SLAC-PUB-2920
May 1982
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COMPOSITE MODELS OF QUARKS AND LEPTONS AND STRONG COUPLING LATTICE GAUGE THEORIES*

Helen R. Quinn, Sidney D. Drell and Subhash Gupta
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We extend a previous variational block spin analysis of the realization of chiral symmetry in a strong coupling lattice gauge theory to models which have been suggested as possibly describing massless composite fermions. In all cases we find massive fermion composites and spontaneous breaking of chiral symmetry. We also discuss the relevance of this result to the continuum limit.

Submitted to Physical Review D

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

I. INTRODUCTION

This paper examines the realization of chiral symmetry in lattice gauge theories at strong coupling. Our purpose is to explore for possible realizations of unbroken chiral symmetry with massless composite fermions particularly in examples in which the 't Hooft anomaly conditions[1] can be satisfied. It is generally believed that such models may allow construction of composite fermions which are candidates for leptons and quarks. In particular such particles must have the property that their masses are very small compared to their inverse radii, and a massless composite is supposed to be a good starting point for such an object. Unfortunately we find a general result that the effective strong coupling theory is antiferromagnetic in character, choosing to realize the chiral symmetry in the Nambu-Goldstone fashion with massless bosons, which are spin-wave-like excitations, and all fermions massive. This result is independent of the gauge group and representation content of the theory.

This work is an extension of, and follows directly, an earlier study,[2] henceforth referred to as SDQW, which performed a variational block spin calculation for strong coupling lattice gauge theories of both the QCD and the Abelian types. Using the long-range SLAC gradient to preserve chiral symmetry and avoid spectrum doubling in the lattice Hamiltonian, SDQW found in these cases that the chiral symmetry of the theory was realized in the Nambu-Goldstone fashion. In the present paper we extend these results to Hamiltonians with any gauge groups and with the fermions in more than one representation of the gauge group. We also consider purely left-handed fermions, in which cases we find spontaneous breaking of the lattice rotational symmetry as well as of the chiral symmetry.

There has been much discussion in the literature about lattice fermions and various choices for the lattice derivative. It has been shown by Nielson and Ninomoya[3] and further discussed by Rabin[4] that no local version of a lattice derivative can simultaneously possess chiral symmetry and avoid spectrum doubling. For this reason we choose a nonlocal formulation. It is of course local in the continuum limit. This formulation is unpopular for various reasons. Firstly, it is extremely inconvenient for most types of calculation (for example in strong coupling perturbation theory it corresponds to an infinite number of terms in the perturbative Hamiltonian). Secondly, it has been specifically criticized on two grounds;[5] namely, that it fails to have any anomalies and that it does not correctly reproduce weak-coupling perturbation theory. Both these critisisms have been refuted. It has recently been shown by Weinstein[6] by a careful examination of regulated axial currents for the Schwinger model that the anomalies do reappear in the continuum limit. This feature can be expected also to be true for higher dimensional theories. Furthermore the work of Rabin[7] has demonstrated that a gauge invariant subtraction prescription can be defined, with which the usual weak coupling perturbation theory is reproduced term by term. Hence we feel that this gradient provides the best candidate for a chiral symmetric lattice theory and thus choose to use it in our analysis.

The derivation of a strong coupling effective Hamiltonian is discussed in Section II for general gauge groups and for any number of two or four component fermions in any representations of the gauge group. In Section III we discuss a construction of the gauge group singlet states at a single site in such theories. Section IV contains a mean-field analysis of the effective strong coupling Hamiltonian. In the SDQW it was observed that this Hamiltonian is a generalized antiferromagnet and hence that one might expect that approximations that give reasonable results for the multi-dimensional Heisenberg antiferromagnet would also work well here. It was then shown by Greensite and Primack[8] that an alternating site mean-field calculation reproduced the physical picture given by the more accurate block-spin procedure of SDQW. Because this calculation is simple and easy to understand we present here the analysis of the general strong-coupling effective Hamiltonian by this method. We stress those features which would survive in a more general ground-state ansatz, such as that constructed by a block-spin procedure.

In Section V(a) we discuss the question of the relevance of our results to the continuum limit, which corresponds to weak rather than strong coupling. We discuss two possibilities, neither of which suggests that these are theories of massless composite fermions. One possibility is that there is a unique phase for all g, in which case our strong-coupling result of spontaneously broken chiral symmetry applies also to the continuum. The other possibility is that there is a phase transition at some finite g and a weak coupling phase with manifest chiral symmetry. However there is no indication that such a weak coupling phase would be a confining phase with small radius composite particles. In Section V(b) we discuss the effect on our results of the addition of an explicit chiral symmetry breaking term. We examine the effect of a quark mass term, and also of the chiral symmetry breaking terms that appear in a Wilson gradient.[9] Finally, to complete our discussion, we show that our results will also apply for a theory using the Kogut-Susskind lattice gradient,[10] which splits a four-component fermion onto two sites and interprets the doubling as multiple flavors. We note one peculiar exception suggested by Banks and Kaplonovsky, who use a variant of a Kogut-Susskind gradient which is only possible for groups with real representations, such as O(n), and with single flavor of fermion. This formulation of the gradient does not allow the construction of a local gauge group singlet bilinear operator and thus evades our general result. These theories would give massive baryons when treated using any of the standard lattice gradients. In Section V(c) we briefly discuss the extension of this work to theories with nonsimple gauge groups. Section V(d) contains a summary of our conclusions.

In an Appendix we discuss explicitly some models which have appeared in the literature as possible examples of preon type theories.

