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BREAKING OF SUPERSYMMETRY AT INTERMEDIATE ENERGY*

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ABSTRACT

We consider the proposal that supersymmetry is broken at a scale μ midway between the Planck scale M and the usual weak scale. We show how a phenomenological explicitly softly broken supersymmetric theory can emerge below scale μ. The characteristic scale for the explicit supersymmetry breaking is of order $\alpha\mu^2/M$. Identifying this with the weak scale ~250 GeV gives $\mu \sim 10^{12}$ GeV.

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1. SUPERSYMMETRY AND THE GAUGE HIERARCHY PROBLEM

The present theory of elementary particles, SU(3)×SU(2)×U(1), contains some 13 independent field multiplets and about 20 free dimensionless parameters. It is widely believed that this theory is at most a low energy remnant of a more symmetrical theory which is manifest at some very high energy, possibly the Planck scale M. Perhaps the most surprising feature of such a theory is the very existence of a low energy world characterized by masses some 17 orders of magnitude smaller than M.[1]

In general such large ratios of scales are not stable. Parameters may be adjusted to obtain these ratios in the classical Lagrangian but in general radiative corrections will upset the delicate adjustments, leading to an order by order readjustment of fundamental parameters to many decimal places.

The only known remedy for this unnatural situation is to have a symmetry which can prevent radiative corrections from spoiling the hierarchy. This can occur if a symmetry prevents some mass term from occurring. For example chiral and gauge symmetries can prevent fermion and gauge boson masses. If such symmetries are violated by very small dimensionless parameters, the resulting masses will remain small. This idea leads to a fundamental requirement of naturalness: For every quantity which almost vanishes, a symmetry should exist which, if unbroken, would require that quantity to exactly vanish. The nonvanishing is caused by small dimensionless parameters which break the symmetry. Although this mechanism does not explain the smallness of such quantities it does provide a framework in which the smallness is stable against radiative corrections.

In the current "standard" theory the masses of quarks, leptons and gauge bosons are all proportional to a single mass parameter $\sim 10^2$ GeV which can be identified with the quadratic (mass) terms in the scalar Higgs potential. Unfortunately the theory contains no symmetry which potentially could keep this scale zero. This fact manifests itself in quadratically divergent radiative corrections to the mass of the Higgs field.[2]

Two ways out have been proposed, both of which require new physics in the TeV region. In one scheme the Higgs scalars are replaced by dynamically bound composites.[2,3] The other scheme introduces supersymmetry (S.S.) in order to control the Higgs mass parameter.[4,5] Indeed, S.S.[6] is the only known symmetry which can keep a scalar mass zero in the presence of interactions. Roughly speaking supersymmetry introduces partners which cancel quadratic divergences in the Higgs mass. Typically the radiative corrections to the Higgs $(\text{mass})^2$ will be $\sim\alpha$ times the typical splitting within a supermultiplet. For example the graph shown in Fig. 1 will be cancelled by a second graph in which the fermionic partners of H and W circulate in the loop. The cancellation is exact in the limit in which the fermions are equal in mass to their bosonic partners. More generally

$$\delta m_H^2 \sim \alpha\, \Delta m^2 \qquad (1.1)$$

where Δm^2 is of order of the supersplitting.

Evidently if the theory is to be free of unnatural adjustments we would want δm_H^2 to be no bigger in order of magnitude than m_H^2 itself. This requires

$$\Delta m \lesssim \frac{1}{\sqrt{\alpha}} \times 100 \text{ GeV} \sim \text{a few TeV} \quad . \qquad (1.2)$$

The obvious conclusion is that the scale of S.S. breaking ought to be of the same general order of magnitude as the weak scale.

In this paper we will argue that the scale for spontaneous breaking of S.S. can be many orders of magnitude higher than the weak scale if the S.S. breaking mechanism is in some sense distant from the ordinary degrees of freedom. Indeed we shall show that the fundamental spontaneous S.S. breaking scale can be as large as $\sim 10^{12}$ GeV without inducing a corresponding splitting among the superpartners responsible for keeping m_H^2 small.

The possibility of supersymmetry breaking at an intermediate scale was raised by Witten[7] and Banks.[8] Recent models by Dine and Fischler,[9] Dimopoulos and Raby[10] and Barbieri, Ferrara and Nanopoulos[11] are also of this type. Many of the features we discuss are also evident in the $M \gg \mu$ limits of the models of Alvarez-Gaume, Claudson and Wise[12] and Dine and Fischler.[13]

In Section 2 we describe a toy model in which S.S. is broken by the O'Raifeartaigh mechanism[14] at an intermediate mass scale μ. The S.S. breaking however is not directly coupled to the ordinary light world. Instead it is coupled to a world of superheavy supermultiplets of mass

$M \gg \mu$. These in turn couple to the light world. This produces an indirect coupling mediated by superheavy intermediate states.

The resulting theory at energies less than μ looks like an explicitly softly broken S.S. characterized by a scale $\alpha(\mu^2/M)$. In particular if a particle such as the Higgs scalar is protected from mass counterterms by S.S. then its mass will be no bigger than $\alpha(\mu^2/M)$. One of the central points of this paper is to determine the stability of such a two stage hierarchy against radiative corrections.

In Section 3 we introduce light gauge fields into the model and discuss new features such a gaugino mass generation. Section 4 is devoted to the physics of the Goldstone multiplet. Section 5 analyzes an example of Witten's inverted hierarchy,[15] showing how it fits into our general framework. In Section 6 we discuss our conclusions and speculate about the influence of gravity on this class of theories.

In Appendix A we show that the one loop gluino mass vanishes in a class of theories. In Appendix B we discuss some features of theories in which the supersymmetry breaking at scale μ is due to the Fayet-Iliopoulos[16] mechanism rather than the O'Raifertaigh mechanism.

2. A TOY MODEL

Our simplest example involves four chiral superfields.[17] Two of them, $\hat{B}$ and $\hat{C}$, are heavy with mass ~M.

$$\hat{B} = B + i\psi_B\theta + \theta\theta F_B$$
$$\hat{C} = C + i\psi_C\theta + \theta\theta F_C \tag{2.1}$$

A superfield $\hat{X}$ describes the Goldstino and its scalar partner

$$\hat{X} = X + i\psi_X\theta + \theta\theta F_X \tag{2.2}$$

S.S. is broken by F_X getting a v.e.v. of order μ^2.

The ordinary light world is replaced by a single superfield $\hat{L}$.

$$\hat{L} = L + i\psi_L\theta + \theta\theta F_L \tag{2.3}$$

The superpotential is

$$W(\hat{B},\hat{C},\hat{X},\hat{L}) = g\hat{X}\left(\hat{B}^2 - \frac{\mu^2}{g}\right) + M\hat{B}^2 + M'\hat{B}\hat{C}$$
$$+ g\hat{B}^3 + g\hat{B}^2L + g\hat{B}L^2 + g\hat{L}^3 \tag{2.4}$$

For simplicity, all dimensionless coupling constants are called g. We assume g is small enough to do perturbation theory and that $M \gg \mu$.

The potential (2.4) leads to spontaneous breaking of S.S. Recall that the equations of motion for F_i are[17]

$$F_i^* = - \frac{\partial W_i}{\partial \hat{\phi}_i} \bigg|_{\hat{\phi}=\phi} \qquad (2.5)$$

which gives

$$- F_X^* = gB^2 - \mu^2 \qquad (2.6)$$

$$- F_C^* = M'B \qquad (2.7)$$

Evidently these cannot both be zero so supersymmetry is broken.

The absolute minimum of the potential

$$V = \sum_i F_i^* F_i$$

occurs at

$$B = C = L = 0 \quad , \qquad (2.8)$$

while X is undetermined. We can always set the v.e.v. of X to zero by adjusting M. We assume that when this is done $M \neq 0$.

