SLAC-PUB-2925
PEP Note-370
May 1982
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EXACT TRANSFER FUNCTIONS FOR THE PEP STORAGE RING MAGNETS AND SOME GENERAL CHARACTERISTICS AND TECHNIQUES*

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I. Introduction

The exact, ion-optical transfer functions for the dipoles, quadrupoles and sextupoles of the PEP standard 'FODO' cell are calculated for any single particle with initial coordinates($\vec{r}_i$, $\vec{p}_i$, $\vec{s}_i$). Modifications resulting from radiative energy loss are also calculated and discussed. These functions allow one to characterize individual magnets or classes of magnets by their aberrations(1) and thereby simplify their study and correction. In contrast to high-energy spectrometers where aberrations are often 'analyzed away', those in storage rings drive series of high order resonances, even for 'perfect' magnets(2), that can produce stop bands and other effects which can seriously limit performance. Thus, one would like to eliminate them altogether or failing this to develop local and global correction schemes. Even then, one should expect higher order effects to influence injection, extraction or single-pass systems either because of orbit distortions or overly large phase space distortions such as may occur in low-beta insertions or any final-focus optics.

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* Work supported by the Department of Energy under contract DE-AC03-76SF00515. This paper was presented at the Brookhaven Workshop on Accelerator Orbit and Particle Tracking.
Published in the Proceedings of the Workshop on Accelerator Orbit and Particle Tracking Programs, Upton, New York, May 3 - 6, 1982.

The term 'exact' means that the results here are based on solving the relativistic Lorentz force equation with accurate representations of measured magnetostatic fields. Such fields satisfy Maxwell's equations and are the actual fields seen by a particle as it propagates around a real storage ring. This is discussed in detail and illustrated with examples that show that this is possible, practical and may even be useful.

This approach is practical because the transfer function, when taken to sufficiently high order, becomes equivalent to the exact solution of the equations of motion from which it is derived and the series converge rapidly. Characterizing the optics of multipoles with the complete transfer function shows clearly how correctors are best employed and also suggests new multipole magnets which serve the same purpose as conventional magnets but produce fewer aberrations as well as fewer symmetry allowed field harmonics. This is tantamount to proving the possibility of aberration free optics and suggests a specific program of 3-dimensional, end-field studies in contrast to the usual expedient of ignoring the longitudinal field component.

Although an optical transfer function may be arbitrarily accurate, the highest order which is retained is determined by the problem e.g. the use of sextupoles implies the need for at least a third order calculation(3). On the other hand, transfer functions arn't really necessary and may not be warranted when systems are not purely deterministic. The underlying approach used here remains applicable to such problems and provides an unambiguous and consistent framework with which to study them.

II. The Possibility and Importance of Exact Calculations

There are a number of reasons why one might question both the possibility and importance of such calculations. Tracing real particles through real fields without significant approximations means you must actually be able to know and represent those fields. This inevitably raises questions concerning the inherent experimental uncertainties in magnetic measurements, magnet alignment as well as more arcane possibilities from the 'Obstructionist's Handbook' such as mechanical or magnetic relaxation processes as well as the various sources of noise.

Rather than recite the complete list, I only comment that the approach used here provides specific, quantitative guidance on such questions within a single, consistent framework. Thus, I have found no significant sensitivity of higher order terms to practically achievable tolerances. However, this deserves a separate, detailed study. The question of whether it is really possible to know the actual magnetostatic fields is discussed in Section IV and Appendix I. This question has been studied in detail for the PEP dipoles(4) with a conditional conclusion. The answer is yes but it is highly unlikely based on the usual techniques employed today. For instance, the use of C-magnets virtually precludes the possibility for large rings.

Any discussion of higher order optical effects must also eventually consider spin and quantized, radiative energy loss which are of increasing interest and may also influence both orbit and optics. The increasing use of rings for producing synchrotron radiation with their unprecedented demands on spot sizes and the possibility of high-power, high-efficiency free electron lasers clearly argues for a more integrated approach to storage ring design. These additional degrees of freedom can be treated in many ways but the most general method, with the fewest approximations, again would seem to be the direct numerical solution of the nonlinear, coupled difference equations. The spin coordinate, $\vec{S}_i$, will be studied in this same framework but in a separate paper dealing solely with it since that treatment should also include the study of other elements required for longitudinal polarization insertions and the use of skew quads in the low-beta insertions where geometric and higher chromatic nonlinearities become increasingly important with decreasing β_y^*.

We are mainly concerned with the optics of perfect multipoles(2). Calculations for all multipoles through octupole are given with an accuracy adequate for virtually any problem. Sextupoles and octupoles are rather short so they may be used to check thin lens and other approximations in other codes(3). The effects of average, incremental energy loss on orbits and transfer functions were also calculated because there are an adequate number of photons radiated to warrant this. As with other effects such as alignment errors, the variations were minor. Thus, it appears possible to improve predictability of beam lifetime, beam distribution function and optimal operating points in tune space as well as give their dependence on magnetic or misalignment errors, RF cavity distributions and other effects which lower the effective and dynamic aperture.

III. Magnet Description

Most of the magnets studied here are described in the PEP design handbook(5). Table 0 lists the relevant parameters used for ray tracing. These have the same meaning as for 'TRANSPORT' except for ones like the fringing field coefficients which are discussed in Appendix I. Figure 1 shows the coordinate systems employed as well as some of the parameters and how they relate to the PEP bend. The fringing field coefficients, pole contours and multipole components describing the field distribution are defined in the effective field boundary(EFB) coordinate systems at entrance and exit of each magnet. Generally, these systems never coincide with the mechanical boundaries of a magnet nor does the orbit on which they're placed correspond to an actual orbit in dipoles(6). However, in all properly aligned and built higher multipoles, the equilibrium orbit and design orbit are the same. None of this changes the outcome of the calculations but is relevant for the alignment of the magnets or knowing the field distribution and alignment of each magnet for accurately predicting the actual equilibrium orbit relative to the magnets.

IV. Field Description

The exactness of any solution ultimately depends on the precision and accuracy of the field description. It is useful to distill the magnetic measurement data into a few parameters which describe the vector field in the volume of interest for all currents with negligible error. A prescription for doing this is discussed in Appendix I where it is applied to all of the PEP multipoles. The success of this venture is assessed by comparing the results to the measurements. The success of the relatively simple model is based on an important phase of magnet design which involves the study of the pole sizes and shapes required to minimize any variations with current such as occur in the fringing fields and/or the multipole content of the internal fields.

Assuming one has designed and fabricated the magnets well, there is still the problem of energizing and measuring them in a reproducible way. This method must obviously be compatible with how the magnet is to be operated in the ring. Likewise, transport, installation, support and alignment procedures must not modify the magnet in any essential way nor should they be incompatible with the methods used during measurements. Proper implementation of this phase is necessary for any exact calculation even though one might try to skip it by attempting to do 'experimental' ray-tracing(7). More detailed information on this aspect which is relevant to the PEP magnets and fields is available(4). The successful implementation of these steps is considered necessary if not sufficient for exact calculations.

V. Transfer Functions

Tables 1-4 give the results of calculations corresponding respectively to the first four orders in the input coordinate variables for various types of magnets. Their characteristics are given in Table 0. They represent the mean values of their particular class of magnet in PEP. The octupole parameters are the same as for the sextupole which is the shortest family of sextupoles(or magnet) in PEP. This should simplify comparison to thin, nonlinear lens models. Units were chosen to simplify the study of the various terms e.g. the relative importance of any two is quickly assessed by comparing their values because the magnitudes of the standard deviations of the variables are roughly comparable in these units(8). Accuracy and precision of the calculations are much better than implied by the number of significant figures given in the tables which were thought sufficient for electron storage rings having typical damping times of a few hundred turns.

First, Table 1 shows that $(x|x)=(x'|x')$ and $(y|y)=(y'|y')$. This is because symmetrical drift lengths have been used, i.e. A=B in Table 0. Furthermore, with the exception of the quad and the vertical plane of the bend, these terms all equal one and these elements act like drift spaces in first order. Since properly aligned and fabricated elements with higher numbers of poles than quadrupoles don't couple in first order, both sextupoles and octupoles are pure drifts in this order having total lengths of $A+L+B=0.7424$m or $(x|x')=(y|y')=0.07424$cm/mr. Appendix III gives a simple but general demonstration of why the bend is a special case of magnet with $(x|x)=(x'|x')=1.0$. It is also shown there that $(x|x^n)=0$ for $n>1$ and that $(x'|x^n)$ $=(x'|x^n x'^m y^o \delta^p)=0$ for all $n>0$. Thus, in the median plane, this bend is a perfect parallel-to-parallel optical system with an infinite total length. This is ideal for storage rings with their naturally small vertical emittance since this result no longer applies in the vertical plane. Adding multipoles to these bends destroys this general property but retains it in first order when only symmetry allowed multipoles of the bend are allowed i.e. N=odd as for sextupoles. In the vertical, the bend acts like a converging thin lens with $(y'|y) < 0$ and $(y|y)=(y'|y')\approx 1.0$. The corresponding horizontal, defocusing action shown in Fig. 2A is cancelled by the natural focusing action of the resulting sector magnet in the bending plane.

Next, one sees from Tables 1-4 that all elements having no field component on their longitudinal axes, i.e. perfect multipoles above dipole, are expected to have $(x|\delta^n)=(x'|\delta^n)=(y|\delta^n)=(y'|\delta^n)=(z|\delta^n)=(\delta|\delta^{n+1})= 0$ for all $n>0$. Appendix IV calls these elements 'non-dispersive' because they have no pure chromatic aberrations. Unfortunately, these are never achromatic even though their leading order optical terms are purely geometric because the next order i.e. their 'leading order aberrations' are mixed chromatic with terms of order n including mixed chromatics of order n-1 in δ. Although this may well tempt one to superimpose quads and dipoles for some applications, this is seldom a good idea when one intends to use iron-dominated magnets(4).

One also sees a natural grouping between the properties of the odd and even magnetic elements. This results directly from the symmetry of the fields and is most easily seen from the leading order geometrics because they follow the leading order spatial dependence of the fields and avoid the higher order complications discussed in the appendices. Some of the general implications of this property are considered below.

Since the leading order of any multipole with no field on its axis produces only geometric terms, no set of multipoles superposed on one of lower order can be expected to fully correct that multipole. The most common technique of dealing with this problem, in lowest order, is to introduce dispersion and the next order multipole. Thus, second order chromatic terms introduced by a quad such as $(x|x\delta)$, $(x|x'\delta)$, $(x'|x\delta)$ and $(x'|x'\delta)$ can be corrected by the leading order geometric terms of the sextupole [$(x|x^2)$, $(x|xx')$, $(x'|x^2)$ and $(x'|xx')$]. A small vertical emittance and/or small vertical spot compared to the horizontal then minimizes the sextupole strengths. The major, leading order aberrations introduced by the sextipole such as $(x|x^2\delta)$ and $(x|xx'\delta)$ are corrected by the major geometrics of the octupole in exactly the same way. Likewise, the higher order geometrics of the sextupole continue to crudely compensate the corresponding significant, mixed chromatic terms of similar order from the quadrupole. Brown has shown how one can virtually eliminate cross-coupling between sextupoles in some circumstances(10).

The addition of successively higher multipoles appears convergent since the series for the individual multipoles is convergent as shown by the Tables. However, this depends on the dispersion at the multipole. Thus, one can correct the leading order chromatics of one multipole with geometrics of the next higher multipole by using dispersion to basically superimpose the original multipole on itself. Convergence of this procedure can be established by relations such as:

$$(x|x^2)_{SF} \approx -(x|x\delta)_{QF}/(x|\delta)_{BB}$$
$$(x|x^3)_{OF} \approx -(x|x^2\delta)_{SF}/(x|\delta)_{BB}$$

which, however, are clearly dependent on the specific optics. However, if $(x|x^3)_{QF}$ or $(x|x^3)_{SF}$ is comparable to $(x|x^3)_{OF}$ at the octupole, then the correction is subverted. This is one of the problems of using only the leading orders in the field expansions for each multipole type. It eliminates aberration terms completely and miscalculates others. One solution to this problem is to be able to completely eliminate such terms in at least one plane at a time so that one can exploit symmetry. A subsequent section gives an example of how this can be done in a physically realizable way by superimposing octupoles and quads. This can be done in a way that allows one to reduce or zero all higher order geometrics proportional to x^n for $n \geqslant 2$. Before discussing that, it is interesting to show some examples to illustrate the importance of higher order effects.

