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A MODEL FOR THE STRUCTURE OF POINT-LIKE FERMIONS, QUALITATIVE FEATURES AND PHYSICAL DESCRIPTION*

by

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ABSTRACT

A model for the structure of point-like fermions as tightly bound composite states is described. The model is based upon the premise that electromagnetism is the only fundamental interaction. The fundamental entity of the model is an object called the vorton. Vortons are semi-classical monopole configurations of electromagnetic charge and field, constructed to satisfy Maxwell's equations. Vortons carry topological charge and one unit each of two different kinds of angular momenta, and are placed in magnetically bound pair states having angular momentum $\ell = 1/2$. The topological charge prevents the mutual annihilation of the vorton pair. The helicity eigenstates of the vortons' intrinsic angular momenta form the basis for a set of internal quantum numbers for the pair which distinguish the different (point-like) pair states. Sixteen four-component spinor states, eight leptonic and eight hadronic, are obtained. Eleven of these are identified with the quantum numbers of the experimentally known particles: e, ν_e, μ, ν_μ, τ, ν_τ; p, n, Λ, Λ_c, and b. Thus one new heavy lepton with its neutrino and three new quark states are predicted. Some possibilities for the extension of this model are discussed.

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I. INTRODUCTION

This paper, concerned mainly with point-like fermions, is a first step in an effort to develop from electromagnetism alone a model for elementary particles and their interactions. Accordingly, the fundamental equations of the model are the well known Maxwell's equations. Since quantum effects are introduced by applying the Bohr-Sommerfeld quantum condition to certain angular momenta, the model is semi-classical in nature. The fundamental entity of the model is an object called the vorton.[(1)] It is proposed that what we now call elementary particles[1] are actually composite bound states of vortons.

This model is most economical at its fundamental level; it has only one fundamental interaction and only one fundamental particle, which is a (static) solution to the equations of that fundamental interaction. All nonfundamental (i.e., nonelectromagnetic) phenomena are to be governed by effective (unrenormalized) equations to be derived from the dynamics at a more basic level. It is postulated that the criterion of self-consistency will enable the determination of the parameters (e.g., masses and coupling constants) in these effective equations.

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[1] In keeping with common usage, the word "elementary" will be used to refer to the (composite) elementary fermions (e.g., muon, electron, etc.), which in this model are elementary in the same sense as the chemical elements are elementary. The word "fundamental" will be used to refer to the basic equations, from which all phenomena (are assumed to) derive, and to the basic building block, the vorton, which is an indivisible entity that is conserved at all interaction vertices (because it carries an absolutely conserved quantity, topological charge).

As background for this model, it is useful to review some history.[2] Dirac[(7)] was the first to speculate about the incorporation of magnetic charge into elementary particle theory. He showed that the presence of magnetic charge would provide a natural explanation for the quantization of electric charge. His quantization condition was

$$\frac{e_0\, g_0}{\hbar c} = \frac{n}{2} \quad , \tag{1}$$

where e_0 and g_0 are respectively the basic electric and magnetic charge quanta and n is an integer. Gaussian units are used here; $\hbar$ and c have their usual significance. As is well known, Eq. (1) implies a very large magnetic coupling constant.

Equation (1) may be viewed as a quantization condition for an angular momentum which may be associated with the simultaneous presence of the two types of charges. That is, if one imagines that e_0 and g_0 are separated by a distance (vector) $\vec{d}$, then it is easy to see that there will be an electromagnetic (angular) momentum[3] circulating around $\vec{d}$. In fact, Saha pointed out[(9)] that Eq. (1) can be derived quite simply by first calculating the Poynting vector and thence the classical angular momentum residing in the (crossed) electric field of a point charge e_0 and magnetic field of a point pole g_0, separated by a distance d, and then quantizing this

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[2] An excellent review article on Dirac monopoles, covering both theory and experiment (up to 1968) has been published by Amaldi.[(2)] Experiment to 1975 is covered by Eberhard et al.[(3)] Stevens[(4)] has compiled a useful bibliography on this subject. Recent experimental searches have been tabulated by Jones,[(5)] and recent theoretical developments are covered by Goddard and Olive.[(6)]

[3] That (classical) angular momentum can reside in the static electromagnetic fields due to electromagnetic sources has been known since Poincaré.[(8)]

angular momentum in units of $\hbar/2$. (The angular momentum is a function of the charges alone and not of d.) This type of angular momentum will be referred to as the Poincaré angular momentum, L_P.

A considerable period of time elapsed before the next step was taken, the consideration of a particle carrying both electric and magnetic charge.(10) Schwinger coined the name "dyon" for this entity of dual charge, and went on to develop "A Magnetic Model of Matter."(11) Schwinger's quantization condition relating electric and magnetic charge is

$$e_1 g_2 - e_2 g_1 = n\hbar c \quad , \tag{2}$$

where the 1 and 2 denote two dyons and n is an integer. In his model, dyons are endowed with spin 1/2. His dyon electric charge assignments parallel those of conventional quark models ($\pm e/3$ and $\pm 2e/3$), and he combines dyons in threes to form the hadronic fermions. Hadrons in his model, then, are magnetically neutral composite particles whose strong interactions derive from the superstrong magnetic force.(12)

A variation of the Schwinger dyon model has been suggested(13) which entails a modification in the relationship between the electric and magnetic SU(3) symmetries. It, like Schwinger's model, uses a total of 18 basic entities (9 dyons and 9 antidyons), combining them in essentially the same way as Schwinger to form the hadronic baryons and mesons.

It is also appropriate to mention the dyon model of Barut.(14) His dyons are spinless and are electrically quite different from the vorton. The major similarity between his model and this one is that he combines dyons in pairs with orbital angular momentum $\ell = 1/2$ to form (the hadronic) fermions. He has no explanation for the hadronic flavors and does not apply his dyon model to the leptons.