The S.S. breaking v.e.v. is $\langle F_X \rangle$ which according to (2.6) is

$$F_X = \mu^2 \qquad (2.9)$$

Taking μ to be as large as 10^{12} GeV might seem to undetermine the original intent of the S.S., namely to keep the radiative corrections to the quadratic light scalar effective potential of order 100 GeV or less. Normally we would assume these would be of order $g^3\mu^2/4\pi^2$. For example consider the ordinary Feynman graph in Fig. 2, where the cross indicates the insertion $gB^2\mu^2$ which appears in the Lagrangian from the term $F_X^*F_X$. On dimensional grounds it is of order $g^3\mu^2/4\pi^2$. Note that even though S.S. breaking is only coupled to L through intermediate superheavies it can potentially split L masses by $\sim\mu^2$.

However by combining S.S. combinations of ordinary Feynman graphs one finds the order μ^2 contributions cancel to all orders in g. For example the graph shown in Fig. 3 is part of the same supergraph as Fig. 2. It exactly cancels Fig. 2. This cancellation is very general and can be expressed as a theorem:

Consider the quadratic contribution to the effective potential V(L) which arises when S.S. is broken at scale μ. It can be written as a series in powers of μ^2/M^2:

$$V_2(L) = L^2\mu^2 \left\{ c_0(g) + c_1(g)\frac{\mu^2}{M^2} + c_2(g)\frac{\mu^4}{M^4} + \dots \right\} \qquad (2.10)$$

The theorem asserts that

$$C_0(g) = 0 \qquad (2.11)$$

To see this we work in a supersymmetric formalism in which S.S. breaking is represented by a "spurion" line in a supergraph. The spurion is F_X which has v.e.v. μ^2. For example Fig. 4 shows the supergraph containing Figs. 2 and 3.

Calculating this graph is equivalent to computing the correction to the term

$$(\hat{L}\ \hat{L}\ \hat{X})_F \qquad (2.12)$$

in the effective action. When $\hat{X}$ is given its v.e.v. ($\langle\hat{X}\rangle = \langle F_X\rangle\theta\theta = \mu^2\theta\theta$) Eq. (2.12) becomes $\mu^2 LL$.

Although dimensional analysis and symmetry considerations allow a log-divergent $(\hat{L}\hat{L}\hat{X})_F$ counterterm it vanishes to all orders by the Grisaru-Rocek-Siegel (GRS) theorem which says that "F terms" are not induced by loop diagrams.[18]

The possibility remains that a "D term" might give rise to a splitting of the L supermultiplet. For example consider the operator

$$(\hat{L}^* \hat{L} \hat{X})_D \tag{2.13}$$

Giving $\hat{X}$ its v.e.v. produces the effective term

$$\mu^2(\hat{L}^* \hat{L})_F \tag{2.14}$$

We shall discuss this operator later. For now we note that it does not contain anything quadrative in the scalar components of L.

Another operator of interest is

$$(\hat{L} \hat{L} \hat{X}^*)_D \tag{2.15}$$

yielding the effective operator

$$\mu^2(\hat{L} \hat{L})_F \tag{2.16}$$

Curiously this operator is supersymmetric and although it can produce boson masses it does so in a supersymmetric manner. We shall return to it later.

The next class of operators contain X quadratically. Consider

$$(\hat{L}^* \hat{L} \hat{X}^* \hat{X})_D \tag{2.17}$$

which gives

$$\mu^4(\hat{L}^* \hat{L})_A = \mu^4 L^* L \tag{2.18}$$

A contributing supergraph is shown in Fig. 5. By dimensional analysis it is of order: $(g^4/4\pi^2)(1/M^2)$ (with the exception of Eq. (5.13) all coefficients are estimates). Thus the induced scalar mass-squared is

$$\frac{g^4}{4\pi^2} \cdot \frac{\mu^4}{M^2} \qquad (2.19)$$

The operator L*L induced by (2.15) splits the L-bosons from their fermionic partners but leaves the scalar and pseudoscalar degenerate.

Similarly the graph in Fig. 6 gives rise to the mass term L^2 from the operator $(\hat{L}\hat{L}\hat{X}\hat{X}^*)_D$. This operator gives equal and opposite contributions to the scalar and pseudoscalar $(\text{mass})^2$. The magnitude is again given by (2.19).

Fermion masses can also be generated. Consider the operator

$$(\hat{L}\ \hat{L}\ \hat{X}^*)_D \qquad (2.20)$$

which is produced by Fig. 7 with the coefficient $(g^3/4\pi^2)(1/M)$. When X is set equal to $\mu^2\theta\theta$ we obtain the effective operator

$$\frac{g^3}{4\pi^2} \cdot \frac{\mu^2}{M} \cdot (\hat{L}\ \hat{L})_F \qquad (2.21)$$

which is supersymmetric. It produces both fermion and boson masses. Supersymmetry violating fermion masses are suppressed by additional powers of μ^2/M^2. (See however Section 3 for gaugino masses.)

Soft supersymmetry violating interactions cubic in boson fields can also occur. The operator

$$(\hat{L}^*\ \hat{L}\ \hat{X}^*)_D \qquad (2.22)$$

(see Fig. 8) yields the breaking

$$\frac{g^3}{4\pi^2} \cdot \frac{\mu^2}{M} \cdot (L^* L)_F \qquad (2.23)$$

which contains

$$\frac{g^3}{4\pi^2} \cdot \frac{\mu^2}{M} \cdot L^* F_L$$

Using the equation of motion for F_L this becomes the sum of two supersymmetry violating terms

$$a L^3 + b L^2$$

$$a \sim \frac{g^4}{4\pi^2} \cdot \frac{\mu^2}{M} \quad ; \quad b \sim \left(\frac{g^3}{4\pi^2} \cdot \frac{\mu^2}{M} \right)^2 \qquad (2.25)$$

Note that so far all the induced dimensional constraints have a common scale proportional to μ^2/M. This circumstance, if general, insures the stability of the hierarchy. However the present model does have an effect which ruins this stability. Consider Fig. 9, this graph induces wave function mixing of the Goldstino multiplet X with L through the supersymmetric operator

$$(\hat{L} \hat{X}^*)_D \qquad (2.26)$$

With coefficient $g^2/4\pi^2$. It gives an effective operator

$$\frac{g^2}{4\pi^2} \mu^2 F_L \qquad (2.27)$$

This is a supersymmetric operator but it causes a shift of L by $\sim(g\mu^2/4\pi^2)^{1/2}$ giving it a mass $\sim(g^3\mu^2/4\pi^2)^{1/2}$. It evidently destroys the two stage hierarchy. If other light fields couple to L they too would get masses of order μ from (2.27) even if they are forbidden to directly mix with X.

A similar, S.S. violating, effect is produced by the graph in Fig. 10 which gives

$$\frac{g^3}{4\pi^2 M} (\hat{L}\ \hat{X}\ \hat{X}^*)_D \qquad (2.28)$$

The low energy effective operator is

$$\frac{g^3}{4\pi^2} \cdot \frac{\mu^4}{M} L \qquad (2.29)$$

This explicitly breaks low energy S.S. by an amount $\gg (\mu^2/M)^3$. This effect will introduce S.S. violating throughout the low energy sector. (It is of course possible that this is the true scale of supersymmetry violation in the real world.)

In realistic theories we can easily avoid this problem by not having neutral chiral fields which couple directly to the light sector. For example the light sector might consist of the minimal supersymmetric extension of quarks, leptons, Higgs bosons and SU(3)×SU(2)×U(1) gauge bosons. All light chiral multiplets are non-neutral under

SU(3)×SU(2)×U(1) and cannot mix with the Goldstino fields, or participate in the graph of Fig. 10.

Thus far we have not considered graphs with internal $\hat{L}$ lines. Consider for example Fig. 11. Power counting reveals that the only significant contribution to this graph occurs when the internal lines carry ℓ^2 of order M^2. When the momentum of an $\hat{L}$ line is of order M it is appropriate to treat it as part of the heavy sector which is integrated out. Thus Fig. 11 is essentially identical to Fig. 5 in its effects.