VI. Low-Beta Insertions and Final Focus Systems

The purpose of the low-beta insertion is to demagnify the beam waist between the symmetry points (located midway between interaction points (IP's)) and the IP's. Generally, the optimum luminosity ($\mathcal{L}$) can be increased by decreasing the vertical beta function at the IP (β_y^*) whenever $\beta_y^* > \sigma_z$. It can also be increased by increasing the horizontal beam size (σ_x) whenever it is below the aperture limit. Either scheme will increase higher order effects. For instance, the betatron amplitude between the IP and the entrance to the first quadrupole varies quadratically with the separation distance, L, i.e.

Appendix I - FIELD DESCRIPTION

Here we consider only the time independent, guide fields of a storage ring. The magnetostatic, scalar potential, $\Phi(x,y,z)$, can be written in the regions of interest as

$$\Phi(x,y,z) = \sum_{\ell=0}\sum_{m=0} \Phi_{\ell m} \frac{x^{\ell}}{\ell!}\frac{y^{m}}{m!} . \qquad (A1)$$

the z-dependence of this function occurs through $\Phi_{\ell m}$ i.e.

$$\Phi_{\ell m}(z) = \left[\frac{\partial^{\ell+m}}{\partial x^{\ell}\partial y^{m}}\Phi(x,y,z)\right]_{x=y=0} . \qquad (A2)$$

Figure A1 shows the measured z-dependence of the PEP magnets in the EFB coordinate system i.e. the EFB is at z=0 for each multipole. For z<<0, we are inside the magnet where the field can be characterized by its primary harmonic magnitude B_o, G_1, or G_2' and only the transverse coordinates (x,y). Outside, the normalized fields all approach zero whenever z/g>>1. This representation is clearly valid for both fringing fields of the PEP bend shown in Fig. 1.

If one assumes that all magnets possess mechanical symmetry about the dispersion plane(y=0) of the dipoles and the potential satisfies the condition $\Phi(x,y,z)=-\Phi(x,-y,z)$, then $B_y(x,0,z)$ is the only component allowed in this plane i.e.

$$B_y(x,o,z) = \left[\frac{\partial}{\partial y}\Phi(x,y,z)\right]_{y=0} = \sum_{\ell=0} \Phi_{\ell 1}(z)\frac{x^{\ell}}{\ell!} . \qquad (A3)$$

along the optic axis(x=y=0), only the dipole terms are nonzero i.e. $\Phi_{\ell 1}=0$ for l>0. Furthermore, specifying $\Phi_{\ell 1}(z)$ for all l specifies the field over the entire volume of interest as one may easily verify by substitution in laplace's equation. Thus, all one requires is an adequate representation for the various multipoles. The one used here which fits the data, as shown in Fig. A1, is:

$$\begin{aligned} &\text{Dipole} &&: \quad \Phi_{01}(z)=B_o/(1+e^{S_0}) \\ &\text{Quadrupole} &&: \quad \Phi_{11}(z)=G_1/(1+e^{S_1}) \\ &\text{Sextupole} &&: \quad \Phi_{21}(z)=G_2'/(1+e^{S_2}) . \end{aligned} \qquad (A4)$$

A modified Fermi-Dirac distribution was chosen for its obvious resemblance to the data as well as its flexibility. It appears that one can then write with sufficient accuracy for any multipole, l:

$$\Phi_{\ell 1}(z) = \left[\frac{\partial^{\ell}}{\partial x^{\ell}} B_y(x,o,z)\right]_{x=0} = G_{\ell}/(1+e^{S_{\ell}(z)}) , \qquad (A5)$$

where G_{ℓ} is the central field strength and $S_{\ell}(z)$ is a polynomial in z normalized to the total gap or bore opening(g) for each multipole. This prescription requires slight modification for curved pole boundaries but this needn't concern us here. The fifth order polynomial

$$S_\ell(z) \equiv \sum_{i=0}^{5} C_i^\ell (z/g)^i \qquad \text{(A6)}$$

is sufficient for our purposes as shown by the least-squares fits in Fig. A1. The coefficients(C_i) are given in Table A1.

Table A1. FIELD COEFFICIENTS

Coefficients	Dipole	Quadrupole	Sextupole
C_0	0.478959	0.296417	0.176559
C_1	1.911289	4.533219	7.153079
C_2	-1.185953	-2.270982	-3.113116
C_3	1.630554	1.068627	3.444311
C_4	-1.082657	-0.036391	-1.976740
C_5	0.318111	0.022261	0.540068

Although the least-squares technique is best suited to problems where the exact functional form of the distribution is known but one has to deal with imprecise data, it is used here because the exact functional form generally isn't known for most cases of practical interest and the data need to be smoothed because of such things as the large magnitude of the field variations, the finite volume of the measurement probes and the complicated spatial dependence of the fields. The range of applicability of these coefficients is $-2<(z/g)<5$.

Appendix II - SOME HIGH ORDER OPTICS CONSIDERATIONS

For any set of initial, particle coordinates (r_i, p_i, s_i) in some coordinate system, A, as shown in Fig. 1, there will be a corresponding set, (r_f, p_f, s_f), in system B. The optic axis shown in Fig. 1, which is used to define such reference coordinate systems is not an actual orbit because of finite fringe fields, radiative energy loss and/or imperfections in the field experienced by the particle. Ignoring such effects, it is clear that one can determine the orbit at any value z_0 in terms of the assumed parameters and initial coordinates(i) using simple geometric constructions. However, as one refines the model towards what they would like to call an exact representation, the methods of solution become more restricted until the only practically viable means would seem to be the direct numerical solution of the differential equations, as, ultimately, there isn't even a direct functional mapping of (i) → (f) for individual particles. In problems of concern here, a correspondence is possible so long as one considers only incremental, average energy loss. Thus, we are able to use conventional Taylor expansions for the transfer function.

Let R, S & T represent the first, second and third order contributions to the total transfer function, in terms of the initial coordinate variables, so the transform from i→o is:

$$\vec{x}_o = R_o^i \cdot \vec{x}_i + S_o^{ij} \cdot \vec{x}_i \cdot \vec{x}_j + T_o^{ijk} \cdot \vec{x}_i \cdot \vec{x}_j \cdot \vec{x}_k + \ldots \quad \text{(A7)}$$

where dot products imply summations over the common components i, j, k, etc. The transform from o to f gives a similar result:

$$\vec{x}_f = R_f^o \cdot \vec{x}_o + S_f^{op} \cdot \vec{x}_o \cdot \vec{x}_p + \ldots \quad \text{(A8)}$$

Substitution of (A7) into (A8) then leads to

$$\vec{x}_f = R_f^i \cdot \vec{x}_i + S_f^{ij} \cdot \vec{x}_i \cdot \vec{x}_j + \ldots$$

$$\vec{x}_f = (R_f^i + (S_f^{ij} + (T_f^{ijk} + \ldots) \cdot \vec{x}_k) \cdot \vec{x}_j) \cdot \vec{x}_i + \ldots \quad \text{(A9)}$$

where

$$R_f^i = R_f^o \cdot R_o^i$$

$$S_f^{ij} = R_f^o \cdot S_o^{ij} + S_f^{op} \cdot R_o^i \cdot R_p^j$$

$$T_f^{ijk} = R_f^o \cdot T_o^{ijk} + S_f^{op} \cdot (S_o^{ij} \cdot R_p^k + R_o^i \cdot S_p^{jk}) + T_f^{opq} \cdot R_o^i \cdot R_p^j \cdot R_q^k \quad \text{(A10)}$$

$$F_f^{ijkl} = R_f^o \cdot F_o^{ijkl} + S_f^{op} \cdot (R_o^i \cdot T_p^{jkl} + S_o^{ij} \cdot S_p^{kl} + T_o^{ijk} \cdot R_p^l) + \ldots$$

there are terms up to sixth order in (A10) which may or may not be dropped based on the supposed adequacy of the original third or higher order approximation for the individual transformations. All terms in any order don't have to be retained i.e. the inner sum may be composed of a significantly reduced selection of higher order terms above second or third based on a row by row assessment of the individual elements in the optical array such as given in Tables 1-4. If the two transforms are for the same magnet or arrays of magnets which have no aberrations e.g. there are only first order terms then the total transform has only first order terms and there will be no cross-coupling.

One sees that terms above second order are generally made up of lower order contributions e.g. the third order transform in Eq. A10 has products of second order terms. If the series for the individual elements is convergent, then the these terms will also be and one thus has a guide on determining which high order terms to retain based on the magnitudes of the lower order products. Notice that the transform in any order is made up of simple products of the basic building blocks so one can easily determine how much additional time is required to compute any problem to any order once they are given the individual terms for each family of magnets. It should be noted that $i<j<k<l$ etc.

Appendix III - SOME HIGH ORDER OPTICS THEOREMS - DIPOLES

For three, parallel, equienergy, equidistant rays with a separation of x at z=0 in the efb system at the entrance of the bend magnet shown in Fig. 2A, the vertical separation of these rays remains constant i.e. $\delta x=\pm x$ for any z, both in and outside of the magnet when $\alpha=\varphi/2$ and the poles are sufficiently wide to provide a straight EFB across the extent of the ray envelop in the fringing fields. It then follows quite generally from the figure that $(x|x^n)=0$ for all n>1 when $\alpha=\beta=\varphi/2$ or $\alpha=\varphi$ and $\beta=0$ or $\alpha=0$ and $\beta=\varphi$ or, by inference, for any combination of angles with $\alpha+\beta=\varphi$. This result also remains true for realistic fringe fields, regardless of their specific form, so long as all iso-induction lines(lines along which $B_y(x,0,z)$=constant) remain parallel with constant z-values for any z.

Similarly, the horizontal focal length is infinite and, in fact, $(x'|x^n)=0$ for all n, regardless of the distances A and B. In fact, a less obvious but more general result from Fig.A2 and the above is that $(x'|x^n x'^m y^\ell \delta^o)=0$ for n>0 and any m,l,o. This follows because any initially equienergy, parallel rays remain parallel, independent of their separation, δx. To summarize the results for any dipole with $\alpha+\beta=\varphi$, one has:

$$
\begin{aligned}
(x|x^n) &= 0 \qquad n>1 \\
(x'|x^n) &= 0 \qquad n>0 \qquad\qquad \text{(A12)} \\
(x'|x^n x'^m y^\ell \delta^o) &= 0 \qquad n>0 \text{ and } m,l,o \text{ arbitrary.}
\end{aligned}
$$

Altogether, this represents more than thirty additional terms through fourth order that arn't allowed in this kind of dipole that will be present for ones like sector magnets. Tables 1-4 verify this for the PEP dipoles where $\alpha=\beta=\varphi/2$.

Finally, we note that because the total path length and bend angles through such magnets are independent of x, one has

$$(\delta|x^n) = 0 \qquad n>0, \qquad\qquad \text{(A13)}$$

when only average radiative energy loss is considered. This is clearly not the case for most dipoles.

Needless to say, such examples are practically useful and also provide good test cases for codes intended to calculate such higher order terms.

Appendix IV - SOME HIGH ORDER OPTICS THEOREMS - QUADRUPOLES

Quadrupoles and elements of higher multipolarity which have no field along there longitudinal axis have no purely chromatic aberrations when their axes coincide with the optic axis of the system, i.e.

$$(x|\delta^n) = (x'|\delta^n) = (y|\delta^n) = (y'|\delta^n) = (z|\delta^n) = (\delta|\delta^{n+1}) = 0. \qquad (A15)$$

Such elements may be called 'non-dispersive' since the first moments of their image are independent of the pure chromatic terms. A major distinguishing characteristic between odd and even magnetic elements is that odd elements such as quads and octupoles are odd functions of the coordinate variables which implies they can't shift the centroids of symmetric beams when properly centered regardless of their strength.

If one computes these pure chromatic terms, they provide another good check on the accuracy of the calculations.

Next, consider the form of the field components for the perfect quadrupole. From Appendix A and some analysis one can show that if

$$\int \left(\Phi_{31} - \tfrac{1}{12}\Phi_{11}'' \right) dz = 0 \qquad (A16)$$

then quads may be made to approimate the ideal quads used in many calculations. This will be discussed more elsewhere.

REFERENCES/FOOTNOTES

(1) Higher order terms in the particle coordinates are called aberrations because they modify the behavior expected from the leading order terms - usually in ways that need to be corrected. For instance, perfect quadrupoles(2) are often used to produce first order correlations between position and angle variables such as $(x'|x)$, but since they couple transverse and longitudinal coordinates via second order terms like $(x'|x\delta)$ or $(y'|y\delta)$, one is then often forced to add sextupoles (or their equivalant) as well as dipoles to correct such chromatic effects. These higher order terms will be called 'natural' or characteristic aberrations to distinguish them from ones resulting from unintended field errors, misalignments or the like. The natural aberrations of an optical array are usually corrected with the basic, pure multipoles whereas field errors and perceived orbit errors are fixed by 'correctors'. The natural aberrations are tabulated for a range of multipoles which can modify the transfer function through third order or less i.e. up to and including octupoles. Such aberrations are usually called either 'geometric' or 'chromatic' if they depend on only transverse variables or also include the momentum, δ. We distinguish the latter as 'pure chromatic' e.g. $(x|\delta^n)$ or 'mixed chromatic' e.g. of the form $(x|x\delta^n)$.

