## TRACKING STUDIES IN PEP AND DESCRIPTION OF THE

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Ubi bene ibi patricia
(there one is happy, where is PATRICIA)

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## 1. CODE PATRICIA AND ITS MODIFICATIONS

In this section, I will describe the code PATRICIA, the system of codes PAQUASEX and the code PNWM.

### 1.1 Code Patricia

PATRICIA (PArticle TRacking In CIrcular Accelerators) is a computer code written by $H$. Wiedemann ${ }^{1,2}$ and designed mainly for the purpose of tracking of particles in an electron storage ring.
1.1.1 What it does

The program fulfills the following calculations:
a) It adjusts horizontal and vertical chromaticities to the values prescribed by the user.
b) It calculates Twiss parameters and eta functions of the latticc.
c) It calculates emittances of the beam and relevant parameters of the ring.
d) It performs harmonic analysis of the particle motion and produces its frequency spectrum.
e) It tracks up to four particles simultaneously through up to onethousand revolutions. The oscillations in all three degrees of motion can be included into calculations, but horizontal and vertical motions are treated independently (no coupling is taken into account besides that which appears from the passage of a displaced particle through a sextupole).

In all these calculations usual $(3 \times 3)$ matrix formalism is used. Sextupoles are treated in thin lens approximation.

P'ATRICIA does not fit parameters of a linear lattice. It requires the input of locations, lengths and strengths of ring elements, comprising the lattice, which has a matched solution found by some other program, for example, by MAGIC.

The program uses the lattice which is supplied to it and attempts to find a periodic solution for the Twiss parameters and the dispersion function. If no periodic solution can be found for the on-momentum particle, the program stops.

If such a periodic solution is found, the program then looks for eta function for an off-momentum particle. The effective bending and focusing action of sextupole magnets are taken into account. These calculations are performed by iterative procedure since the value of the eta function in any sextupole depends on the locations and the strengths of all the other sextupoles.

If no periodic solution is found in twenty iterations, the program says so and goes on.

From the lattice functions and particle energy the integrals which determine the synchrotron radiation, damping constants and horizontal beam emittance $\varepsilon_{x}$ are found. The standard horizontal beam width $\sigma_{x}$ at the location with $\beta_{x}$ is then $\sqrt{\varepsilon_{x} \beta_{x}}$. The vertical standard beam height is calculated assuming full ( $100 \%$ ) coupling $\sigma_{y}=\sqrt{\varepsilon_{x} \beta_{y}}$.

The frequency of the synchrotron oscillation is calculated assuming that the quantum lifetime should be not less than 50 hours. From here the bucket size, the RF voltage and the synchronous phase are found (one needs certainly the $R F$ harmonic number for these calculations). From these data the standard energy spread $\sigma_{E}$ is found.

Particle tracking is accomplished by the proper linear transformations of particle coordinates between the sextupoles and by the proper change of particle transverse momenta at each sextupole ("kick"). For an off-momentum particle there are two options:
a) Tracking of particles with two fixed momentun deviations $\pm \delta_{\text {MAX }}(\delta=\Delta \mathrm{p} / \mathrm{p})$. Parameter $\delta_{\text {MAX }}$ is given in the input (DELMAX). In this case corresponding eta-functions and matrices for these values of $\delta$ are used to track particles.
b) Tracking of particles performing the synchrotron oscillations. The energy deviation of the particle is changed after each superperiod by the amount

$$
\delta=\delta_{\mathrm{MAX}} \sin 2 \pi \Omega_{\mathrm{s}} \frac{\mathrm{n}}{\mathrm{~N}}, \quad \mathrm{n}=1,2, \ldots
$$

where $N$ is the number of superperiods in the ring, $\Omega_{s}$ is the tune of the synchrotron oscillations, and $\delta_{\text {MAX }}$ is the amplitude of the synchrotron (energy) oscillations.

Proper transformation matrices from one sextupole to another are evaluated in the following way. Eta-functions $\eta$ and matrices $W$ for five equally spaced different values of $\delta$ in the interval $[$-DELMAX, DELMAX $]$ are stored for each linear piece of the ring. For each given $\delta$ the corresponding matrix and eta-function are found by interpolation with the help of a third-order (in power of $\delta$ ) polynomial. For example,

$$
\begin{aligned}
n(\delta) & =\eta_{0}+n_{1} \delta+n_{2} \delta^{2}+n_{3} \delta^{3} \\
w_{i k}(\delta) & =w_{i k}^{0}+w_{i k}^{1} \delta+w_{i k}^{2} \delta^{2}+w_{i k}^{3} \delta^{3}
\end{aligned}
$$

These values then are used in tracking calculations.

### 1.1.2 Input

All data input is done in subroutine READIN. An example of input stream is given in Table 1 . It has the following structure.