Sometimes loop integrations involving $\hat{L}$ lines will diverge logarithmically when $M \to \infty$. In this case significant contributions come both from $\ell^2 \sim M^2$ and $\ell^2 \ll M^2$. For example see Fig. 12. Power counting shows that the left loop is of order

$$\int \frac{d^4\ell}{\ell^4} \qquad (2.30)$$

for all $\ell^2 \ll M^2$.

A convenient way to separate the low energy and high energy contributions is to subtract out the value of the right loop at zero external momentum. This is shown schematically in Fig. 13. The first pieces now behave like

$$\int \frac{d^4\ell}{\ell^2} \qquad (2.31)$$

and contributes only for $\ell \sim M$. This can be considered as part of the integration of the large mass degrees of freedom. The second part involves only light lines and is logarithmically divergent. It is just the contribution in the effective light theory of a previously computed S.S. breaking operator. This procedure can be generalized. The resulting logs can be treated by the usual renormalization group method.

Power counting shows that the effective low energy theory has no quadratic divergences. The effective supersymmetry breaking operators we have found all on Girardello and Grisaru's list of soft breakings.[19] The only logarithmic divergences of the low energy theory are supersymmetric wave function renormalization and renormalization of the effective supersymmetry violating interactions.

The models we have thus far considered have no particles with mass $\sim\mu$. Generalizations can contain intermediate mass particles. In particular in Witten's inverted hierarchy scheme[15] particles of mass $\sim\mu$ couple directly to the light and heavy sectors but not to the supersymmetry breaking. These particles do not affect the above analysis. To see this first consider graphs with internal intermediate mass lines involving only $\hat{L}$ lines externally. These graphs are supersymmetric and by the GRS theorem only renormalize the wave functions of the low mass fields.

Graphs which couple to external F_X are small as before. For example consider the graph in Fig. 14 involving intermediate mass particles I. This makes a mass renormalization for L from the operator $(\hat{X}^*\hat{X}\hat{L}^*\hat{L})_D$. This graphs can be analyzed by the same method as Fig. 12. Its coefficient is of order

$$\frac{g^6}{(4\pi^2)^2 M^2} \ \ell n \left[\frac{M^2}{\mu^2} \right] \qquad (2.32)$$

Apart from the log this has the same order of magnitude as other two loop diagrams involving only heavy lines. The logs can again be summed with the renormalization group.

Another mass term of order μ that might be present (and generally is in inverted hierarchy models) is the mixing between the heavy and light fields: $\mu\hat{B}\hat{L}$. Figure 15 gives rise to the supersymmetric mass term $(\mu^2/M)(\hat{L}\hat{L})_F$. This is an F-term but is not forbidden by the GRS theorem because it is a tree level graph. Figure 16 induces $g(\mu^2/M^2)(\hat{L}\hat{L}\hat{X})_F$ which leads to the mass term $g(\mu^4/M^2)L^2$. This is the same mass term induced at one loop in Fig. 6, but the coefficient here is larger unless the mixing is small.

The mass term $\mu\hat{B}\hat{L}$ could have been absorbed by a small redefinition of $\hat{B}$ and $\hat{L}$. Then Fig. 16 appears explicitly in the Lagrangian as a small $[O(\mu^2/M^2)]$ Yukawa coupling between the light fields and Goldstino field.

3. EXAMPLES WITH GAUGE FIELDS

A simple model involving light gauge fields utilizes heavy adjoint fields $\hat{B}$, $\hat{C}$ and a singlet Goldstino field $\hat{X}$. The light fields are the gauge field $\hat{V}$ and some matter multiplets $\hat{L}$ in the fundamental and $\hat{K}$ in the antifundamental. The superpotential is

$$\mathrm{Tr}\left\{ g\hat{X}\left(\hat{B}^2 - \frac{\mu^2}{g} \right) + M\hat{B}^2 + M'\hat{C}\hat{B} + g\hat{B}^3 \right\} + g\hat{K}\hat{B}\hat{L} \qquad (3.1)$$

In addition the action involves the usual gauge couplings to the gauge bosons and fermions. The gauge coupling is denoted e. Our goal is a low energy theory containing the superfields $\hat{V}$, $\hat{L}$, and $\hat{K}$. As before there will be soft explicit violations of supersymmetry. Much of the discussion is the same as Section 2.

In particular the scalar masses LL, LL* and the interaction LF* are induced with the same order of magnitude. The dangerous terms linear in $\hat{L}$ are now excluded by the unbroken gauge symmetry. If there were no direct couplings $\hat{K}\hat{B}\hat{L}$ (as may be the case in left-right asymmetrical theories) scalar masses would still be generated by two loop graphs such as Fig. 17. The resulting order of magnitude of their masses is

$$\frac{\mu^2}{M} \cdot \frac{e^2 g}{4\pi^2} \qquad (3.2)$$

In this diagram the significant contribution occurs when all lines have momenta ~M. Accordingly it is treated as part of the high energy integration.

A phenomenologically important question is the gaugino mass. Since this necessarily breaks S.S. it must proceed via heavy intermediaries as in Fig. 18. The resulting operator is

$$\frac{e^2 g}{4\pi^2 M} (\hat{X}\ \hat{V}\ D\ \bar{D}^2\ D\ \hat{V})_D \tag{3.3}$$

where the D's inside parentheses denote covariant derivatives.[17]

Giving X its v.e.v. yields

$$\frac{e^2 g}{4\pi^2} \cdot \frac{\mu^2}{M} \bar{\lambda}\ \lambda \tag{3.4}$$

Here λ = gaugino field. However, careful inspection shows that the actual coefficient of this graph is zero. In Appendix A we have proved that the one-loop contribution to gaugino masses vanishes identically in a class of theories of this type. This is not so in theories with superheavy vectors - see Section 5. Two loop diagrams give gaugino masses of order

$$\frac{e^2 g^3}{(4\pi^2)^2} \cdot \frac{\mu^2}{M} \tag{3.5}$$

Finally if the low energy gauge group contains U(1) then the Fayet-Iliopoulos D-term V_D can be induced. It is not produced directly as a counterterm in the supersymmetric Lagrangian. However the operator

$$(D^\alpha\ \hat{W}_\alpha\ \hat{X}^*\ \hat{X})_D \tag{3.6}$$

is not excluded by any general theorem. Giving $\hat{X}$ its v.e.v. yields

$$(\hat{V})_D \tag{3.7}$$

with coefficient $\sim(eg^2/4\pi^2)(\mu^4/M^2)$.

To summarize, we get the same general pattern here as in Section 2. Integrating out the heavy fields and replacing $\hat{X}$ with its v.e.v. leads to a variety of supersymmetric and nonsupersymmetric effective interactions. The coefficient of the effective interaction is always μ^2/M to a power, times coupling constants (except for the case of the light singlet matter field).

4. THE GOLDSTINO SECTOR

The operators which induce supersymmetry breaking in the low energy theory also determine the couplings of the Goldstino. As an example consider the operator

$$\frac{g^4}{4\pi^2 M^2} (\hat{X}^* \hat{X} \hat{L}^* \hat{L})_D \qquad (4.1)$$

which was produced by Fig. 5. The term proportional to $F_X{}^*F_X$ gave a mass splitting $\delta m^2 \sim g^4\mu^4/4\pi^2M^2$. The term proportional to $F_X{}^*\psi_X$ is

$$\frac{g^4\mu^2}{4\pi^2 M^2} \psi_X \psi_L L^* \sim \frac{\Delta m^2}{\mu^2} \psi_X \psi_L L^* \qquad (4.2)$$

This is just what the Ward identities of broken supersymmetry require. The coupling of the Goldstino ψ_X is proportional to the effective supersymmetry violation Δm^2, and inversely proportional to the supersymmetry breaking v.e.v. $F_X = \mu^2$.