More recently, there has been considerable effort investigating the monopole solutions found by 't Hooft(15) and Polyakov.(16) It is important to note that the monopole nature of these solutions derives from the non-Abelian character of their associated gauge theories, while the vorton is a constructed solution to the (Abelian) Maxwell's equations.[4] Hence, there are essential differences between the 't Hooft-Polyakov monopoles and the vorton.

In the following sections, the vorton and the magnetically bound vorton pair state are described. This tightly bound pair state is identified with the (elementary) point-like fermions. Since this model uses the same basic structure for the point-like aspects of both leptonic and hadronic fermions, a consistent, unified picture is obtained. While in this model, the point-like leptonic fermions are identified directly with the physical leptons (e, μ, τ, etc.), the point-like hadronic fermions are identified with the point-like spin 1/2 partons(17) as the elementary fermionic constituents of the hadrons, which are in turn composite structures of a more complicated nature. These partons carry the various quanta of flavor which are attributed to the quarks. The structuring of the hadrons into the SU(4) mesonic and baryonic multiplets analogous to those of the quark model thus requires an additional level of dynamics which is not discussed in this paper.

Finally, a brief discussion of particle interactions is included. Self-interactions are designated as the (dynamical) source of particle masses. While it is evident that QED can be (phenomonologically)

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[4]While the 't Hooft-Polyakov monopoles need not have an associated electromagnetic charge as such, but appear to have one as a result of topological properties of the non-Abelian vector potential A^a_μ, the vorton derives its monopole character specifically from the presence of the postulated electromagnetic charge distribution.(1)

incorporated directly into this model, only qualitative arguments are given on how the other interactions might obtain.

At present, it is a matter of subjective judgement whether or not having simplicity of foundation and a qualitative outline of particle multiplet structure furnishes enough motivation to tolerate the obscurity due to the complications of quantitative calculation. While it is clear that much work remains to be done, the results obtained to date are cause for some optimism. And, although certain aspects of it are speculative, it seems timely to present this paper with the aim to stimulate a broader interest in this approach.

II. THE VORTON

The fundamental building block of this model for point-like fermions is an object called the vorton. Its analytical basis and salient features are reviewed in this section; further details of its construction and underlying analysis may be found in Ref. (1). Quantum mechanical corrections, not covered in Ref. (1), are also estimated (in Appendix A).

The vorton is a static semiclassical configuration of electromagnetic charge and associated fields constructed to satisfy generalized Maxwell's equations. This generalization, which derives from a symmetry first pointed out by Rainich,[18] is one which treats the electric and magnetic parts of the field tensor on an equal footing,[19] and the straightforward extension to include both electric and magnetic charge.[11,20,21] Thus a vorton has an electromagnetic charge of magnitude Q at an angle Θ which specify its electric and magnetic components of charge by

$$Q \sin \Theta, \quad Q \cos \Theta \tag{3}$$

respectively. Θ is essentially the dyality angle employed by Han and Biedenharn,[21] and the symmetry with respect to Θ will be called the dyality symmetry.

As will be seen, different vortons can have different values of Q and Θ. These values of Q will not, in general, satisfy Eqs. (1) or (2) with any observed value of electric charge; thus the expected value of the Poincaré angular momentum (measured by n) will be nonintegral. While this might appear to be a problem, in fact, the rigorous requirement of a quantum condition upon the magnetic monopole strength [i.e., Eq. (1) or (2)] is subject to serious reservations.[22] The view adopted here is that the relationship of free vortons to other particles is non-stationary, and analogous to the orbital angular momentum of ordinary scattering states, L_P need not be quantized;[5] i.e., the Q of free vortons is unrestricted by a quantum condition involving other particles. In the context of our semiclassical approach, one notes that for free vortons there are no cyclic action variables to which one could apply the Bohr-Sommerfeld quantum condition. On the other hand, as discussed in Sec. III, the bound vorton pair is a stationary state which does have such a cyclic variable and to which it is assumed that Eq. (2) does apply.

The reference direction for Θ is furnished by the electrical charge of physical fermion states, but since fermions in this model are composed of bound vortons, the selection of this reference direction is, in fact, a spontaneous breaking of the dyality symmetry. One expects this symmetry breaking to lead to a Goldstone boson,[(24)] which it seems appropriate to call the Rainon.

Rainons are oscillations or waves in the vacuum dyality angle Θ about its quiescent or ground state reference direction Θ_0 (since we define fermions to be electrically charged, $\Theta_0 = \pi/2$). Physically, a Rainon is a collective excitation of the vacuum (Dirac sea fermions), very much analogous to a phonon, which is a collective excitation of lattice nuclei.

The possibility exists that via a Higgs mechanism[(25)] the Rainon and the photon may combine into a three component vector particle, or physical photon of very small mass, which could then permit the non-conservation of electromagnetic charge. In this model a nonzero $\delta\Theta \equiv \Theta - \Theta_0$ (associated with the Rainon) would enable the vacuum (fermions)

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[5]Recall, for example, that a plane wave has an undefined orbital angular momentum about an arbitrary point, but is expandable as a sum of partial waves[(23)] each of which carries an integral quantum of angular momentum.

to carry incremental values of electromagnetic charge, facilitating such nonconservation of charge. While it has been shown that vertices at which charge conservation is violated by a finite amount have a vanishing probability, the possibility of an infinitesimal violation was left open.[26] The vorton solution and the fermions constructed from vorton pairs offer this latter possibility, but the details are not explored here. The experimental upper limit on the photon mass is quite small,[27] but, of course, it is not possible to determine experimentally that the photon mass is identically zero; at present, this question remains moot.

