SLAC-PUB-2935
June 1982
(T/E)

# SINGLE LOGARITHM EFFECTS IN ELECTRON-POSITRON ANNIHILATION*

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ABSTRACT

We show that the inclusion of the single logarithmic terms in the perturbative treatment of the energy-energy correlation at large collinearity angles makes the introduction of nonperturbative hadronization effects crucial to describe the experimental data at the present energies. It is unlikely that further perturbative corrections will change this result.

Submitted to Physics Letters B

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* Work supported in part by the Department of Energy, contract number DE-AC03-76SF00515.

† Work supported by the Japan Society for the Promotion of Science through the Joint Japan-U.S. Collaboration in High Energy Physics.

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A particularly interesting class of processes in perturbative Quantum Chromodynamics is represented by the semi-inclusive semi-hard processes characterized by the presence of two large but very different mass scales [1]. Examples are the cross section electron-positron → A + B + X with A and B hadrons at a relative transverse momentum $Q_T^2 \ll Q^2$ the total center-of-mass energy [$Q_T^2 = Q^2 \sin^2(\theta/2)$ collinearity angle $\theta \leq 180^o$] and the Drell-Yan cross section with the lepton pair transverse momentum $Q_T^2$ much smaller than the pair mass $M^2$, $Q_T^2 \ll M^2$. Both from theoretical and experimental point of view these processes represent an important step toward the understanding of the structure of the Quantum Chromodynamic theory in the perturbative phase. A common feature of these processes is the appearance of an effective form factor built up by the ressummation of large corrections arising from soft gluon effects [2,3] to all orders of the perturbative expansion.

We have recently carried a systematic study [4,5,6] of the role played by leading and subleading contributions to the effective quark form factor at two loop level. We have considered in particular the effect of the corrections in the case of the energy-energy correlation cross section [7] in electron-positron annihilation [8]. By using the unintegrated parton densities $D(Q^2,p_T,x)$ [9] one has*

$$\frac{1}{\sigma_{tot}} \frac{d\Sigma}{d^2Q_T} = \frac{1}{4} \sum_{AB} \int dx_A \, x_A \int dx_B \, x_B \sum_{q\bar{q}} \int d^2p_T^A \, d^2p_T^B \, d^2p_T^s$$

$$\times \, \delta^{(2)}\left(Q_T - \frac{p_T^A}{x_A} - \frac{p_T^B}{x_B} - p_T^s\right) D_q^A\left(Q^2,p_T^A,x_A\right) D_{\bar{q}}^B\left(Q^2,p_T^B,x_B\right) S\left(Q^2,p_T^s\right) \quad (1)$$

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* Our normalization here differs by a factor of two from the one in refs. [4,5,6].

as represented in fig. 1. By solving the equations governing the evolution of the parton densities and the expression for the central blob $S(Q,p_T^s)$ in impact parameter space one has [4],[6]

$$\frac{1}{\sigma_{tot}} \frac{d\Sigma}{d\cos\theta} = \frac{Q^2}{16\pi}\int d^2b_T\; e^{ib_TQ_T}\; e^{T(b)}$$

$$\times \sum_A \int dx_A\; x_A\; D_q^A\left(\frac{1}{b^2},b,x_A\right) \sum_B \int dx_B\; x_B\; D_{\bar{q}}^B\left(\frac{1}{b^2},b,x_B\right) \qquad (2)$$

where the relation $Q_T^2 = Q^2\sin^2\frac{\theta}{2}$ has been used. The $\exp\{T(b)\}$ is the effective quark form factor. We have [4,6]

$$T(b) = -\frac{C_F}{\pi}\int_{1/b^2}^{Q^2} \frac{dq^2}{q^2}\left[\alpha_s(q^2)\,\ell n\frac{Q^2}{q^2} + \frac{K}{2\pi}\,\alpha_s^2(q^2)\,\ell n\frac{Q^2}{q^2}\right.$$

$$\left. + 2\ell n\,\frac{e^{\gamma_E}}{2}\,\alpha_s(1/b^2) - \frac{3}{2}\,\alpha_s(q^2)\right] \qquad (3)$$

with $\gamma_E$ the Euler constant and $K = C_G[(67/18) - \pi^2/6] + N_FT_F(-10/9)$ the group factor due to the insertion of the two loop contributions. In eq. (3) the first term in the integrand is the leading double logarithmic contribution [2,3] which in impact parameter space corresponds to the sum of the infinite set of terms of the type $B(B/L)^n$ $(n \geq 1)$ with $B = \ell n\, Q^2b^2$ and $L = \ell n\, Q^2/\Lambda^2$ [3,4,6]. All the other terms take into account the entire class of the subleading single logarithmic terms $(B/L)^n$. The neglected terms are contributions of the type $\frac{1}{L}(B/L)^n$ and power corrections [4,6].†

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†The expression given for the form factor in ref. [10] agrees with our eq. (3). One difference is however the coefficient of the $\alpha_s^2(q^2)$ term, $\gamma_k^{(2)}$ in the notation of Collins and Soper, which has not yet been evaluated by those authors. Their expression reproduces our eq. (3) if $\gamma_k^{(2)} = C_FK$ with $C_1^2 = C_2^2 = e^{\gamma_E}/4\pi$.