It is interesting to consider also operators containing only X. The diagrams in Fig. 19 give rise to the operators

$$\frac{g^2}{4\pi^2} (\hat{X}^* \hat{X})_D \qquad (4.3a)$$

$$\frac{g^3}{4\pi^2 M} (\hat{X}^* \hat{X} \hat{X})_D \qquad (4.3b)$$

$$\frac{g^4}{4\pi^2 M^2} (\hat{X}^* \hat{X}^* \hat{X} \hat{X})_D \qquad (4.3c)$$

$$\frac{g^4}{4\pi^2 M^2} (\hat{X}^* \hat{X} \hat{X} \hat{X})_D \qquad (4.3d)$$

Inserting the v.e.v. of F_X, (4.3a) becomes simply $g^2\mu^4/4\pi^2$. It is a correction to the vacuum energy. The method used in this paper is easily applied to the calculation of the vacuum energy (effective potential) in these models. This will be published separately.[20]

At $F_X = \mu^2$, $(\hat{X}^*\hat{X}\hat{X})_D$ becomes $\mu^4 X$. It represents a shift of the scalar component of X. In the state which minimizes the effective potential, the coefficient of this operator must vanish.

At tree level the Goldstino superfield $\hat{X}$ is massless. The Goldstino itself remains massless (ignoring gravity). The operators (4.3c) and (4.3d) give rise to

$$\frac{g^4}{4\pi^2} \cdot \frac{\mu^4}{M^2} X^* X \qquad (4.4)$$

and

$$\frac{g^4}{4\pi^2} \cdot \frac{\mu^4}{M^2} X^2 \qquad (4.5)$$

which are masses for the X scalars. They are of the same order as the masses in the low energy sector.

In all examples so far, the Goldstino was part of a chiral superfield $\hat{X}$. In Appendix B we briefly consider models in which S.S. is broken by a D-term, so that the Goldstino is part of a gauge multiplet.[16]

5. THE INVERTED HIERARCHY

Witten's inverted hierarchy model[15] provides us with an especially interesting example, in which the superheavy scale M is spontaneously induced by radiative effects. The particular example is due to Ginsparg[21] and consists of an SU(2) gauge theory with gauge superfield V_a ($a = 1,2,3$) coupled to adjoint chiral fields Y_a and B_a, and a singlet Z; the superpotential for the chiral fields is

$$\lambda\mu\ \hat{B}\cdot\hat{Y} + g\hat{Z}\ \hat{B}\cdot\hat{B} - g\hat{Z}\ \mu^2 \qquad (5.1)$$

with $2g > \lambda$. The gauge interactions of the chiral fields are contained in the gauge invariant kinetic energy,

$$(\hat{B}^* e^{e\hat{V}}\ \hat{B} + \hat{Y}^* e^{e\hat{V}}\ \hat{Y} + \hat{L}^* e^{e\hat{V}}\ \hat{L} + \hat{Z}^*\ \hat{Z})_D \qquad (5.2)$$

where $\hat{V}$ is a matrix in the appropriate representation.

The full scalar potential is minimized at

$$\begin{aligned} \langle B_3\rangle &= \mu \left(1 - \frac{\lambda^2}{2g^2} \right)^{1/2} \\ \langle B_{1,2}\rangle &= 0 \\ \langle Y_3\rangle &= -\frac{2g}{\lambda} \left(1 - \frac{\lambda^2}{2g^2} \right)^{1/2} \langle Z\rangle \\ \langle Y_{1,2}\rangle &= 0 \end{aligned} \qquad (5.3)$$

The expectation value of Z is undetermined at tree level. Following Witten we assume that quantum corrections produce a minimum at a value

$\langle Z\rangle$ such that

$$\ln (\langle Z\rangle/\mu) \sim 1/g^2$$

which for small couplings makes $\langle Z\rangle$ many orders of magnitude larger than μ. The nonvanishing auxiliary fields are

$$\begin{aligned} \langle F_Z\rangle &= \frac{\mu^2\lambda^2}{2g} \\ \langle F_{Y_3}\rangle &= -\mu^2\lambda \left[1 - \frac{\lambda^2}{2g^2} \right]^{1/2} \end{aligned} \qquad (5.4)$$

The supersymmetry is broken at order μ.

One linear combination of $\hat{Z}$ and $\hat{Y}_3$ has a vanishing expectation value for both scalar and auxiliary components - we call it $\hat{U}$:

$$\hat{U} = \hat{Y}_3 \cos\theta - \hat{Z} \sin\theta \qquad (5.5)$$

where

$$\cos\theta = \lambda(4g^2 - \lambda^2)^{-1/2} \qquad (5.6)$$

The other linear combination

$$\hat{T} = \hat{Y}_3 \sin\theta + \hat{Z} \cos\theta \qquad (5.7)$$

has

$$\begin{aligned} \langle T\rangle &= \langle Z\rangle \left[\frac{4g^2}{\lambda^2} - 1 \right]^{1/2} \equiv M \\ \langle F_T\rangle &= \lambda\mu^2 \left[1 - \frac{\lambda^2}{4g^2} \right]^{1/2} \equiv f \end{aligned} \qquad (5.8)$$

The expectation value of Y_3 breaks the gauge symmetry to U(1) at the large scale M. To see the spectrum it is convenient to use a unitary gauge

$$\hat{Y}_1 = \hat{Y}_2 = 0 \tag{5.9}$$

and to shift away the scalar field expectation values

$$\begin{aligned} \hat{X} &= \hat{T} - M \\ \hat{C}_a &= \hat{B}_a - \delta_{3a} \langle \hat{B}_3 \rangle \end{aligned} \tag{5.10}$$

The superpotential becomes

$$\begin{aligned} & g \cos\theta \, \hat{X}\hat{C}\cdot\hat{C} - \hat{X}f + gM \cos\theta \, \hat{C}\cdot\hat{C} \\ & + \lambda\mu \cos\theta \, \hat{U}\hat{C}_3 - g \sin\theta \, \hat{U}\hat{C}\cdot\hat{C} \end{aligned} \tag{5.11}$$

This has the same general form as the superpotentials studied in previous sections. The important piece from the kinetic Lagrangian is that for the large fields Z and Y_3. In terms of the new fields it is

$$\{ \,|\, \hat{X}\cos\theta - \hat{U}\sin\theta + M\cos\theta \,|^2$$

$$+ (e^{e\hat{V}})_{33} \,|\, \hat{X}\sin\theta + \hat{U}\cos\theta + M\sin\theta \,|^2 \}_D$$

$$= \{ \hat{X}^*\hat{X} + \hat{U}^*\hat{U} + 2M^2 e^2 \sin^2\theta \, (\hat{V}_1{}^2 + \hat{V}_2{}^2) \}_D$$

$$+ \text{interactions} \qquad (5.12)$$

Here $\hat{V}$ is in the adjoint representation:

$$(\hat{V})_{bc} = -2i\hat{V}_a \, \epsilon_{abc} \qquad (5.13)$$

The only supersymmetry breaking expectation value is F_X. From (5.11), (5.12) and (5.13) we see that all vertices involving $\hat{X}$ also include $\hat{V}_1$, $\hat{V}_2$, or $\hat{C}$. These fields are all superheavy. Thus, to bridge between the supersymmetry breaking and the light fields always requires intermediate states with superheavy fields. This comes about because the auxiliary field expectation value and the undetermined scalar expectation value were in the same superfield $\hat{T}$. This is a fairly general property of O'Raifertaigh supersymmetry breaking.

This model does contain a light neutral field $\hat{U}$ which can mix with $\hat{X}$. This is not dangerous because $\hat{U}$ couples to the other light fields only through superheavies.

