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THE TRITON AS A THREE NUCLEON-ONE MESON PROBLEM*

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ABSTRACT

The standard method for basing nuclear physics on elementary particle physics is to first derive a "potential" and then use this interaction in the nonrelativistic Schroedinger equation for the nucleonic degrees of freedom. Unfortunately there has never been a consensus as to how to perform the first step. Currently we have dispersion-theoretic models and one-boson-exchange models which contain much the same physics, but which differ in detail; more "modern" approaches start from quark bags, but again there is no consensus as to whether the bag should be large or small. In this paper we offer an alternative approach in which the mesonic and nucleonic degrees of freedom are put on the same footing.

The basic relativistic three particle Faddeev equations for separable two particle amplitudes were given long ago by Lovelace, and more recently by Brayshaw. Lindesay has shown that if we specialize the input to a two particle amplitude containing only the term generated by two particles m_i, m_j forming a bound state of mass μ_{ij} - a relativistic generalization of the scattering length model - that the resulting integral equations can be easily solved numerically and give unitary and covariant results. In particular they reproduce the logarithmic accumulation of nonrelativistic three particle bound states predicted by Efimov when the scattering length goes to infinity. If this model is still further specialized by assuming the system to consist of two scalar particles of mass m_1 and m_2 and a meson of mass m_Q with no scattering between the particles and bound state of mass $\mu_{iQ} \equiv m_i$ physically indistinguishable from the particles which never come apart (i.e., there is no $m_i + m_Q$ scattering for physical momenta) except at short distance, one can calculate a fully covariant and unitary off-shell amplitude for the scattering of m_1 and m_2 caused by repeated exchanges of m_Q.

At this point we could simply use the amplitude so generated in a three nucleon Faddeev equation, or use it to compute a "potential" from the Low equation. We think a more interesting alternative is to write down and solve the Faddeev-Yakubovsky four particle equations - which have been derived for this type of "zero range" theory - for a system containing $m_1 m_2 m_3$ and m_Q. Then our restrictions reduce the 12 (3,1) configurations to six amplitudes and eliminate the (2,2) configurations entirely. Thus the difference of including the mesonic degree of freedom is that there are six rather than three amplitudes and that the equations have internally a four particle rather than a three particle propagator. Explicit equations will be given.

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The basic assumptions on which our three and four body covariant theory of single meson exchange rest are that the only elementary process is the (s-channel) absorption and re-emission of a meson by a nucleon, and that there is no direct nucleon-nucleon scattering. This is a covariant generalization of the scattering length, or zero range "bound state," model of nonrelativistic quantum mechanics (i.e., $e^{i\delta} \sin\delta/q = (-1/a - iq)^{-1}$, $a > 0$). We use i,j,k,... to distinguish nucleons and Q to distinguish the meson; generalization of the model is briefly discussed at the end of the paper. For simplicity we confine ourselves here to scalar nucleons and scalar mesons, following an old tradition of attempts to understand nuclear forces in a covariant context; again, brief discussion of the removal of this restriction will follow.

Once a separable, unitary and covariant two particle amplitude is postulated, Faddeev equations for the relativistic three particle problem follow and predict unitary, covariant, and time reversal invariant three particle amplitudes, as has been proved several times.[1,2,3] Although we restrict ourselves here to two or three nucleons and one meson, subsequent work will relax both restrictions, and include degrees of freedom corresponding to antiparticles. Our restriction at each level of approximation to a finite number of particulate degrees of freedom and exact unitarity differentiates our approach from the S-matrix program in a subtle way. S-matricists who follow Chew add to unitarity the postulate of "crossing" which necessarily brings in an infinite number of degrees of freedom as customarily employed. We clearly differ from theories based on the second quantization of the matter field, which bring in an infinite number of degrees of freedom in another way. What has recently been discovered[4] is that by concentrating on two particle scattering as the basic concept we can go to a nonrelativistic limit corresponding to "two body interactions" in a Hamiltonian theory without ambiguity, but need not take that limit; our theory is both covariant and unitary at any energy.