(2) A 'perfect' magnet or multipole, as used here, is defined as one whose central field has only a single harmonic and satisfies Maxwell's equations everywhere that particles go. Sufficiently short magnets, e.g. L<2-3 bore diameters, may approximate thin lenses but aren't perfect nor is any ideal magnet which only satisfies Maxwell's equations in leading order such as generally assumed in present tracking codes. Real magnets can be perfect when 2-D simulations are done properly but perfect magnets can not be pure multipoles. Failure to consistently satisfy Maxwell in this way then implies that such codes neither predict the closed orbit nor specify alignment criteria properly. Similarly, they are not adequate for either optimizing or checking such things as chromatic corrections.

(3) I know of no codes other than 'TRANSPORT' which calculates the transfer function beyond first order except for 'GIOS' which is presently under development. Other ray-tracing codes which compute higher order terms usually do so only for point sources under the assumption of 'median-plane' symmetry - none of which is assumed here. Thus, as far as I am aware, this is the only code which computes all terms through third order.

(4) J.E.Spencer, "Harmonic Strengths of PEP Dipoles and Some Related Effects and Lessons", PEP Note-367, Sept.1981.

(5) PEP Design Handbook, SLAC and LBL publication, Nov.1973.

(6) H.A.Enge, Rev. Sci. Inst. 35(1964)278.

(7) J.E.Spencer and H.A.Thiessen, "High Resolution Techniques for Use With Negative Ion Beams", Proceedings 1972 Proton Linear Accel. Conf., Los Alamos, N.M., Oct.1972, LA-5115. See also: G.Moritz, U.Czok and H.Wollnik, "Measurements of Second Order Aberrations of a Sector Magnet", Nucl. Instr. and Meth. 187(1981)75.

(8) Ideally, the units would be $\langle\sigma_x\rangle$, $\langle\sigma_{x'}\rangle$, etc. as determined over the region of interest so that a term's comparative importance would be based on whether it was greater than or comparable to one. Our choice of the transverse unit of length was a tradeoff between beam size and a magnet's bore size.

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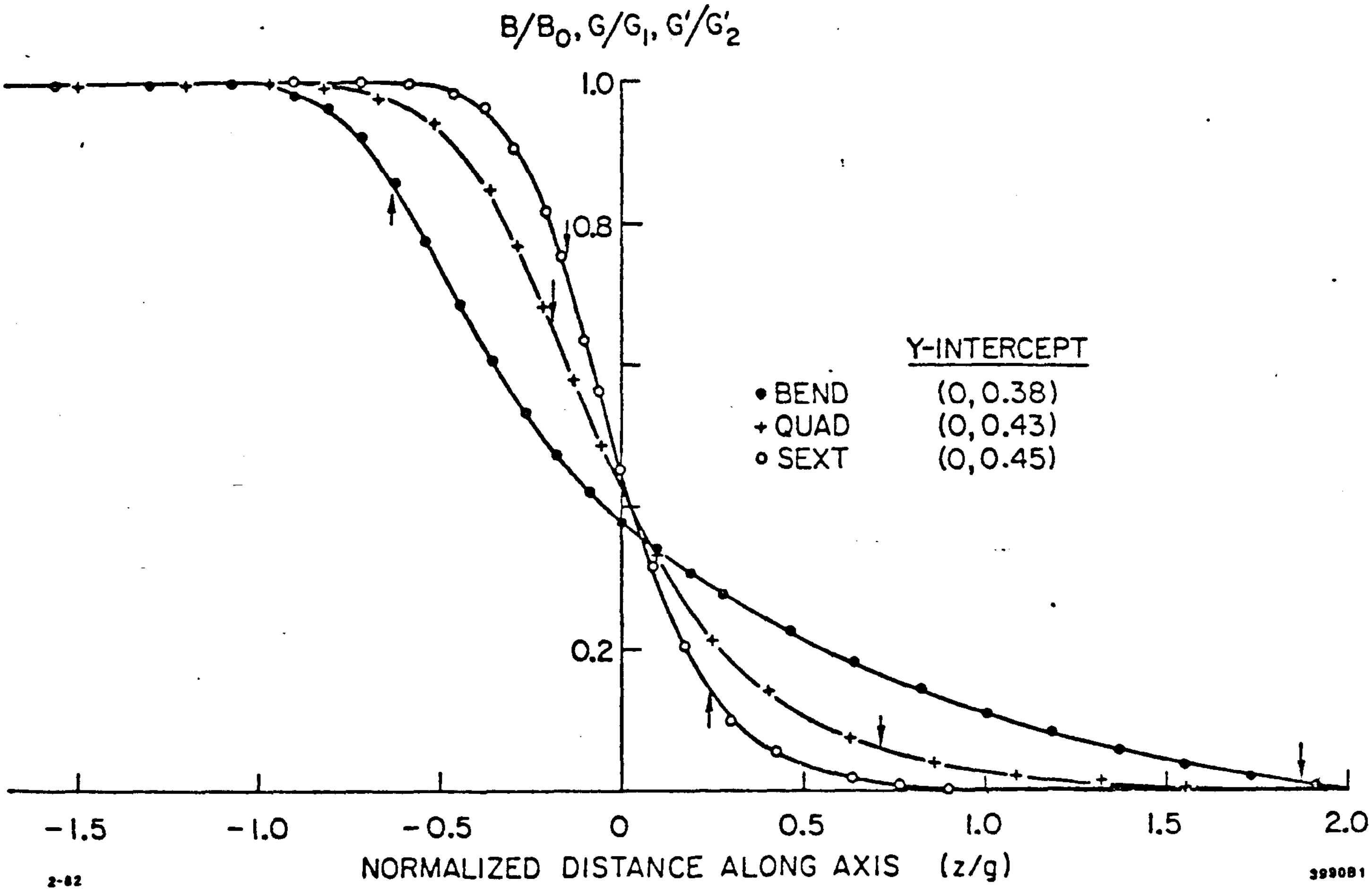

Fig. A1: Dipole, quadrupole and sextupole field distributions normalized to their pure harmonic, central field values versus distance along the longitudinal axis normalized to their respective gap openings. The dots (•), pluses (+) and circles (o) are measured field data for the PEP standard bends, quads and sextupoles. The solid curves are fits corresponding to the coefficients of Table A1. The two arrows associated with each field distribution correspond to the locations of the iron-coil boundary and the outer boundary of the coils.

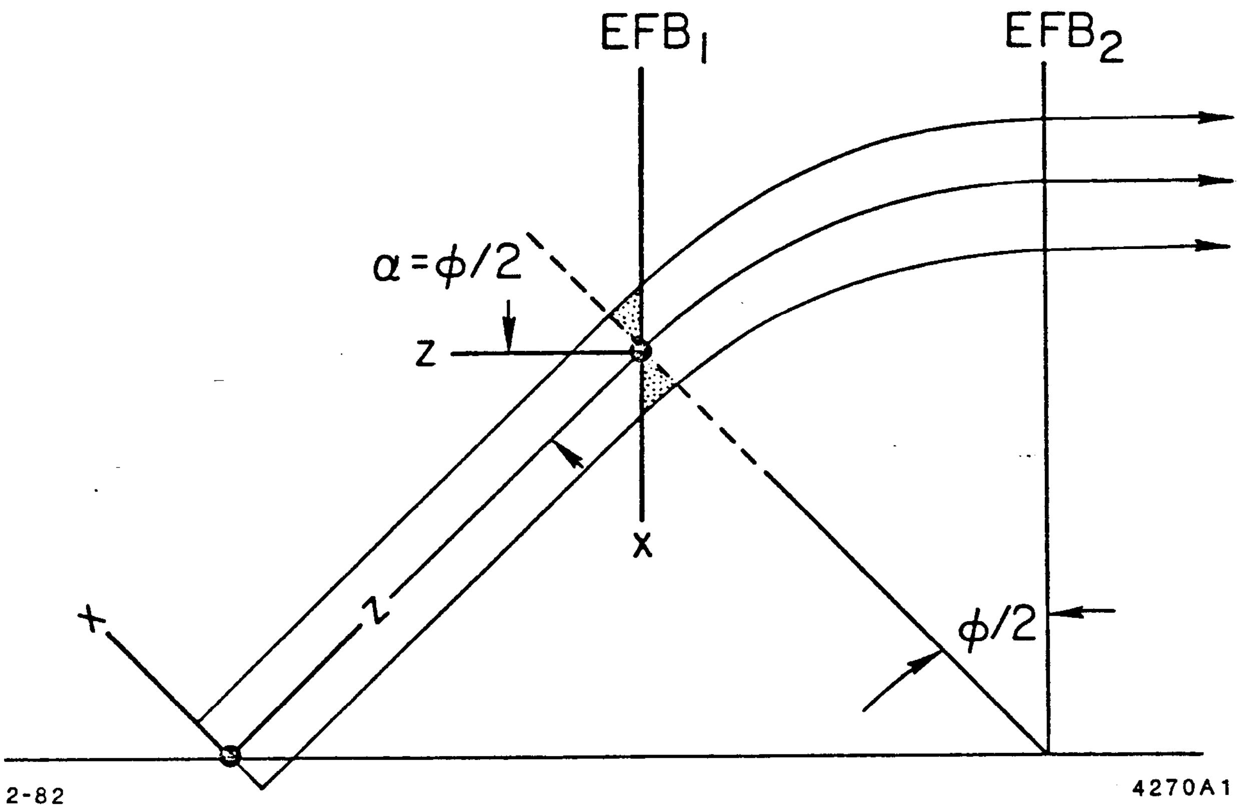

Fig. A2: Optics of the PEP bends and other dipoles having parallel effective field boundaries at entrance and exit. This view represents the entrance half of the dipole shown in Fig. 1.

Table 0: Average magnet parameters used for calculating the transfer coefficients for the PEP standard cell magnets given in Tables 1-4. Parameters are either defined in fig. 1 or the appendices.

	DIPOLE	QUADRUPOLE	SEXTUPOLE	
A =	20.000000	25.000000	25.000000	cm
B =	20.000000	25.000000	25.000000	cm
L =	540.000000	71.634400	24.240000	cm
g =	7.012200	10.005200	11.400000	cm
BF =	0.351354	0.568207	0.556512	T
AH1 =	0.0	0.0	0.0	rad
AH2 =	0.0	0.0	0.0	1/cm
AH3 =	0.0	0.0	0.0	1/cm2
AH4 =	0.0	0.0	0.0	1/cm3
AH5 =	0.0	0.0	0.0	1/cm4
AH6 =	0.0	0.0	0.0	1/cm5
Z11 =	20.000000	25.000000	15.000000	cm
Z12 =	-13.000000	-15.000000	-9.000000	cm
Z21 =	-13.000000	-15.000000	-9.000000	cm
Z22 =	20.000000	25.000000	15.000000	cm
C01 =	0.478959	0.296417	0.176659	
C02 =	1.911289	4.533219	7.153079	
C03 =	-1.185953	-2.270982	-3.113116	
C04 =	1.630554	1.068627	3.444311	
C05 =	-1.082657	-0.036391	-1.976740	
C06 =	0.318111	0.022261	0.540068	
C11 =	0.478959	0.296417	0.176659	
C12 =	1.911289	4.533219	7.153079	
C13 =	-1.185953	-2.270982	-3.113116	
C14 =	1.630554	1.068627	3.444311	
C15 =	-1.082657	-0.036391	-1.976740	
C16 =	0.318111	0.022261	0.540068	
PHI =	1.869270			deg
ALPH=	0.934635			deg
BETA=	0.934635			deg
RAP1=	0.0			1/cm
RAP2=	0.0			1/cm
RW1 =	0.0			cm
RW2 =	0.0			cm
BR1 =	0.0			T
BR2 =	0.0			T
CAT1=	0.0			1/cm2
CAT2=	0.0			1/cm2
XCR1=	0.0			cm
XCR2=	0.0			cm
CFV1=	0.0			1/cm3
CFV2=	0.0			1/cm3
CNN1=	0.0			1/cm4
CNN2=	0.0			1/cm4

Table 1: First order coefficients of the exact transfer functions for the PEP storage ring magnets calculated for their respective standardization currents of 1250,200 and 200A and an energy of E=17.43451GeV. The magnet parameters are given in Table 0. All elements are positive with respect to one another i.e. horizontally focusing.