The construction of the vorton was motivated by the invariance properties of Maxwell's equations. In addition to having the dyality symmetry mentioned above, it is well known that Maxwell's equations are invariant under the operators of the conformal group, a group with 15 generators. These include the ten generators of the Poincaré group, $M_{\mu\nu}$ and P_μ, where $\mu,\nu = 0,1,2,3$, as well as five additional ones: K_μ, the generators of special conformal transformations, and D, the generator of dilitations. The conformal group in Minkowski space has been shown to be isomorphic to the group O(4,2) in a six dimensional pseudo-Euclidean space.[28]

The vorton charge distribution is constructed to be invariant under the subgroup of the conformal group generated (in the t = 0 Euclidean 3-space) by the six operators L_i and X_i, where $i = 1,2,3$. $L_i = M_{jk}$, i,j,k cyclic, are the generators of ordinary rotations and $X_i = (K_i - P_i)/2$ generate in the t = 0 Euclidean 3-space what might be called toroidal rotations, resembling the motion of a smoke ring. L_i and X_i obey commutation relations isomorphic to those of O(4). Since $[L_i, X_i] = 0$, no summation over i, one can simultaneously diagonalize L_3 and X_3. The eigenvalues of L_3 and X_3 have been shown to be both integer or both half-integer.[(29)] The eigenvalues, respectively labelled m_ϕ and m_ψ, denote the projections of ordinary angular momentum and toroidal angular momentum on the z-axis. The vorton is assumed to be in a state of "double rotation," and thus it carries quanta of the two different kinds of angular momenta, the usual angular momentum, which might be called spin, and a toroidal angular momentum.

In keeping with the semiclassical nature of this model, it will be assumed that m_ϕ and m_ψ are integral.[6] When m_ϕ, $m_\psi \neq 0$, the vorton carries a nonzero topological or Hopf charge[(30)] Q_H. Specifically,

$$Q_H = m'_\phi m_\psi = m_\phi m'_\psi = C m'_\phi m'_\psi \quad , \tag{4}$$

where C is the largest common factor in m_ϕ and m_ψ; i.e., m'_ϕ and m'_ψ are relatively prime. Equation (4) shows that the Q_H of the vorton can take on an infinite set of possible values (the integers), indicating that it is additive modulo infinity. (Recall that the topological charge of the monopole in 't Hooft's SO(3) model[(15)] was additive module 2.)

The condition

$$|m_\phi| = |m_\psi| = 1 \tag{5}$$

defines the "ground state" vorton, the one with the lowest value of Q having also a nonzero value of Q_H. Equation (5) then specifies

$$Q_H = \pm 1 \quad , \tag{6}$$

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[6] One could not define a topological charge for half-integral m_ϕ or m_ψ. Also, see footnote 9, below.

the sign being determined by the relative sense of m_ϕ and m_ψ. The ground state vorton will be used as the fundamental particle of this model.

From the assumption that mass is dynamically generated by self-interactions, the "mass" of the vorton is taken to be equal to the total energy (over c^2) contained in the electromagnetic field, which is given by

$$W_T = \frac{5Q^2}{2\pi a} \quad , \tag{7}$$

where a sets the scale of the vorton's toroidal coordinate system.[31] Equation (7) means that vorton mass goes inversely with size, very small vortons being very massive; there is no intrinsic scale or mass associated with the semiclassical vorton.

In Appendix A is given a recapitulation of the analysis in Ref. (1), which shows that in semiclassical approximation Q satisfies

$$Q^2 = \pi\sqrt{\frac{6}{5}}\sqrt{m_\phi^2 + m_\psi^2}\ \hbar c \quad , \tag{8}$$

independent of the vorton scale a. Using Eq. (5) and $e^2/\hbar c = \alpha \cong 1/137$ yields $Q_0^2 \cong 4.87\ \hbar c$ and

$$Q_0 \cong 25.8e \quad , \tag{9}$$

where the subscript on Q_0 indicates that this is the asymptotic value of Q for a ground state vorton valid for $a >> \lambda\!\!\!^-_e$, the (reduced) Compton wavelength of the election; e is the positron charge.

Once $a < \lambda\!\!\!^-_e$, one expects that there should be quantum mechanical corrections which would cause the Bohr-Sommerfeld quantum condition to entail an a dependence. This a dependence, which turns out to be an important feature in this model is investigated in Appendix A. Thus one writes

$$Q^2 = Q^2(a) \quad . \tag{A-1}$$

It was found that initially $Q^2(a)$ tends to drop slowly (logarithmically) and then at extremely small scale can be expected to rise fairly rapidly.

In the range $\bar{\lambda}_\mu < a < \bar{\lambda}_e$

$$Q^2(a) \cong \frac{Q_0^2}{1 + R_{11}(a)} \quad , \tag{A-18}$$

where $R_{11}(a)$ is a Uehling potential function due to electrons in a vacuum polarization loop (Fig. 1). When one goes to sufficiently small scale, such that all N_f point-like fermions that electromagnetically couple to the photon are active in the vacuum polarization loops (and sums over all concatenations of such loops, Fig. 2), then one has the approximation

$$Q^2(a) \cong Q_0^2 \left(1 - \frac{2N_f\alpha}{3\pi} \ln \frac{\Lambda}{2\bar{m}c}\right) \quad , \tag{A-23}$$

where $\Lambda = \hbar/a$ is the momentum associated with the scale a, and $\bar{m}$ is the r.m.s. mass of the N_f fermions. (As will be seen below, this model predicts that $N_f = 8$.) Finally when at extremely small scale, beyond the Landau singularity[32] at P_L, where P_L is defined by

$$\frac{N_f\alpha}{3\pi} \ln \frac{P_L^2}{\bar{m}^2c^2} = 1 \quad , \tag{A-24}$$

the photon propagator cuts off,[7] and one has

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[7] The cutoff at P_L is a crucial feature of this model. The assumptions which lead to this consequence are listed in Appendix A; the reasoning behind them is covered in more detail in Ref. (33).

$$Q^2(a) \sim \frac{Q_0^2\hbar}{aP_L} \quad . \tag{A-33}$$

$Q^2(a)$ as given by Eqs. (A-18, 24, and 33) is depicted in Fig. 3.

