overline{\nu W_i} = S_i(x)(1-x)^3 + T_i(x) \frac{(1-x)}{Q^2} + U_i(x) \frac{(1-x)^{-1}}{Q^4} + \ldots \qquad (3)$$

where the remaining x dependence in S, T and U is logarithmic in the variable $m^2/[\Lambda_{mom}^2(1-x)] = \xi$. There is additional logarithmic Q^2 behavior in all amplitudes except S_2 and T_2. There are also terms with higher (1-x) powers at every level which we do not examine here. We have retained in the matrix elements only those terms which contribute to the leading 1-x power forms listed above. The amplitudes S, T and U are integrals over terms proportional to (roughly) $\alpha^4[(m^2+\ell_t^2)/(1-x)]$. The 3-quark initial wave function employed is that determined by Brodsky et al [7] in obtaining an overall description of the proton/neutron form factors and $\psi \to 3g \to p\bar{p}$ decay. We have tested for sensitivity to this initial wave function by comparing to weak binding (where all incoming quarks carry 1/3 of the proton "+" - momentum) and to the fully evolved

form $(x_1 x_2 x_3$ - see [7] for notation). Essentially, only the overall normalization of S_2 changes and by less than a factor of 3. The ratios quoted in the following are insensitive to the initial wave function choice. Our explicit calculations will be for S_2, T_2 and the $Q^2 \to \infty$ limit of S_L.

In axial gauge the contributions to S_2 and T_2 arise only from the (+-+) and (++-) initial state helicity configurations. In addition only three diagrams contribute to each; for instance for the initial helicity configuration (+-+) these are the diagrams of Fig. 1(c). In principle there are two sources of the T_2 term: explicit non-scaling terms arising in the matrix elements; and kinematic corrections arising from energy conservation. In the proton target calculation only the latter terms occur, in contrast to the pion case where both types are present. We present our results in Fig. 2 where $m^4 S_2$ is plotted as a function of the variable ξ defined earlier. The values were computed by numerical integration over 5 parameters: the transverse momenta of the two spectator quarks and the fraction, z, of longitudinal momentum carried by one relative to their combined longitudinal momentum, $(1-x)p^+$. The plots are given in the range $\xi > 10$ for which the internal $\alpha(k^2)$'s are in their perturbative range. The results for T_2 are easily summarized as

$$T_2/S_2 \approx - 6m^2 \quad , \tag{4}$$

which should be compared to the result for the average transverse momentum squared of the struck quark

$$K_T^2 \approx 3m^2 \quad . \tag{5}$$

It is clear that the calculation is sensitive to the non-perturbative scale, m, of the quark mass, which can be thought of as being determined by K_T^2. Indeed our calculations are infrared divergent in the limit $m \rightarrow 0$. However, once m^2 is fixed all relative normalizations and x dependences are determined given a choice for Λ_{mom}.

The surprising feature of this calculation is the negative leading $x \rightarrow 1$ higher twist coefficient, Eq. (4). This result is easily understood. The leading higher twist correction comes, in the proton target case, entirely from the phase space restrictions implicit in energy conservation. These inevitably reduce the cross section at subasymptotic Q^2. It should be recalled that the phenomenological fits of Barnett et al. [1] and by Duke and Roberts [1], which incorporate higher twist terms, find a negative coefficient for their leading $1/Q^2$ correction to scaling. Taking into account the fact that they assumed a form $\sim (1-x)^2/Q^2$, i.e., more strongly damped in (1-x), a value of m in the range of 0.1 GeV to 0.2 GeV would yield a reasonable size for our T_2/S_2 and would, at the same time, give a sensible value for K_T^2. The normalization of νW_2 itself depends on the value of Λ_{mom}. We adopt the approximate determinations of $\Lambda_{mom} \sim 0.1$ GeV from the high Q^2 muon scattering experiments [8]. For these choices $\xi > 10$ requires $x > 0.9$. Thus, to decide whether our normalization for νW_2 is reasonable for the above range of m, we must extrapolate our curves for S_2 back to $x \sim 0.65$ where data exist; this extrapolation yields between 25% and 100% of the observed normalization. Note that S_2, which is the coefficient of the leading $(1-x)^3$ power law behavior in $m^4 \nu W_2$, increases substantially as x

decreases; this is due to the effects of the moving coupling constants in our calculation, which have arguments proportional to ξ.

For νW_L we have computed only the scaling term S_L since r has an explicit $1/Q^2$ factor. Many diagrams and all helicity configurations contribute to the large Q^2 limit of S_L. In Fig. 2 we plot $m^2 S_L / m^4 S_2$ and find

$$\frac{S_L}{S_2} \sim 3 \times 10^4 \; m^2 \tag{6}$$

which leads to

$$r \sim 10^5 \; \frac{m^2}{Q^2} \tag{7}$$

roughly independent of x as x → 1.

The large relative normalization for $m^2 S_L / m^4 S_2$ has five distinguishable sources:

(1) An explicit factor of $(1/2)^4$.

(2) A factor of approximately $(7)^2$ due to there being nearly seven times as many terms contributing to S_L as to S_2.