The first FORTRAN line is a title of the problem (i.e., any text can be written in this line). Then follow two NAMELISTS. NAMELIST/PARAM/ contains a set of main lattice and particle parameters:

ENERGY Particle energy in GeV .
ENO Initial particle energy in GeV.
DEN Energy step in GeV (relevant ring parameters will be calculated for nine different energies: ENERGY, ENO, ENO+DEN, ..., ENO +7 *DEN) .

CHROMX ${ }^{-}$Desired linear chromaticities in horizontal and vertical planes CHROMY (linear chromaticity is defined as $[v(\delta)-v(0)] \not f \delta$, where $\delta=\Delta p / p)$.

NHAR Number of harmonics used to calculate analytic approximation for tune dependence on $\delta$ due to the influence of sextupoles. The higher this number is the more accurate is the analytical computation of the tunes versus energy as compared to the computation from matrix multiplication.

NSUP Number of Superperiods
SYM Logical variable. If its value is $T$ (true) the program assumes that the superperiod of the machine is mirror symmetric. In this case the lattice of a half of the superperiod should be given below. If its value is $F$ (false) no symmetry of the superperiod is assumed and data for full period should be given.

DELMAX Maximum range of the momentum deviation in percents for which the Twiss parameters of the lattice should be calculated

HARM Harmonic number of the ring.

The next NAMELIST/TRP/ contains some parameters relevant for particle tracking:

TURNS Maximum number of revolutions for which particle should be tracked.

TRACK

TRSYN Logical variable which switches on and off the tracking with synchrotron oscillations. If TRSYN=T all particles are assumed to have initial momentum deviation equal to SIE times $\sigma_{E} / E$ ( $\sigma_{E}$ is the standard momentum spread of a bunch at the energy = ENERGY).

OFFEN Logical variable which allows (if it is True):

1) Calculation and printout of the lattice parameters for off-momentum particles with $\delta= \pm \operatorname{SIE}^{*} \sigma_{\mathrm{E}} / \mathrm{E}$;
2) Tracking off-momentum particles with the same as in (1) fixed energy deviations (i.e., without synchrotron oscil-lations).

PLOTS Positive real number scaling the phase space plot of any particle. Besides points representing coordinates of the particle (marked $x$ for horizontal and $y$ for vertical plane),
the plot is supplied by the beam-stay-clear ellipse defined by $10 * \operatorname{SIGMA}(X, Y)+10 * E T A * \sigma_{E} / E$. For PLOTS $=1$, the total size of the graph is just as big as this ellipse. If the plot size should be larger, choose PLOTS>1.

SCALEB Positive real number scaling the frequency spectrum of transverse particle motion. Since the amplitude of the signal on the betatron frequency is much larger then that at any other frequency, to make the satellites stand out of the noise it is advisable to choose $\operatorname{SCALEB}=10$. That will suppress all signals with amplitudes larger than $10 \%$ of the main signal.

The next line containes an array of flags KW which control print and in some cases calculation at different steps of the program.
$K W(i)=0 \quad$ Suppresses printout.
$\mathrm{KW}(1)=1 \quad$ Prints structure of the lattice.
$K W(2)=1 \quad$ prints results of the calculations for beam parameters and some radiation integrals.

KW(3) $=1$ Prints Twiss parameters and eta-functions at the exit of each element.

KW(4) $=1 \quad$ Calculates and prints information on the tune shift as a function of amplitude.

KW(5) $=1 \quad$ Calculates, prints and plots dependencies of tunes, beta- and eta-functions on momentum ( $\delta$ in percent).
$K W(6)=1 \quad$ Calculates and prints some information on harmonics of the chromaticity functions.

KW(7) $=1$ Prints average values for beta- and eta-functions over each element. Also linear derivatives of these functions in respect
to energy are printed. This output is obtained only if also $\operatorname{KW}(6)=1$.
$\mathrm{KW}(8)$ - KW(20) are not used in PATRICIA. (Some of them are used in other PATRICIA modifications, see below).

The next two lines specify the initial amplitudes for all four particles. The five numbers in the first line give the numbers of horizontal standard deviations for all the four particles and the number of energy standard deviations. The four numbers in the second line give the numbers of vertical standard deviations for initial displacements in the vertical plane. Initial transverse momenta deviations are assumed to be zero.

Then follows the list of element types, which can consist of up to fifty different elements. Each element is characterized by:

NAM Name of the element (any name excluding CELL and END).

ICD Integer code for magnet type:
ICD=1 drift space

ICD=2 dipole magnet
$I C D=3$ quadrupole magnet
$I C D=4 \quad$ sextupole magnet
$I C D=5$ is vacant (but is used in PNWM)
ICV Integer code of the bend plane:
ICV $=0 \quad$ horizontal bend (default)

ICV $=1$ vertical bend

FL Length of the element in meters (the length of the sextupole should be zero).

FS Strength of the element:
$1 / \rho$ for dipole in $\mathrm{m}^{-1}$,
$K$ for quadrupole in $\mathrm{m}^{-2}$,
$M$ for sextupole in $\mathrm{m}^{-2}$ (integrated strength).