III. VORTON PAIRS

A. General Description

In this model, the magnetically bound vorton pair state is the basic structure for all point-like fermions. The half unit of spin of these states derives from an angular momentum $\ell = 1/2$ to be described further below. This unified structure leads to the same quantization condition on electrical charge (0, ±1) for all elementary particles, accounting for the exact equality of the electric charges on leptons and hadrons. Thus, leptons and the (point-like) spin 1/2 hadronic partons are bound vorton pairs. The quantum numbers (e.g., flavors) associated with these point-like fermion states will be associated with the structure of the helicity eigenstates of the (bound) vortons. The hadronic fermions (e.g., protons, neutrons, etc.) are more complicated objects to be described more fully in a later paper.[8]

The mutual annihilation of the vortons in these pair states is precluded by the conservation of topological charge. This will come about if both vortons of the pair have the same Q_H. Thus $Q_H = +2$ for fermions and -2 for antifermions (or vice versa). Fermion and antifermion (of the same flavor), having a total $Q_H = 0$, can mutually annihilate.

B. Electrical Charge

It is assumed that the bound vorton pair is represented by a stationary state eigenfunction in which the electromagnetic charges

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[8]The quarks will be described as coherent collective states of the integrally charged partons, accounting for quark confinement inside hadrons as analogous to the confinement of phonons inside solids.

satisfy Eq. (2); i.e., consistent with a semiclassical view, $L_P = n\hbar$ is assumed to be integral rather than half-integral.[9]

Since the bound pair is expected to be magnetically neutral, one may write

$$g_2 = -g_1 \equiv g_0 \quad . \tag{10}$$

Substituting Eq. (10) into Eq. (2) yields

$$(e_1 + e_2)g_0 = n\hbar c \quad . \tag{11}$$

Now $(e_1 + e_2)$ is just the total electric charge of the pair. Since the bound vorton pair is to be identified with the elementary fermions, it is natural to set

$$|e_1 + e_2| = e \text{ or } 0 \quad . \tag{12}$$

The quantity $(e_1 + e_2)$ is so far specified only up to a sign. This ambiguity will be resolved below through a detailed consideration of the structure of the $\ell = 1/2$ state.

The electrical charge exhibited by the fermions in this model derives directly from the electromagnetic charge Q (at some angle Θ) carried by the constituent vortons, which, using Eq. (3) gives

$$g_0 = Q_B \cos \Theta_B \quad , \tag{13}$$

and

$$e = 2Q_B \sin \Theta_B \quad , \tag{14}$$

where the subscript B denotes the final bound state configuration after the collapse process described in the next section has been completed. Using Eqs. (11) through (14) with n = 1, one finds that

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[9] As is logically simpler, only one physical aspect of the model (i.e., the $\ell = 1/2$) is associated with half-integral angular momenta.

$$\frac{Q_B^2}{\hbar c} = \frac{1}{\alpha} + \frac{\alpha}{4} = 137.038 \tag{15}$$

and

$$\Theta_B = \frac{1}{2} \text{ arc sin } \frac{\hbar c}{Q_B^2} = 0.209^\circ \quad . \tag{16}$$

We observe that as required by Eq. (15), $Q_B^2/\hbar c \cong 137$ is indeed available at scales on the other side of the Landau singularity (see Fig. 3). Equation (16) shows that only a slight shift of Θ away from the pure magnetic values, 0 and π, suffices to give us electrically charged fermions with the appropriate electrical charge; for the neutral fermions, of course, $n = 0$ with $\Theta = 0$ or π. In this way both electrically charged and neutral (point-like) fermions have the same basic structure and are magnetically bound by (essentially) the full value of Q_B. It will be argued below that only $n = 0, \pm 1$ are allowed in the $\ell = 1/2$ bound pair states.

C. Spatial Scale

When two vortons of opposite electromagnetic charge enter into a bound state, one expects it to be tightly bound; Q_0 is already a large charge. In fact, since $Q_0^2/\hbar c = 4.87 > 1$, one expects from general arguments that this bound state will collapse.[34] As the state collapses, the reduction in scale would apply to the vortons themselves as well as to the intervorton distance. As the collapse proceeds, then, this reduction in scale is assumed to induce, due to the quantum mechanical corrections discussed in Appendix A, a change in the vorton charges. The ultimate bound state configuration has $Q = Q_B$, the scale of which is given by the Landau length ℓ_L.

It is not clear what mechanism would stop this collapse process at $\sim \ell_L$. One possibility is that a new phenomenon, not included in this model could enter the picture. As one looks at smaller and smaller scales to find this new phenomenon, the possibility of the Planck length $\ell_P = \left(\hbar k/c^3\right)^{1/2} = 1.616 \times 10^{-33}$ cm, where k is the gravitational constant, suggests itself. This would imply that the gravitational force may play a role at a very small scale(32); ℓ_P is ~20 orders of magnitude smaller than a hadron (and possibly related to ℓ_L). Discussions of possible phenomena at this scale may be found in the literature.(35)

Another possibility, which is more in keeping with the precepts of this model, is that the limit of collapse arises from phenomena already contained in the model. Since an ultimate resolution of this question by the latter means is not essential to the other aspects of the model discussed in this paper, this topic will not be taken up here.