The light sector in this model consists of the light matter field $\hat{L}$ and the unbroken Abelian gauge field $\hat{V}_3$. The L scalars get supersymmetry breaking masses from graphs mediated by heavy vector fields such as Fig. 20. This induces

$$\frac{e^4}{4\pi^2 (eM \sin\theta)^2} (\hat{L}^* \hat{L} \hat{X}^* \hat{X})_D \qquad (5.14)$$

The photinos can get mass from loops of heavy gauge and heavy $\hat{C}$ fields, as shown in Figs. 21 and 22. These graphs have been calculated in the supersymmetric R_ξ gauge of Ref. 22. The photino mass is

$$\left(\frac{2e^2 f^2}{8\pi^2 M} \right)_{\text{gauge loop}} + \left(- \frac{e^2 f^2}{8\pi^2 M} \right)_{\hat{C}\ \text{loop}} = \frac{e^2 f^2}{8\pi^2 M}$$

and the sort of cancellation found in Appendix A does not quite occur. This mass can also be obtained from a low energy theorem for spontaneously broken R-symmetry.[20]

6. CONCLUSIONS

What we have shown in this paper is that a class of model exists in which supersymmetry is broken at a scale μ intermediate between a superheavy mass M and a light scale μ^2/M. The breaking of S.S. manifests itself in the light world through explicit violations with a strength characteristic of the light scale. Most of the machinery of the supersymmetry breaking mechanism is hidden at the superheavy scale. In practice, this suggests a phenomenological explicitly softly broken low energy theory, which could contain nothing more than the quarks, leptons, $SU(3)\times SU(2)\times U(1)$ gauge fields, and two Higgs doublets, plus their supersymmetric partners. Explicit masses of order μ^2/M would be:

1. Supersymmetric mass terms for the Higgs doublets.
2. Supersymmetry violating gaugino mass.
3. Supersymmetry violating scalar masses.

The only supersymmetry violating interaction is a trilinear scalar interactions from $[\hat{L}^*\hat{L}]_F$. Curiously, the constraints on the explicit breakings are the same as in Ref. 19.

To calculate the actual value of the S.S. violating parameters requires a detailed knowledge of the superheavy sector, perhaps including gravity. Exchange of gravitons and gravitinos of momentum near the Planck scale should induce the same operators we have discussed with M_P in place of the superheavy scale. Unless the superheavy scale is significantly smaller than M_P, gravity cannot be ignored. In fact, if all other interactions connecting the Goldstino to normal matter were turned off or made very small, gravity would still connect the two.

Indeed, one definite effect of gravity is to combine the Goldstino and the gravitino into a spin 3/2 particle of mass $(4\pi/3)^{1/2}\,\mu^2/M_P$.[23]

The fact that the Goldstino and its scalar partners are massive has cosmological implications, as noted by Weinberg[24] and Hung and Suzuki.[25] Assuming that they are in thermal equilibrium in the early universe, they would dominate the mass of the universe at helium synthesis temperatures unless they had already decayed. This gives a lower bound on the S.S. breaking scale of $\sim 10^{11}$ GeV, consistent with our S.S. breaking scale.

ACKNOWLEDGEMENTS

We would like to thank Tom Banks, Mike Dine, Willy Fischler, Stuart Raby, Steven Weinberg and Mark Wise for useful discussions. This work was supported by the Department of Energy, contract DE-AC03-76SF00515.

APPENDIX A

THE ONE LOOP GAUGINO MASS

Consider a Lagrangian of the form

$$\hat{X}(\hat{B}_i\hat{B}_j g_{ij} - \mu^2) + M_{ij}\hat{B}_i\hat{B}_j + 2N_{ij}\hat{B}_i\hat{C}_j + P_{ij}\hat{C}_i\hat{C}_j \qquad (A.1)$$

where $\hat{B}_i$ and $\hat{C}_i$ are a collection of chiral fields ($i = 1,\ldots,n$). For convenience we take them all in a real representation of the gauge group G - the argument is trivially extended to arbitrary representations. The condition for S.S. to be broken is

$$P_{ij} = 0 \quad ; \quad N_{ij} \text{ nonsingular} \qquad (A.2)$$

The mass matrix then has the form

$$\begin{array}{c} \\ B \\ \\ C \end{array}\begin{array}{c} \begin{array}{cc} B & C \end{array} \\ \left(\begin{array}{c|c} M & N \\ \hline N^+ & 0 \end{array}\right) \end{array} = \mathcal{M} \qquad (A.3)$$

The coupling matrix of the Goldstino field to $\hat{B}$ and $\hat{C}$ is

$$\left(\begin{array}{c|c} g & 0 \\ \hline 0 & 0 \end{array}\right) = \mathcal{G} \qquad (A.4)$$

Let $\mathcal{L}$ be a matrix which diagonalizes M and define

$$\mathcal{L}\, \mathcal{M}\, \mathcal{L}^{-1} = \mathcal{M}' \,, \quad \text{diagonal} \tag{A.5}$$

$$\mathcal{L}\, G\, \mathcal{L}^{-1} = G' \tag{A.6}$$

In the one loop gluino mass graph, Fig. 18, the gauge vertices are always diagonal. By dimensional analysis the total contribution is

$$\sum_i G'_{ii} \frac{1}{\mathcal{M}'_i} = \mathrm{Tr}\,[\, G' \,(\mathcal{M}')^{-1} \,]$$

$$= \mathrm{Tr}\,[\, G\, \mathcal{M}^{-1} \,] \tag{A.7}$$

From (A.3) it follows that

$$\mathcal{M}^{-1} = \left(\begin{array}{c|c} 0 & (N^{+})^{-1} \\ \hline N^{-1} & -N^{-1}\, M (N^{+})^{-1} \end{array} \right) \tag{A.8}$$

and from (A.8) and (A.4) we see that (A.7) vanishes.

This cancellation is curious. It takes place only at zero gluino momentum, and the condition that it take place is precisely the condition, (A.2), that supersymmetry be broken. In general it appears to be accidental (though for a special case, see Ref. 20) and we know of no reason for the two loop graph to vanish. In the case that some eigenvalues of N are small (order μ), some mass eigenvalues are order μ^2/M. The cancellation then appears after adding the effect of the light loops to the effective operators from the heavy loops.

APPENDIX B

SUPERSYMMETRY BREAKING BY D-TERMS

In this Appendix we consider some general features of models in which the S.S. breaking at scale μ is due to the Fayet-Iliopoulos mechanism.[16] A toy model has a low energy gauge symmetry G and an additional U(1)′ gauge symmetry, with superfields $\hat{V}^a$ and $\hat{V}'$ and couplings e and e′ respectively. There are heavy matter superfields B, with (G representation, U(1)′ charge) = (R,+1), and C, with ($\bar{R}$,−1), and light superfields L with (r,0) and K with ($\bar{r}$,0). The Lagrangian consists of the usual kinetic terms with minimal coupling, plus

$$M(\hat{B}\hat{C})_F + M(\hat{B}^*\hat{C}^*)_{F\star} + \mu^2(\hat{V}')_D \qquad \text{(B.1)}$$

The last term is the Fayet-Iliopoulos D-term for the U(1)′ gauge symmetry. The auxiliary fields for $\hat{V}'$, $\hat{B}$, and $\hat{C}$ are given by

$$\begin{aligned} D' &= -\mu^2 - e'(B^*B - C^*C) \\ F_B{}^* &= -\,MC \\ F_C{}^* &= -\,MB \end{aligned} \qquad \text{(B.2)}$$

These cannot all vanish. For $\mu \ll M$, the scalar potential is minimized at B = C = 0, and the only S.S. breaking v.e.v. is $D' = -\mu^2$. The gauge symmetries are unbroken (we assume L = K = 0, since this is undetermined at tree level). Because the U(1)′ gauge invariance is unbroken, effective operators must be gauge invariant. Since all light fields are neutral under U(1)′, the effective operators can only depend on the field strength

$$\hat{W}'_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha \hat{V}' = -\theta_\alpha \mu^2 \qquad (B.3)$$

The light gauge and matter fields couple to D′ only through the heavy fields $\hat{B}$ and $\hat{C}$. The graph in Fig. 23 induces

$$\frac{e^2 e'^2}{4\pi^2 M^4} (\hat{W}^{a\alpha} \hat{W}'_\alpha \hat{W}_{\dot\beta}{}^a \hat{W}'^{\dot\beta})_D \qquad (B.4)$$

Inserting (B.3) this becomes a supersymmetry violating wave function renormalization, with negligible dimensionless coefficient of order $(\mu/M)^4$. The operator $[\hat{W}^{a\alpha} \hat{W}_\alpha{}^\alpha \hat{W}'^\beta \hat{W}'_\beta]_F$, which leads to a gauge fermion mass, is excluded by an R invariance. If additional fields are added to break the R invariance, a two loop gaugino mass of order

$$\frac{g^2 e^2 e'^2}{(4\pi^2)^2} \cdot \frac{\mu^4}{M^3} \qquad (B.5)$$

can be produced.