The covariant three particle model explored by Lindesay[3] is extremely simple. We simply take the invariant two particle amplitudes in the space of three particles of mass m_a, m_b and m_c represented in the coordinate system in which $\underline{k}_a + \underline{k}_b + \underline{k}_c = 0$ and the invariant four-momentum $M = \varepsilon_a + \varepsilon_b + \varepsilon_c$ to be

$$t_{ab} \equiv t_c(\underline{k}_c, \underline{k}_c'; M) = \sqrt{s_{ab}}\ (2\pi)^{-2}\, \varepsilon_c\, \delta^3(\underline{k}_c - \underline{k}_c')\ D_{ab}^{-1}(s_{ab}) \tag{1}$$

where

$$D_{ab}(s) = \left[-q_{ab}^2(\mu_{ab})\right]^{\frac{1}{2}} - \left[-q^2(s)\right]^{\frac{1}{2}} \tag{2}$$

and

$$q_{ab}^2(s) = \left[s - (m_a + m_b)^2\right]\left|s - (m_a - m_b)^2\right|/4s \tag{3}$$

with

$$\varepsilon_a = \varepsilon_{m_a}(k_a) = (m_a^2 + k_a^2)^{\frac{1}{2}} \quad . \tag{4}$$

In this model we see that the entire dynamics is specified by the "bound state mass" μ_{ab} where the two particle amplitude has an "s-channel pole." Since s_{ab} is the invariant four momentum squared of the ab pair, the requirement that the two particle scattering be unaffected by the "spectator" m_c in our basic description (the "cluster property") tells us that in coordinate system where $s_{ab} = q_{ab}^2 + \mu_{ab}^2$ (the ab zero momentum system), the momentum of the spectator k_c^{ab} ranges from zero to infinity. Making a Lorentz transformation of these limits to the three particle zero momentum system, we find that k_c lies between zero and $(M^2 - m_c^2)/2M$. We emphasize this point since it is not always understood that the asymptotic variables in a relativistic three particle problem are bounded once M is fixed, and hence in a separable model lead to finite integral equations. Given that the two particle input is unitary, as holds for this model, the form of these equations guarantees the unitarity of the three particle amplitudes computed from them, as was proved by Freedman, Lovelace and Namyslowski,[1] by Brayshaw[2] and again by Lindesay.[3]

For the two nucleon one meson model we first consider, we call the nucleons m_1 and m_2 and the meson m_Q, and make two postulates: (a) there is no elementary nucleon-nucleon scattering (i.e., $t_{12} \equiv t_Q = 0$) and (b) that the bound state mass $\mu_{iQ} \equiv m_i$ and is asymptotically indistinguishable physically from the particle m_i. The equations then describe a model for single particle exchange and production which is still covariant and unitary, as has been claimed previously.[4,5] For the current application to nuclear physics we specialize the model still further by postulating (c) that the meson never appears as a free asymptotic particle. That we can achieve "confinement" in this sense without destroying covariance or unitarity may not be immediately obvious, so we will spell out the steps with some care.

Our first step, in the two-nucleon, one meson space, is to note that under postulates (a) and (b) we have only two input amplitudes $t_{iQ} \equiv t_j$ with $i,j \in 1,2$ and that in the three particle zero momentum system $s_{iQ} = (\varepsilon_i + \varepsilon_Q)^2 - (\underline{k}_i + \underline{k}_Q)^2 = (M - \varepsilon_j)^2 - k_j^2 = M^2 + m_j^2 - 2M\varepsilon_j$. Further, if we start with a spectator m_j with momentum $\underline{k}_j^{(0)}$ the "bound state" (with mass $\mu_{iQ} \equiv m_i$) has momentum $-\underline{k}_j^{(0)}$ and $M = \varepsilon_i^{(0)} + \varepsilon_j^{(0)}$. Thus if we rationalize the denominator in Eq. (1) and define $t_j = \varepsilon_j \delta^3 \tau_j$ we find that

$$\tau_j(k_j,k_j^{(0)};M) = -\frac{\Gamma_j^2}{2M(\varepsilon_j-\varepsilon_j^{(0)})} + \hat{\tau}_j = -\frac{\Gamma_j^2}{2M} P_j(k_j,k_j^{(0)}) + \hat{\tau}_j \qquad (5)$$

where $\hat{\tau}_j$ contains the branch cut in s_{iQ} starting at $m_i + m_Q$ which in the two particle space describes meson-nucleon scattering. With these initial conditions all we need do to prevent meson production is to define $M_{ij} = t_i\delta_{ij} + \tau_i Z_{ij}\tau_j$, iterate the covariant Faddeev equations once to obtain an equation for Z_{ij}, and take $\hat{\tau} = 0$ closing the production channel. For reasons we discuss below, the physical amplitudes for $m_1 + m_2$ scattering are not Z_{ij} but $\Gamma_i Z_{ij} \Gamma_j \equiv K_{ij}$. Following the above procedure we find that they satisfy the coupled equations