II. EFFECTIVE HAMILTONIAN FOR STRONG COUPLING

First we consider the problem of a simple gauge group, say SU(N), with fermions (preons) assigned to some set of representations R of dimension d_R. We denote the number of flavors in represention R by f_R. The four-component fermion fields $\psi_R^{\alpha a}(\vec{j})$ thus carry site labels $\vec{j}$ denoting location on a three-dimensional (spatial) lattice, color labels, α, which run from 1 to d_R; and flavor labels, a, which run from 1 to f_R. We use the usual Wilson notation for the gauge field operators: ${}^{\alpha}U_{\beta}^{R}(\vec{j},\mu)$ denotes an operator which acts on the link from $\vec{j}$ to $\vec{j}+\hat{\mu}$ and transforms as the representation R at the site $\vec{j}$ and the conjugate representation $\bar{R}$ at site $\vec{j}+\hat{\mu}$. The conjugate representation $\bar{R}$ is denoted by the lowered gauge label β and the obvious contraction of R and $\bar{R}$ to a color singlet object is to be assumed when color labels are suppressed.

We use the long range form of the lattice gradient operator, which for an infinite volume lattice is

$$\partial_\mu \psi(\vec{j}) \equiv \sum_{\vec{j}} \left(\sum_{\hat{n} \neq \hat{\mu}} \delta_{j_{\hat{n}} j'_{\hat{n}}} \right) \frac{(-1)^{(j_\mu - j'_\mu)}}{j_\mu - j'_\mu} \psi(\vec{j}') \tag{2.1}$$

in order to explicitly maintain chiral symmetry without the fermion "doubling" problem. The Hamiltonian in $A^0 = 0$ gauge is thus

$$H = \frac{1}{a} \left[\sum_{\text{links}} g^2 E^2 + \frac{1}{g^2} \sum_{\text{plaquettes}} \sum_R \left\{ \operatorname{Tr}_{\Box} \left(U^R U^R U^{R\dagger} U^{R\dagger} \right) + \text{h.c.} \right\} \right.$$

$$\left. + \sum_R \sum_{a=1}^{f_R} \sum_{\vec{j},\ell,\hat{\mu}} \left\{ \frac{(-1)^\ell}{\ell} \bar{\psi}^a_R(j)\, \alpha_\mu \prod_{\vec{j}'=\vec{j}}^{\vec{j}+(\ell-1)\hat{\mu}} U^R(\vec{j}',\hat{\mu})\, \psi^a_R(\vec{j}+\ell\hat{\mu}) \right\} \right] \tag{2.2}$$

where the lattice spacing a is the only dimensionful quantity and α_μ is the Dirac matrix $\gamma_0\gamma_\mu$. This Hamiltonian has an explicit chiral symmetry

$$S_{\text{chiral}} = \prod_R \left\{ SU(f_R)_{\text{right}} \otimes SU(f_R)_{\text{left}} \otimes U(1)_{\text{vector},R} \otimes U(1)_{\text{axial},R} \right\} \tag{2.3}$$

It is also useful to remark that all the terms of H except the fermion terms corresponding to even lattice separations have an even larger symmetry

$$S_{nn} = \prod_R \left\{ SU(4f_R) \otimes U(1)_R \right\} \tag{2.4}$$

(S_{nn} stands for nearest-neighbor symmetry.) Clearly if we drop all terms except the ℓ=1 term, (2.2) retains only the nearest-neighbor form of the derivative and suffers the usual additional degeneracies in the fermion spectrum—which we here identify as a property of the spurious S_{nn} symmetry of that term.

The generators of the S_{nn} can most readily be identified by introducing redefined fermion fields

$$\tilde{\psi}_R(\vec{j}) = \alpha_x^{j_x} \alpha_y^{j_y} \alpha_z^{j_z} \psi_R(\vec{j})$$
$$\tilde{\psi}^{\alpha a}_R = \begin{pmatrix} b^{\alpha a}_R \\ d^{\dagger\alpha a}_R \end{pmatrix} \tag{2.5}$$

where b and $d^\dagger$ are two-component spinors. The U(1) charge is then

$$Q_R(j) = b_R^\dagger(\vec{j})\, b_R(\vec{j}) - d_R^\dagger(\vec{j})\, d_R(\vec{j}) = \tilde{\psi}_R^\dagger(\vec{j})\, \tilde{\psi}_R(\vec{j}) - 2f_R d_R$$

$$Q_R = \sum_j Q_R(\vec{j}) \tag{2.6}$$

and the generators of the $SU(4N_f)$ are

$$Q_R^k(\vec{j}) = \tilde{\psi}_R^\dagger(\vec{j})\, M^k\, \tilde{\psi}_R(\vec{j})$$

$$Q_R^k = \sum_j Q_R^k(\vec{j}) \tag{2.7}$$

where the M^k are the usual $(4f_R \times 4f_R)$ traceless Hermitian unitary matrix representation of $SU(4f_R)$. In our notation these can be constructed (up to a normalization factor) from the tensor products of the 4×4 Dirac matrices with the $SU(f_R)$ flavor symmetry generators, which we denote by τ. One can readily verify that all the Q_R^k and Q_R commute with all odd-ℓ terms in H. However only those generators

$$M^k \propto \gamma_0(1 \pm \gamma_5) \times \tau \qquad , \qquad \gamma_0(1 \pm \gamma_5) \times I \tag{2.8}$$

which are to the generators of S_{chiral} commute also with the even-ℓ terms.

To derive a strong-coupling effective Hamiltonian from H we follow the procedure of SDQW. States containing any non-vanishing color flux have energies of order g^2, and hence at large g^2 these have very high energy. The prescription is thus to separate H into the leading term H_0 plus a correction V, where

$$H_0 = \sum g^2\, \vec{E}^2 \tag{2.9}$$

$$V = H - H_0$$

and to perform degenerate perturbation theory in the sector of flux free states. In this sector the non-Abelian equivalent of Gauss' law, which must be imposed as a superselection rule in this gauge, requires that the fermion state at every site is a gauge-group singlet. We will describe the construction and classification of such states in the following section. Here it is sufficient to note that there are many of them, and that they fall into multiplets of S_{nn}.