The light matter superfields couple to the supersymmetry breaking only at two loops, through light gauge bosons. Direct coupling of the light matter fields to $\hat{B}$ or $\hat{C}$ is absent in this model, because it would restore unbroken supersymmetry. Figure 24 gives rise to the operator

$$\frac{e^4 e'^4}{(4\pi^2)^2 M^6} (\hat{L}^* \hat{L} \hat{W}^{\alpha\prime} \hat{W}'_\alpha \hat{W}'_{\dot\beta} \hat{W}'^{\dot\beta})_D \qquad (B.6)$$

which, using (B.3), becomes a light scalar mass of order

$$\frac{e^2\, e'^2}{4\pi^2} \cdot \frac{\mu^4}{M^3} \qquad (B.7)$$

The masses (B.5) and (B.7) are much smaller, for given μ and M, than those found in models with O'Raifeartaigh breaking. The S.S. breaking scale μ can be much closer to M than in those models.

Other effective breakings can be induced if fields are added to break the R-symmetries of this model. The coefficients depend on how this is done, but they are generally comparable to (B.4)-(B.7). One interesting operator, which can occur if there is a light field $\hat{L}^a$ in the adjoint representation of G, is $(\hat{W}^\alpha, \hat{W}_\alpha{}^a\, \hat{L}^a)_F$

$$(\theta^\alpha\, \hat{W}_\alpha{}^a\, \hat{L}^a) \;=\; \frac{i}{\sqrt{2}}\, \lambda^a\, \psi_L{}^a + D^a\, L^a \qquad (B.8)$$

It gives a Dirac mass, mixing the light gauge and matter fermions, plus a mixing between matter scalar and gauge auxiliary fields. Power counting and spurion analysis[19] shows that it is soft in the sense of Girardello and Grisaru, whereas the two pieces $\lambda\psi$ and DL are separately hard.

Actually if G has a U(1) factor, there is one more operator which dwarfs all other effects. The graph of Fig. 25 gives

$$\frac{ee'}{4\pi^2}\, [W^\alpha, W_\alpha]_F \;\rightarrow\; \frac{e^2\, e'^2\, \mu^2}{4\pi^2}\, D \qquad (B.9)$$

which is a Fayet-Iliopoulos term for the low energy U(1), with large coefficient. Such a large D term for ordinary hypercharge is not acceptable. To make a realistic model one has to imbed hypercharge in a semisimple group at low energy, or exclude the operator (B.9) by a symmetry.

The small size of (B.4)-(B.7) came about because U(1)′ gauge invariance forced the effective operators to be of high dimension. More general models could be built if the U(1)′ gauge invariance had been broken at the large scale M. Is it possible for the D term of a gauge symmetry broken at scale M to break the supersymmetry by the Fayet-Iliopoulos mechanism at a much smaller scale? No, it is not. The tree level vacuum minimizes the energy

$$E = \sum_i F_i^* F_i \qquad (B.10)$$

The energy is stationary under all variations, including the complex extension of the gauge group,[22]

$$\delta\phi_i = g^a \tau_{ij}^a \phi_j \quad \text{(without the usual } i\text{)} \qquad (B.11)$$

under which

$$\begin{aligned} \delta^a F_i &= g^a \tau_{ij}^a \phi_j \\ \delta^a D^b &= -(M^2)^{ab} + i g^a f^{abc} D^c \end{aligned} \qquad (B.12)$$

where $(M^2)^{ab}$ is precisely the vector boson mass matrix. Going to a basis in which (M^2) is diagonal, $\delta^a E(f_i, D^b) = 0$ implies

$$D^a = 2g^a \frac{\sum_{i,j} F_i^* \tau_{ij}^a F_j}{(M^a)^2} \qquad (B.13)$$

Since the total energy density is $O(\mu^4)$, $F_i \lesssim \mu^2$, and for heavy gauge fields

$$D^a \lesssim \frac{\mu^4}{M^2} \ll \mu^2 \qquad (B.14)$$

and the S.S. breaking must be due to some other auxiliary field. This satisfies our intuition that all components of a multiplet of mass M, including the auxiliary, should decouple from the physics at much lower scales.[26]

The model of Ref. 11 is not a counterexample to this. There, the D term of a U(1) broken at high energy has a v.e.v. of order 1 TeV. As pointed out in Ref. 11, though, there is also an auxiliary field, F_{R^c}, with an intermediate v.e.v.. This is an interesting variation on our models. The most direct connection between R^c and the low energy world is exchange of a single heavy gauge boson. Inserting the v.e.v. for F_{R^c}, this becomes a D term for the heavy U(1).

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FIGURE CAPTIONS

Fig. 1. Renormalization of the Higgs boson mass.

Fig. 2. Graph contributing to the mass of the L scalar. The X represents the order μ^2 splitting of the B multiplet.

Fig. 3. Another graph contribution to the mass of the L scalar.

Fig. 4. Supergraph containing Figs. 2 and 3.

Fig. 5. Supergraph inducing $(\hat{L}^* \hat{L} \hat{X}^* \hat{X})_D$.

Fig. 6. Supergraph inducing $(\hat{L} \hat{L} \hat{X}^* \hat{X})_D$.

Fig. 7. Supergraph inducing $(\hat{L} \hat{L} \hat{X}^*)_D$.

Fig. 8. Supergraph inducing $(\hat{L}^* \hat{L} \hat{X}^*)_D$.

Fig. 9. Supergraph inducing $(\hat{L} \hat{X}^*)_D$.

Fig. 10. Supergraph inducing $(\hat{L} \hat{X}^* \hat{X})_D$.

Fig. 11. Supergraph with a light internal line.

Fig. 12. Two loop supergraph with light internal lines.

Fig. 13. Decomposition of graph of Fig. 12 into a high energy piece and a low energy piece. The heavy circle represents the value of the right, heavy, loop at zero external momentum.

Fig. 14. Supergraph with intermediate mass lines which are designated I.

Fig. 15. Supergraph inducing $(\hat{L} \hat{L})_F$. The small circle is the mass term $\mu\hat{B}\hat{L}$.

Fig. 16. Supergraph inducing $(\hat{L} \hat{L} \hat{X})_F$.

Fig. 17. Supergraph inducing $(\hat{L}^* \hat{L} \hat{X}^* \hat{X})_D$ via gauge lines.

Fig. 18. Supergraph inducing $(\hat{W}^\alpha \hat{W}_\alpha \hat{X})_F$. This is shown to vanish in Appendix A.

Fig. 19. Supergraphs inducing operators involving $\hat{X}$ only:

(a) The operator $(\hat{X}^* \hat{X})_D$.

(b) The operator $(\hat{X}^* \hat{X} \hat{X})_D$.

(c) The operator $(\hat{X}^* \hat{X}^* \hat{X} \hat{X})_D$.

(d) The operator $(\hat{X}^* \hat{X} \hat{X} \hat{X})_D$.

Fig. 20. Supergraph inducing $(\hat{L}^* \hat{L} \hat{X}^* \hat{X})_D$ via heavy gauge bosons.

Fig. 21. Supergraph inducing $(\hat{W}_3{}^\alpha \hat{W}_{3\alpha} \hat{X})_F$ via heavy gauge loop.

Fig. 22. Supergraph inducing $(\hat{W}_3{}^\alpha \hat{W}_{3\alpha} \hat{X})_F$ via heavy matter loop.

Fig. 23. Supergraph inducing $(\hat{W}^{a\alpha} \hat{W}'_\alpha \hat{W}_{\dot{\beta}}{}^a \hat{W}'^{\dot{\beta}})_D$.

Fig. 24. Supergraph inducing $(\hat{L}^* \hat{L} \hat{W}'^\alpha \hat{W}'_\alpha \hat{W}'_{\dot{\beta}} \hat{W}'^{\dot{\beta}})_D$.