In eq. (2) the residual densities $D(1/b^2,b,x)$, due to the choice of $1/b^2$ as a starting value of the perturbative evolution, can be expanded in terms of $\alpha_s(/b^2)$ [for $\alpha_s(1/b^2) << 1$] to give $D(1/b^2,b.x) = D(1/b^2,x) + \mathcal{O}[\alpha_s(1/b^2)]$. The distributions $D(1/b^2,x)$ satisfy the sum rule $\sum_A dx_A x_A D(1/b^2,x) = 1$. By substituting in eq. (2), one has for the cross section

$$\frac{1}{\sigma_{tot}} \frac{d\Sigma}{d\cos\theta} = \frac{Q^2}{8} \int b\, db\, J_0 [bQ \sin(\theta/2)]\, e^{T(b)} \quad . \qquad (4)$$

In this paper we analyze the recent experimental data for the energy-energy correlation cross section of the CELLO Collaboration at PETRA in DESY [11]. The comparison of eq. (4) with the data at 22 and 34 GeV for the center-of-mass energy Q is given by the dashed curve in figs. 2(a) and 2(b). We have chosen the value of $\Lambda_{\overline{MS}}$ = 100 MeV and $N_F = 5$.

By varying Λ no better agreement can be found [5]. As we have noticed before [5] the inclusion of the entire class of single logarithms $(B/L)^n$ in the effective form factor $\exp\{T(b)\}$ considerably affects the behavior of the cross section given by using only the double logarithmic approximation. The dotted line shown in figs. 2(a) and 2(b) correspond to the inclusion of the double logarithmic contributions [2,3] only [first term in the integrand of eq. (3)] in the form factor $\exp\{T(b)\}$ in eq. (4). The dotted curves are the best fits with Λ = 338 MeV at 22 GeV and Λ = 264 MeV at 34 GeV. In the double logarithmic approximation the pure perturbative expression is sufficient to give agreement with the data. The origin of the large difference between the double logarithmic and the single logarithmic approximations can be understood by looking at the various terms in Eq. (3). It is the last term, proportional to $\frac{3}{2}\,\alpha_s(q^2)$, which gives the largest correction. The effect of this term is in fact a milder suppression of the region of large values of b, $b \simeq \mathcal{O}(1)$ in $\exp\{T(b)\}$ as compared with the stronger suppression in the form factor if only the first (double logarithmic) term is included [5,6].‡ The second and third term on the contrary give only small corrections. In particular the term proportional to $\alpha_s^2(q^2)$ coming from the two loop insertion gives only a small effect. This is shown in fig. 3(a) where we have plotted the cross section eq. (4) with (solid line) and without (dashed line) the second order term in the form factor $\exp\{T(b)\}$. The dash-dotted line represents their ratio.

The less stronger suppression of the large values of b, $b \sim \mathcal{O}(1)$ in the form factor, enhancing the importance of nonperturbative effects $[\alpha_s(1/b^2) \simeq 1]$, is responsible for the disagreement with the data of the pure perturbative cross section eq. (4) [5]. In this region in fact nonperturbative effects, related to the mechanism of hadronization, play a relevant role. In order to take into account such effects we have used the fragmentation function of quarks into hadrons $\sum_A D_q^A(k_T,x_A) = \left[3(1-x_A)^2/x_A\right]\left[e^{-k_T^2/2\langle k_T\rangle^2}/2\pi\langle k_T\rangle^2\right]$ with $\langle k_T\rangle$ being the intrinsic transverse momentum of the partons with respect to the hadrons which satisfies the sum rule $\sum_A \int_0^1 dx_A\, x_A \int d^2k_T\, D_q^A(k_T,x_A) = 1$. By taking the Fourier transform into inpact parameter space $D_q^A(b,x_A) \equiv \int d^2k_T\, e^{-ik_T b/x_A} D_q^A(k_T,x_A)$ and substituting $D(b) = D_{q(\bar{q})}(b) = \sum_{A(B)} \int dx_{A(B)}\, x_{A(B)}\, D_{q(\bar{q})}^{A(B)}\left(b,x_{A(B)}\right)$ to the