Since every term in V contains flux-creating operators acting on such a state, it takes the system out of the sector of flux-free states. V may act any number of times again before returning the system to the flux-free sector. Since intermediate states containing flux give energy denominators of order g^2, perturbation theory becomes a power series expansion in $1/g^2$ for the effective Hamiltonian in the flux free sector. The leading term is of order $1/g^2$ and arises when the fermion term of V acts twice, exciting flux and then annihilating it on any given segment of the lattice.

This gives

$$H_{eff} = \frac{1}{g^2} \sum_R \sum_{\vec{j},\ell,\hat{\mu}} \psi^{\dagger a}_{R\alpha}(\vec{j})\, \alpha_\mu\, \psi^{\beta a}_R(\vec{j}+\ell\hat{\mu})\, \psi^{\dagger b}_{R\beta}(\vec{j}+\ell\hat{\mu})\, \alpha_\mu\, \psi^{\alpha b}_R(\vec{j})$$

$$\times \left(\frac{-1^\ell}{\ell}\right)^2 \frac{N_R}{g^2 c_R \ell} + \mathcal{O}\left(\frac{1}{g^4}\right) \qquad (2.10)$$

where

$${}^{\alpha}U^R_\mu \left({}^{\sigma}U^R_\nu\right)^\dagger = N_R\, \delta^\alpha_\sigma\, \delta^\nu_\mu + \text{non-singlet plieces}$$

and $g^2 c_R \ell$ is the energy denominator from H_0 corresponding to a string of length ℓ in representation $R\text{-}\bar{R}$. When fermions are assigned to representations R which are self-conjugate or to any two representations R and R' such that $R \times R'$ contains a singlet then there are additional terms in H_{eff} at order $1/g^2$. However, (2.10) is completely general when all representations R can be formed from products of less than N fundamental representations; we deal with the exceptional cases in Section II(c).

By performing a Fierz transformation and using the definitions (2.6) and (2.7), we can rewrite (2.10) in a more compact form.

$$H_{eff} = \frac{1}{g^2} \sum_R \sum_{\vec{j},\ell,\hat{\mu}} \frac{N_R}{4f_R c_R} \left\{ Q_R(\vec{j})\; Q_R(\vec{j}+\ell\hat{\mu}) + x^2 \sum_k (\eta^{\mu k})^{\ell+1}\; Q_R^k(\vec{j})\; Q_R^k(\vec{j}+\ell\hat{\mu}) \right\} \frac{1}{\ell^3} \tag{2.11}$$

where

$$\alpha_\mu\, M^k\, \alpha_\mu = \eta^{\mu k}\, M^k \quad (\eta^{\mu k} = \pm 1) \quad . \tag{2.12}$$

and x is a normalization factor for the charges Q_R^k in terms of the Dirac matrices. The $\eta^{\mu k}$ for the various M^k are shown in Table I. The importance of S_{nn} becomes quite obvious in this strong coupling H_{eff}. Because the interactions fall off rapidly, as $(\ell)^{-3}$, the odd-neighbor terms, which include ℓ=1, provide the dominant part of H_{eff}, the smaller even-neighbor terms provide the symmetry breaking perturbations. The odd-ℓ terms are of the form

$$x^2 \sum_k Q_R^k(\vec{j}) \quad Q_R^k\left(\vec{j} + (2n+1)\hat{\mu}\right) + Q_R(\vec{j})\; Q_R\left(\vec{j} + (2n+1)\hat{\mu}\right)$$

and hence are antiferromagnetic in character, tending to anti-align $SU(4f_R) \times U(1)$ spins on sites separated by odd numbers of lattice spacings. The even-ℓ terms tend to reinforce this pattern by lowering the energy for spins separated by even distances, provided those spins are aligned in those $SU(4f_R)$ directions which correspond to all $\eta^{\mu k} = -1$. However, they give additional energy for spins aligned in (any) those $SU(4f_R)$ direction for which $\eta^{\mu k}$ is positive (or) as well as in the U(1) direction. These remarks and the conclusion that fermions are massive in order $1/g^2$ will be made more explicit in the detailed analysis of Section IV.

(b) Two-Component Preons

The preceeding discussion can easily be repeated for the case in which ψ is a two-component spinor, say purely left-handed. The changes are simply that the Dirac matrices α_μ are replaced by spin matrices σ_μ and the charges become

$$\begin{aligned} Q_R(\vec{j}) &= \tilde{\psi}^\dagger{}_R(\vec{j})\ \psi_R(\vec{j}) - 2f_R \\ Q_R^k(\vec{j}) &= x\ \tilde{\psi}_R^\dagger(\sigma\times\tau)^k\ \tilde{\psi}_R \end{aligned} \tag{2.13}$$

so that the symmetry S_{nn} is

$$S_{nn} = \prod_R \left\{ SU(2f_R)\times U_R(1) \right\} \quad . \tag{2.14}$$

The theory is again antiferromagnetic in character, leading to massive fermions. The case when the same number f_R of flavors of left-handed spinors are placed in both the representation R and the representation $\bar{R}$ is entirely equivalent to the four-component theory with $SU(4f_R)\times U(1)$ symmetry.

(c) Theories with Color Singlet Diquarks

If the fermions of the lattice theory belong to two representations R and R' such that $R\times R'$ contains the singlet representation then the terms of order $1/g^2$ in H_{eff} include additional terms of the form

$$\begin{aligned} \Delta H_{eff} = \frac{1}{g^2 a} \sum_{\vec{j},\ell,\hat{\mu}} \frac{1}{\ell^3} \Bigg\{ \Big[& \psi^{\dagger a}_{R\alpha}(\vec{j})\ \alpha_\mu\ \psi^{a\beta}_{R'}(\vec{j}+\ell\hat{\mu})\ \psi^{\dagger b}_{R'\sigma}(\vec{j})\ \alpha_\mu\ \psi^{b\nu}_{R}(\vec{j}+\ell\hat{\mu}) \\ & \times c^{\alpha\sigma}\ c_{\beta\nu}\ \frac{N_{RR'}}{c_R} \Big] + \text{h.c.} \Bigg\} \end{aligned} \tag{2.15}$$

where

$$\alpha_{U^R_\beta}\ \sigma_{U^{R'}_\nu} = c^{\alpha\sigma}\ c_{\beta\nu}\ N_{RR'}\ I + \text{non-singlet pieces}$$

and $c^{\alpha\sigma}$ ($c_{\beta\nu}$) are contractions of R with $\bar{R}'$ ($\bar{R}$ with R') to a SU(N) singlet.

