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GAUGE FERMION MASSES IN SUPERSYMMETRIC HIERARCHY MODELS*

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ABSTRACT

The gauge fermion mass is found to be generally nonzero in three scale supersymmetric hierarchy models. In a large class of models, the mass can be obtained from a simple current algebra argument for an anomalous R-symmetry.

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A recent paper[1] [to be referred to as (I)] with L. Susskind analyzes a class of supersymmetric models in which there are three scales: a heavy scale M, an intermediate scale μ, and a low energy scale μ^2/M. Supersymmetry is to be broken at the scale μ, but this breaking is supposed to reach the light fields only through the exchange of heavy fields. In (I), a very general and surprising cancellation was found in the one loop gaugino mass: it appears to vanish in all theories without heavy gauge fields. This paper reports on the one loop gaugino mass in theories which have heavy gauge fields. In general it is nonvanishing.

Consider first the simple inverted hierarchy model[2] studied in Ref. 3 and in (I). The Lagrangian is

$$\mathcal{L} = \frac{1}{4} (\hat{W}^{\alpha} \hat{W}_{\alpha})_F + \text{h.c.} + (\hat{A}^{+} e^{eV} \hat{A} + \hat{Y}^{+} e^{eV} \hat{Y} + \hat{Z}^{+} \hat{Z})_D$$

$$+ (g\hat{Z}\hat{A}\cdot\hat{A} - g\mu^2 \hat{Z} + \lambda\mu\, \hat{A}\cdot\hat{Y})_F + \text{h.c.} \qquad (1)$$

The gauge group is SO(3); $\hat{A}$ and $\hat{Y}$ are adjoint chiral superfields and $\hat{Z}$ is a singlet chiral superfield.[4] Additional light fields may be added, coupling to $\hat{A}$ and the gauge field. For $2g > \lambda$, SO(3) breaks at tree level to U(1) and the following fields have v.e.v.'s:

$$\langle A_3 \rangle = \mu \left(1 - \frac{\lambda^2}{2g^2} \right)^{1/2}$$

$$\langle Y_3 \rangle = \langle T \rangle \sin\theta \quad ; \quad \langle F_{Y_3} \rangle = f \sin\theta \qquad (2)$$

$$\langle Z \rangle = \langle T \rangle \cos\theta \quad ; \quad \langle F_Z \rangle = f \cos\theta$$

where $\hat{T} = \hat{Z}\cos\theta + \hat{Y}_3 \sin\theta$, $\cos\theta = \lambda(4g^2 - \lambda^2)^{-1/2}$, and $f = \lambda\mu^2(1 - \lambda^2/4g^2)^{1/2}$. Phases may be chosen to make all couplings and expectation values real. $\langle T\rangle$ is undetermined at tree level, but assumed to be $\sim M \gg \mu$. Because the F expectation values have the same ratio as the large scalar expectation values (and so can be put in the single superfield $\hat{T}$), the F expectation value couples only to massive fields and we have the sort of three scale model considered in (I).

S.S. breaking comes from effective operators coupling $\hat{X} = \hat{T} - \langle T\rangle$ to the light fields. For example, the lowest dimensional operator producing a mass for the U(1) gauge fermion is

$$k(\hat{X}\ \hat{W}_3{}^{\alpha}\ \hat{W}_{3\alpha})_F + k^*(\hat{X}^+\ \hat{W}_{3\dot{\alpha}}\ \hat{W}_3{}^{\dot{\alpha}})_{F*} \tag{3}$$

where $\hat{W}_3{}^{\dot{\alpha}}$ is the S.S. U(1) field strength and k is a coefficient of order $1/\langle T\rangle$. The graphs which contribute to k in an R_ξ-gauge[5] are shown in Figs. 1 and 2. They have been calculated and found to give

$$k = \left(\frac{e^2}{8\pi^2\langle T\rangle}\right)_{fig.1} - \left(\frac{e^2}{16\pi^2\langle T\rangle}\right)_{fig.2} = \frac{e^2}{16\pi^2\langle T\rangle} \tag{4}$$

and the cancellation found in (I), Appendix A, does not quite occur.