COEFF.	PEP BEND (BB)	PEP QUAD (QF)	PEP SEXT. (SF)	OCTUPOLE (OF)	PEP BEND (ENERGY LOSS)	UNITS
(1|1)	= 1.00000	0.91592	1.00000	1.00000	1.00000	cm/cm
(1|2)	= 0.57990	0.11712	0.07424	0.07424	0.57990	cm/mr
(1|3)	= 0.0	0.0	0.0	0.0	0.0	cm/cm
(1|4)	= 0.0	0.0	0.0	0.0	0.0	cm/mr
(1|5)	= 0.0	0.0	0.0	0.0	0.0	cm/cm
(1|6)	= 9.4605D-02	0.0	0.0	0.0	9.4605D-02	cm/%
(2|1)	= 0.0	-1.37541	0.0	0.0	0.0	mr/cm
(2|2)	= 1.00000	0.91592	1.00000	1.00000	1.0	mr/mr
(2|3)	= 0.0	0.0	0.0	0.0	0.0	mr/cm
(2|4)	= 0.0	0.0	0.0	0.0	0.0	mr/mr
(2|5)	= 0.0	0.0	0.0	0.0	0.0	mr/cm
(2|6)	= 0.32628	0.0	0.0	0.0	0.32628	mr/%
(3|1)	= 0.0	0.0	0.0	0.0	0.0	cm/cm
(3|2)	= 0.0	0.0	0.0	0.0	0.0	cm/mr
(3|3)	= 0.99943	1.08611	1.00000	1.00000	0.99943	cm/cm
(3|4)	= 0.57998	0.12623	0.07424	0.07424	0.57998	cm/mr
(3|5)	= 0.0	0.0	0.0	0.0	0.0	cm/cm
(3|6)	= 0.0	0.0	0.0	0.0	0.0	cm/%
(4|1)	= 0.0	0.0	0.0	0.0	0.0	mr/cm
(4|2)	= 0.0	0.0	0.0	0.0	0.0	mr/mr
(4|3)	=-1.9518D-03	1.42301	0.0	0.0	-1.9518D-03	mr/cm
(4|4)	= 0.99943	1.08611	1.00000	1.00000	.99943	mr/mr
(4|5)	= 0.0	0.0	0.0	0.0	0.0	mr/cm
(4|6)	= 0.0	0.0	0.0	0.0	0.0	mr/%
(5|1)	=-3.2628D-02	0.0	0.0	0.0	-3.2628D-02	cm/cm
(5|2)	=-9.4605D-03	0.0	0.0	0.0	-9.4606D-03	cm/mr
(5|3)	= 0.0	0.0	0.0	0.0	0.0	cm/cm
(5|4)	= 0.0	0.0	0.0	0.0	0.0	cm/mr
(5|5)	= 1.00000	1.00000	1.00000	1.00000	1.00000	cm/cm
(5|6)	=-9.5834D-04	0.0	0.0	0.0	-9.5835D-04	cm/%
(6|1)	= 0.0	0.0	0.0	0.0	0.0	%/cm
(6|2)	= 0.0	0.0	0.0	0.0	0.0	%/mr
(6|3)	= 0.0	0.0	0.0	0.0	0.0	%/cm
(6|4)	= 0.0	0.0	0.0	0.0	0.0	%/mr
(6|5)	= 0.0	0.0	0.0	0.0	0.0	%/cm
(6|6)	= 1.00000	1.00000	1.00000	1.00000	1.00000	%/%

Table 2: Second order coefficients of the exact transfer function for the PEP storage ring magnets corresponding to the calculation for Table 1.

COEFF.	PEP BEND (BB)	PEP QUAD (QF)	PEP SEXT. (SF)	OCTUPOLE (OF)	PEP BEND (ENERGY LOSS)	UNITS
(1|11) =	0.0	0.0	-2.650Q-03	0.0	0.0	cm/cm2
(1|12) =	3.263Q-05	0.0	-1.886Q-04	0.0	3.263Q-05	cm/cm-mr
(1|13) =	0.0	0.0	0.0	0.0	0.0	cm/cm2
(1|14) =	0.0	0.0	0.0	0.0	0.0	cm/cm-mr
(1|15) =	0.0	0.0	0.0	0.0	0.0	cm/cm2
(1|16) =	5.323Q-06	8.308Q-04	0.0	0.0	5.323Q-06	cm/cm-%
(1|22) =	4.730Q-06	0.0	-3.500Q-06	0.0	4.730Q-06	cm/mr2
(1|23) =	0.0	0.0	0.0	0.0	0.0	cm/mr-cm
(1|24) =	0.0	0.0	0.0	0.0	0.0	cm/mr2
(1|25) =	0.0	0.0	0.0	0.0	0.0	cm/cm-mr
(1|26) =	2.875Q-06	4.471Q-05	0.0	0.0	2.875Q-06	cm/mr-%
(1|33) =	3.183Q-08	0.0	2.650Q-03	0.0	-7.782Q-09	cm/cm2
(1|34) =	-3.261Q-05	0.0	1.886Q-04	0.0	-3.262Q-05	cm/cm-mr
(1|35) =	0.0	0.0	0.0	0.0	0.0	cm/cm2
(1|36) =	0.0	0.0	0.0	0.0	0.0	cm/cm-%
(1|44) =	-1.419Q-05	0.0	3.500Q-06	0.0	-1.419Q-05	cm/mr2
(1|45) =	0.0	0.0	0.0	0.0	0.0	cm/mr-cm
(1|46) =	0.0	0.0	0.0	0.0	0.0	cm/mr-%
(1|55) =	0.0	0.0	0.0	0.0	0.0	cm/cm2
(1|56) =	0.0	0.0	0.0	0.0	0.0	cm/cm-%
(1|66) =	-9.458Q-04	0.0	0.0	0.0	-9.458Q-04	cm/%2
(2|11) =	0.0	0.0	-7.139Q-02	0.0	0.0	mr/cm2
(2|12) =	0.0	0.0	-5.300Q-03	0.0	0.0	mr/cm-mr
(2|13) =	0.0	0.0	0.0	0.0	0.0	mr/cm2
(2|14) =	0.0	0.0	0.0	0.0	0.0	mr/cm-mr
(2|15) =	0.0	0.0	0.0	0.0	0.0	mr/cm2
(2|16) =	0.0	1.352Q-02	0.0	0.0	0.0	mr/cm-%
(2|22) =	-1.631Q-05	0.0	-1.025Q-04	0.0	-1.631Q-05	mr/mr2
(2|23) =	0.0	0.0	0.0	0.0	0.0	mr/mr-cm
(2|24) =	0.0	0.0	0.0	0.0	0.0	mr/mr2
(2|25) =	0.0	0.0	0.0	0.0	0.0	mr/mr-cm
(2|26) =	-5.323Q-06	8.308Q-04	0.0	0.0	-5.323Q-06	mr/mr-%
(2|33) =	-8.893Q-11	0.0	7.139Q-02	0.0	-1.372Q-07	mr/cm2
(2|34) =	3.179Q-08	0.0	5.300Q-03	0.0	2.374Q-08	mr/cm-mr
(2|35) =	0.0	0.0	0.0	0.0	0.0	mr/cm2
(2|36) =	0.0	0.0	0.0	0.0	0.0	mr/cm-%
(2|44) =	-1.630Q-05	0.0	1.025Q-04	0.0	-1.630Q-05	mr/mr2
(2|45) =	0.0	0.0	0.0	0.0	0.0	mr/mr-cm
(2|46) =	0.0	0.0	0.0	0.0	0.0	mr/mr-%
(2|55) =	0.0	0.0	0.0	0.0	0.0	mr/cm2
(2|56) =	0.0	0.0	0.0	0.0	0.0	mr/cm-%
(2|66) =	-3.264Q-03	0.0	0.0	0.0	-3.264Q-03	mr/%2
(3|11) =	0.0	0.0	0.0	0.0	0.0	cm/cm2
(3|12) =	0.0	0.0	0.0	0.0	0.0	cm/cm-mr
(3|13) =	-3.184Q-08	0.0	5.300Q-03	0.0	-3.184Q-08	cm/cm2
(3|14) =	3.261Q-05	0.0	1.886Q-04	0.0	3.261Q-05	cm/cm-mr
(3|15) =	0.0	0.0	0.0	0.0	0.0	cm/cm2
(3|16) =	0.0	0.0	0.0	0.0	0.0	cm/cm-%

(3|22)=	0.0	0.0	0.0	0.0	0.0	cm/mr2
(3|23)=	-3.265Q-05	0.0	1.886Q-04	0.0	-3.265Q-05	cm/mr-cm
(3|24)=	9.455Q-06	0.0	7.000Q-06	0.0	9.455Q-06	cm/mr2
(3|25)=	0.0	0.0	0.0	0.0	0.0	cm/mr-cm
(3|26)=	0.0	0.0	0.0	0.0	0.0	cm/mr-%
(3|33)=	0.0	0.0	0.0	0.0	0.0	cm/cm2
(3|34)=	0.0	0.0	0.0	0.0	0.0	cm/cm-mr
(3|35)=	0.0	0.0	0.0	0.0	0.0	cm/cm2
(3|36)=	5.998Q-06	-8.713Q-04	0.0	0.0	5.998Q-06	cm/cm-%
(3|44)=	0.0	0.0	0.0	0.0	0.0	cm/mr2
(3|45)=	0.0	0.0	0.0	0.0	0.0	cm/mr-cm
(3|46)=	1.393Q-06	-4.636Q-05	0.0	0.0	1.393Q-06	cm/mr-%
(3|55)=	0.0	0.0	0.0	0.0	0.0	cm/cm2
(3|56)=	0.0	0.0	0.0	0.0	0.0	cm/cm-%
(3|66)=	0.0	0.0	0.0	0.0	0.0	cm/%2
(4|11)=	0.0	0.0	0.0	0.0	0.0	mr/cm2
(4|12)=	0.0	0.0	0.0	0.0	0.0	mr/cm-mr
(4|13)=	0.0	0.0	0.14280	0.0	0.0	mr/cm2
(4|14)=	-3.184Q-08	0.0	5.300Q-03	0.0	-3.184Q-08	mr/cm-mr
(4|15)=	0.0	0.0	0.0	0.0	0.0	mr/cm2
(4|16)=	0.0	0.0	0.0	0.0	0.0	mr/cm-%
(4|22)=	0.0	0.0	0.0	0.0	0.0	mr/mr2
(4|23)=	3.030Q-14	0.0	5.300Q-03	0.0	8.805Q-10	mr/mr-cm
(4|24)=	3.263Q-05	0.0	2.049Q-04	0.0	3.263Q-05	mr/mr2
(4|25)=	0.0	0.0	0.0	0.0	0.0	mr/mr-cm
(4|26)=	0.0	0.0	0.0	0.0	0.0	mr/mr-%
(4|33)=	0.0	0.0	0.0	0.0	0.0	mr/cm2
(4|34)=	0.0	0.0	0.0	0.0	0.0	mr/cm-mr
(4|35)=	0.0	0.0	0.0	0.0	0.0	mr/cm2
(4|36)=	3.903Q-05	-1.447Q-02	0.0	0.0	3.903Q-05	mr/cm-%
(4|44)=	0.0	0.0	0.0	0.0	0.0	mr/mr2
(4|45)=	0.0	0.0	0.0	0.0	0.0	mr/mr-cm
(4|46)=	1.665Q-05	-8.713Q-04	0.0	0.0	1.665Q-05	mr/mr-%
(4|55)=	0.0	0.0	0.0	0.0	0.0	mr/cm2
(4|56)=	0.0	0.0	0.0	0.0	0.0	mr/cm-%
(4|66)=	0.0	0.0	0.0	0.0	0.0	mr/%2
(5|11)=	0.0	-4.635Q-05	0.0	0.0	0.0	cm/cm2
(5|12)=	-5.323Q-07	7.885Q-05	0.0	0.0	-5.323Q-07	cm/cm-mr
(5|13)=	0.0	0.0	0.0	0.0	0.0	cm/cm2
(5|14)=	0.0	0.0	0.0	0.0	0.0	cm/cm-mr
(5|15)=	0.0	0.0	0.0	0.0	0.0	cm/cm2
(5|16)=	0.0	0.0	0.0	0.0	0.0	cm/cm-%
(5|22)=	-2.900Q-04	-5.645Q-05	-3.712Q-05	-3.712Q-05	-2.900Q-04	cm/mr2
(5|23)=	0.0	0.0	0.0	0.0	0.0	cm/cm-mr
(5|24)=	0.0	0.0	0.0	0.0	0.0	cm/mr2
(5|25)=	0.0	0.0	0.0	0.0	0.0	cm/cm-mr
(5|26)=	-9.460Q-05	0.0	0.0	0.0	-9.460Q-05	cm/mr-%
(5|33)=	-9.767Q-07	-4.893Q-05	0.0	0.0	-9.763Q-07	cm/cm2
(5|34)=	5.322Q-07	-9.159Q-05	0.0	0.0	5.323Q-07	cm/cm-mr
(5|35)=	0.0	0.0	0.0	0.0	0.0	cm/cm2
(5|36)=	0.0	0.0	0.0	0.0	0.0	cm/cm-%
(5|44)=	-2.899Q-04	-6.557Q-05	-3.712Q-05	-3.712Q-05	-2.899Q-04	cm/mr2
(5|45)=	0.0	0.0	0.0	0.0	0.0	cm/cm-mr
(5|46)=	0.0	0.0	0.0	0.0	0.0	cm/mr-%
(5|55)=	0.0	0.0	0.0	0.0	0.0	cm/cm2
(5|56)=	0.0	0.0	0.0	0.0	0.0	cm/cm-%
(5|66)=	-1.058Q-06	0.0	0.0	0.0	-1.058Q-06	cm/%2

Table 3: Third-order, transverse coefficients of the exact transfer function for the PEP storage ring magnets corresponding to the calculations for Tables 1&2 based on the assumption of perfect mechanical symmetry of each magnet type about the horizontal bending plane, i.e. all unlisted permutations can be taken to be zero.