APX Aperture of element in millimeters, in horizontal and vertical APY planes. Optional value of the gradient in $\mathrm{m}^{-2}$ for combined function bending magnet.

The type list is terminated by NAM=END.

Then follows the lattice structure. The lattice is given as a series of element names. Up to 3000 elements ( 600 quads among them) may be used in a superperiod or half superperiod if it has mirror symmetry. The lattice is terminated by the element END, which should be the first one in a line. Some elements may be BLANK. After each line of elements then follows a line of integer code numbers. A code number specifies the length of a drift space which should preceed the element positioned just above it. The code number I identifies the I'th real number in the list of the driftspace lengths which follows below.

If the first name in a line is CELL, the program expects the following parameters of a repeating part of the lattice:

IREP Number of cells.
IMAG Number of elements in a cell (up to 18).
then Sequence of names of elements,
and Sequence of spacedrift codes in the next line.

The lattice structure is terminated by the statement END.

The last follows the list of drift space lengths. There can be up to 50 different lengths in the list.

### 1.1.3 Chromaticity correction

Normally PATRICIA uses sextupole distribution derived by some other tools, e.g., by code HARMON. ${ }^{3}$ However, there is a possibility of using PATRICIA to improve old or even to develop new sextupole configurations. This can be done by using the information produced by PATRICIA on the magnitudes of amplitudes of different harmonics in the expansion of the tunes as a function of $\delta$ and on the contribution of different individual sextupoles into each of these harmonics. By making successive changes in the strengths of the most effective sextupoles, the variations of the tunes with momentum may be decreased. Certainly one should take into consideration also the variations of the tunes on betatron amplitudes, as well as the variations of the beta- and etafunctions at the interaction point with momentum.

With the set-up of the sextupole magnets in the ring, the following calculations are performed by PATRICIA:
a) Tune dependence on momentum.
b) Beta- and eta-function values at interaction point as functions of momentum.
c) Tune dependence on betatron amplitudes.

The usual matrix formalism for various momenta $\delta$ is used to find the tunes, the beta- and eta-functions. Sextupole is considered in thin lens approximation. Its action on transverse motion is replaced by a "kick" and its focusing action is represented by effective quadrupole of strength $K=\tilde{M n}(\delta) \delta$, where $K$ is the ratio of the integrated field
gradien't to the particle rigidity, $M$ is integrated sextupole strength, $\tilde{n}(\delta)$ is dispersion function for the particle with the relative momentum shift $\delta=\Delta \mathrm{p} / \mathrm{p}$.

All sextupoles are devided into two groups: special (any number of families or power supplies in this group) and regular, consisting of two families - one focusing and one defocusing. The strengths of the special sextupoles are considered as constants and will not be changed by the program. The strengths of the regular sextupoles are considered as variables for the program. It chooses them to adjust both linear chromaticities of the ring to values CHROMX and CHROMY, determined by the user in the NAMELIST/PARAM/. Actually, there is even no need to give the strengths of the regular sextupoles, but their types ought to be specified.

### 1.1.4 Output

The output of PATRICIA depends on values of parameters given in NAMELIST/TRP/ and print control code numbers KW . In the case where all the KW are put to 1 , the following printouts will result:

1) Input stream. Input data are reproduced (NAM=END is suppressed).
2) Structure of a superperiod (or half superperiod if $S Y M=T$ ). This table contains in natural order for all elements (including driftspace) : the name, the strength, the length and the position of the element in meters.
3) Mean values. Table of some radiation integrals and main integral ring parameters, such as momentum compaction factor. Table of main radiation characteristics (such as damping constants) for nine different particle energies is produced.
4) Lattice functions. Table of beta-, alpha-, phi (phase/2T)- and etafunctions for horizontal and vertical planes for on-momentum particle at the exit of each element. Sequential number and the name of element are given also. Both tune values conclude the table. If OFFEN=T, two more tables of lattice functions for particles with $\pm \delta\left(=\right.$ SIE* $\left.\sigma_{E} / E\right)$ are printed.
5) Tune shift with amplitude. Coefficients of both tune shift dependencies on both amplitudes.
6) Chromatic corrections, Table of all the sextupoles contains the sequential number, the number in the lattice, beta-, phi- and etafunctions at the sextupole and the strength (found by the program if it belongs to regular group). Full number of all sextupoles in the superperiod (or half the superperiod) can be found at the beginning of the table. Then follow values of natural chromaticity, chromaticity resulting from special sextupoles, chromaticity resulting from all sextupoles and the asked chromaticity for both planes.
7) Momentum dependencies. Table and plots of tunes, $\Delta \beta_{x} / \beta_{x}, \Delta \beta_{y} / \beta_{y}$ and $\Delta \eta_{x} / \eta_{x}$ at the interaction point as functions of $\delta$.
8) Harmonic analysis of chromaticity functions. Contribution of each sextupole into the amplitude of the given harmonic for all the harmonics specified by parameter NHAR.
9) Average values of beta- and eta-functions over each element and their linear derivatives with respect to $\delta$.
10) Results of tracking of the on-momentum particles. Initial coordinates ( $x, x^{\prime}, y, y^{\prime}$ ) and coordinates after the first revolution for all the four particles (in mm and mrad, correspondingly). Table of
the numbers of turns which each particle survives. Values of $\sigma_{x}$ and $\sigma_{y}$ in millimeters. Plot of the betatron frequency spectra for $x$ and $y$. Plots of projections of the Poincare section on ( $x, x^{\prime}$ ) and ( $y, y^{\prime}$ ) planes for each particle which survived full number of revolutions (TURNS in NAMELIST/TRP/).
11) Results of tracking particles performing synchrotron oscillations. Output looks the same as for on-momentum particle. The value of $\sigma_{E}$ is also given.