Rather than giving the explicit calculation, there is a simple current algebra argument which accounts for (4). The operator (3) contains several pieces, two of which are:

$$- F_X\ (k\ \lambda^{\alpha}\ \lambda_{\alpha} + k^*\ \lambda_{\dot{\alpha}}\ \lambda^{\dot{\alpha}}) \tag{5a}$$

$$- \frac{1}{\sqrt{2}}\ B_X\ [\mathrm{Re}(k)\ F\tilde{F} - \mathrm{Im}(k)\ F^2] \tag{5b}$$

where $F_{\mu\nu}$ and λ^{α} are the U(1) field strength and gaugino, and

$B_X = \sqrt{2}\,\mathrm{Im}(X)$ is a real (pseudo) scalar field. Equation (5a) gives rise to the gauge fermion mass $m = 2f|k| \sim \mu^2/\langle T\rangle$. Equation (5b) represents a coupling between B_X and two photons. Both effects are governed by the same coefficient k.

The model (1) has an R-symmetry[4] - $\hat{Z}$ and $\hat{Y}$ have R = 1, and $\hat{A}$ and $\hat{V}$ have R = 0. In terms of component fields:

$$\begin{aligned} R(Z,\ \psi_Z,\ F_Z) &= (2,\ 1,\ 0) \\ R(Y,\ \psi_Y,\ F_Y) &= (2,\ 1,\ 0) \\ R(A,\ \psi_A,\ F_A) &= (0,\ -1,\ -2) \\ R(F_{\mu\nu},\ \lambda,\ D) &= (0,\ 1,\ 0) \end{aligned} \qquad (6)$$

The large scalar v.e.v. is seen to break the R-symmetry, and the associated Goldstone boson is $\sqrt{2}\,\mathrm{Im}(T) = \sqrt{2}\,\mathrm{Im}(X) = B_X$, with $f_{B_X} = 2\sqrt{2}\,\langle T\rangle$. Thus the amplitude $B_X \to \gamma\gamma$ is fixed in terms of the anomaly in the R-symmetry. The argument is the standard one for $\pi^0 \to \gamma\gamma$.[6] It is convenient to review here the form of the argument for exact chiral symmetry. Consider the matrix element for the R-current to connect the vacuum to two photons:

$$\langle 0|\,j_R^{\mu}\,|\alpha,p;\beta,q\rangle = \frac{i f_{B_X}(p+q)^{\mu}}{(p+q)^2}\,\langle B_X|\alpha,p;\beta,q\rangle + \Gamma^{\mu\alpha\beta}(p,q) \qquad (7)$$

The first piece on the R.H.S. is the contribution of the B_X pole; suppose that the remainder $\Gamma^{\mu\alpha\beta}$ is analytic at p = q = 0.[7] Then Bose symmetry and gauge invariance for the photons forces the term in $\Gamma^{\mu\alpha\beta}$ which is linear in p and q to vanish and $\Gamma^{\mu\alpha\beta}$ is at least cubic in the momenta (the standard argument[6] uses parity, but this is not necessary). Contracting with $(p+q)^\mu$, we then have

$$f_{B_X}\langle B_X|\alpha,p;\beta,q\rangle = \langle 0|\partial_\mu j_R{}^\mu|\alpha,p;\beta,q\rangle + O(p^X q^{4-X})$$

$$= \frac{e^2}{16\pi^2}\, A_R\, \langle 0|F\tilde{F}|\alpha,p;\beta,q\rangle + O(p^X q^{4-X}) \qquad (8)$$

Here A_R is the coefficient of the Adler anomaly,[8]

$$A_R = \sum_i R_i\, q_i{}^2 = \sum_r R_r\, T(r) \qquad (9)$$

where the sums are over all left-handed fields; the second sum is written in terms of representations of the unified group.

For the model (1), $\Gamma^{\mu\alpha\beta}$ is certainly analytic at one loop, as all fields with U(1) charge are massive. A_R is

$$T(adj)(R_{\psi_Y} + R_{\psi_B} + R_{\lambda_V}) = T(adj) = C_2(G) = 2 \qquad (10)$$