$$K_{ij}(\underline{k}_i,\underline{k}_j';M) + \bar{\delta}_{ij}\,\Gamma_i\, R(\underline{k}_i,\underline{k}_j';M)\,\Gamma_j \qquad (6)$$

$$= \frac{1}{2M}\int_0^{(M^2-m_k^2)/2M} \frac{d^3k_k''}{\varepsilon_k''}\,\bar{\delta}_{ik}\,\Gamma_i\, R(\underline{k}_i,\underline{k}_k'';M)\,\Gamma_k\, P_k(\underline{k}_k'',k_k')\;K_{kj}(\underline{k}_k'',k_j';M)$$

$$= \frac{1}{2M}\int_0^{(M^2-m_k^2)/2M} \frac{d^3\varepsilon_k''}{\varepsilon_k''}\,K_{ik}(\underline{k}_i,\underline{k}_k'';M)\;P_k(\underline{k}_k,\underline{k}_k'')\,\Gamma_k\, R(\underline{k}_k'',k_j';M)\,\Gamma_j\,\bar{\delta}_{kj}$$

where

$$R^{-1}(k_j,k_j';M) = \varepsilon_{ij}(\varepsilon_{ij} + \varepsilon_i + \varepsilon_j) \quad ; \quad \bar{\delta}_{ij} = 1 - \delta_{ij} \qquad (7)$$

and

$$\varepsilon_{ij} = \left[m_Q^2 + (\underline{k}_i + \underline{k}_j)^2\right]^{\frac{1}{2}} \quad . \qquad (8)$$

We see immediately that these are covariant coupled channel equations of the Lippmann-Schwinger type and hence define unitary amplitudes. Further, the unitarity is independent of the (finite) value of $\Gamma_1\Gamma_2$, a point which we exploit below.

In order to understand why the residue at the pole which, if we start from Eq. (1), would appear to be fixed by two particle unitarity can be treated as an arbitrary parameter in our context, it is convenient to isolate the primary singularities by using Eq. (5), which gives

$$M_{ij} = t_i\,\delta_{ij} + \left(\frac{\Gamma_i^2}{s_{iQ} - m_j^2} + \hat{\tau}_i\right) Z_{ij} \left(\frac{\Gamma_j^2}{s_{iQ} - m_j^2} + \hat{\tau}_j\right) \tag{9}$$

and compare this with the relation between the Faddeev amplitudes and the physical amplitudes whose squares are related to cross sections as given by Osborn and Bollé,[6] Eq. (IV.7), which in our notation is

$$M_{ij} = t_i\,\delta_{ij} + F_{ij} + G_{ij}\,\phi_j (s_{iQ} - m_j^2)^{-1} + \phi_i (s_{iQ} - m_i^2)^{-1}\,\tilde{G}_{ij}$$
$$+ \phi_i (s_{iQ} - m_i^2)\; K_{ij} (s_{iQ} - m_j^2)\,\phi_j \quad . \tag{10}$$

This shows us that if the bound state wave functions ϕ_i are identified, as they should be in our s-channel (or zero range, or on shell) model, with the asymptotic normalization of the bound state Γ_i, the 3-3 amplitude is $F_{ij} = \hat{\tau}_i\,Z_{ij}\,\hat{\tau}_j$, the amplitude from which breakup can be computed (cf. OB Eq. (I.2)) $G_{ij} = \hat{\tau}_i\,Z_{ij}\,\Gamma_j$, the amplitude from which coalesence can be computed $\tilde{G}_{ij} = \Gamma_i\,Z_{ij}\,\hat{\tau}_j$, and the elastic scattering and rearrangement amplitudes are given by $K_{ij} = \Gamma_i\,Z_{ij}\,\Gamma_j$, as asserted without proof above. But then we see that indeed if we take $\hat{\tau} = 0$, the only scattering processes are elastic and rearrangement scattering. As is well known in nuclear physics, the asymptotic normalization of the bound state wave function, or "reduced width," need not correspond to what we would compute from a zero range theory. Thus in a sense it is model dependent, but in fact it can be determined by experiments which break up a two particle bound state and shown to be independent of the particle used for the breakup. Thus, so long as we confine our theory to the region below meson production threshold, as is appropriate for our discussion of nuclear physics, we can treat this parameter as empirical without interfering with the flux conservation of the nucleonic degrees of freedom. A similar freedom was used by Amado in his "nonrelativistic field theory" for n-d scattering[7] in which he treats the residue at the np-d vertex as an adjustable parameter. We return to the consideration of this constant as measuring how much of the nucleon is "composite" and how much "elementary" in our final discussion.