COEFF.	PEP BEND (BB)	PEP QUAD (QF)	PEP SEXT. (SF)	OCTUPOLE (OP)	PEP BEND (ENERGY LOSS)	UNITS
(1|111) =	0.0	-8.182D-08	1.351D-06	-4.649D-04	0.0	cm/cm3
(1|112) =	0.0	-5.556D-07	1.391D-07	-4.963D-05	0.0	cm/cm2-mr
(1|116) =	0.0	0.0	2.650D-05	0.0	0.0	cm/cm2-%
(1|122) =	2.661D-10	-1.521D-07	4.779D-09	-1.842D-06	2.661D-10	cm/cm-mr2
(1|126) =	-3.634D-17	0.0	1.886D-06	0.0	-1.746D-15	cm/cm-mr-%
(1|133) =	-1.446D-15	-1.140D-06	1.351D-06	1.395D-03	-2.238D-12	cm/cm3
(1|134) =	1.557D-12	-2.447D-07	-1.886D-06	9.925D-05	1.334D-13	cm/cm2-mr
(1|144) =	-7.981D-10	-9.312D-08	2.242D-09	1.842D-06	-7.981D-10	cm/cm-mr2
(1|166) =	-5.324D-08	-8.208D-06	0.0	0.0	-5.324D-08	cm/cm-%2
(1|222) =	1.932D-07	3.212D-08	2.480D-08	1.058D-09	1.932D-07	cm/mr3
(1|226) =	1.418D-07	0.0	3.500D-08	0.0	1.418D-07	cm/mr2-%
(1|233) =	4.881D-09	-7.178D-07	2.381D-08	4.963D-05	4.875D-09	cm/cm2-mr
(1|234) =	1.733D-09	-1.110D-08	2.538D-09	3.684D-06	1.733D-09	cm/cm-mr2
(1|244) =	1.380D-10	-3.358D-09	5.521D-11	7.107D-08	1.380D-10	cm/mr3
(1|266) =	3.159D-09	-4.430D-07	0.0	0.0	3.159D-09	cm/mr-%2
(1|336) =	-1.591D-10	0.0	-2.650D-05	0.0	-1.593D-10	cm/cm2-%
(1|346) =	3.262D-07	0.0	-1.886D-06	0.0	3.262D-07	cm/cm-mr-%
(1|446) =	1.420D-07	0.0	-3.500D-08	0.0	1.420D-07	cm/mr2-%
(1|666) =	9.459D-06	0.0	0.0	0.0	9.459D-06	cm/%3
(2|111) =	0.0	-8.227D-06	4.425D-05	-1.253D-02	0.0	mr/cm3
(2|112) =	0.0	-1.037D-06	4.637D-06	-1.395D-03	0.0	mr/mr-cm2
(2|116) =	0.0	-2.222D-10	7.139D-04	0.0	0.0	mr/cm2-%
(2|122) =	0.0	-3.229D-08	1.618D-07	-5.393D-05	0.0	mr/mr2-cm
(2|126) =	0.0	0.0	5.300D-05	0.0	0.0	mr/cm-mr-%
(2|133) =	0.0	-2.774D-05	4.425D-05	3.758D-02	0.0	mr/cm3
(2|134) =	-2.888D-15	-6.189D-06	-4.961D-05	2.790D-03	-4.477D-12	mr/cm2-mr
(2|144) =	5.186D-13	-1.627D-06	7.650D-08	5.393D-05	3.873D-13	mr/mr2-cm
(2|166) =	0.0	-1.326D-04	0.0	0.0	0.0	mr/cm-%2
(2|222) =	2.661D-10	-2.823D-08	1.897D-09	-7.204D-07	2.662D-10	mr/mr3
(2|226) =	1.633D-07	-5.235D-14	1.025D-06	0.0	1.633D-07	mr/mr2-%
(2|233) =	1.911D-12	-2.234D-07	7.678D-07	1.395D-03	2.191D-12	mr/mr-cm2
(2|234) =	-1.950D-09	1.151D-06	8.528D-08	1.079D-04	-1.950D-09	mr/mr2-cm
(2|244) =	-8.321D-10	-1.165D-08	1.897D-09	2.161D-06	-8.321D-10	mr/mr3
(2|266) =	1.065D-07	-8.208D-06	0.0	0.0	1.065D-07	mr/mr-%2
(2|336) =	2.443D-12	0.0	-7.139D-04	0.0	3.139D-12	mr/cm2-%
(2|346) =	-1.272D-09	0.0	-5.300D-05	0.0	-1.272D-09	mr/mr-cm-%
(2|446) =	1.628D-07	0.0	-1.025D-06	0.0	1.628D-07	mr/mr2-%
(2|666) =	3.265D-05	0.0	0.0	0.0	3.265D-05	mr/%3
(3|113) =	0.0	-1.004D-06	1.351D-06	1.395D-03	0.0	cm/cm3
(3|114) =	-5.195D-13	6.905D-07	2.380D-08	4.963D-05	-5.195D-13	cm/cm2-mr
(3|123) =	-5.195D-13	-2.360D-07	1.153D-07	9.925D-05	-5.051D-13	cm/cm2-mr
(3|124) =	5.317D-10	-1.018D-08	2.538D-09	3.684D-06	5.317D-10	cm/cm-mr2
(3|136) =	6.368D-10	0.0	-5.300D-05	0.0	6.368D-10	cm/cm2-%
(3|146) =	3.695D-10	0.0	-1.886D-06	0.0	3.695D-10	cm/cm-mr-%
(3|223) =	-9.002D-10	7.726D-08	2.242D-09	1.842D-06	-9.002D-10	cm/mr2-cm
(3|224) =	2.060D-10	2.459D-09	5.521D-11	7.107D-08	2.060D-10	cm/mr3
(3|236) =	3.266D-07	0.0	-1.886D-06	0.0	3.266D-07	cm/mr-cm-%
(3|246) =	9.479D-08	0.0	-7.000D-08	0.0	9.479D-08	cm/mr2-%
(3|333) =	-1.089D-07	2.445D-08	1.351D-06	-4.649D-04	-1.089D-07	cm/cm3
(3|334) =	-1.671D-08	8.665D-07	1.391D-07	-4.963D-05	-1.671D-08	cm/cm2-mr
(3|344) =	-5.962D-09	1.902D-07	4.779D-09	-1.842D-06	-5.962D-09	cm/cm-mr2
(3|366) =	-6.334D-08	8.815D-06	0.0	0.0	-6.334D-08	cm/cm-%2
(3|444) =	1.921D-07	5.010D-08	2.480D-08	1.058D-05	1.921D-07	cm/mr3
(3|466) =	-5.452D-09	4.678D-07	0.0	0.0	-5.452D-09	cm/mr-%2
(4|113) =	0.0	-2.363D-05	4.425D-05	3.758D-02	0.0	mr/cm3
(4|114) =	0.0	3.381D-08	7.678D-07	1.395D-03	0.0	mr/cm2-mr
(4|123) =	0.0	-5.861D-06	3.869D-06	2.790D-03	0.0	mr/cm2-mr
(4|124) =	-5.195D-13	-1.618D-06	8.528D-08	1.079D-04	-5.051D-13	mr/cm-mr2
(4|136) =	0.0	0.0	-1.428D-03	0.0	0.0	mr/cm2-%
(4|146) =	6.368D-10	0.0	-5.300D-05	0.0	6.368D-10	mr/cm-mr-%
(4|223) =	-5.857D-09	1.179D-06	7.650D-08	5.393D-05	-5.857D-09	mr/mr2-cm
(4|224) =	-1.433D-09	-1.113D-08	1.897D-09	2.161D-06	-1.433D-09	mr/mr3
(4|236) =	-2.229D-09	0.0	-5.300D-05	0.0	-2.229D-09	mr/mr-cm-%
(4|246) =	-8.813D-10	0.0	-2.049D-06	0.0	-8.813D-10	mr/mr2-%
(4|333) =	-3.753D-07	-8.895D-06	4.425D-05	-1.253D-02	-3.753D-07	mr/cm3
(4|334) =	-3.265D-07	-1.107D-06	4.637D-06	-1.395D-03	-3.265D-07	mr/cm2-mr
(4|344) =	-1.737D-07	-3.431D-08	1.618D-07	-5.393D-05	-1.737D-07	mr/cm-mr2
(4|366) =	-5.856D-07	1.471D-04	0.0	0.0	-5.856D-07	mr/cm-%2
(4|444) =	-3.254D-08	2.848D-08	1.897D-09	-7.204D-07	-3.254D-08	mr/mr3
(4|466) =	-2.233D-07	8.815D-06	0.0	0.0	-2.233D-07	mr/mr-%2

Table 4: Some higher order coefficients of the exact transfer function corresponding to the calculations for tables 1,2&3.

COEFF.	PEP BEND (BE)	PEP QUAD (QF)	PEP SEXT. (SF)	OCTUPOLE (OF)	PEP BEND (ENERGY LOSS)	UNITS
(1|1111)=	0.0	0.0	-5.941D-10	0.0	0.0	cm/cm4
(1|1122)=	0.0	0.0	-3.979D-09	0.0	0.0	cm/cm2-mr2
(1|1133)=	0.0	0.0	-5.444D-09	0.0	0.0	cm/cm4
(1|1144)=	1.693D-17	0.0	-3.969D-09	0.0	4.248D-18	cm/cm2-mr2
(1|1166)=	0.0	0.0	-2.650D-07	0.0	0.0	cm/cm2-%2
(1|2222)=	-3.912D-13	0.0	-7.584D-12	0.0	-3.912D-13	cm/mr4
(1|2233)=	5.575D-14	0.0	1.331D-09	0.0	-2.302D-14	cm/cm2-mr2
(1|2244)=	-1.658D-11	0.0	8.666D-12	0.0	-1.658D-11	cm/mr4
(1|2266)=	-1.417D-09	0.0	-3.500D-10	0.0	-1.417D-09	cm/mr2-%2
(1|3333)=	4.596D-11	0.0	-1.047D-10	0.0	-1.235D-09	cm/cm4
(1|3344)=	-6.578D-12	0.0	1.325D-09	0.0	-1.206D-11	cm/cm2-mr2
(1|3366)=	-4.776D-12	0.0	2.650D-07	0.0	-4.776D-12	cm/cm2-%2
(1|4444)=	-3.887D-12	0.0	6.213D-12	0.0	-3.941D-12	cm/mr4
(1|4466)=	-1.423D-09	0.0	3.500D-10	0.0	-1.423D-09	cm/mr2-%2
(1|5555)=	0.0	0.0	0.0	0.0	0.0	cm/cm4
(1|6666)=	-9.462D-08	0.0	0.0	0.0	-9.462D-08	cm/%4
(2|1111)=	0.0	0.0	-2.135D-08	0.0	0.0	mr/cm4
(2|1122)=	0.0	0.0	-3.583D-08	0.0	0.0	mr/mr2-cm2
(2|1133)=	0.0	0.0	-1.275D-07	0.0	0.0	mr/cm4
(2|1144)=	-2.364D-20	0.0	-3.624D-08	0.0	-3.651D-17	mr/cm2-mr2
(2|1166)=	0.0	0.0	-7.139D-06	0.0	0.0	mr/cm2-%2
(2|2222)=	-6.804D-12	0.0	-1.195D-10	0.0	-6.804D-12	mr/mr4
(2|2233)=	-1.611D-16	0.0	1.065D-07	0.0	-1.373D-13	mr/cm2-mr2
(2|2244)=	4.079D-11	0.0	5.087D-10	0.0	4.079D-11	mr/mr4
(2|2266)=	-1.637D-09	0.0	-1.025D-08	0.0	-1.637D-09	mr/mr2-%2
(2|3333)=	1.372D-10	0.0	-3.847D-09	0.0	-4.329D-09	mr/cm4
(2|3344)=	1.439D-10	0.0	1.070D-07	0.0	8.397D-11	mr/cm2-mr2
(2|3366)=	-6.790D-14	0.0	7.139D-06	0.0	-8.228D-14	mr/cm2-%2
(2|4444)=	1.649D-12	0.0	2.219D-10	0.0	-7.727D-13	mr/mr4
(2|4466)=	-1.622D-09	0.0	1.025D-08	0.0	-1.622D-09	mr/mr2-%2
(2|5555)=	0.0	0.0	0.0	0.0	0.0	mr/cm4
(2|6666)=	-3.267D-07	0.0	0.0	0.0	-3.267D-07	mr/%4