There is information on the CPU time used by the program at various steps of its performance.

All calculations are done with double precision.

### 1.2 Code PAQUASEX

A modification of the program PATRICIA is a part of a system of codes PAQUASEX ${ }^{4}$ which is essentially a combimation of the three codes PATRICIA (named here PATMOD), QUADS ${ }^{5}$ and MICROSEX. ${ }^{6}$ The system is designated for configuration survey over a grid of points in the space of main configuration parameters $\nu_{x}, \nu_{y}, \beta_{x}^{*}, \beta_{y}^{*}$ and $\eta_{x}^{*}$ (the star means the value of a parameter at the interaction point).

The system starts by preparing with the help of PATMOD input data decks for QUADS and MLCROSEX, i.e., target values of desired parameters. One option prepares a deck for a grid of $5 \times 5$ points in $\nu_{x}, \bar{\nu}_{y}$ space. The other option prepares five sets of five points. Each set of five points are increments in one of the five above mentioned parameters (keeping all others fixed). These options are selected by means of the control code number $\mathrm{KW}(16)$.

Each deck consequently is then run by QUADS, which finds quadrupole strengths, and MICROSEX to find appropriate sextupole solutions, by simple adjustment to the strength of sextupoles to maintain the linear chromaticity constant over the range of configuration parameters.

A data set is produced which is comprised of a sequence of PATRICIA input data ready to track particles for each point of the grid independently. Each data deck is supplied by JCL cards, including JOB=HOLD instruction for use in an automatic job release mechanism. The first job must be released manually, then as each job reaches a successful conclusion it will release the next job in the sequence. Such a job release procedure: a) prevents other users from being blocked out of the computer system by a string of production jobs and b) prevents large amounts of computing time from being wasted if fault conditions appear.

The output of PAQUASEX is put on a set of microfiches (the results of PATRICIA run for each point of the grid is put on a separate microfiche).

### 1.3 Code PNWM

The original beam dynamics and tracking program PATRICIA includes the influence of sextupole magnets both in the computation of chromatic eta-functions and in particle tracking. To investigate the invluence of higher multiple fields in different elements of a machine, an optional version of PATRICIA under the name PNWM can be used. The action of the nonlinear field in a given element is approximated by an effective integrated nonlinear "kick". The longitudinal position of the kick is at the discretion of the user. For placing the kick inside a lattice element one should prepare the lattice, in which the given element is split into two parts.

The non-1inear kick is assumed to have the following form:

$$
\begin{align*}
& \Delta x^{\prime}=K \sum_{n=1}^{N} A_{n} r^{n} \cos \left(n \theta+\beta_{n}\right)  \tag{1}\\
& \Delta y^{\prime}=-K \sum_{n=1}^{N} A_{n} r^{n} \sin \left(n \theta+\beta_{n}\right)
\end{align*}
$$

where $r=\left(x^{2}+y^{2}\right)^{1 / 2}, x=r \cos \theta, y=r \sin \theta, N$ is the maximum multipole number to be included into calculations. The coefficient $K$ is the integrated strength of the element, where the kick (1) is produced. For quadrupoles it should be expressed in $\mathrm{m}^{-1}$, for sextupoles in $\mathrm{m}^{-2}$, and for dipoles it should be dimensionless. The coefficient $K$ is proportional to the excitation.

To take into account the case of manufacturing errors in which all the values of amplitudes $A_{n}$ and phases $\beta_{n}$ are the same for similar elements, the list of different possible names of nonlinear kicks is inserted into the input data. The corresponding value of $K$ should be put in the place of the "strengths" of each kick included in the lattice.

For example, if the nonlinear kick MP1 is supposed to be placed in the quadrupole Q1, then the "strength" of the kick MP1 in the list should be $K=K_{Q 1} L_{Q 1}$.

The values of amplitudes $A_{n}\left(i n m^{-(n-1)}\right)$ and phases $\beta_{n}$ (in degrees) follow immediately after all the input cards for PATRICIA. The format of each card is 8 D 10.4 . The coefficient $A_{1}$ should always be equal to 0 .