D. Angular Momentum Eigenfunctions

The fact that in the tightly bound state, the vorton sizes are expected to be comparable to their separation furnishes an additional degree of rotational freedom; in classical terms, there are three significant (and comparable) moments of inertia. As a consequence, the bound vorton pair is appropriately analyzed as an object in a state of general rotation[10] requiring for a proper description all three Euler angles, $\alpha\ \beta\ \gamma$, rather than in one of simple orbital motion, which requires only

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[10] It is assumed that there is a rest frame in which to describe this rotation. Such a frame becomes conceptually possible through the existence of a very small bare mass m_0 which is associated with each point-like bound $\ell = 1/2$ vorton pair. See Sec. V below.

two. To obtain this description, one writes the eigenvalue equation for a general angular momentum[(36-38)]

$$L^2\psi = -\hbar^2 \left[\frac{\partial^2}{\partial\beta^2} + \cot\beta \frac{\partial}{\partial\beta} + \frac{1}{\sin^2\beta}\left(\frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\gamma^2}\right) - \frac{2\cos\beta}{\sin^2\beta}\frac{\partial^2}{\partial\alpha\partial\gamma}\right]\psi$$

$$= \ell(\ell+1)\psi \quad , \tag{17}$$

where α and β can be thought of as equivalent to the usual angles ϕ and θ of spherical coordinates and γ is the angle of (body) rotation about the figure axis.

It is well known that Eq. (17) is the quantum mechanical equation for the symmetric top[(36,39)] which has for solutions the matrix elements $\mathscr{D}^{\ell}_{m'm}(\alpha,\beta,\gamma)$ of the general rotation operator $D(\alpha,\beta,\gamma)$.[11] In likening the bound vorton pair to a symmetric top, one observes that the location of the two vortons will define the axis of symmetry, or figure axis, for the configuration.

The $\mathscr{D}^{\ell}_{m'm}$ are irreducible representations of the rotation group SO(3)[12] [as well as its covering group SU(2)] and are also known as the generalized spherical functions[(38)] $T^{\ell}_{m'm}$ of order ℓ, where

$$\begin{aligned} &\ell = 0,\ 1/2,\ 1,\ 3/2,\ \ldots\ , \\ &-\ell < m < \ell, \text{ and } -\ell < m' < \ell \quad . \end{aligned} \tag{18}$$

[11] This result has been demonstrated as also applicable to the relativistic symmetric top.[(40)] Therefore, while Eq. (17), strictly speaking, is non-relativistic, its solutions are valid in the relativistic domain as well -- an important point, since in Sec. IVB a "fully" relativisitic (internal) motion is attributed to the magnetically bound vorton pair.

[12] In consonance with Wigner's classification scheme,[(41)] one wants to use (for an elementary particle of nonzero mass) one of the irreducible representations of the rotation group SO(3).

The quantum number m is associated with the usual projection of ℓ upon the z-axis and m' with the projection of ℓ upon the figure axis (of the rotating entity). Of course, m and m' are integral if ℓ is integral and are half-integral if ℓ is half-integral. It is of interest to note that[42]

$$\mathscr{D}^{\ell}_{0m}(\phi,\theta,\gamma) = (-1)^m \left(\frac{4\pi}{2\ell+1}\right)^{1/2} Y^m_\ell(\theta,\phi) \qquad ; \tag{19}$$

the $\mathscr{D}^{\ell}_{m'm}$ reduce to the Y^m_ℓ when $m' = 0$.

In this model, the half unit of angular momentum[13] carried by fermions derives from the assignment of a quantum mechanical $\ell = 1/2$ to our symmetric top. Therefore, for the quantum mechanical description of the $\ell = 1/2$ rotational motion of our tightly bound vorton pair, we shall use the functions $\mathscr{D}^{\frac{1}{2}}_{m'm}$ rather than the Y^m_ℓ, which have a long history of controversy[36,43-46] associated with $\ell = 1/2$. For example, Schwinger[36] observed as an argument against $\ell = 1/2$, that to use Y^m_ℓ we must, by Eq. (19) set $m' = 0$, but that by Eq. (18) $m' = 0$ is incompatible with $\ell = 1/2$. In this model, however, $m' = \pm 1/2 \neq 0$, eliminating that difficulty.[14]

The $\mathscr{D}^{\ell}_{m'm}$ are of the form[47]

$$\mathscr{D}^{\ell}_{m'm} = e^{im\alpha}\, d^{\ell}_{m'm}(\beta)\, e^{im'\gamma} \qquad . \tag{20}$$

Substituting Eq. (20) into Eq. (17) yields

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[13] The notion that strong magnetic binding leads naturally to an $\ell = 1/2$ ground state has already been proposed[34] and discussed. A key element in that argument was the preclusion by a strong Lamb shift of an S-state as the ground level. Since such a (dominant) Lamb shift is absent in nuclear and atomic physics, in these cases the S-state is observed as the energetically favored ground level (for example, the deuteron or hydrogen).

[14] It is also of interest to refer to the paper by Whippman,[44] who refutes other arguments against the notion of $\ell = 1/2$.

$$\left[\frac{d^2}{d\beta^2} + \cot\beta\frac{d}{d\beta} + \ell(\ell+1) - \frac{m^2 - 2\,mm'\cos\beta + m'^2}{\sin^2\beta}\right] d^{\ell}_{m'm}(\beta) = 0 \quad . \tag{21}$$

The solutions to Eq. (21) are tabulated.