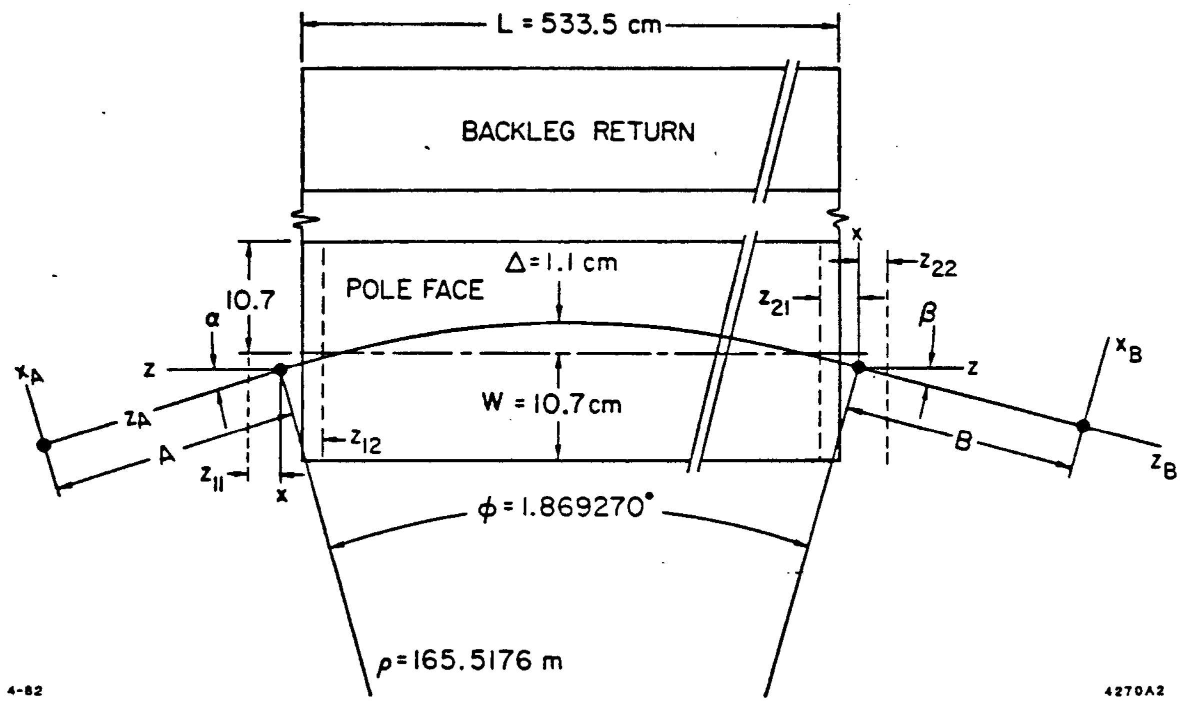

Fig. 1: Cross-section through the median plane of a standard PEP dipole showing the various coordinate systems used for ray tracing. L is the average bend angle in degrees i.e. the average dipole strength. All magnet definition is done in terms of the effective field boundary coordinate systems at entrance and exit.

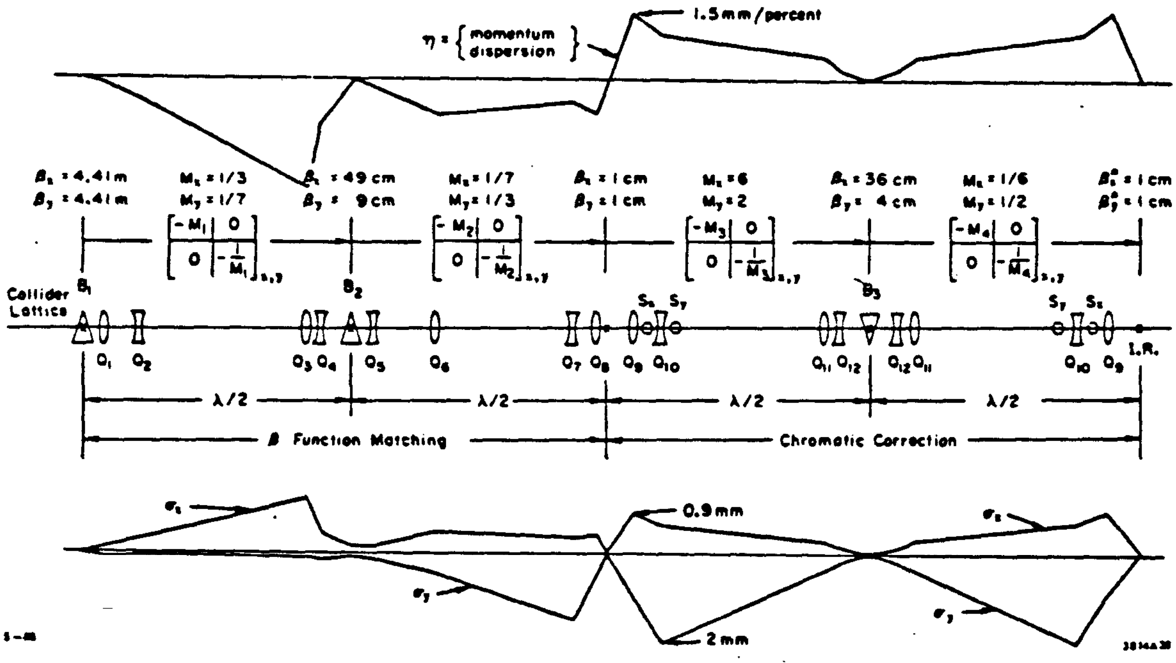

Fig. 2: Optical layout of the FFS showing the dispersion function (η_x) and betatron amplitudes ($\beta_{x,y}$). This is described in detail in Ref. 10.

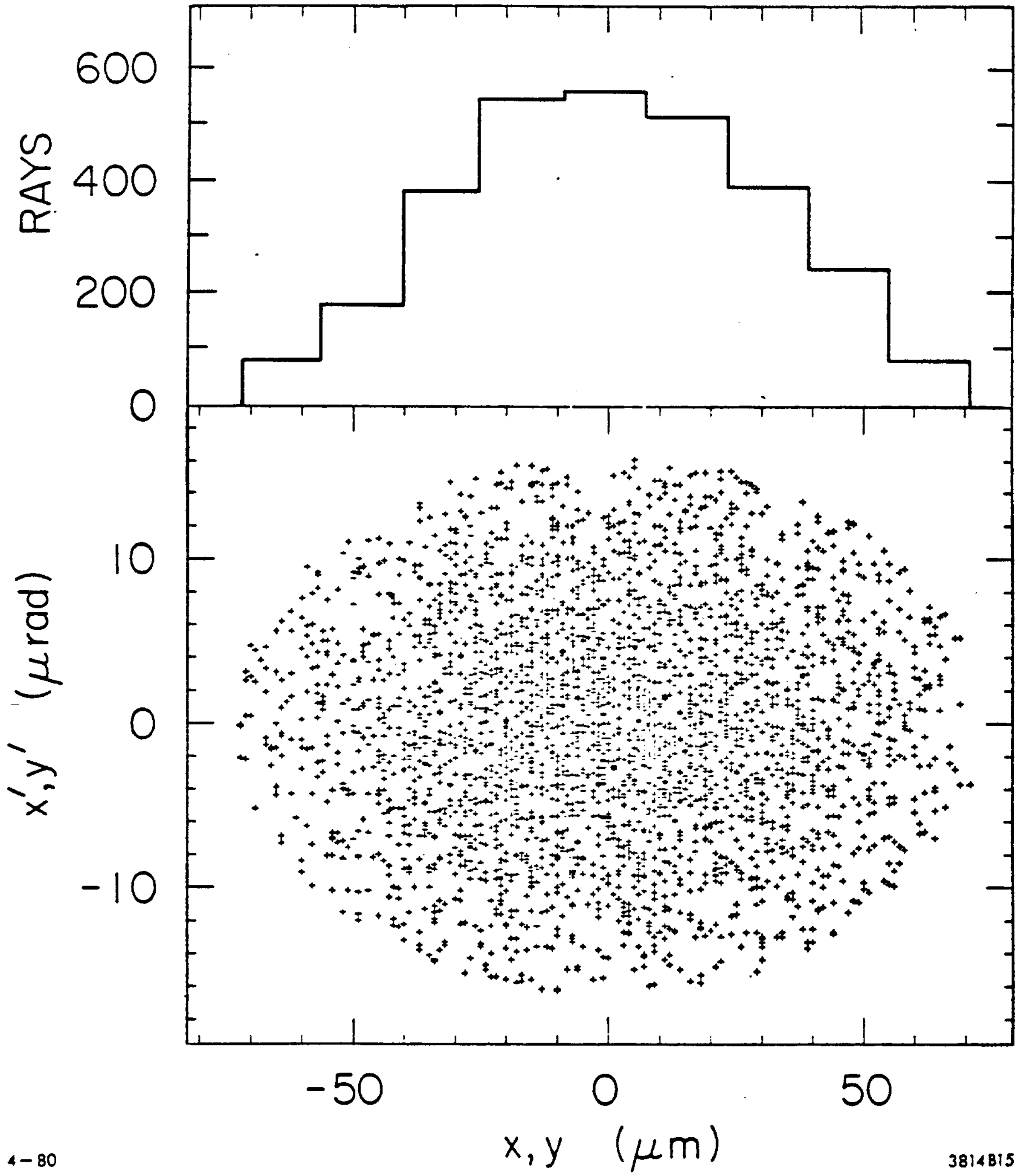

Fig. 3: Scatter plot of 3000 randomly generated rays at the input of the FFS.

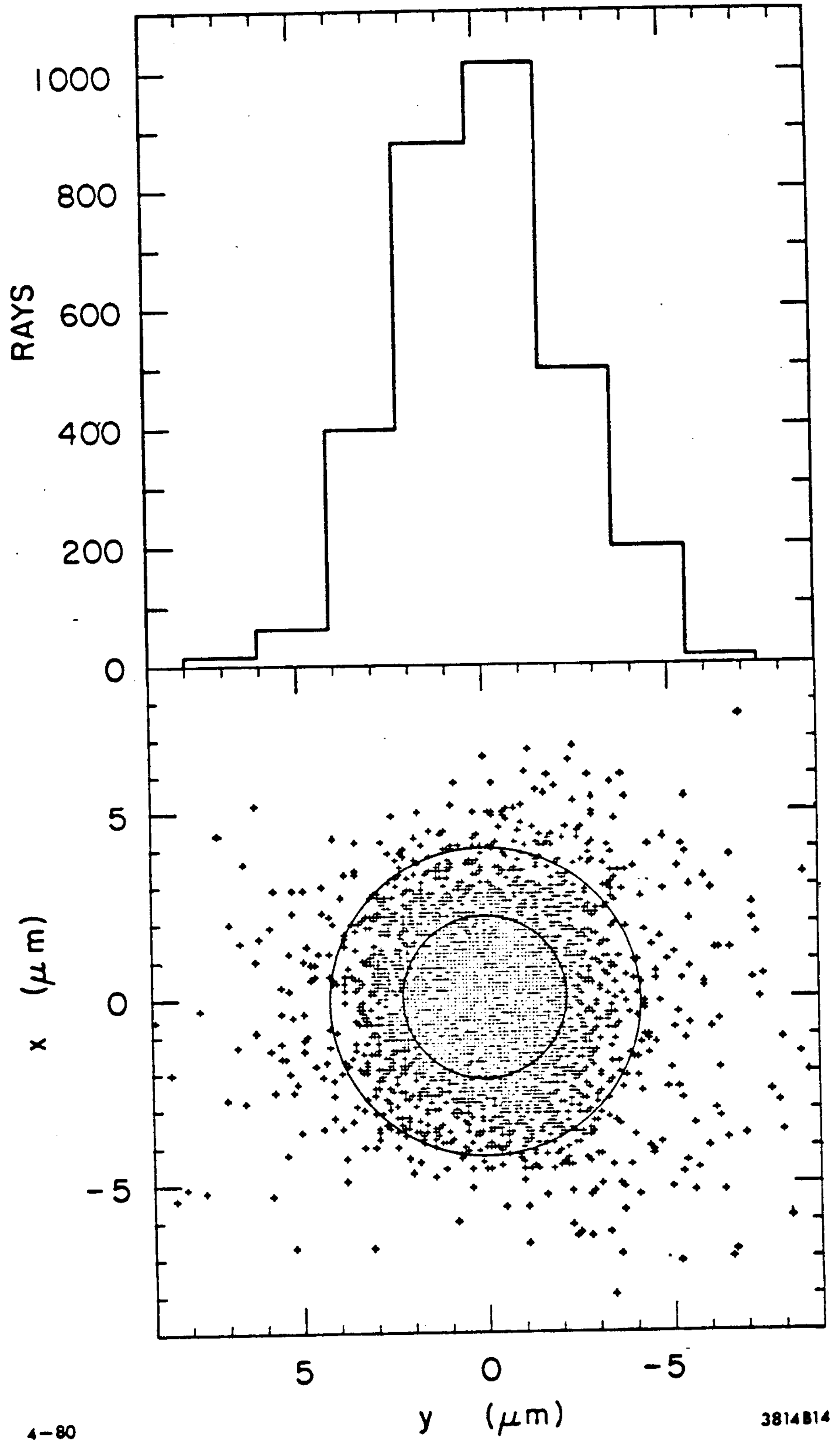

Fig. 4: Scatter plot of 3000 random rays traced through the FFS. The circles are drawn with radii of one and two standard deviations which contain more than 50% and 90% of the beam, respectively.