As can be seen from (1), the complex kick

$$
\begin{equation*}
\Delta z^{\prime}=k \sum_{n} A_{n}(\bar{z})^{n} e^{-i \beta_{n}} \tag{2}
\end{equation*}
$$

may be introduced, where $\bar{z}=x$-iy. This form is very useful for particle tracking since the computation consumes much less time than that spent using trigonometric functions.

The code PNWM allows us to include high multipole fields components in different lattice elements without any restrictions on the symmetry of their distribution around the ring. The maximum number of different types of elements which PATRICIA allows is fifty. Since the types of multipoles are included in the same list, the maximun number of the different types of elements in which multipoles can be included is the difference between fifty and the number of all other types of elements in the lattice. The maximum number of different field multipoles in the given element is forty. The full number $N_{\text {mult }}$ of the lattice elements in which high multipole fields are included should be given in the NAMLIST/PARAM/ in the form NMLT $=\mathrm{N}_{\text {mult }}$.

The price for such freedom of including the multipoles is that the parameters NSUP and SYM are restricted to values 1 and $F$ only. That means that the whole lattice of the ring should be prepared without the consideration of its symmetry. There is only one option to make this task easier. Namely, one can limit oneself to half a ring in the case if the ring including all types of multipoles has a symmetry point. In this case it is enough to list only half of the lattice, setting at the same time the value of $\mathrm{KW}(14)$ to one

For each multipole listed in the lattice (including those in the second half of it if one uses the above mentioned option) one needs to give forty numbers of the multipole relative amplitudes (format 8D10.3) following the list of the drift lengths and forty numbers of the multipoles tilts in degrees (the same format) following the list of amplitudes.

Let us consider now field errors in quads. Suppose one is supplied with measured relative field multipole coefficients at the radius $r_{0}$ of a quadrupole bore. Using the expression (1) for the kick one gets the multiple coefficients according to the following formula:

$$
\begin{equation*}
A_{n}=\frac{C_{(n)} / C_{0}}{r_{0}^{n-1}} \quad, \quad n=2,3, \ldots \tag{3}
\end{equation*}
$$

where $C(n) / C_{0}$ are corresponding relative measured field components. Here $r_{0}$ is in $m$, and $A_{n}$ in $m^{-(n-1)}$.

The code PNWM has two additional options for preparing random distribution of error fields in quads. To use these options onc should set the flag $K W(15)$ equal to 2 (all multipoles randomized) or 1 (all but allowed multipoles, i.e., with numbers $K=4,8,12,16 \ldots$ randomized). The amplitudes $A_{n, k}$ of higher multipoles in this case are chosen uniformly distributed in the range

$$
\frac{A_{n, k}^{o}}{2} \leq A_{n, k} \leq \frac{3 A_{n, k}^{o}}{2},
$$

where $A_{n, k}^{0}$ is the value of the amplitude from the input data. The phases $B_{n, k}$ are chosen uniformly in the interval $(-\pi, \pi)$.

For the case of field errors in bending magnets similar considerations bring to the following connection between relative field components measured at the distance $x_{0}$ from equilibrium orbit and coefficients $A_{n}$

$$
\begin{equation*}
A_{n}=\frac{c_{(n)} / C_{0}}{x_{0}^{n}} \quad, \quad n=1,2, \ldots \tag{4}
\end{equation*}
$$

The connection between the integer number $n$ in formulae (1-4) and the number $\ell$ of poles for corresponding multipole field is the following:

$$
\begin{equation*}
\ell=2 *(n+1) \tag{5}
\end{equation*}
$$

For example, $\mathrm{n}=2$ corresponds to $\ell=6$, i.e., sextupole field component.
The output of the code PNWM is essentially the same as that of PATRICIA.

## 2. Tracking studies in PEP

Tracking method in PEP is used both to study the influence of a sextupole scheme for a given lattice configuration and to evaluate the quality of the measured fields in different particular magnets via the influence of the field errors on the particle motion. For the last task the code PNWM is used.

The usual procedure is:

1) to find a satisfactory sextupole scheme which provides stability of the motion of the particle with initial coordinates $x=8 \sigma_{x}, y=8 \sigma_{y}$ and $\delta=6 \sigma_{E} / E$. The fulfillment of this condition is checked by means of the code PATRICIA.
2) to investigate the machine acceptance in five dimensional space of the lattice parameter $v_{x}, v_{y}, \beta_{x}^{*}, \beta_{y}^{*}$, and $\eta_{x}^{*}$. Such a survey for each given configuration is performed with the help of the code system PAQUASEX.