Our final step at the two nucleon, one meson level is to note that since $\underline{k}_i = -\underline{k}_j$ in either the initial or the final state, and our postulate (c) does not allow us to distinguish the "bound state" μ_{iQ} from the "bare nucleon" m_i, we only have one amplitude $T_{12}(\underline{k},\underline{k}';M) = K_{11}(\underline{k},\underline{k}';M) + K_{21}(-\underline{k},\underline{k}') = K_{12}(\underline{k},-\underline{k}';M) + K_{22}(-\underline{k},-\underline{k}';M)$ where we have taken as our reference direction the direction of the momentum $\underline{k}'$ of particle 1 in the initial state, and the equality of the two forms expresses time reversal invariance, as guaranteed by the equivalence of the two forms of Eq. (6). To relate our theory to more familiar modes we note that with the arbitrary parameter g_1g_2 replacing $\Gamma_1\Gamma_2$ the equations do indeed reduce in the nonrelativistic kinematic region (where $M = \varepsilon_1 + \varepsilon_2$) to the Lippmann-Schwinger equation of scattering by a Yukawa potential since $\Gamma_1\Gamma_2\ R(\underline{k},\underline{k}';M) \to g_1g_2[m_Q^2 + (\underline{k}-\underline{k}')^2]^{-1}$ and P_k goes to the usual nonrelativistic propagator. But of course we need not take this limit. The T_{12} we have obtained is a fully covariant off shell two particle amplitude which can be used, for example, in covariant three particle Faddeev equations, or their nonrelativistic limit.

We believe the above suggested use of the amplitude we have obtained is a better way to do nuclear physics that using a "potential" for the n,n,p system, but until we have shown how to include Coulomb effects in this covariant description, the construction of the corresponding potential will have its importance. This can easily be accomplished, since we showed long ago[8] that the Low equation can be used as a defining equation given a half off shell two particle amplitude consistent with time reversal invariance. Explicitly for the case at hand the nonrelativistic energy parameter $z = M - m_1 - m_2$, and we can write

$$V(k,k') = T(k,k';m_1 + m_2 + z) - \sum\!\!\!\!\!\!\int_0^\infty q^2\,dq\,\frac{T(k,q;m_1+m_2+\tilde{q}^2)\;T^*(q,k';m_1+m_2+\tilde{q}^2)}{\tilde{q}^2 - z} \tag{11}$$

where we have for simplicity confined ourselves to s-waves and the Σ superimposed on the integral is supposed to remind us to include any $m_1 m_2$ bound states predicted by the model in our calculation. Since the z-dependence of the right hand side is not necessarily negligible (except for high energy where we go to the Born approximation), this calculation will tell us to what accuracy and over what energy range our model can indeed be represented by a static potential.

Another application of our approach looks interesting. Instead of using relativistic Faddeev equations and T_{ij} computed from the two nucleon one meson system to calculate the three nucleon system, we can formulate relativistic four particle Faddeev-Yakubovsky equations for three nucleons and one meson and use our same postulates to restrict the asymptotic degrees of freedom to the three nucleons. The zero range nonrelativistic equations are easy to derive[9] and the generalization of these to separable relativistic two particle driving terms is equally straightforward. For our special model the fact that we allow no elementary particle-particle scattering immediately eliminates the (2,2) configurations. Normally there are 4×3 = 12 (3,1) configurations, but since Q is not allowed to be a spectator either of the three particle systems or within the subsystems, this reduces the number of amplitudes to 3×2 = 6. We symbolize these by F^i_j where the superscript i labels the spectator of the two nucleon one meson subsystem and the subscript j labels the spectator in that subsystem. Since $j \neq i$, the inclusion symbol of the Faddeev-Yakubovsky hierarchy[10] can in our case simply be replaced by a $\bar{\delta}_{ij}$, and our four particle amplitudes with $i,j,k,\ell,m \in 1,2,3$ satisfy the six coupled equations

$$F^i_j(\underline{k}_i,\underline{k}_j;M_4) + M^{(i)}_{jj}(\underline{k}_j,\underline{k}_j^{(0)};M_3^{(i)}) = -\sum_k \bar{\delta}_{ik} \sum_m \bar{\delta}_{im} \sum_\ell \bar{\delta}_{k\ell}$$

$$\times \int_0^{(M_4^2-m_k^2)/2M} \frac{d^3k_k}{\varepsilon_k} \int_0^{(M_3^{(\ell)2}-m_\ell^2)/2M} \frac{d^3k_\ell}{\varepsilon_\ell} \; M^{(i)}_{jm}(\underline{k}_j,\underline{k}_k;M_i^{(3)})$$