The role of these additional terms is to provide a kinetic energy for the gauge-group singlet diquark states which exist in such a theory. By arguments parallel to those for composites with odd numbers of fermions these diquark states are also massive in order $1/g^2$ and their existence does not alter our previous discussion.

In a theory with two left-handed quarks in representations R and $\bar{R}$, after having included all such terms, one can make a redefinition of the $\bar{R}$ antiparticles as the right-handed part of a four component spinor in representation R. Then one recovers the form of a four-component theory in representation R. [Note that the single site state $|\chi\rangle$ annihilated by all ψ_R and $\psi_{\bar{R}}$'s is thus reidentified correctly as the state with the maximum possible number of $d^\dagger$'s acting on it.]

III. GAUGE GROUP SINGLET STATES AT A SINGLE SITE

Because the strong coupling H_{eff} is so simply expressed in terms of the local S_{nn} charges it is convenient to classify the gauge group singlet states at a single site under this symmetry. The prescription for doing this is straightforward, though in general the group theory can become quite tedious:

(1) Denote by $|0\rangle$ the state annihilated by all b's and d's.

(2) Construct the state $|\chi\rangle$ with minimum value of all $\langle Q_R\rangle$

$$\text{for four-component spinors: } \langle Q_R\rangle^{min} = -2d_R f_R \;;\; |\chi\rangle = \prod_{\alpha,a,i} d_i^{\dagger\alpha a}|0\rangle$$

$$\text{for two component spinors: } \langle Q_R\rangle^{min} = -d_R f_R \;;\; |\chi\rangle = |0\rangle \quad . \tag{3.1}$$

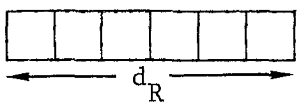
This state is readily classified as a singlet under all $SU(nf_R)$'s where $n = 2$ or 4 for two- or four-component spinors.

$$|\chi\rangle = \left|\prod_R \left(1\,;-\tfrac{n}{2}d_R f_R\right)\right\rangle \quad . \tag{3.2}$$

(3) Construct all possible gauge group singlet operators which can be made from products of the $\psi_R^\dagger$'s or from some $\psi_R^\dagger$'s and some ψ_R's. Those with odd numbers of fermion fields are the operators identified as the creation operators for composite fermions when counting states in the theory for the 't Hooft anomaly cancellation. This will be true here, once the vacuum sector has been correctly identified. We remark that there will always be a set of composite operators of the type

$$B_A^\dagger(R) = \left(\psi_R^\dagger\right)^{d_R}_{\substack{\text{antisymmetrized}\\ \text{gauge group indices}}} \quad . \tag{3.3}$$

Since the fermion field is in the fundamental representation of the $SU(nf_R)$ symmetry, this operator must belong to the representation of the $SU(nf_R)$ symmetry given by



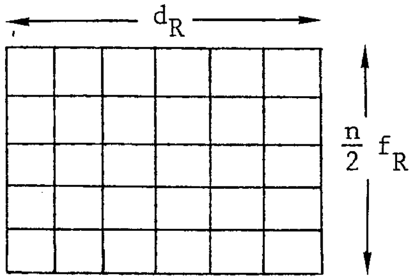
$$\text{dimension} = \frac{(d_R + nf_R - 1)!}{(nf_R - 1)!\; d_R!} \tag{3.4}$$

In general there may be many other gauge group singlet operators.

(4) All possible gauge group singlet states can be constructed by acting on the state $|\chi\rangle$ with products of the various singlet operators. In particular the states with maximal $SU(nf_R)$ representations, and all $\langle Q_R\rangle = 0$, can be constructed from the state χ as follows

$$\begin{aligned} |M\rangle &= \prod_R \left(B_A^\dagger(R)\right)^{nf_R/2} |\chi\rangle \\ &= \left|\prod_R \left(D_R\,;0\right)\right\rangle \quad . \end{aligned} \tag{3.5}$$

The representation of the $SU(nf_R)$ symmetry is then of the form



Dimension = D_R . (3.6)

We will call these states the maximal states; this representation contains the largest $SU(4f_R)$ weight of any single-site color singlet state in such a theory. We will show that the ground state of the strong coupling effective Hamiltonian always lies in the sector of states for which every site of the lattice is in a maximal state.

As a simple example of this general discussion consider the usual QCD theory. The gauge group is SU(3) and there are f flavors of quarks in the fundamental representation. The state χ is thus

$$|\chi\rangle = \prod_{\alpha,a,i} d_i^{\dagger\alpha a} |0\rangle = |1\,;-6f_R\rangle \quad . \tag{3.7}$$

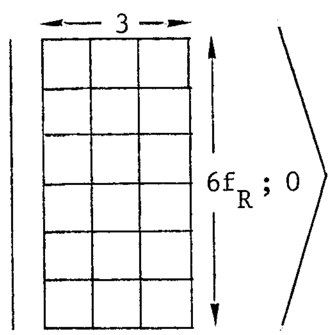
The color singlet operators are the generators Q_R^k and Q_R and

$$B_{ijk}^{\dagger abc} = \varepsilon^{\alpha\beta\gamma} \psi_{\alpha i}^{a\dagger} \psi_{\beta j}^{\dagger b} \psi_{\gamma k}^{\dagger c} \quad . \tag{3.8}$$

The $B^\dagger$ belong to the totally symmetrized 3-index $SU(4f_R)$ representation, and the generators of course belong to the adjoint representation.