### 2.1 Survey of mini-beta-6 configurations

As an example of tracking results obtained with the codes PATRICIA and PAQUASEX, I present here the results of the tracking study of several
mini－beta－6 configurations．Linear optics for these configurations were produced by R．Helm．${ }^{8}$

Modification of the PEP ring for decreasing $\beta_{y}^{*}$ while maintaining 6－fold symmetry of the ring（mini－beat－6）demands，among other things， developing new sextupole corrections．This stems not only（and mainly not）from the linear optic changes connected with the inward shift of the Q1 quadrupoles，${ }^{9}$ but also from the cojoint request to eliminate two sextupoles families，positioned in the ring where dispersion function is zero．

The procedures of developing sextupole corrections by means of computer program HARMON ${ }^{3}$ and of surveying the sextupole configurations thus obtained by means of the program package PAQUASEX are described in detail above．

Table 2 contains the main parameters of several configurations．The last line gives the maximum initial amplitude of a particle which was found stable after performing $\sim 800$ revolutions．The synchrotron oscil－ lations with amplitude $6 \sigma_{\mathrm{E}} / \mathrm{E}$ are also taken into account．Table 3 gives quadrupole and sextupole strengths for each configuration．Figures 1 and 2 illustrate the vertical tune and $\beta_{y}^{*}$ dependence on relative particle momentum．Figure 3 gives as an example results of tracking in the vicinity of an operational point in the tune diagram for the configura－ tion $⿰ ⿰ 三 丨 ⿰ 丨 三 一 2$ from Table 2.

Contours on these diagrams divide regions of stability for different initial transverse amplitudes for a particle performing the synchrotron oscillations with the initial longitudinal amplitude $6 \sigma_{E} / E$ ．

### 2.2 Influence of magnet field imperfections

The real fields produced by different magnets of the lattice deviate from the nominal desired fields due to finite geometrical pole sizes, manufacturing errors, edge effects, etc. The deviation of the field could be expanded into series of higher multipoles. The presence of those multipoles produces perturbation in particle motion, shifts the frequencies of betatron oscillations, induces the changes in closed orbits and produces a lot of nonlinear resonances. As a net result, all this could reduce the acceptance of the machine.

To estimate the influence of higher multipoles in different elements of the ring on the particle motion, we have used the program PNWM. To have a reference frame, several runs were made for the ideal machine. ${ }^{11}$ The particular configuration is a standard one with $\beta_{x}^{*}=3.0 \mathrm{~m}, \beta_{y}^{*}=.11 \mathrm{~m}$, $n^{*}=-.513 \mathrm{~m}, \nu_{x}=21.75$ and $\nu_{y}=18.75$.

Figure 4 illustrates results. The dashed curve is for the ideal machine. The solid curve is drawn between points represented stable (•) and unstable (+) initial particle coordinates measured in corresponding standard deviations $\sigma_{\mathrm{x}, \mathrm{y}}$.

For all cases in Fig. 4, the particle is executing synchrotron oscillations with amplitude $6 \sigma_{E} / E$ and with frequency $\Omega_{s}=.04784$. The standard deviations are those at 15 GeV with a coupling $\mathrm{k}=.50$, i.e., $\sigma_{x}^{*}=.609 \mathrm{~mm}, \sigma_{y}^{*}=.0862 \mathrm{~mm}$ and $\eta_{x}^{*} \sigma_{E} / E=-.530 \mathrm{~mm}$.

To estimate the acceptance of the machine with errors in ring quadrupole magnets, the results of ring quad measurements ${ }^{10}$ were used. Table 4 summarizes the measured value of high multipole fields as well as the amplitudes $A_{n}$ of $n$-pole used in the progran PNWM. The ratio of
high multipole amplitudes to quadrupole field were assumed to be the same in all quads, the value of the kick being proportional to the quad strength.

In Figure 5 the dashed curve 2 shows the change in the horizontal tune (the change in the vertical tunes are too small to be seen in this scale). In Fig. 6 we plot the acceptance curve for the case where the insertion quads have measured higher multipole fields. For the comparison the curve (dashed curve) for the ideal machine including sextupoles to make the chromaticity zero is also shown. As could be clearly seen, the machine acceptance is reduced significantly.

One more example of the use of PNWM gives investigation of the influence of higher multipole fields in the sextupole field of a high $\beta$ sextupole magnet. Corresponding values of the amplitudes $A_{n}$ of $n$-poles can be found in Table 4. Figure 7 presents the results of tracking particles with different initial coordinates as well as the dashed curve for the ideal machine.