(3) The fact that half the terms contributing to S_L are weighted by initial state convolution integrals which are a factor of 5 larger than those appearing in S_2, yielding enhancement by a factor of approximately $9 = [(5/2)+(1/2)]^2$.

(4) Cancellations between terms contributing to S_2. [For example, the sign of the first two diagrams in Fig. 1(c) is opposite to that of the third.] Such cancellations do not occur in the expression for S_L, yielding a relative enhancement of S_L by a factor of ~16.

(5) A factor of more than 10 coming from the fact that the final state integral for S_L converges only logarithmically through the moving coupling constants whereas the integral for S_2 is power law convergent. The nearly divergent pieces arise from graphs in which the struck quark and one spectator quark interact in a net helicity 0 configuration via one gluon exchange at large relative momenta immediately prior to the photon attachment. Since large relative momenta correspond to small separation, these configurations can be interpreted as spin 0 diquark structure. Note that T_2, which has the same dimensions as S_L, is nonetheless power law convergent. For νW_2 it is only at the level of U_2 that one first encounters slow convergence.

These crude estimates, which one can approximately isolate on the computer, yield a factor in reasonable agreement with the full result, 30000.

As already mentioned S_L, T_L and $U_{2,L}$ must really be considered as asymptotic series in inverse $\ln Q^2$ powers. One expands the energy conservation delta function and computes the coefficients of the various $1/Q^2$ powers as integrals over transverse momenta of the spectator quarks. However portions of these integrals for the coefficients of $1/Q^2$ for νW_L and of $1/Q^6$ for νW_2 quadratically diverge. This divergence promotes these terms by a factor of

$$Q^2(1-x)^2 \left[\frac{\alpha(Q^2)}{\alpha[(m^2+\ell_t^2)/(1-x)]} \right]^2 \tag{8}$$

to the level of U_2 and S_L respectively. A similar promotion occurs for still higher terms. It is these corrections to S_L and the explicitly higher-Q^2-power terms in the νW_L series (e.g., T_L and U_L) that must restore positivity to σ_T at moderate Q^2. When such corrections in $1/\ell n\ Q^2$ are present they serve as a warning that the asymptotic result for the coefficient function in question (e.g., S_L or U_2) will only be dominant at very large Q^2. We have seen this explicitly in the case of S_L for which the asymptotic result violates positivity until Q^2 is larger than $10^5\ m^2$.

At this point it is useful to compare the present results for the proton to those for the pion. We first note that the coefficient T_2 for the pion target is positive due to the presence of matrix element as well as delta function $1/Q^2$ corrections. A second difference is that T_2 for the pion exhibits very slow convergence just like U_2 and S_L in the proton case. Berger and Brodsky [4] did not explicitly perform the necessary integrals involving moving coupling constants. They instead parameterize their result for the pion T_2 in terms of an $\langle K_T^2 \rangle$. Their preferred value of $\langle K_T^2 \rangle$ (of order 1 GeV^2) is not unreasonably large in light of the above discussion for S_L, item 5). Indeed it could easily be an underestimate. Of course, as in the case of the proton S_L, the pion T_2 is expected to have potentially large $1/\ell n\ Q^2$ corrections to its asymptotic limit.

If $m \approx \Lambda_{mom} \approx 0.1$ GeV, then our perturbative calculation should be accurate for $x > 0.9$; thus it is not possible to say for certain that those features which are calculated at large x should characterize the

data in the range x < 0.7. However, in the large x domain we have a reliable gauge invariant description of the proton wave function and deep inelastic amplitudes, including possibilities for interference and other subtle effects. In this domain there is a significant negative higher twist $1/Q^2$ term and a result for r, Eq. (7), which suggests that r cannot be reliably obtained as an asymptotic power law series until m^2/Q^2 is smaller than 10^{-5} or, for the approximate value of m^2 discussed ($m^2 \approx 0.1$ GeV^2), $Q^2 > 1000$ GeV^2. However, we regard this calculation as giving support for the magnitude and sign of the leading higher twist terms needed phenomenologically for νW_2 and as a warning that the standard QCD leading log component of R [9] may be much smaller than that coming from higher twist effects. The higher twist component will yield a roughly x independent result for R at large x with unexpectedly slow Q^2 dependence, not inconsistent with current experimental determinations. We eventually hope that data in the x near 1 domain will become available so that a direct test of this perturbative calculation and the underlying theoretical techniques can be made [10].
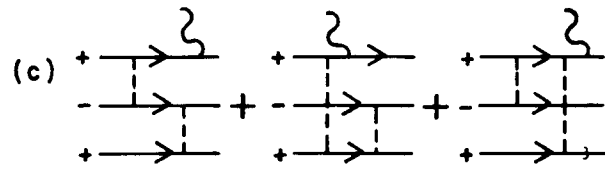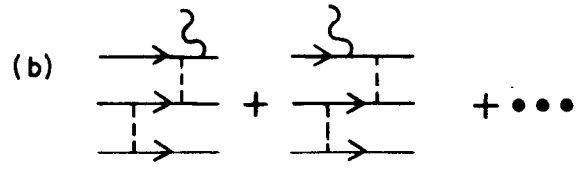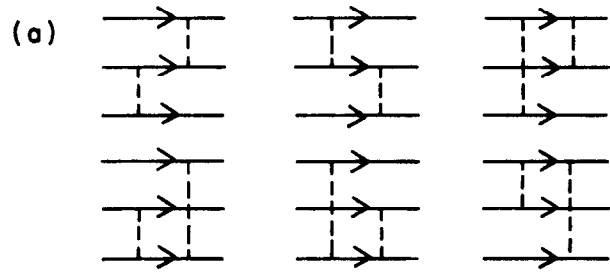
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[10] Using the form and sign of our results, a reasonable fit to the present data can be achieved (Michael Barnett, private communication).