‡ We do not agree with the objections made in ref. 12 about the resulting shape. These originate from the inappropriate procedure of exponentiating the cross section to obtain the effective form factor. See for example refs. [1,6,10].

corresponding expressions in eq. (2) we get§

$$\frac{1}{\sigma_{tot}} \frac{d\Sigma}{d\cos\theta} = \frac{Q^2}{8} \int b\, db\, J_0\left[bQ\sin(\theta/2)\right] e^{T(b)} \left[D(b)\right]^2 . \qquad (5)$$

The comparison with the data of eq. (5) is shown by the solid lines in figs. 2(a) and 2(b) where we have chosen $\Lambda$ to be $\Lambda_{\overline{MS}}$ = 100 MeV. $\langle k_T \rangle$ being the only additional parameter, the best fit gives for $\langle k_T \rangle$ the values $\langle k_T \rangle$ = 427 ± 12 MeV and $\langle k_T \rangle$ = 514 ± 19 MeV at 22 and 34 GeV, respectively. Also if we have found that a determination of $\Lambda$ is not possible since changes of $\Lambda$ can be compensated by changes of $\langle k_T \rangle$ we have observed that the values of the intrinsic transverse momentum are rather stable with respect to $\Lambda$ as long as $\Lambda \leq 150$ MeV. Changing $\Lambda$ from 150 MeV to 50 MeV we obtain that $\langle k_T \rangle$ changes from 422 ± 13 MeV to 436 ± 13 MeV at 22 GeV and from 509 ± 20 MeV to 516 ± 18 MeV at 34 GeV. We have also considered the inclusion of the pure perturbative contributions from ref. [7], by adding the cross section of ref. [7] to our eq. (5). This contribution can give small corrections at intermediate angles as shown in fig. 3(b). We find that the impact on our analysis is quite undramatic and does not affect our conclusions. We have in fact that for $\Lambda_{\overline{MS}}$ = 100 MeV the best fit gives the values of $\langle k_T \rangle$, $\langle k_T \rangle$ = 386 ± 10 MeV at 22 GeV and $\langle k_T \rangle$ = 459 ± 16 MeV at 34 GeV.

In conclusion, from our analysis emerges that nonperturbative effects still play a crucial role to describe energy-energy correlation at large collinearity angles at present energies, a feature already known for intermediate angles [14]. This results from the more complete

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§ For a different treatment of nonperturbative effects, see ref. [13].

perturbative treatment [eq. (3)] which includes the subleading single logarithmic contributions. We want to stress the fact that it is unlikely that additional perturbative corrections neglected in our approximation may have a sizeable numerical impact on our results. The neglected corrections can be considered as really perturbative ones [1,4,6]. For this reason, even if we expect a decreasing importance of nonperturbative effects at higher energies, only a more detailed analysis of nonperturbative contributions can give new insights.

We would like to thank M. Barnett for his help during the numerical analysis. We are grateful to S. Drell for the warm hospitality we have received at the SLAC Theory Group. One of us (L.T.) would like to thank also L. Susskind for having made his stay at Stanford possible.

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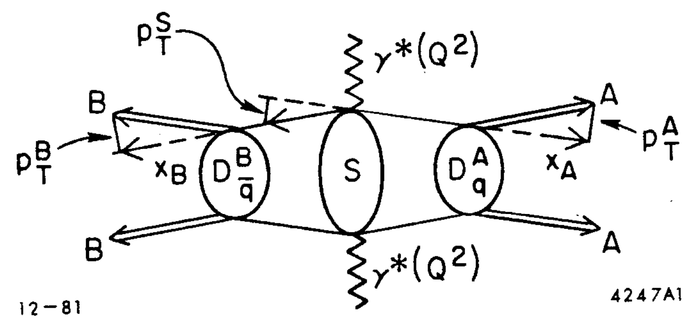
# FIGURE CAPTIONS

Fig. 1. The process $e^+e^- \to \gamma^* \to A + B + X$ with kinematics. Single (double) lines represent quarks and antiquarks (hadrons).

Fig. 2. $(1/\sigma_{tot})(d\Sigma/d\cos\theta)$ compared with the CELLO data at a) 22 GeV and b) 34 GeV, without fragmentation [eq. (4)] (dashed lines) $\Lambda_{\overline{MS}} = 0.1$; with fragmentation [eq. (5)] $\Lambda = 0.1$, $\langle k_T \rangle = 427$ MeV at 22 GeV and $\langle k_T \rangle = 514$ MeV at 34 GeV. The dotted lines represent the leading double logarithmic contribution of ref. [3] [first term in eq. (3) for T(b) in eq. (4)], $\Lambda = 0.1$.