Other examples of the use of the codes PATRICIA and PNWM can be found in several internal PEP reports. ${ }^{12-19}$

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TABLE 1


TABLE 2: Main parameters of different configurations

| \# | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Conf. | 212215 | 212211 | 212Z09 | 212 Z 07 | 217211 | 222Z11 | 222209 |
| $\nu_{\mathrm{x}}$ | 21.27 | 21.27 | 21.27 | 21.27 | 21.87 | 22.27 | 22.27 |
| $\nu_{y}$ | 18.20 | 18.20 | 18.20 | 18.20 | 18.70 | 19.20 | 19.20 |
| $\beta_{x}^{*} \mathrm{~m}$ | 2.40 | 1.76 | 1.80 | 1.60 | 1.76 | 1.76 | 1.66 |
| $\hat{\beta}_{x} \mathrm{~m}$ | 350. | 468. | 459. | 514. | 468. | 468. | 437. |
| $\beta_{\mathrm{y}}^{*} \mathrm{~m}$ | 0.15 | 0.11 | 0.09 | 0.07 | 0.11 | 0.11 | 0.09 |
| $\hat{\beta}_{y} \mathrm{~m}$ | 314. | 427. | 522. | 671. | 427. | 427. | 445. |
| $\eta_{x}^{*} \mathrm{~m}$ | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| $\hat{\eta}_{\mathrm{x}} \mathrm{m}$ | 1.38 | 1.39 | 1.41 | 1.41 | 1.30 | 1.25 | 1.25 |
| $\xi_{\mathrm{x}}$ | -47.8 | -58.2 | -57.5 | -62.4 | -59.1 | -59.8 | -62.3 |
| $\xi_{y}$ | -74.8 | -95.6 | -113.6 | -141.4 | -96.3 | -97.0 | -114.3 |
| $\sigma_{x} \mathrm{~m}$ | 0.50 | 0.43 | 0.45 | -- | 0.40 | 0.41 | 0.38 |
| $\sigma_{y} \mathrm{~mm}$ | 0.090 | 0.072 | 0.061 | -- | 0.072 | 0.072 | 0.067 |
| $\sigma_{\mathrm{E}} \mathrm{mm}$ | 0.006 | 0.002 | 0.008 | -- | 0.004 | 0.024 | 0.021 |
| $\hat{N}_{\sigma}$ | 8 | 8 | 7 | No Stable Solution | 8 | 8 | 8 |

TABLE 3: Quadrupole and Sextupole Strengths

|  | \# | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N | Config. | 212215 | 212211 | 212209 | 212 Z 07 | 217211 | 222211 | 222209 |
| Quadrupoles (Strengths in $\mathrm{m}^{-2}$ ) |  |  |  |  |  |  |  |  |
| 1 | Q1 | . 1072369 | . 1078637 | . 1080972 | . 1082978 | . 1078868 | . 1079100 | . 1080518 |
| 2 | Q2 | -. 0744027 | -. 0752630 | -. 0754172 | -. 0756825 | -. 0753523 | -. 0753543 | -. 0756059 |
| 3 | Q3 | . 0926554 | . 1040188 | . 1137074 | . 1201018 | . 1116388 | . 1144953 | . 1257062 |
| 4 | QF1 | -. 1139333 | -. 1119642 | -. 1179723 | -. 1180428 | -. 1190178 | -. 1224639 | -. 1270189 |
| 5 | QD1 | . 2273302 | . 2129494 | . 2071517 | . 2028293 | . 2230332 | . 2335272 | . 2278653 |
| 6 | QF2 | -. 1788230 | -. 1772496 | -. 1763517 | -. 1759856 | -. 1844021 | -. 1892684 | -. 1885936 |
| 7 | QF3 | -. 2249990 | -. 2189361 | -. 2166563 | -. 2149418 | -. 2238350 | -. 2283592 | -. 1163574 |
| 8 | QF | -. 1734549 | -. 1730224 | -. 1722640 | -. 1719395 | -. 1788064 | -. 1831356 | -. 1824782 |
| 9 | QD | . 1829656 | . 1848309 | . 1843534 | . 1841484 | . 1884141 | . 1910415 | . 1906462 |
| $-10$ | QD8 | . 1895568 | . 1875802 | . 1873765 | . 1872878 | . 1891189 | . 1902750 | . 1900802 |
| 11 | QF9 | -. 1302707 | -. 1294146 | -. 1290746 | -. 1289292 | -. 1320442 | -. 1340443 | -. 1337387 |
| Sextupoles (Strengths in $\mathrm{m}^{-2}$ ) |  |  |  |  |  |  |  |  |
| 1 | SF | -. 50495 | -. 56753 | -. 59847 | -. 71092 | -. 78705 | -. 84710 | -. 94222 |
| 2 | SD | . 36361 | . 51219 | . 61461 | . 71621 | . 70526 | . 67307 | 1.07487 |
| 3 | SD5 | . 64402 | . 37362 | . 10984 | . 20000 | . 93782 | 1.07584 | 1.16719 |
| 4 | SD6 | . 99842 | 1.83180 | 2.15303 | 2.46034 | 1.30871 | 1.43045 | 1.60867 |
| 5 | SD7 | 2.06786 | 2.22140 | 2.35271 | 2.46183 | 2.14921 | 2.21123 | 1.45817 |
| 6 | SF9 | -. 06899 | -. 21352 | -. 33725 | -. 72463 | -. 00945 | -. 10000 | -. 10000 |
| 7 | SF6 | -. 96073 | -1.23577 | -. 99623 | -. 20000 | -. 82569 | -. 49838 | -. 42672 |