Fig. 25. Supergraph inducing $(\hat{W}^\alpha \hat{W}'_\alpha)$.

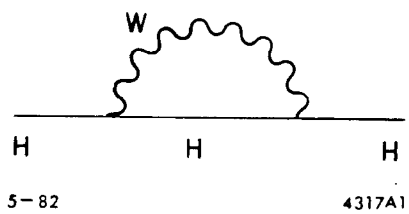
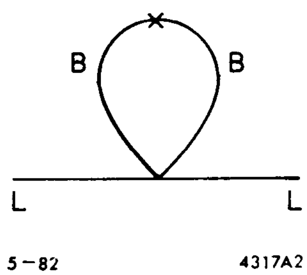


Fig. 1



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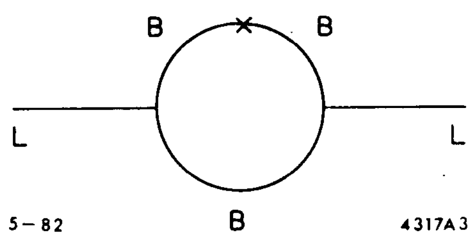

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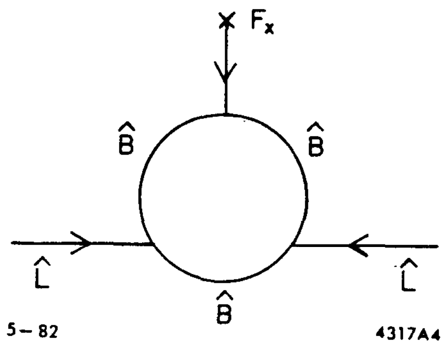

Fig. 4

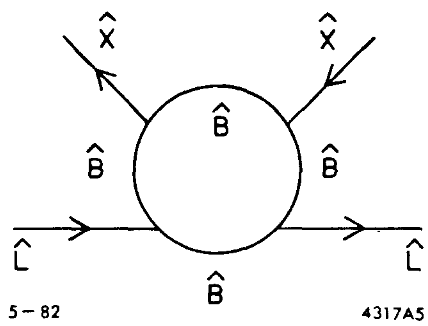

Fig. 5

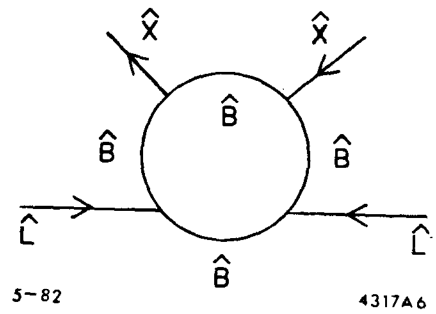

Fig. 6

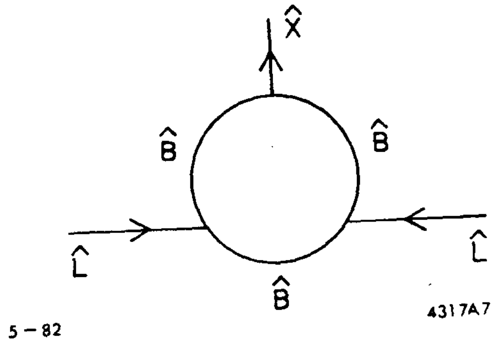

Fig. 7

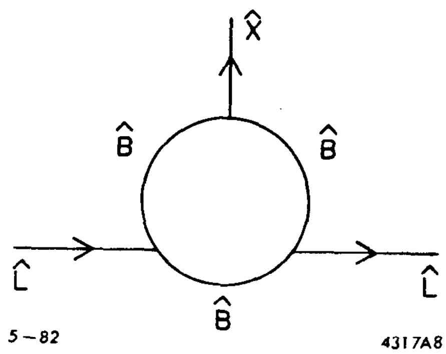

Fig. 8

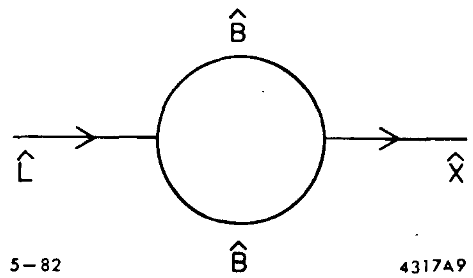

Fig. 9

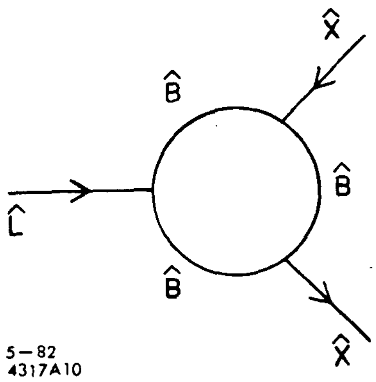

Fig. 10

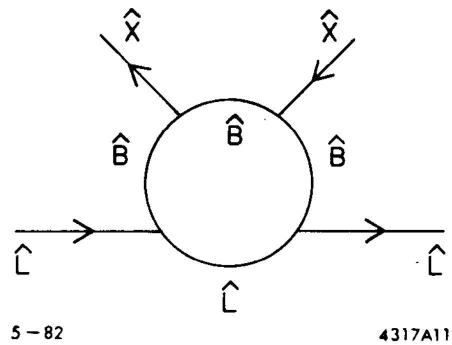

Fig. 11

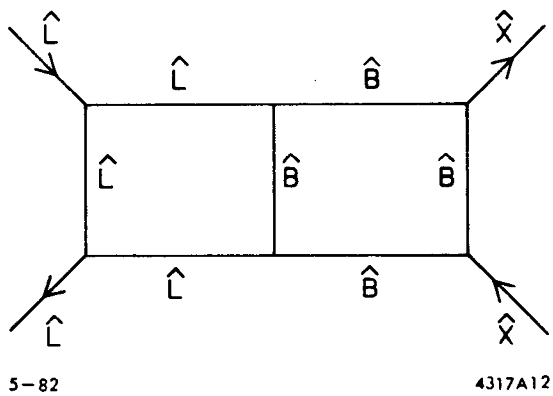

Fig. 12

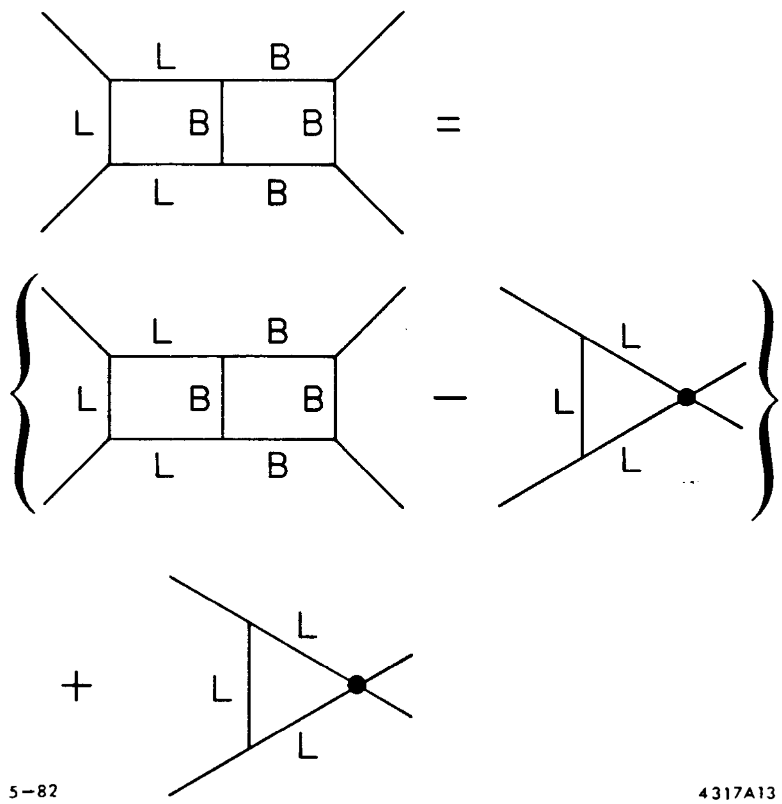

Fig. 13

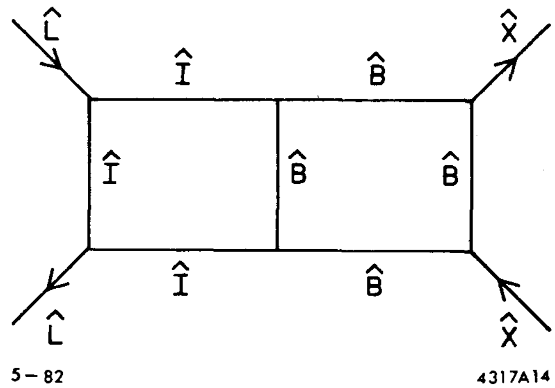

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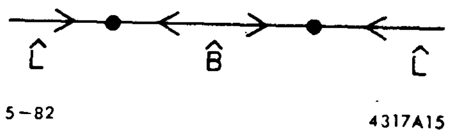
Fig. 14