Since we are interested in $\ell = 1/2$ we record[48]

$$d^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}}(\beta) = d^{\frac{1}{2}}_{-\frac{1}{2}-\frac{1}{2}}(\beta) = \cos\beta/2 \quad , \tag{22}$$

and

$$d^{\frac{1}{2}}_{\frac{1}{2}-\frac{1}{2}}(\beta) = -d^{\frac{1}{2}}_{-\frac{1}{2}\frac{1}{2}}(\beta) = \sin\beta/2 \quad , \tag{23}$$

which, using Eq. (20) yields four linearly independent $\ell = 1/2$ symmetric top eigenfunctions:

$$\mathscr{D}^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}}(\phi,\theta,\gamma) = e^{i\phi/2}\cos\theta/2\; e^{i\gamma/2} \tag{24}$$

$$\mathscr{D}^{\frac{1}{2}}_{\frac{1}{2}-\frac{1}{2}}(\phi,\theta,\gamma) = e^{-i\phi/2}\sin\theta/2\; e^{i\gamma/2} \tag{25}$$

$$\mathscr{D}^{\frac{1}{2}}_{-\frac{1}{2}\frac{1}{2}}(\phi,\theta,\gamma) = -e^{i\phi/2}\sin\theta/2\; e^{-i\gamma/2} \tag{26}$$

$$\mathscr{D}^{\frac{1}{2}}_{-\frac{1}{2}-\frac{1}{2}}(\phi,\theta,\gamma) = e^{-i\phi/2}\cos\theta/2\; e^{-i\gamma/2} \quad , \tag{27}$$

where the angles ϕ and θ of the standard spherical coordinate system have been substituted for α and β, respectively. It is no surprise that these four functions are at once recognized as (essentially) the same as the four spinor functions[49] which comprise the Dirac 4-component spinor. The only difference is the sense of ϕ, which comes about because we are considering these functions to represent the wavefunction of a physically rotating body rather than a result of rotating a coordinate frame, and the $e^{\pm i\gamma/2}$ dependence, which is customarily discarded as an "irrelevant phase."[49]

E. Wavefunction Interpretation

The four functions $\mathscr{D}^{1/2}_{m'm}$ will be interpreted as wavefunctions furnishing the quantum mechanical description of the rotational motion of the bound vorton pair, i.e., the point-like spin 1/2 fermion. As a wavefunction a $\mathscr{D}^{1/2}_{m'm}$ specifies by the angles ϕ,θ,γ the probability amplitude for the orientation of the body coordinate system of the rotating bound vorton pair. This is the natural generalization of a Y^m_ℓ which specifies by the angles θ and ϕ the probability amplitude for the direction of the vector spearation of an orbiting pair. (The Y^m_ℓ has no angular momentum about this vector separation, whereas the $\mathscr{D}^\ell_{m'm}$ does.) As with the Y^m_ℓ, the sense of the direction of rotation associated with the $\mathscr{D}^\ell_{m'm}$ (in both the lab system x,y,z and in the body system x',y',z') is displayed when one introduces the usual $e^{-i\omega t}$ time dependence. The θ dependence of the $\mathscr{D}^\ell_{m'm}$ gives the probability amplitude describing the direction of the z' axis when viewed in the lab frame. Using this wavefunction interpretation, we see that the probability of a spin direction measurement for a pair state will equal $(\cos\beta/2)^2$, where β is the angle between the maximum of the probability distribution and the quantization direction which is being interrogated, as one would expect from conventional spinor theory.[50]

Representations of the four $\mathscr{D}^{1/2}_{m'm}$ functions are depicted in Fig. 4, in which the origin of the (rotating) body coordinate frame x'y'z' is placed at the origin of the lab frame (x,y,z), and the z' axis is oriented in its most probable position. The direction of rotation associated with the $e^{-i\omega t}$ time dependence is also indicated.

IV. MULTIPLET STRUCTURE

A. Isospin

Isospin structure in the elementary particle table derives from the fact that the observed elementary fermions fall naturally into a sequence of pairs or isodoublets, one charged and one neutral. The sets in this sequence are now known as families or generations. This structure is particularly clear in weak interaction phenomena. To see how such doublet structure derives from this model, we consider the placement of the vortons into the body frame x'y'z'.

Since the figure axis, or z'-axis, is defined by the vorton locations, L_P is quantized along that axis; according to Eq. (2), $n = 0, \pm 1, \pm 2 \ldots$. It is easy to see that the sign of n will be given by

$$s_n = s_e s_g \quad , \tag{28}$$

where s_e is the sign of the electric charge of the pair and s_g is the sign of the intrinsic fermion moment due to magnetic charge. (When north is placed on $z' > 0$, $s_g = +1$.) This intrinsic moment will be present in all $\mathcal{D}^{\frac{1}{2}}_{m'm}$ bound pair functions and will be denoted by μ_U (U for universal).

We now recall that m' is also quantized along the z'-axis. Therefore, in view of Eq. (18), and noting that $\ell = 1/2$, it is reasonable to assume that

$$|n + m'| \leq \ell = 1/2 \quad , \tag{29}$$

yielding the restriction $n = 0, \pm 1$. Furthermore, by Eq. (29) and depending upon m', only one sign of $n = \pm 1$ will be allowed for each $\mathcal{D}^{\frac{1}{2}}_{m'm}$.

The (electrically charged) configurations for $|n| = 1$ allowed by Eqs. (28) and (29) are given in Table I. Using the geometry of Fig. 4 one can determine that for these configurations, the sense of μ_U will always be opposite to that of the Bohr moment μ_B.

Table I shows that when the electric and magnetic charges are taken into account, there are eight distinct spinor functions (or components), four electrically positive and four electrically negative. (The state counting associated with these eight states will be discussed below.) Because each of these electrically charges state functions will have a companion neutral state formed by setting n = 0, one sees that Eq. (29) dictates that the point-like fermions of this model exhibit an isodoublet charge structure. Since the $\ell = 1/2$ magnetically bound vorton pair structure of this model is to apply to both leptonic and hadronic point-like fermions, this model explains at once the similarities in the leptonic and hadronic generations.

This result implies that, like the charged fermions, the neutral fermions, and in particular the neutrinos, are basically four-component objects. The notion that neutrinos might be four-component objects has been suggested before.[(51)] Such uniformity of description affords some esthetic appeal, and indeed, is a more consistent view since all of the quarks are also thought to comprise four components. The two component appearance of observed neutrinos, then, would have to derive from the (dynamical) structure of the weak interaction rather than be an intrinsic characteristic of neutrinos.