TABLE 4

|  |  | Insertion Quadrupoles$r_{0}=0.1047 \mathrm{~m}$ |  |  | Ring Quadrupoles$r_{0}=0.04494 \mathrm{~m}$ |  |  | High Sextupole $\mathrm{r}_{0}=0.08 \mathrm{~m}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \& | n | Field | $A_{n}$ | $B_{\square}^{0}$ | Field | $A_{n}$ | $B_{n}^{\circ}$ | Field | $A_{n}$ | $B_{n}^{0}$ |
| 4 | 1 | 1. |  | 0 | 1. |  | 0 |  |  |  |
| 6 | 2 | $0.11 \times 10^{-2}$ | $0.105 \times 10^{-1}$ | 180 |  |  |  | 1. |  | 0 |
| 8 | 3 | $0.09 \times 10^{-2}$ | $0.821 \times 10^{-1}$ | 0 |  |  |  |  |  |  |
| 10 | 4 | $0.01 \times 10^{-2}$ | $0.871 \times 10^{-1}$ | 0 |  |  |  |  |  |  |
| 12 | 5 | $0.38 \times 10^{-2}$ | $0.316 \times 10^{2}$ | 180 | $2.5 \times 10^{-3}$ | $6.13 \times 10^{2}$ | 0 |  |  |  |
| 14 | 6 |  |  |  |  |  |  |  |  |  |
| 16 | 7 |  |  |  |  |  |  |  |  |  |
| 18 | 8 |  |  |  |  |  |  | $1.94 \times 10^{-2}$ | $7.40 \times 10^{4}$ | 180 |
| 20 | 9 | $0.13 \times 10^{-2}$ | $0.899 \times 10^{5}$ | 180 | $3.0 \times 10^{-3}$ | $1.80 \times 10^{8}$ | 180 |  |  |  |
| 22 | 10 |  |  |  |  |  |  |  |  |  |
| 24 | 11 |  |  |  |  |  |  |  |  |  |
| 26 | 12 |  |  |  |  |  |  |  |  |  |
| 28 | 13 | $0.22 \times 10^{-2}$ | $0.126 \times 10^{10}$ | 180 | $2.7 \times 10^{-3}$ | $3.98 \times 10^{13}$ | 180 |  |  |  |
| 30 | 14 |  |  |  |  |  |  | $1.47 \times 10^{-3}$ | $2.14 \times 10^{10}$ | 0 |
| 32 | 15 |  |  |  |  |  |  |  |  |  |
| 34 | 16 |  |  |  |  |  |  |  |  |  |
| 36 | 17 |  |  |  |  |  |  |  |  |  |
| 38 | 18 |  |  |  |  |  |  |  |  |  |
| 40 | 19 |  |  |  |  |  |  |  |  |  |
| 42 | 20 |  |  |  |  |  |  | $1.40 \times 10^{-4}$ | $7.78 \times 10^{15}$ | 180 |

## FICURE CAPTIONS

Fig．1．Variation of fractional vertical tune（ $\Delta \nu_{y}$ ）with momentum deviation $(\delta=p / p$ in $\%$ ）。 Numbers on the curves correspond to configuration numbers in Table 2.

Fig．2．Variation of vertical beta－function at the interaction point $\left(\beta_{y}^{*}\right)$ with momentum deviation $\delta$ ．Numbers on the curves correspond to configuration numbers in Table 2 ．

Fig．3．Stability of the configuration $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ 2 in $\left(\nu_{x}, \nu_{y}\right)$ plane．The three digit numbers are used to index the tracking runs recorded on microfiche：$\quad * 8 \sigma_{x}, 8 \sigma_{y}, 6 \sigma_{E}$ is stable
$\odot 7 \sigma_{x}, 7 \sigma_{y}, 6 \sigma_{E}$ is stable
（－ $6 \sigma_{x}, 6 \sigma_{y}, 6 \sigma_{E}$ is stable．

Fig．4．Acceptance of the ring with higher multipoles in ring quads． Sextupoles are adjusted for zero chromatisities：
＋particle unstable
－particle stable for 750 revolutions
The dashed curve is for ideal machine．

Fig．5．Tune dependence on momentum．
1）$v_{x}$ for ideal ring quadrupole field．
2）$v_{x}$ for measured ring quadrupole field．
3）$\nu_{y}$ for both cases．

Fig．6．Acceptance of the ring with the measured multipoles in the insertion quads．Dashed curve for ideal machine．

Fig。 7. Acceptance of the ring with higher multipoles in the high-beta sextupole magnet. Dashed curve is for ideal machine.


Fig. 1


Fig. 2


Fig. 3


Fig. 4


Fig. 5

$N \sigma_{x}$
4314 A 2

Fig. 6


Fig. 7


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