Fig. 3. a) $(1/\sigma_{tot})(d\Sigma/d\cos\theta)$ eq. (4) with (solid line) and without (dashed line) the second order term $\alpha_s^2$ in T(b) and their ratio (dash-dotted line) for $\Lambda = 0.1$ at 34 GeV.
b) $(1/\sigma_{tot})(d\Sigma/d\cos\theta)$ eq. (5) with (solid line) and without (dashed line) the intermediate angles contribution of ref. [7] and their ratio (dash-dotted line), $\Lambda = 0.1$ at 34 GeV.



$p_T^S$
$\gamma^*(Q^2)$
B
A
$p_T^B$
$x_B$
$D_{\bar{q}}^B$
S
$D_q^A$
$x_A$
$p_T^A$
B
A
$\gamma^*(Q^2)$
12-81
4247A1


Fig. 1

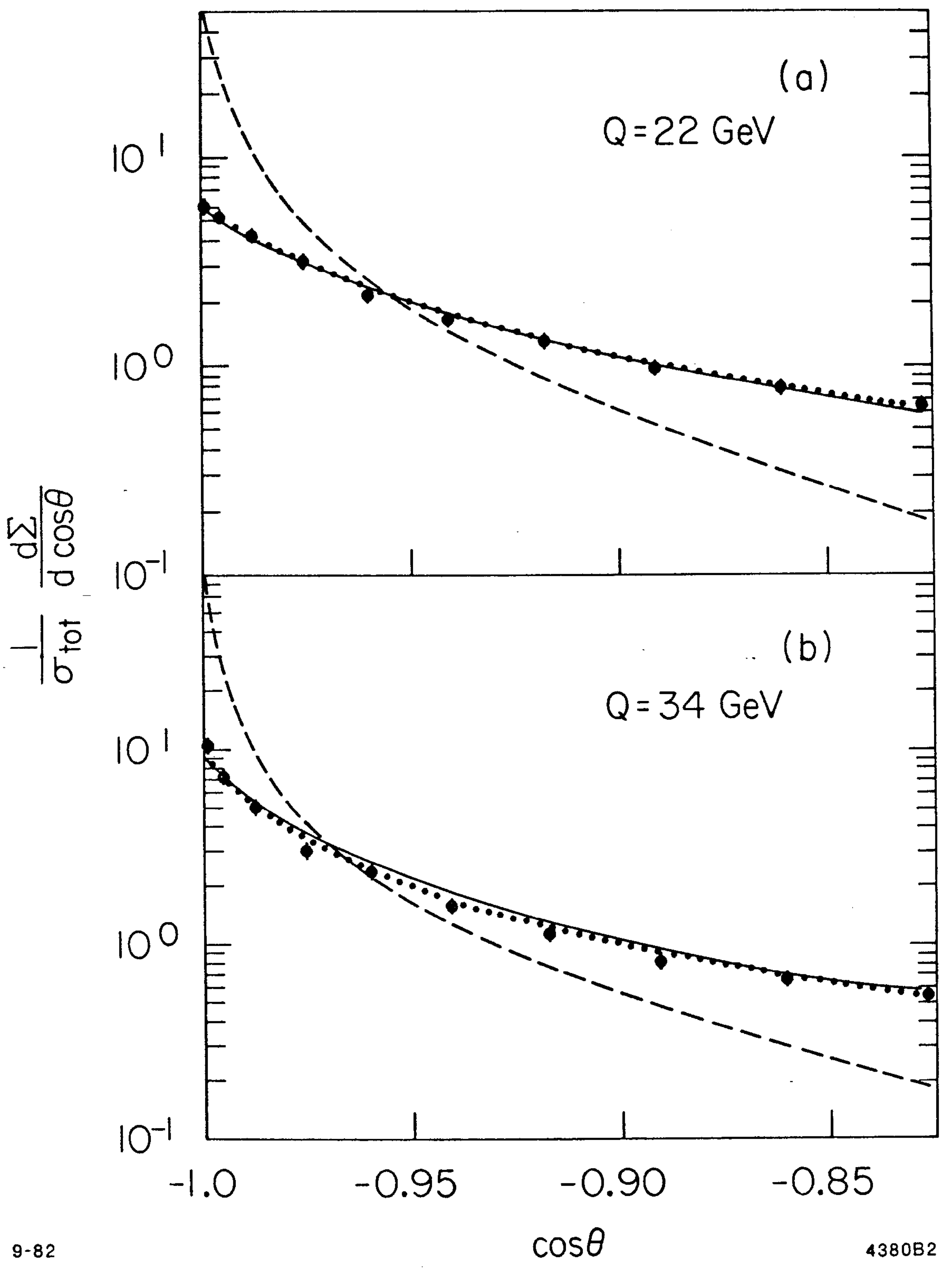



Fig. 2

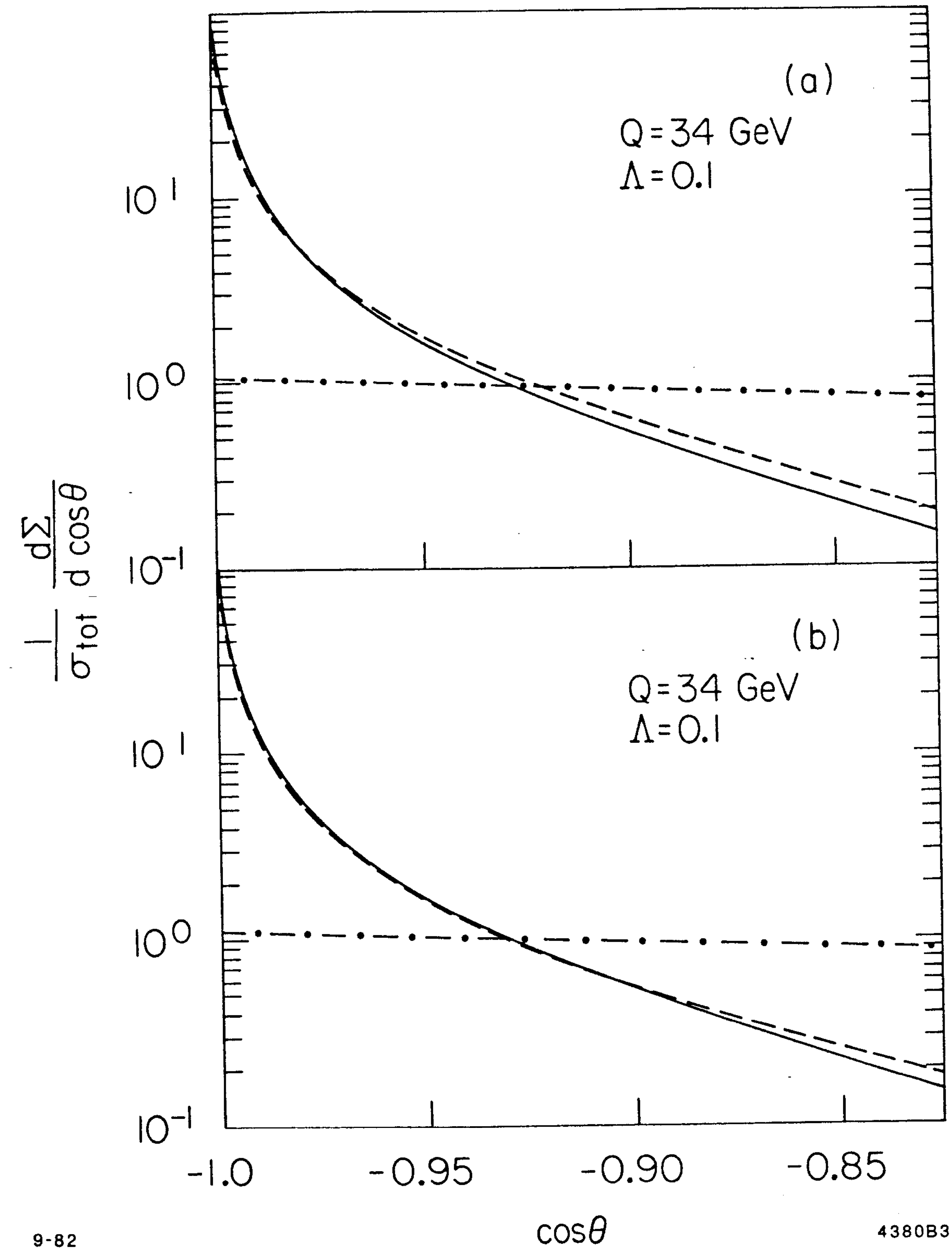

(a)
Q=34 GeV
Λ=0.1
(b)
Q=34 GeV
Λ=0.1
$10^1$
$10^0$
$10^{-1}$
$\frac{1}{\sigma_{tot}} \frac{d\Sigma}{d\cos\theta}$
-1.0
-0.95
-0.90
-0.85
cosθ
9-82
4380B3

Fig. 3