B. Vorton Eigenstates

The vorton eigenstates are labelled by the eigenvalues belonging to the operators L_i and X_i. In looking for a role in this model for these eigenvalues to play, one is led by the appeal of economy to suggest that they become internal variables of the bound vorton pair state, and that as such they are conserved quantities appropriate to label the particle types (e.g., electron, muon, etc.). While some arguments supporting this suggestion are developed below, the reader will bear in mind that a better understanding of vorton dynamics is required before it has the persuasive support of a proper mathematical analysis behind it.

The operators L_i and X_i satisfy the commutation relations of the generators of SO(4). The SO(4) reduces to SO(3) $\otimes$ SO(3) using the change in basis[52]

$$H_i^{(+)} = \frac{L_i + X_i}{2} \tag{30}$$

$$H_i^{(-)} = \frac{L_i - X_i}{2} \quad , \tag{31}$$

where the $H_i^{(\alpha)}$ satisfy the usual angular momentum commutation relations.

$$\left[H_i^{(\alpha)}, H_j^{(\alpha')}\right] = i\hbar\, \delta_{\alpha\alpha'}\, \epsilon_{ijk}\, H_k^{(\alpha)} \quad ; \tag{32}$$

$\delta_{\alpha\alpha'}$ is the Kronecker delta and ϵ_{ijk} is the totally antisymmetric tensor.

It follows from Eqs. (5), (30), and (31) that vortons have either one unit of $\vec{H}^{(+)}$ or one unit of $\vec{H}^{(-)}$. Consequently, there is a maximum of six distinct "angular momentum" eigenstates for each vorton; three will be associated with $Q_H = +1(\vec{H}^{(+)} \neq 0,\ \vec{H}^{(-)} = 0)$ and three with $Q_H = -1(\vec{H}^{(+)} = 0,\ \vec{H}^{(-)} \neq 0)$. One also sees that the $\vec{L}$ and $\vec{X}$ vectors are either parallel or antiparallel, clarifying the nature of the vortons double rotation. (The direction of the z-axis is, of course, arbitrary.) This colinearity of $\vec{L}$ and $\vec{X}$ must remain true for all time because Q_H is invariant as a function of time.[53]

Now

$$\begin{aligned} \left[P_0, H_i^{(\pm)}\right] &= \frac{1}{2}\left[P_0, \pm X_i\right] = \frac{1}{4}\left[P_0, \pm K_i\right] \\ &= \pm\frac{i\hbar}{2}\,(g_{0i}\, D - M_{0i}) \neq 0 \quad , \end{aligned} \tag{33}$$

where Eqs. (30) and (31), the definition of X_i, and the commutation relations for the conformal group[29] have been used. Equation (33) implies that one cannot diagonalize the quantities $H_i^{(\pm)}$ and energy at the same time. It follows, then, that the eigenstates of $\vec{H}^{(\pm)}$ do not couple as an ordinary angular momentum in the usual way to the other angular momenta in the configuration, in particular to the $\ell = 1/2$ orbital angular momentum. Consequently, we do not include the $\vec{H}^{(\pm)}$ in the total angular momentum vector for the bound pair, which we characterize by the $\ell = 1/2$ alone. (And we assume that energy is sharp.)

This special character of the "angular momenta" $\vec{H}^{(\pm)}$, implied by Eq. (33), and the fact that the vortons have become massless objects in the strongly bound states (their mass has been "consumed" by the binding force) leads us to look to the helicity formalism[54] for the specification of the (stationary) vorton eigenstates. While in general, one would expect six such helicity states for each vorton (labelled by $\lambda^{(+)}$, $\lambda^{(-)}$ = 0, ±1), it is well known that (free) massless particles move with the velocity of light and have only two projections of helicity, and we assume that this result also obtains for the relativistic vortons in the bound pair state.[15] They might therefore be called "fully" relativistic.

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[15] That the helicity eigenvalues appropriately label the (stationary) bound vorton eigenstates is the major assumption in this section. The assumption that the six states reduces to four is less critical because, should this assumption prove false, additional particle states are predicted by this model. Since they would be the ones with $\lambda^{(\pm)} = 0$, they would be qualitatively different from those characterized by $\lambda^{(\pm)} = \pm 1$. For example, due to the extreme relativistic nature of the vortons in the strongly bound pair, couplings to these possible additional states could be exceedingly small, making their observation difficult.

Since in this picture the $\vec{H}^{(\pm)}$ are aligned (or anti-aligned) with the vorton momentum, so are the $\vec{L}$ and $\vec{X}$ and we revert to the latter basis, which is more convenient. To enumerate the states, the symbols R, L. and r, ℓ for $\vec{L}$ and $\vec{X}$ helicities, respectively, are used. If the (instantaneous) z-axis is chosen to be along the vorton momentum vector, Eq. (5) will still obtain.

We now note that since Q_H must be conserved, both the $\vec{X}$ and $\vec{L}$ helicities would have to flip at the same time. But since none of the force carrying bosons of this model (see Sec. V) carry the $\vec{X}$ type angular momentum, one expects such transitions to be forbidden; once established in a given bound pair state, these helicities should therefore remain as constants of the motion.

Due to the fully relativistic nature of the vortons, the same helicities will be manifest in all Lorentz frames. However, the expected value of the projection of these helicities upon any of the lab coordinate axes will be null. As a consequence, (a kind of) rotational invariance is maintained, but the vorton helicity eigenstructure will have the character of a Lorentz scalar.

Taking the helicity eigenstates comprising this structure to be internal (and conserved) variables of the bound vorton pair, we shall associate them with the labels or quantum numbers of the different particle types of this model (e.g., electron, proton, etc.). The 16 helicity states that may be assigned to a pair of fully relativistic vortons are listed in Table II. Also included in Table II are the total Q_H of the pair and whether or not the electric moments μ_E and μ_t (due to the ϕ and ψ rotation, respectively, of the magnetic charge) of the two vortons add (yes) or cancel (no). (It is assumed that the vorton momenta are equal and opposite in direction.)