The maximal states are given by

$$(B^\dagger)^{6f_R} |\chi\rangle =$$

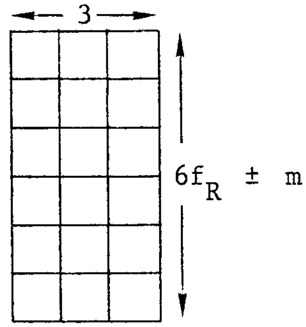


. (3.9)

All possible gauge group singlet states can be obtained by

$$(B^\dagger)^m |M\rangle \quad \text{or} \quad (B) |M\rangle \tag{3.10}$$

and these are states with quantum numbers $Q = \pm 3m$ and SU(4f) representations given by



. (3.11)

The generators simply transform any state to another state within the same representation, or annihilate it.

We present some further explicit examples in the Appendix.

IV. MEAN FIELD ANALYSIS OF H_{eff}

The essential physics of H_{eff} is contained in the observation that the odd-neighbor terms dominate and that they are anti-aligning in character. The even neighbor terms then either reinforce or compete with this anti-alignment depending on whether the sign $\eta^{\mu k}$ is negative or positive. Thus the Hamiltonian is a generalized anti-ferromagnet and we can expect to learn much about its physics by using methods which are known to work well for the Heisenberg anti-ferromagnet. Based on the work of SDQW and the subsequent simplified (but cruder) analysis of Greensite and Primack[8] we argue that we can find the correct ground state sector using an alternating site mean field ansatz. The realization of chiral symmetry and the existence of masses for composite fermions will depend only on very general properties of this ground state which, we will argue, would be retained even in a more sophisticated analysis. We will point out at various stages how results of a more general variational treatment would differ from the mean-field results.

In an alternating site mean-field approach one makes an ansatz for a trial state of the form

$$|\phi\rangle = \prod_{\substack{\text{sites}\\ j_x+j_y+j_z\\ \text{even}}} |\phi_e(j)\rangle \prod_{\substack{\text{sites}\\ j_x+j_y+j_z\\ \text{odd}}} |\phi_0(j)\rangle \quad . \tag{4.1}$$

Thus one divides the lattice into two sub-lattices and assumes all sites of a sub-lattice are in the same state. The states ϕ_e and ϕ_0 are then chosen so as to minimize the energy density

$$\mathscr{E} = \langle\phi|H|\phi\rangle/\text{volume} \quad . \tag{4.2}$$

Let us denote, for any single site operator X

$$\begin{aligned} \langle\phi_e|X|\phi_e\rangle &= \langle X\rangle_e \\ \langle\phi_0|X|\phi_0\rangle &= \langle X\rangle_0 \quad . \end{aligned} \tag{4.3}$$

Then the energy density in this state is

$$\begin{aligned} \mathscr{E} = \sum_R \Bigg\{ &\sum_{\ell>0} \left[\langle Q_R\rangle_e \langle Q_R\rangle_0 + x^2 \sum_k \langle Q_R^k\rangle_e \langle Q_R^k\rangle_0\right] \frac{6}{(2\ell-1)^3} \\ &+ \sum_{\ell>0} \left[\left(\langle Q_R\rangle_e^2 + \langle Q_R\rangle_0^2\right) 3 + x^2 \sum_{k\hat{\mu}} \eta^{\mu k} \left(\langle Q_R^k\rangle_e^2 + \langle Q_R^k\rangle_0^2\right)\right] \frac{2}{(2\ell)^3} \Bigg\} . \end{aligned} \tag{4.4}$$

It is immediately clear that the contribution of the odd-distance terms is minimized by choosing

$$\langle Q_R\rangle_e = -\langle Q_R\rangle_0 \quad \text{and} \quad \langle Q_R^k\rangle_e = -\langle Q_R^k\rangle_0 \tag{4.5}$$

and then choosing states that maximize

$$\sum_R \left\{\langle Q_R\rangle_e^2 + x^2 \sum_k \langle Q_R^k\rangle_e^2\right\} \quad . \tag{4.6}$$

In fact this set of states is highly degenerate, because of the large $SU(4f_R)$ multiplets that exist as single site states. The mean field approximation may in fact give some spurious degeneracy for the odd-neighbor terms because states with different values of $\langle Q_R \rangle$ may give the same result for (4.6). In a more general ground state the energy will depend on the quadratic Casimir $\langle \sum Q_R^k Q_R^k \rangle$ rather than the maximum eigenvalue $\sum_k \langle Q_R^k \rangle^2$. Since the quadratic Casimir is larger than the maximum weight squared this will mean that the degeneracy of (4.6) is lifted and the lowest lying states will occur for $Q_R = 0$, large maximum weight representations on every site. The block-spin calculations of SDQW bear out this statement.

Even in the mean-field approximation, when the effect of the even-separation terms is included, much of the possible degeneracy of (4.6) is lifted. Clearly from (4.4) one sees that these states for which the maximal contribution to (4.6) comes from terms $\langle Q_R^k \rangle$ with k's such that all $\eta^{\mu k}$ are negative are lowered in energy relative to the others that are degenerate with them as far as the odd-neighbor terms are concerned. Any state with an expectation value for any of these operators with $\eta_{\mu k} = -1$ is rotated to another such state by S_{chiral}. Clearly any ground state of this type breaks the chiral symmetry of the theory.

In our mean field calculation we notice that the states that develop an expectation value for $M^k = \gamma_0$, that is for the operator

$$\bar{\psi}(\vec{j})\ \psi(\vec{j}) = (-1)^{j_x + j_y + j_z}\ \tilde{\psi}^{\dagger}(\vec{j})\ \gamma_0\, \tilde{\psi}(\vec{j}) \quad , \qquad (4.7)$$

are among the degenerate set of possible ground states. Any infinitesimal mass term added to the Hamiltonian will select this chiral symmetry breaking state as the state about which the mass acts as a perturbation. The axial $SU(f_R) \times U(1)$ charges generate spin-wave-like excitations about this mean field state, which are massless in the limit of fermion mass going to zero, since as stated above these charges rotate this state into some other direction which also has $\eta^{\mu k} = -1$.