FIGURE CAPTIONS

Fig. 1.   (a) Gluon Born graphs for the transition matrix elements;

(b) The two photon attachments arising from the first gluon

diagram in (a); and (c) The three diagrams in axial gauge that

contribute to S_2 and T_2 in the initial helicity state (+-+).

Fig. 2.   Numerical results at large Q^2 for W_2 and W_L as a function of

$\xi^{-1} = (1-x)\Lambda_{mom}^2/m^2$.

(a)

(b)

(c)

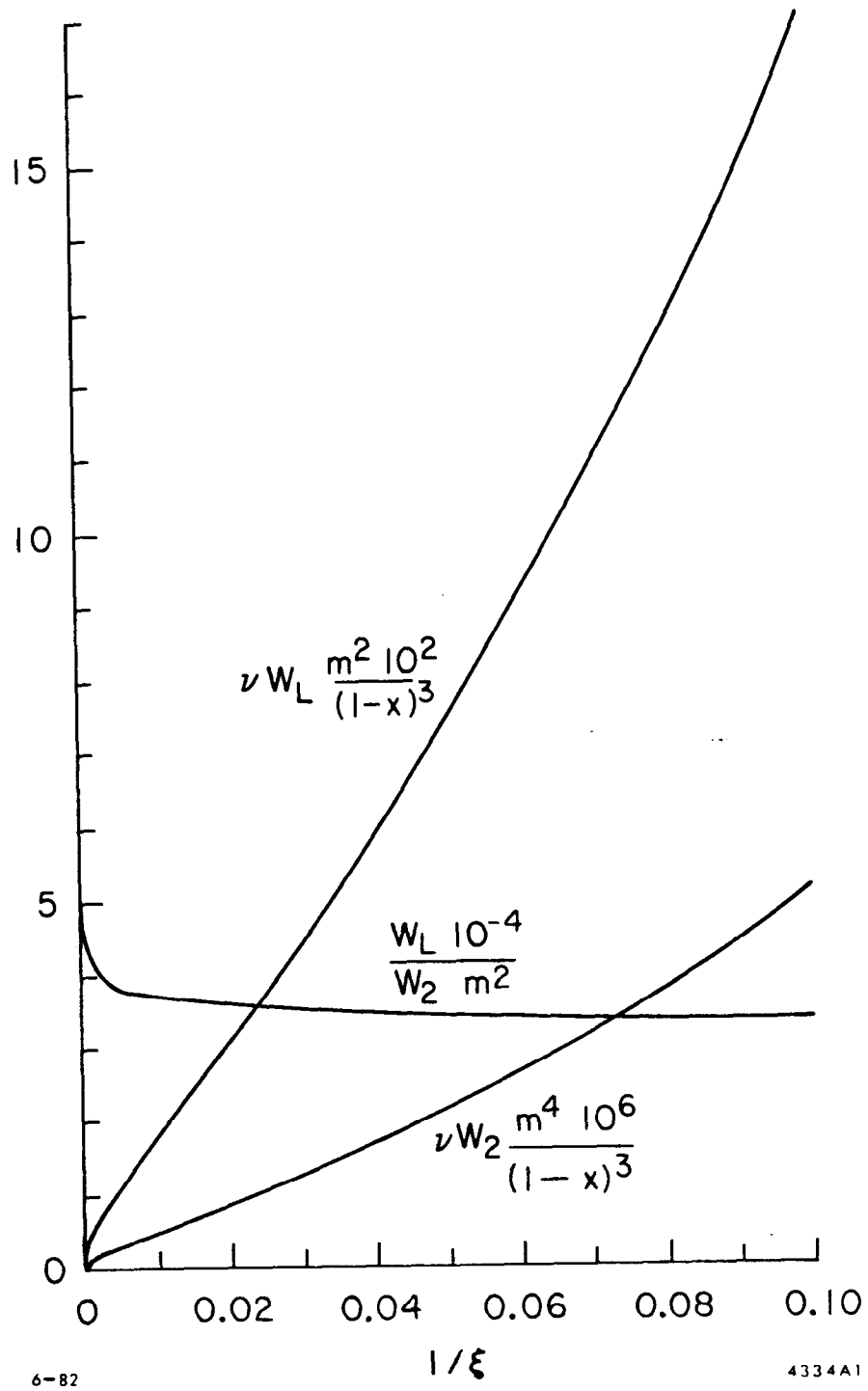6 − 82   4334A2

Fig. 1

Fig.2