We may now compare the order p^1q^1 term in $\langle B_X|\alpha,p;\beta,q\rangle$ from (5b) and (8) to get

$$\mathrm{Re}(k) = \frac{\sqrt{2}}{f_{B_X}} \cdot \frac{e^2 A_R}{16\pi^2} = \frac{e^2\, C_A}{32\pi^2\langle T\rangle}$$

$$\mathrm{Im}(k) = 0 \qquad (11)$$

This is the same as given in (4); the contributions of Figs. 1 and 2 are exactly proportional to the total anomalies of the fields circulating, since $\hat{V}$ includes the $\hat{Y}$ degrees of freedom. The gaugino mass is

$$m_\lambda = \frac{e^2 f C_A}{16\pi^2\langle T\rangle} \tag{12}$$

This argument is readily applied to other models. Consider a simple $SU(5) \to SU(3) \times SU(2) \times U(1)$ example.[2] The superpotential is

$$\lambda \, \mathrm{tr}(\hat{A}^2\hat{Y}) + g\hat{Z}\, \mathrm{tr}(\hat{A}^2) - g\,\mu^2\,\hat{Z} \tag{13}$$

with $\hat{Z}$ an SU(5) singlet and $\hat{Y}$ and $\hat{B}$ in the adjoint representation. Again there is an R-symmetry: $R_Z = R_Y = 1$, $R_A = 0$. The large scalar v.e.v. breaks it and the Goldstone boson is a scalar in the Goldstino superfield. The anomalies of ψ_A and ψ_Y cancel, and only the anomaly from the gauge fermions remains. There is one new feature: for the non-Abelian [SU(2) and SU(3)] subgroups, the remainder $\Gamma^{\mu\alpha\beta}(p,q)$ is <u>not</u> analytic at zero momentum. It has a singularity from the contribution of the light gauge fermions to the anomaly (we are working only to first order in the S.S. breaking, so the gauge fields in loops are massless). Allowing for this, the gaugino mass for an unbroken subgroup H of the unified group G is

$$m_\lambda(H) = \frac{e^2 f}{16\pi^2\langle T\rangle}\left[C_A(G) - C_A(H)\right] \tag{14}$$

Equation (14) is valid in most examples of the inverted hierarchy model. The vertices which enter at one loop satisfy an R-symmetry even when the full Lagrangian does not, and the undetermined scalar field has $R = 2$. (The one exception occurs when there are multiple undetermined fields, and another $R \neq 0$ field has a large v.e.v. also.) The anomalies cancel between the Y-type fermions, with R = 1, and the A-type fermions, with R = -1, leaving only the gauge fermion anomaly.

For the model of Eq. (1) with $\lambda > 2g$, the large v.e.v. is pure Z, leaving SO(3) unbroken and all gauge fields massless. Equation (14) now reads $m_\lambda = 0$. This is the cancellation found in (I), Appendix A, for a single $\hat{B}$ and a single $\hat{C}$ field ($\hat{A}$ and $\hat{Y}$ here, respectively). It is still a mystery that the cancellation found in (I) was so universal, as the general model does not appear to have a spontaneously broken symmetry.

Before trying to apply Eq. (14), we should consider the scale at which the gauge coupling is to be evaluated, as well as other large renormalization effects. Equation (5) (or the Ward identities of broken S.S.) relates the gluino chirality-flip amplitude at low energy (which is the gluino mass) to the $B_X \to \gamma\gamma$ amplitude at that energy. The operators and couplings in Eq. (8) should then be normalized at low energy, so that $F\tilde{F}$ creates the $\gamma\gamma$ state with approximately unit amplitude. The one loop anomalous dimension of j_R^μ vanishes, as its only divergence is anomalous - there is then no large effect from renormalizing j_R^μ and F_{B_X} at low energy. The conclusion is that all large renormalization effects are taken into account by evaluating e^2 at low energy (say 100 GeV):

$$m_\lambda(H) = \frac{\alpha_H(100\ \mathrm{GeV})f}{4\pi\ \langle T\rangle}\,[C_A(G) - C_A(H)] \qquad (14')$$

A more standard analysis of the gauge fermion mass renormalization is given in the Appendix, with the same result.