The above discussion is completely general and does not change for any choice of fermion representation content and gauge group, for four-component fermions. The same analysis can be applied to the two-component fermion theories with very similar results, except that in this case there are no operators which have all $\eta^{\mu k}$ negative. Instead the operators

$$M^k = (\sigma_i \times \tau) \quad ; \quad \sigma_i \times I \tag{4.8}$$

have the property that

$$\eta^{\mu k} = (2\delta_{i\mu} - 1) \tag{4.9}$$

whereas the operators

$$M^k = 1 \times \tau \tag{4.10}$$

have all $\eta^{\mu k}$ positive. The net effect of $\eta^{\mu k}$ of the form (4.9) is to lower the energy density of those states for which $\vec{\sigma} \cdot \hat{\eta}$ acquires a vacuum expectation value for some fixed direction $\hat{\eta}$. This spontaneously breaks <u>both</u> the chiral symmetry and the lattice rotation invariance. In effect these models are too much like a true antiferromagnet in this strong coupling limit.

Once we have found a choice for $|\phi_e\rangle$ and $|\phi_0\rangle$ which minimizes the mean-field energy density we can then ask for what the lowest lying fermion excitation of that state may be. We can add a composite fermion by acting on any site of the lattice with one of the composite operators $B^\dagger$ described in the previous section, and then calculate the energy gap from the mean-field state to the fermionic state. The preceeding discussion found a minimum energy when both $|\phi_e\rangle$ and $|\phi_0\rangle$ lie in the representation sector with $\langle Q_R\rangle = 0$ and a large $SU(4f_R)$ corresponding to the maximum value of $\sum_k |\langle Q_R^2\rangle|^2$. Any baryon creating operator acting on such a state takes the system to a state with $\langle Q_R \rangle \neq 0$. Hence by definition of the maximal state, it will in general also be true that, for at least some k,

$$|\langle B Q_R^k B \rangle| < |\langle Q_R^k \rangle| \tag{4.11}$$

(Since the flavor groups are of the SU(nf) type there are no accidental degeneracies of maximum weights in different representations.) It is therefore clear that for any such state the energy gap from the mean-field state is positive and of order $1/g^2$. This result is not changed by the kinetic energy terms that move fermions made of three or more preons from one lattice site to another. Such terms in H_{eff} involve at least two energy denominators with states of nonvanishing flux and hence are to leading order, $\propto 1/g^4$. They can therefore be neglected to the order of the present calculation, $\sim 1/g^2$, thus to the accuracy of the present calculation we cannot alter the result by making a zero momentum superposition of local baryon states.

Note that our result depends only on the fact that the ground state lies in the sector for which every site is occupied by a maximal state and any baryon number nonzero state has at least one site occupied by a nonmaximal state. Hence the conclusion that all baryons have masses at order $1/g^2$ is much more general than the mean-field calculation described above, as has been shown in some special cases by the variational block spin approach of SDQW.

For a purely left-handed theory it may not at first seem that the choice (2.12) for the definition of the U(1) charge is natural. However, the mean-field analysis justifies this definition by finding the ground state in the sector $\langle Q_R(j) \rangle = 0$ for all j. This choice corresponds to the same number of occupied states as the filled fermi sea of a free field theory. $\langle Q_R \rangle$ thus provides a measure of deviations in quark number from the vacuum value, which is the only physically meaningful definition of quark num-er. As in the four-component case, all states with a nonvanishing value of $\langle Q_R \rangle$ are higher in energy than the ground state by an amount of order $1/g^2$.

We stress the fact that the conclusion that all fermions are massive is not just a peculiarity of the mean-field approximation. We have presented the mean-field discussions because it is very straightforward and easily understood. However the analysis of SDQW shows that adding refinements such as spin-wave corrections to mean-field, or a more general variational ground state ansatz, will not alter the conclusion that all fermions acquire a mass at order $1/g^2$, and that there are massless Goldstone boson excitations of a spin-wave nature. This conclusion requires only that the ground-state lie in the sector of states where every site is occupied by a state of the type which we have called maximal, that the average interaction energy between any pair of sites in this ground state is negative and that the fermion creating operators are not $SU(nf_R)$ singlets. For a Hamiltonian of the type (2.11) this will always be so. [We have also explored more general Hamiltonians by arbitrarily changing the strength of the parameter x in (2.11). For sufficiently small x the ground state lies in the sector with maximal $|Q_R|$ and SU(nf) singlet at every site. The opposite Q_R of the even and odd sublattices means that such a ground state still has net fermion number zero, and any state of nonzero fermion number is massive in this situation also.]

V. CONTINUUM LIMIT

The result of a strong coupling lattice theory are of course always subject to the criticism that we do not know how to take a continuum limit. However, in this case we are studying a question which in any one phase of the theory should have a unique answer — i.e., the question of the realization of the chiral symmetry. Our results suggest two choices for the continuum limit: either the chiral symmetry is realized in the Nambu-Goldstone fashion, with massive composite fermions, or there is a phase transition at some finite coupling.

If there is a phase transition at finite coupling then the desirable property of asymptotic freedom and confinement in a single phase of the theory is lost. In the strong coupling phase we have <u>massive</u> composite fermions with no possibility of using asymptotic freedom to probe their structure in high q^2 experiments. In the weak coupling phase we may have asymptotic freedom, but no argument whatever for confinement with very small composites in such a phase. Neither phase is an attractive model for the physics of composite quarks and leptons.

This argument applies as well to the theories with purely left-handed fermions. Here unfortunately we find no phase which could possibly be interesting for real world physics. As in the above discussion if there is a phase transition at finite coupling we have lost either asymptotic freedom or confinement — or both. One other hand if there is no phase transition then the strong coupling results indicate spontaneous breaking of Lorentz invariance, which is also not a satisfactory model for the physical world. We also find our results are in direct contradiction to the the picture derived using the MAC tumbling scenario.