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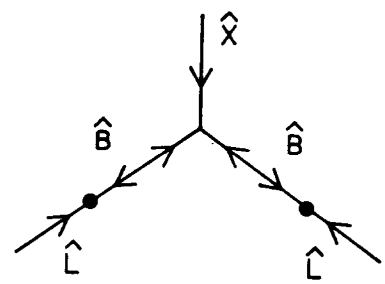

Fig. 16

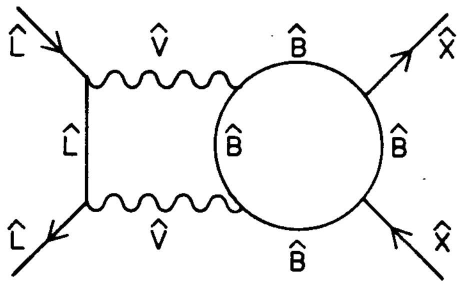
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Fig. 17

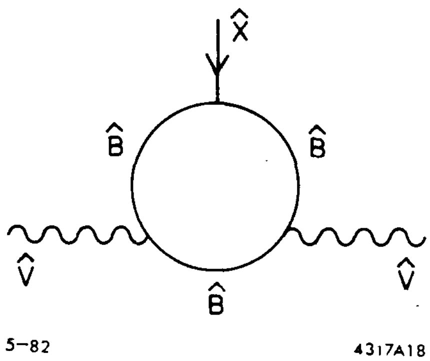


Fig. 18

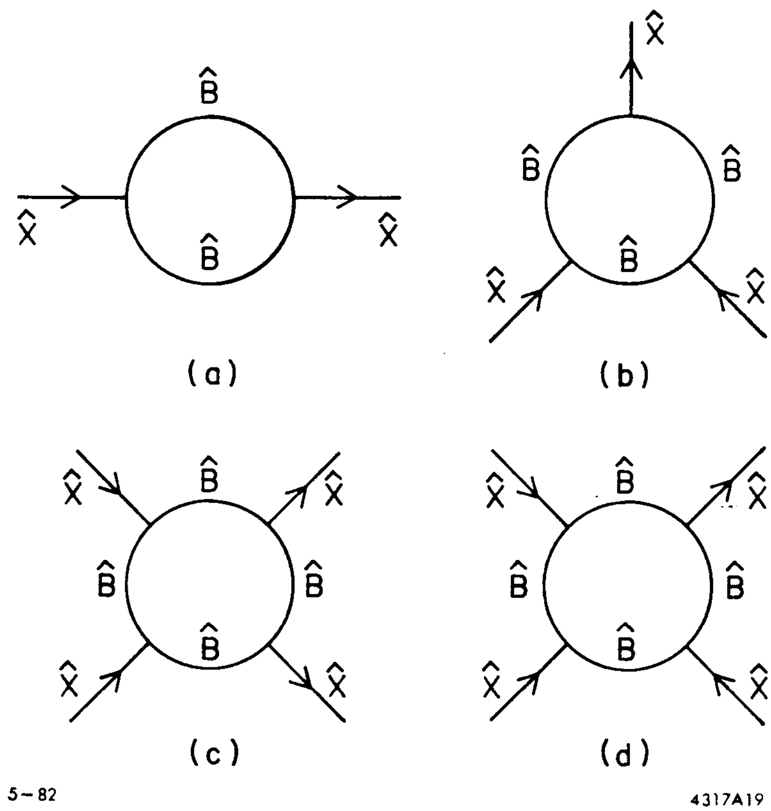


Fig. 19

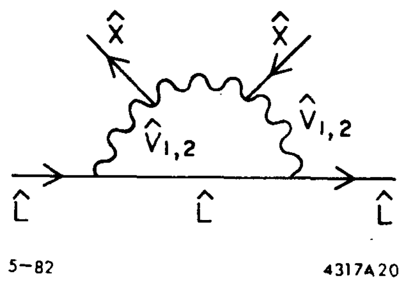


Fig. 20

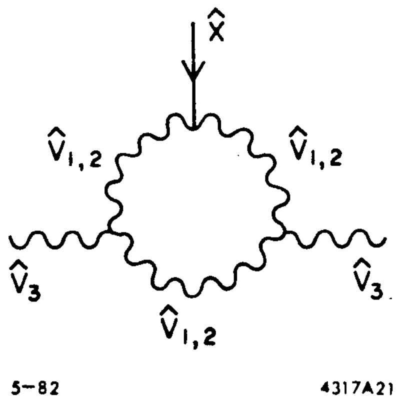


Fig. 21

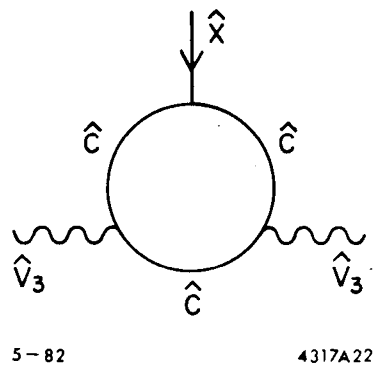


Fig. 22

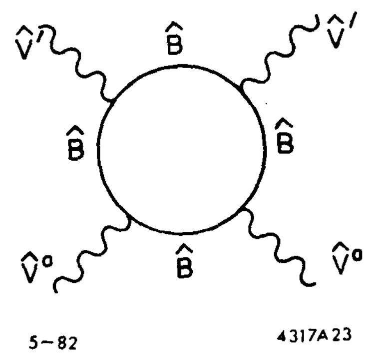
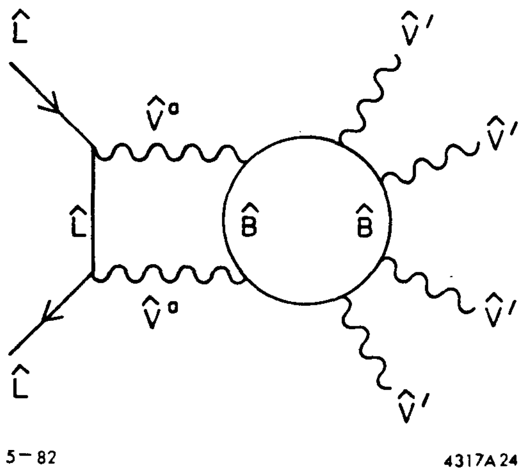


Fig. 23



$\hat{L}$
$\hat{V}^a$
$\hat{V}'$
$\hat{V}'$
$\hat{L}$
$\hat{B}$
$\hat{B}$
$\hat{V}^a$
$\hat{V}'$
$\hat{L}$
$\hat{V}'$
5−82
4317A24

Fig. 24

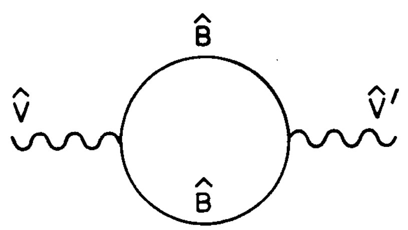


Fig. 25