$$\times R^{ik}_{m\ell}\, F^k(\underline{k}_k,\underline{k}_\ell;M_4) \tag{12}$$

where

$$M_3^{(i)}(k_i,M_4) = M_4^2 + m_i^2 - 2M_4\varepsilon_i \tag{13}$$

and

$$(R^{ik}_{m\ell})^{-1} = \varepsilon^{Q}_{ik\ell}\ \varepsilon^{m_i}_{k\ell}(\varepsilon_k + \varepsilon_\ell + \varepsilon^{m_i}_{k\ell} + \varepsilon^{Q}_{ik\ell} - M_4 - i0^+)$$

$$\varepsilon^{Q}_{ik\ell} = \left[m_Q^2 + (\underline{k}_i + \underline{k}_k + \underline{k}_\ell)^2\right]^{\frac{1}{2}}$$

$$\varepsilon^{m_i}_{k\ell} = \left[m_i^2 + (\underline{k}_k + \underline{k}_\ell)^2\right]^{\frac{1}{2}} \quad . \tag{14}$$

In contrast, if we do not include the mesonic degree of freedom in the internal dynamics, we will have only three coupled equations for the three Faddeev components ${}^{(3)}F_i(\underline{k}_i,\underline{q}_i;M)$ given by the three coupled equations

$$\begin{aligned} &{}^{(3)}F_i(\underline{k}_i,\underline{q}_i;M) + T_i(\underline{q}_i,\underline{q}_i^{(0)};M)\ \delta_{ii_o}\,\delta^3(\underline{k}_i - k^{(0)}_{i_o}) \\ &= -\sum_j \bar{\delta}_{ij} \int_0^{(M^2-m_j^2)/2m_j} \frac{d^3k_j}{\varepsilon_j} \int \frac{d^3q_j}{\varepsilon_j}\ T_j(\underline{q}_i,\bar{\underline{q}}_i;M) \\ &\times\ \delta^3(\bar{\underline{q}}_i + \underline{k}_i + \underline{k}_j)\ R_{ij}(\underline{k}_i,\underline{q}_j;M)\ F_j(\underline{k}_j,\underline{q}_j;M) \end{aligned} \tag{15}$$

where

$$R^{-1}_{ij} = \varepsilon_{ij}(\varepsilon_{ij} + \varepsilon_i + \varepsilon_j)$$

$$\varepsilon_{ij} = \left[m_k^2 + (\underline{k}_i + \underline{k}_j)^2\right]^{\frac{1}{2}}$$

$$\varepsilon_i = (m_i^2 + k_i^2)^{\frac{1}{2}}$$

$$\varepsilon_j = (m_j^2 + k_j^2)^{\frac{1}{2}} \quad . \tag{16}$$

Since we have seen above that $T_i \equiv T_{jk} = M_{jj} + M_{jk}$, the Faddeev equation can be derived from the four particle equation by forming ${}^{(3)}F_i = F^i_j + F^i_k$ where the approximation will be good so long as the significant momenta in the equations are all small compared to $m_Q c$. Quantitative investigation of this approximation will then reveal to what extent the usual nuclear physics approximation is valid, and where the internal dynamics ("three body force" and modification of the two body off shell effects) becomes important in the three nucleon problem in the one meson exchange approximation.

We anticipate that to the extent that a one meson exchange model gives a reasonable description of nucleon-nucleon scattering, these mesonic effects will be small at low energy. But this does not mean that they are physically unimportant for nuclear physics. For an adequate test of this question we must go beyond the one meson exchange approximation and formulate the nucleon-nucleon problem itself as a two-nucleon two-meson system using Faddeev-Yakubovsky equations. To be realistic we must of course use a pseudoscalar pion and also include π-π scattering and the nucleon-antinucleon channels. In this way we might hope to unify the one-boson exchange models and the dispersion-theoretic models, provided our description of $\pi\pi$ scattering produces the ρ and we can couple in the ω phenomenologically. At this point we must also face the problem mentioned above of precisely how our replacement of $\Gamma_i\Gamma_j$ by g_ig_j treats the nucleon as partly composite and partly bare. The three nucleon problem then becomes a five body problem, but judging by the simplifications our model led to in the case discussed in this paper, we can hope that this will still prove tractible. Our conclusion is that this approach could provide a systematic way to investigate mesonic degrees of freedom in nuclei in a systematic way using finite and controlled approximations at each step.

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