Hence these strong coupling results suggest that models for composite quarks and leptons based on gauge theories which confine preons are not realistic, independent of whether 't Hooft's anomaly conditions can be satisfied.

(b) Chiral Symmetry Breaking terms

The effect of adding a small preon mass term can readily be included in this analysis. The situation is just like that of adding a weak alternating applied field to a Heisenberg antiferromagnet. The set of degenerate mean field states is split by a contribution proportional to m in the energy density. The lowest mean-field state in the presence of a mass is the state for which

$$\langle\bar{\psi}_R\psi_R\rangle = \frac{1}{N}\sum_{\vec{j}} \bar{\psi}_R(\vec{j})\;\psi_R(\vec{j}) = \frac{1}{N}\sum_{\vec{j}} (-1)^{j_x+j_y+j_z}\;\tilde{\psi}_R^\dagger(\vec{j})\,\gamma_0\,\tilde{\psi}(\vec{j}) \qquad (5.1)$$

takes its maximally negative value, where N is the total number of lattice sites. Only on this state can the effect of adding a small quark mass be treated as a perturbation, and it is a perturbation which doew not alter the ground state.

The situation is slightly more complicated when a more realistic ground state is considered, such as that constructed by a block spin procedure. In such a state the mass term is a perturbation which modifies the ground state[1] until at sufficiently large mass the state becomes the mean-field state with maximum negative $\langle\bar{\psi}_R\psi_R\rangle$.

It is interesting in this context to examine the Wilson gradient, which corresponds to keeping only the $\ell = 1$ terms of the Hamiltonian (2.11) and adding both chiral symmetry breaking terms of the form

$$\Delta H_W = \frac{1}{6}\sum_{\mu}\left\{\psi_R^\dagger(\vec{j})\,\gamma_0\,\psi_R(\vec{j}+\hat{\mu}) - \psi_R^\dagger(\vec{j})\,\gamma_0\,\psi_R(\vec{j}-\hat{\mu}) + \frac{1}{K}\,\psi_R^\dagger(\vec{j})\,\gamma_0\,\psi_R(\vec{j})\right\}. \quad (5.2)$$

where K is the Wilson hopping parameter. A strong coupling analysis of this Hamiltonian is being pursued by Christine Di'Lieto.[12] However for our purpose it is convenient to note that for sufficiently small K the mean-field ground state of this theory will correspond to the mean-field ground state of the long-range H in the presence of a mass term, and the analysis presented above will apply. That is to say the result that all baryons are massive will be reproduced for the Wilson gradient with sufficiently small K. It is also obvious that a Kogut-Susskind formulation, which has only nearest-neighbor terms in the gradient, will also give all massive baryons as a result of this strong-coupling analysis.

There exists in the literature one counter example to our general result, presented by Banks and Kaplunovsky[11] and analyzed by them in terms of the strong coupling effective Hamiltonian. The peculiarity of this example depends on having a gauge group with real representations, such as O(2n+1). The fermions are introduced as a single flavor of <u>single component</u> Clifford variables in some representation R on each site, with nearest-neighbor gauge couplings. The eightfold degeneracy of this fermion is interpreted, Kogut-Susskind fashion, as two flavors of a two-component complex fermion field. This construction is crucial to their conclusion and its difference from our results. There is no gauge-group singlet local fermion bilinear operator in Banks and Kaplunovsky, and hence there is no term of order $1/g^2$ in H_{eff}. The gauge group representation can be chosen such that the leading term in $1/g^2$ expansion is the kinetic energy term for a fermionic composite of order $1/g^4$. Banks and Kaplunovsky then show that the strong coupling effective Hamiltonian for this theory looks like massless composite fermions with weak four-fermion interactions.

It is important for their result that they choose to put only a single flavor of a single-component fermion on each site, a formulation which can only be used for gauge groups such as O(N) with real fermion representations. The same theory treated with any of the standard gradient formulations, with two component fermions on a site, will have local gauge singlet fermion bilinear operators and will be included in our general result. Furthermore even in their formulation, following the usual Nambu-Jona-Lasinio argument one would expect a phase transition at some finite g^2, as the multifermion coupling terms become important, to a chiral symmetry breaking theory. Although Banks and Kaplunovsky suggest that this may not occur, we do not find their arguments convincing.[13] Even using their peculiar one-component-per-site formulation, one loses the massless baryon results if additional fermion flavors are introduced.

(c) Nonsimple Gauge Groups

Finally we can also analyze a theory which has an additional gauge symmetry under which the composite fermions are nonsinglet. For example consider a theory of the type discussed by Harari and Seiberg,[14] the rishon theory. This theory has a gauge group $SU_c(3) \times SU_h(3)$ (color with coupling g_c and hypercolor with coupling g_h) and contains fermions (T and V) which belong to the (3,3) and $(\bar{3},3)$ representations respectively. The models we have analyzed above correspond to setting the coupling g_c to zero and analyzing the theory for strong g_h. In this limit we find the spectrum of states consists of a set of massive states with quantum numbers of the usual quarks and leptons.

The locally hypercolor-singlet fermion bilinear operators that appear in H_{eff} in the rishon model are not all color singlet quantities. One can introduce nonzero g_c effects in the effective Hamiltonian; this adds a term $g_c^2 E_c^2$ that is of zero order in a/g_h^2, and color flux creating operators between the color nonsinglet local fermion bilinear operators. For small g_c it is readily seen that although this somewhat alters the ground state its effect is not strong enough to render massless the quark and lepton composites which were all massive ag $g_c = 0$. Harari and Seiberg speculate that the finite g_c theory will indeed differ dramatically from the $g_c = 0$ theory, but we see no indication for this in the large g_h treatment.