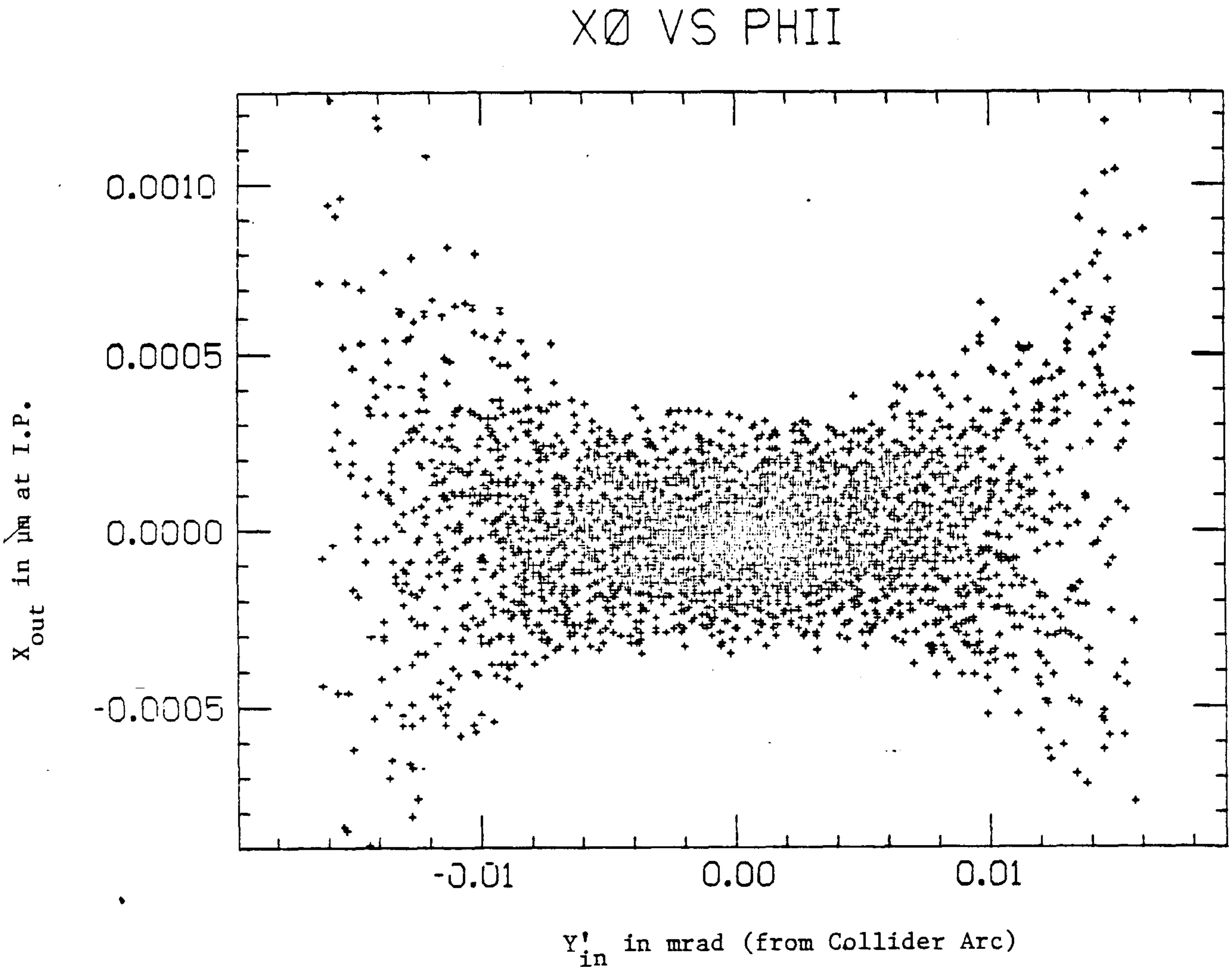

Fig. 5: Results of ray-tracing showing the correlation between the output horizontal position (x_o) at the IR and input vertical angle (y'_i) from the collider lattice.

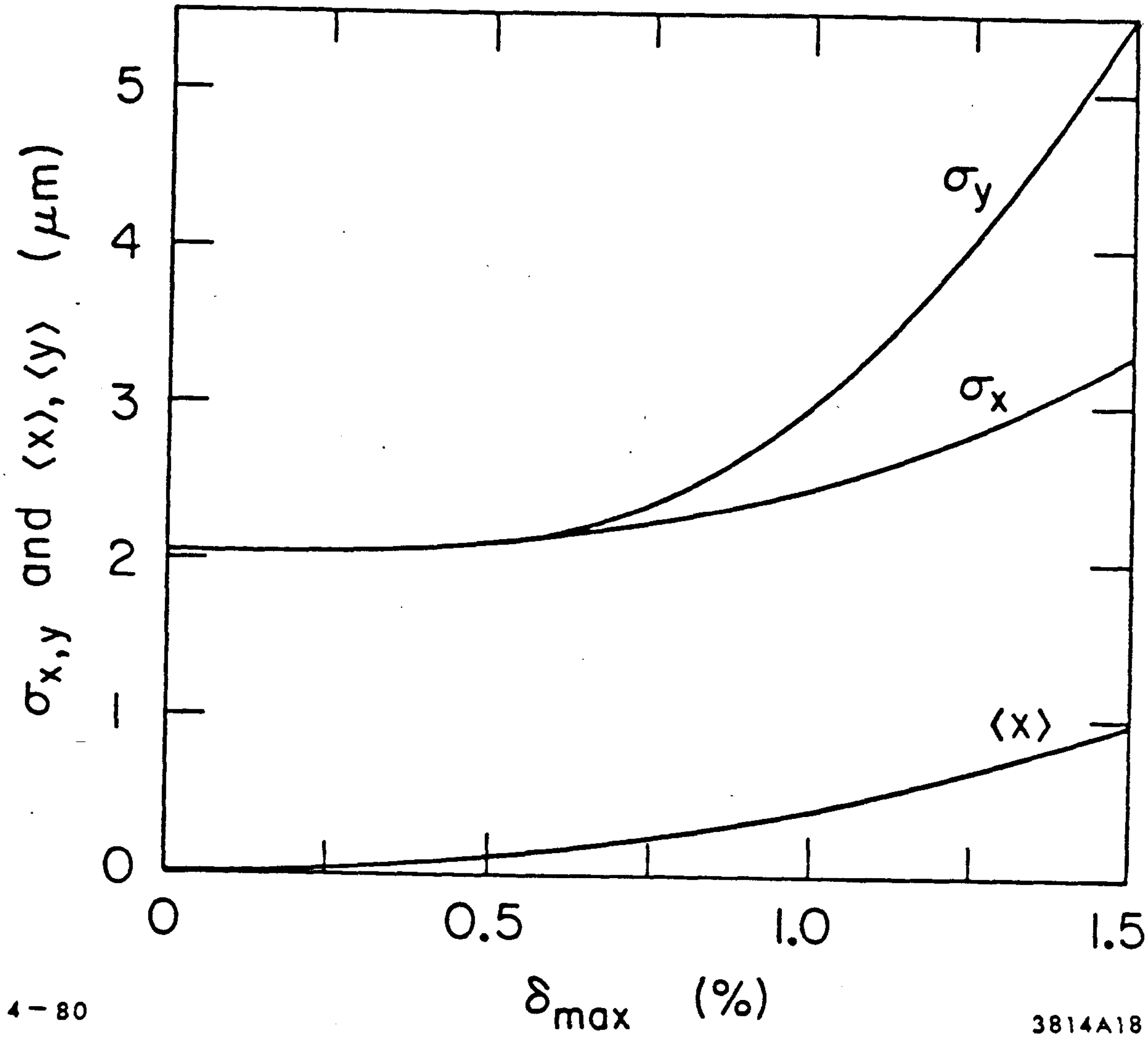

Fig. 6: Predicted beam position and spread at the interaction region as a function of maximum incident momentum δ_{max}. The mean value of vertical position <y> is very small because there are no vertical bending magnets in the system, i.e., $(y|\delta^n)$ are negligible so long as there are no significant rotational misalignments.

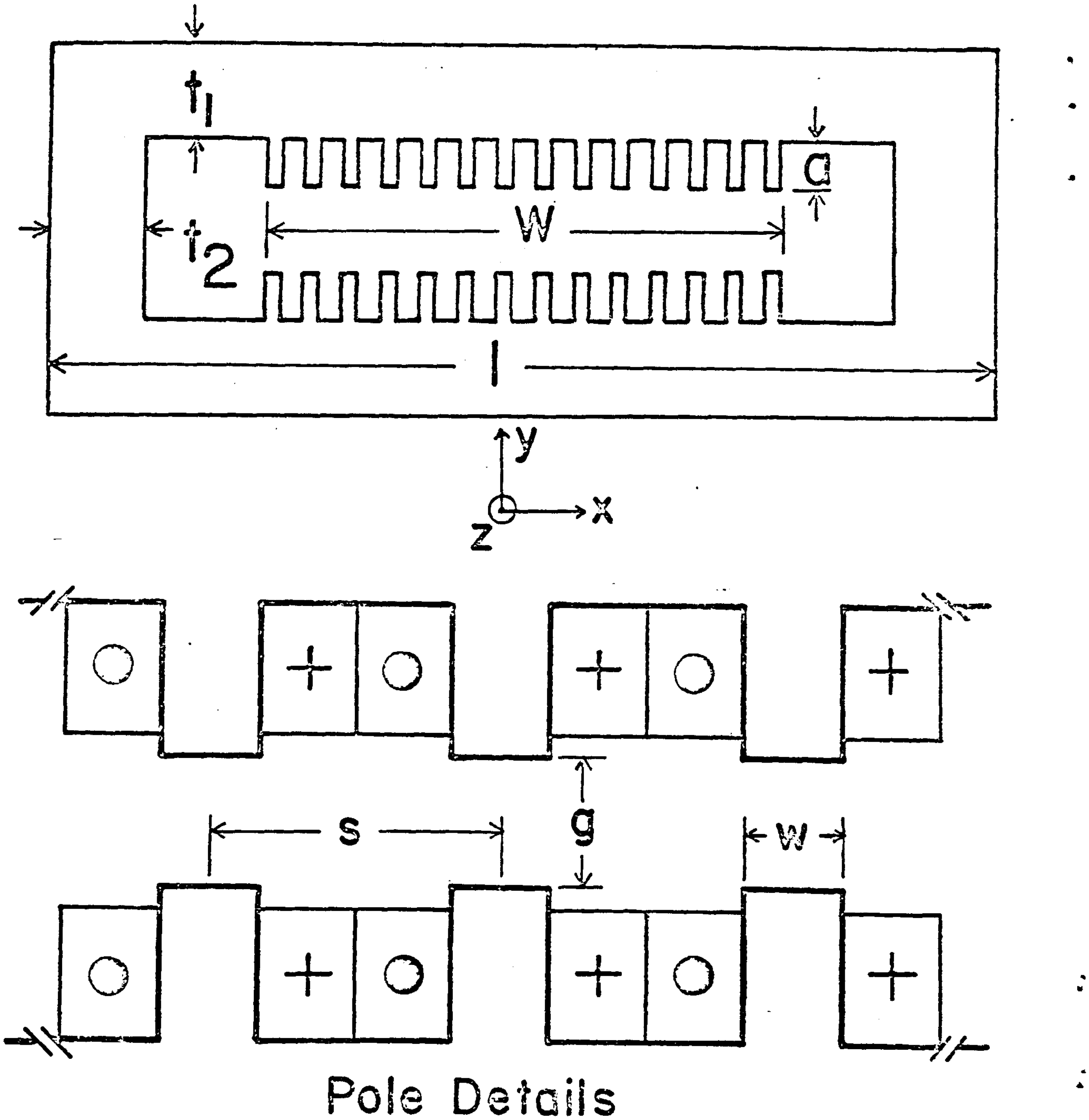

Fig. 7: (Top) Transverse cross section of the iron geometry of an 'active' or variable corrective element capable of an arbitrary number of simultaneous multipole fields. (Bottom) Pole-coil configuration. Further details are available in Ref. 11.