To discuss the magnitude of the gauge fermion masses, it is convenient to trade f for the gravitino mass $m_G = (4\pi/3)^{1/2} f/M_{PL}$, with M_{PL} the Planck mass:[9]

$$m_\lambda(H) = \left[\frac{3}{64\pi^3} \right]^{1/2} m_G \, \alpha_H(100 \text{ GeV}) \frac{M_{PL}}{\langle T \rangle} [C_A(G) - C_A(H)] \qquad (15)$$

For $\langle T \rangle \sim 3\times10^{16}$ GeV, a typical unified scale in minimal S.S. models,[10] and for G = SU(5), $m_{bino} \sim 1.5 m_G$, $m_{wino} \sim 2m_G$, and $m_{gluino} \sim 5m_G$. Recalling the cosmological bound $m_G \gtrsim$ several TeV,[11] these masses are quite large. If $\langle T \rangle = M_{PL}$, $m_{bino} \sim 3\times10^{-3}\, m_G$, $m_{wino} \sim 5\times10^{-3}\, m_G$, and $m_{gluino} \sim 10^{-2}\, m_G$. The light adjoint matter fields in inverted hierarchy models tend to raise the unification scale relative to minimal models, so the gauge fermion mass could be anywhere from a few GeV on up.

For broken gauge symmetries these masses must be added to the rest of the fermion mass matrix. For example, the matrix for the charge wino and higgsino is

$$\begin{bmatrix} m_\lambda[SU(2)] & M_W \\ M_W & m_H \end{bmatrix} \qquad (16)$$

Here m_H is the Majorana mass for the higgsino. There is no effective S.S. breaking term of this form,[1] so it must originate from a supersymmetric Higgs mass term.[12] When $m_\lambda[SU(2)]$ is large, the smaller eigenvalue of (16) is controlled by the unknown mass m_H.

Relations similar to (14) and (15) will hold in any three scale hierarchy model in which one of the partners of the Goldstino is a

Goldstone boson. In fact, these relations are characteristic of the one loop heavy gauge boson contribution, and are approximately true in any three scale hierarchy model, if one replaces ⟨T⟩ with $M_G/e_{GUM}\sin\theta$. Here M_G is the heavy gauge boson mass and $e_{GUM} \sim 1$; $\sin\theta$ is the proportion of G-nonsinglet in the Goldstino superfield $\hat{X}$ ($\hat{X}$ is generally a mixture of G-singlet and G-nonsinglet). For $\sin\theta = O(1)$ and $M_G \sim 10^{16}$-10^{17} GeV, the gauge fermion masses are again in the TeV range. This can be reduced by raising the unification scale, or by making $\hat{X}$ predominantly G-singlet.[13]

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APPENDIX

ONE LOOP RENORMALIZATION OF THE GAUGE FERMION MASS

For the U(1) subgroups of both models (1) and (13), there are no light charged fields and no renormalization. For the SU(2) and SU(3) subgroups in (13) there is one subtlety. The graphs contributing to the gaugino masses involve loops of heavy gauge bosons, with [SU(3),SU(2)] content (3,2), heavy (3,2) $\hat{A}$ fields, heavy (8,1) and (1,3) $\hat{A}$ fields, and light (mass μ^2/M) (8,1) and (1,3) $\hat{Y}$ fields. The total contribution of the $\hat{Y}$ (8,1) fields vanishes, as does that of the (1,3) fields, but only as a cancellation between a "pointlike" mass from the heavy loop and an explicit light field loop. The latter contribution does not violate the point of view in (I), that all S.S. breakings in the low energy theory should be from pointlike effective interactions. Rather, there is a splitting of the light $\hat{Y}$ superfield from $(\hat{X}\ \hat{Y}\ \hat{Y})_F \to (\hat{Y}\ \hat{Y})_A$ (see I, discussion of Fig. 16). The light loop then feeds this to the gluino mass.

[An aside: Since momenta of order μ^2/M dominate the light loop, multiple insertions of $(\hat{Y}\ \hat{Y})_A$ carry no large penalty. This is an expansion in powers of $(\Delta m_Y/m_Y)^2$ (odd powers of the splitting do not contribute). For SU(2) this is $(1/9)^2$ and for SU(3) it is $(3/8)^2$, or a 10-20% effect. This is the size of the error made by working to first order in the S.S. breaking.]