(d) Reprise

The results of this paper can be stated in a very general fashion. Spontaneously broken chiral symmetry is a general property of all strong coupling lattice gauge theories with two or four component fermions. The strong-coupling effective Hamiltonian in the flux-free sector, (2.11), is precisely the lattice version of the physics problem first discussed by Nambu and Jona-Lasinio — an effective four-fermion interaction in the presence of a momentum cut-off. [Their problem of removing a cut-off is our question of taking the continuum limit.] Our results for this theory are nothing new or surprising. We discuss two choices: phase transition or no phase transition. Neither of these produces a continuum theory with all the generally desirable features for a theory of the structure of massless quarks and leptons in terms of confined massless preons, with asymptotic freedom allowing perturbative preon-parton results at extremely large Q^2.

This work was supported by the Department of Energy, contract DE-AC03-76SF00515.

APPENDIX: SPECIFIC EXAMPLES

In this appendix we will present results for some models that have been proposed in the literature as examples which may have light (massless in the chiral limit) composite fermions. The first example of such a model was presented by Dimopoulos, Raby and Susskind.[15] Their analysis was based on the complimentarity of the Higgs and confining phases for theories with fundamental representation scalars and the MAC hypothesis of dynamical breaking of gauge symmetry. Our results for their model disagree both with the MAC picture and with the conclusion of light composite fermions. These results are given in Table 2.

Subsequent examples have attempted to present realistic sets of quarks and leptons and are much more complicated. Table 3 describes an example proposed by Preskill.[16]

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13. The argument of Banks and Kaplunovsky is essentially that weak coupling perturbation theory indicates massless fermions in the spectrum as does the strong coupling analysis, thus suggesting a single phase for all g. However the massless fermions in the weak coupling analysis contain a single preon (screened by glue presumably), whereas at strong coupling the massless states are

three-preon composites. Hence the local operators which create the massless states are quite different in the two regions and correlations between them can be used as an order parameter which distinguishes them. This, along with the Nambu-Jona Lasinio analysis of interacting massless fermions, suggests to us that there probably is a phase transition at some finite g in these theories.

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TABLE 1

M^k		$\eta^{\mu k}$
$\gamma_0 \times \tau$	$\gamma_0 \times I$	-1
$\gamma_0\gamma_5 \times \tau$	$\gamma_0\gamma_5 \times I$	-1
$\gamma_5 \times \tau$	$\gamma_5 \times I$	$+1$
$I \times \tau$		$+1$
$\gamma_i \times \tau$	$\gamma_i \times I$	$-(2\delta_{\mu i} - 1)$
$\gamma_0\gamma_i \times \tau$	$\gamma_0\gamma_i \times I$	$2\delta_{\mu i} - 1$

TABLE 2

Dimopoulos-Raby-Susskind Example; G = SU(5)

<u>Representations</u>		$\underline{d_R}$	$\underline{f_R}$
(2-component fermions)	χ_i □	5	1
	χ_{ijk} □ □ □	10	1

$$S_{nn} = \left[SU(2) \times U(1)\right]_5 \times \left[SU(2) \times U(1)\right]_\chi$$

$$|0\rangle = |1;\ -5;\ 1;\ -5\rangle$$

Maximal Representation $|M\rangle = \left(\chi^\dagger\right)^5 \left(\chi^\dagger\right)^{10} |0\rangle = |5/2;\ 0;\ 5;\ 0\rangle$

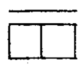
Color Singlet Composite Operators (other than Q's)

fermions	bosons
$\epsilon\psi\psi\psi\psi\psi$	$\psi\psi\psi\chi^\dagger$
$\epsilon\epsilon\epsilon\chi\chi\chi\chi\chi$ [a]	$\epsilon\epsilon\psi\chi\chi\chi$
$\epsilon\chi\psi\psi$ [a]	
$\epsilon\psi\chi^\dagger\chi^\dagger$ [a]	

[a] In a tumbling scenario the massless composite is created by a linear combination of these three operators.

TABLE 3

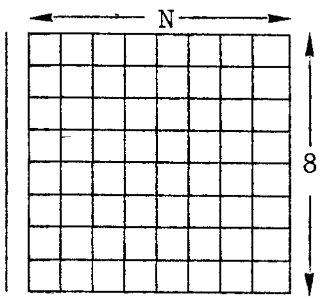
Preskill Example; $G = SU(N)$

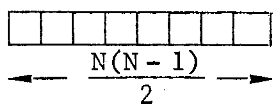
Representations		d_R	f_R	Number of Components
	ψ_i 	N	8	2
	χ_{ij} 	$N(N-1)/2$	1	2
	φ^{ij} 	$N(N+1)/2$	1	2

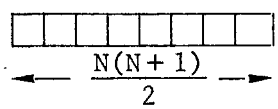
$$S_{nn} = \left[SU(16) \times U(1)\right]_\psi \times \left[SU(2) \times U(1)\right]_\chi \times \left[SU(2) \times U(1)\right]_\varphi$$

$$|0\rangle = |1;\ -8N;\ 1;\ -N(N-1)/2;\ 1;\ -N(N+1)/2\rangle$$

Maximal State $|M\rangle = \left(\psi^\dagger\right)^{8N} \left(\chi^\dagger\right)^{N(N-1)/2} \left(\varphi^\dagger\right)^{N(N+1)/2} |0\rangle$

= | 

8; 0; 

; 0; 

; 0⟩

Color Singlet Composite Operators (other than Q's)

$\psi_i\ \varphi^{ij}\ \chi_j$	$\left(\chi^\dagger\right)^{N(N-1)/2}$
$\psi_i\ \varphi^{ij}\ \chi_{jk}\ \varphi^{k\ell}\ \chi_\ell$	$\varepsilon\varepsilon\left(\varphi^\dagger\right)^N$
$\vdots$	
$\psi_i\ \varphi^{ij};\ \left[(\chi\varphi)^m\right]^\ell{}_\ell$	$\varepsilon\left(\psi^\dagger\right)^N$
$\vdots$	
fermions	bosons or fermions depending on N