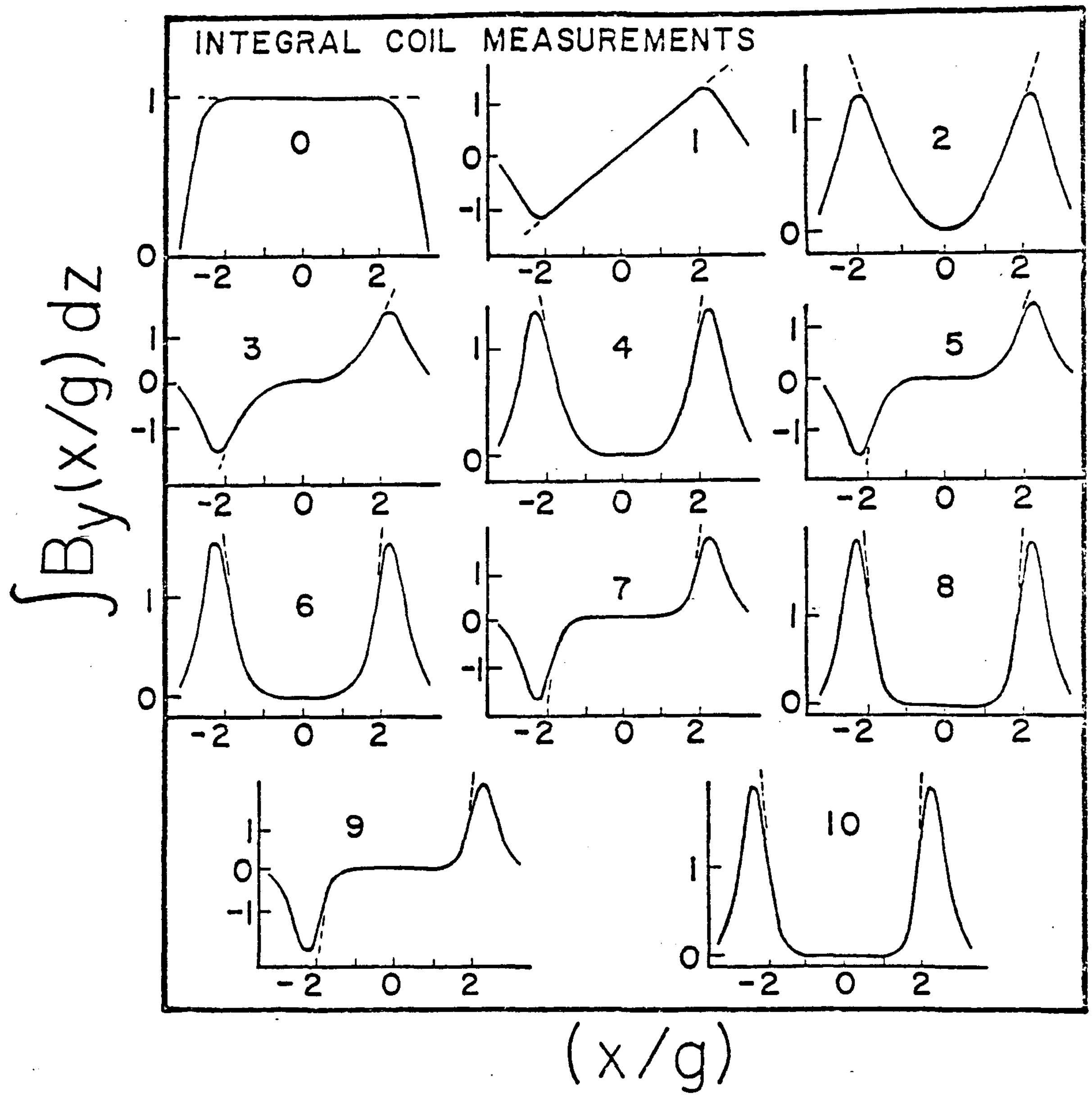

Fig. 8: The solid lines are integral field measurements of the magnet shown in Fig. 7 and the dashed lines are curves of pure multi-polarity $(x/g)^i$ where i is the harmonic number given by each curve.

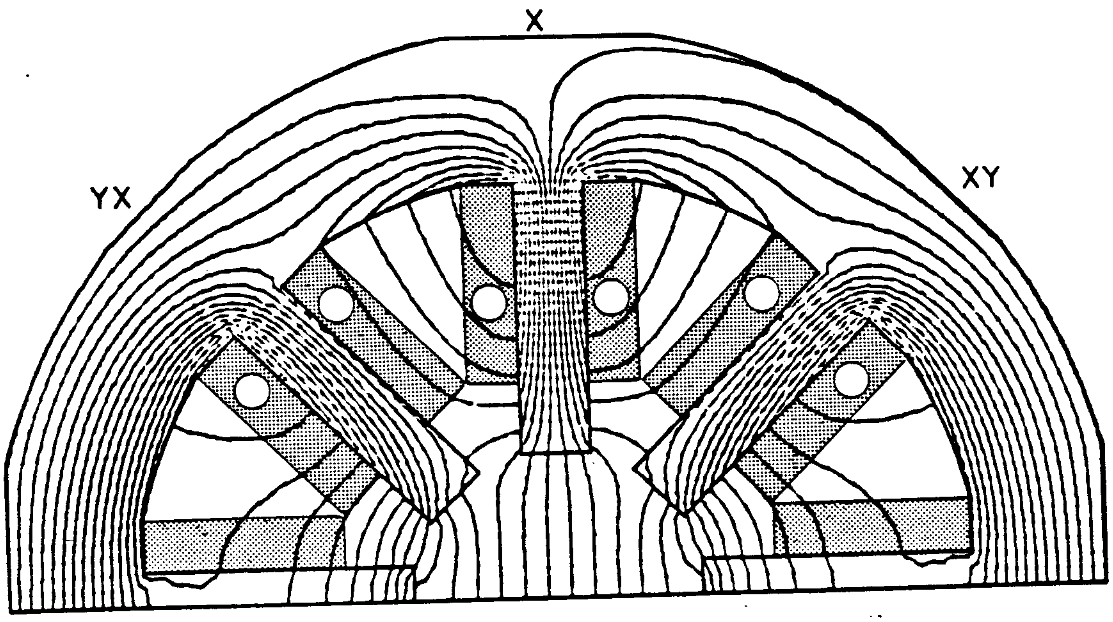

Fig. 9: Transverse cross section of the upper half of a POISSON simultation for the general purpose magnet of Ref. 12. The coils are excited here to give a dipole distribution.

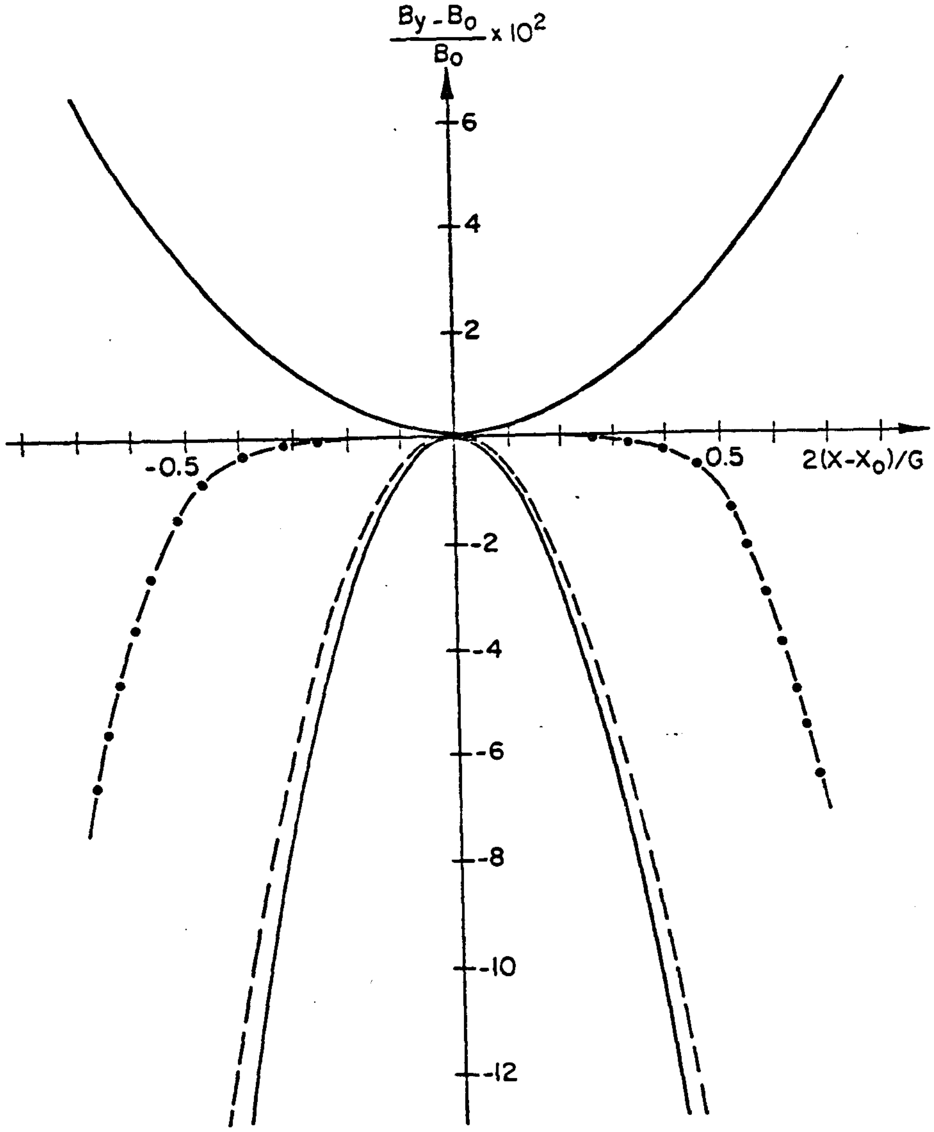

Fig. 10: Results of POISSON calculations for different types and geometries of dipole magnets compared to the dipole field of the 8-pole magnet shown as the dot-dash line. The upper curve is representative of a window-frame for the same aperture and the lower curves of H-magnets as described in Ref. 12.

Figure Captions

Fig. 1: Cross-section through the median plane of a standard PEP dipole showing the various coordinate systems used for ray tracing. L is the average bend angle in degrees i.e. the average dipole strength. All magnet definition is done in terms of the effective field boundary coordinate systems at entrance and exit.

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Fig. 8: The solid lines are integral field measurements of the magnet shown in Fig. 7 and the dashed lines are curves of pure multipolarity $(x/g)^i$ where i is the harmonic number given by each curve.

Fig. 9: Transverse cross section of the upper half of a POISSON simultation for the general purpose magnet of Ref. 12. The coils are excited here to give a dipole distribution.

Fig. 10: Results of POISSON calculations for different types and geometries of dipole magnets compared to the dipole field of the 8-pole magnet shown as the dot-dash line. The upper curve is representative of a window-frame for the same aperture and the lower curves of H-magnets as described in Ref. 12.

Fig. A1: Dipole, quadrupole and sextupole field distributions normalized to their pure harmonic, central field values versus distance along the longitudinal axis normalized to their respective gap openings. The dots (•), pluses (+) and circles (o) are measured field data for the PEP standard bends, quads and sextupoles. The solid curves are fits corresponding to the coefficients of Table A1.

Fig. A2: Optics of the PEP bends and other dipoles having parallel effective field boundaries at entrance and exit.