The effective pointlike contribution is proportional to the total anomaly of the heavy fields:

$$m_\lambda(\text{heavy}) = \frac{e^2 f}{16\pi^2 \langle T \rangle} [C_A(G) - 2C_A(H)] \qquad (A.1)$$

It is renormalized by using $e^2(M)$ in (A.1) and multiplying by[14]

$$\left(\frac{e^2{}_H(\mu^2/M)}{e^2(M)} \right)^{\gamma_{\gamma\gamma} e/2\beta_H} \qquad (A.2)$$

Here $\gamma_{\lambda\lambda}$ is the one loop anomalous dimension of the mass operator $\lambda^\alpha\lambda_\alpha$, β_H is the one loop β function for the subgroup H, and $e_H{}^2$ is the running coupling. One finds

$$\gamma_{\lambda\lambda} = \frac{e^2}{16\pi^2} [-6C_2(H) + 2 \sum_i T(r_i)]$$

$$\beta_H = \frac{e^3}{16\pi^2} [-3C_2(H) + \sum_i T(r_i)] \qquad (A.3)$$

where the possibility of arbitrary additional matter fields has been included in the sum. Comparing (A.1), (A.2) and (A.3), one sees that this part of the mass is renormalized simply by evaluating e^2 in (A.1) at the low energy scale.

To renormalize the remaining contribution to the gaugino mass,

$$m_\lambda(\text{light}) = \frac{e^2 f}{16\pi^2 \langle T \rangle} C_A(H) \qquad (A.4)$$

we must first run the various couplings down to μ^2/M. The relevant terms are $m_1(\hat{Y}\ \hat{Y})_F$, $m_2(\hat{Y}\ \hat{Y})_A$, and the gauge coupling $e_H{}^2$. The terms $(\hat{Y}^+\ \hat{Y})_F$ and $(\hat{Y}^+\ \hat{Y})_A$ do not mix in at this order. The gaugino mass depends on these parameters in the combination

$$m_\lambda(H) \sim e_H{}^2 \frac{m_2}{m_1} \qquad \text{(A.5)}$$

The nonrenormalization theorem for F-terms[15] implies that $(\hat{Y}\,\hat{Y})_F$ and $(\hat{Y}\,\hat{Y})_A \sim (\hat{Y}\,\hat{Y}\,\hat{X})_F$ are affected only by wave function renormalization. $\hat{X}$ is not renormalized below M, as it couples only to heavy fields, so the ratio m_2/m_1 is controlled by $2\gamma_Y - 2\gamma_Y = 0$. Then from (A.5), this part of the mass is also renormalized simply by using the running coupling.

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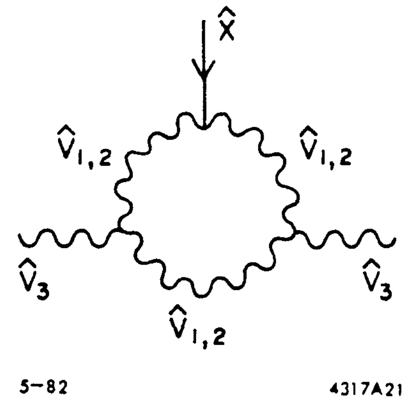
FIGURE CAPTIONS

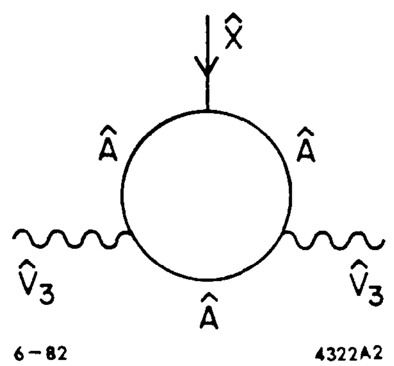
Fig. 1. Heavy gauge loop contributing to $[\hat{X}\,\hat{W}^{\alpha}\,\hat{W}_{\alpha}]_F$. The $\hat{X}\hat{V}\hat{V}$ vertex arises from the $\hat{Y}$ kinetic term.

Fig. 2. Heavy $\hat{A}$ loop contributing to $[\hat{X}\,\hat{W}^{\alpha}\,\hat{W}_{\alpha}]_F$.



$\hat{X}$
$\hat{V}_{1,2}$
$\hat{V}_{1,2}$
$\hat{V}_3$
$\hat{V}_3$
$\hat{V}_{1,2}$
5-82
4317A21

Fig. 1



$\hat{X}$
$\hat{A}$
$\hat{A}$
$\hat{V}_3$
$\hat{V}_3$
$\hat{A}$
6-82
4322A2

Fig. 2