Half of the 16 states listed in Table II have $Q_H = 0$. These (eight) states will not be stable against self-annihilation and are therefore eliminated from further consideration as stable fermion states.

Of the states with $|Q_H| = 2$, half of them have $\mu_E = \mu_t = 0$ and the other half have $\mu_E, \mu_t \neq 0$. We make the assumption that this feature separates leptons from hadrons, hadrons being of the latter type. As discussed below, the motivation for this assumption is that $\mu_E, \mu_t \neq 0$ can then furnish a coupling source for the strong interactions of these particles. Using this feature, the states in Table II are partitioned into hadrons and leptons; h_i stands for hadrons and ℓ_i for leptons,[16] while $\bar{h}_i$ and $\bar{\ell}_i$ are the respective antiparticle labels. The i = 1 or 2 relationships between fermion and antifermion were chosen to yield consistency under the CPT symmetry [see e.g., Eq. (49)].

C. C, P, T, M Transformations

It is of interest to investigate the discrete transformations of the various states which have been constructed in the above sections.

To obtain the transformation of the $\mathcal{D}^{\frac{1}{2}}_{m'm}$ functions under the operation of a parity inversion P, the substitutions,[(56)]

$$\begin{aligned} \phi &\to \phi' = \phi + \pi \\ \theta &\to \theta' = \pi - \theta \\ \gamma &\to \gamma' = \pi - \gamma \quad . \end{aligned} \tag{34}$$

are relevant. Substituting Eqs. (34) into Eq. (24) through (27) yields

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[16] The baryons and leptons have arbitrarily been assigned $Q_H = \pm 2$, respectively. This choice will be discussed further below.

$$P\,\mathcal{D}^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}} = \mathcal{D}^{\frac{1}{2}}_{-\frac{1}{2}\frac{1}{2}}$$

$$P\,\mathcal{D}^{\frac{1}{2}}_{\frac{1}{2}-\frac{1}{2}} = \mathcal{D}^{\frac{1}{2}}_{-\frac{1}{2}-\frac{1}{2}} \tag{35}$$

$$P\,\mathcal{D}^{\frac{1}{2}}_{-\frac{1}{2}\frac{1}{2}} = -\mathcal{D}^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}}$$

$$P\,\mathcal{D}^{\frac{1}{2}}_{-\frac{1}{2}-\frac{1}{2}} = -\mathcal{D}^{\frac{1}{2}}_{\frac{1}{2}-\frac{1}{2}} \quad ;$$

the $\mathcal{D}^{\frac{1}{2}}_{m'm}$ functions are not eigenstates of parity. Eqs. (35) may be condensed using the notational shorthand:

$$\begin{pmatrix}1\\0\end{pmatrix}_+ \equiv N\,\mathcal{D}^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}}, \quad \begin{pmatrix}0\\1\end{pmatrix}_+ \equiv N\,\mathcal{D}^{\frac{1}{2}}_{\frac{1}{2}-\frac{1}{2}} \tag{36}$$

$$\begin{pmatrix}1\\0\end{pmatrix}_- \equiv N\,\mathcal{D}^{\frac{1}{2}}_{-\frac{1}{2}\frac{1}{2}}, \quad \begin{pmatrix}0\\1\end{pmatrix}_- \equiv N\,\mathcal{D}^{\frac{1}{2}}_{-\frac{1}{2}-\frac{1}{2}} \tag{37}$$

where the $\pm$ subscript denotes the sign of m', and N is the normalization constant $(2\pi)^{-1}$. The location of the 1 indicates spin up or spin down in the usual way.

Eqs. (35) and the notation of Eqs. (36) and (37) yield

$$P(\ )_\pm = \pm(\ )_\mp \quad . \tag{38}$$

It follows from Eq. (38) that the application of P^2 to the $\mathcal{D}$ spinor functions will yield the eigenvalue -1 and P^4 will yield +1. These results are in accord with the established behavior of spinors.(57)

It is straightforward to use the $(\ )_{\pm}$ spinors to form spinors which are eigenstates of parity: one defines[17]

$$\xi \equiv \frac{1}{\sqrt{2}}\left[\left(\ \right)_{+} - i\left(\ \right)_{-}\right] \tag{39}$$

and

$$\eta \equiv \frac{1}{\sqrt{2}}\left[\left(\ \right)_{+} + i\left(\ \right)_{-}\right] \quad . \tag{40}$$

Using Eqs. (38) through (40), we see that

$$P\xi = i\xi \tag{41}$$

and

$$P\eta = -i\eta \quad ; \tag{42}$$

the ξ and η have imaginary parity, enabling the naive extension of the parity rule for the Y_ℓ^m, i.e., parity $= (-1)^\ell$.

The above results show that the four linearly independent $\mathscr{D}^{\frac{1}{2}}_{m'm}$ functions are appropriately viewed as two distinct Pauli spinors. Of course, it has been known for a long time that there are two different spinors with "opposite" parity,[(58)] and that the Dirac four-component spinor is a joint combination of these two two-component spinors.[(59)] The ξ and η spinors, then, are analogous to the isotropic spinors employed, for example, by Brinkman[(60)] or to the u and v spinors conventionally employed in quantum electrodynamics,[(61)] and we shall employ below them to construct our point-like fermion particle states.

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[17] While these ξ and η are eigenfunctions of L_z, Eqs. (39) and (40) show that they are not eigenfunctions of $L_{z'}$. For these eigenfunctions m' is not sharp and $[P, L_{z'}] \neq 0$. Upon examination of Table I and Fig. 4, one sees that this has the interesting consequence that in order to obtain a uniform electrically charged state for the ξ and η spinors as defined by Eqs. (39) and (40), the vorton placed on the +z'-axis must be an equal mixture of magnetic north and south. Another way to view this situation is to say the the vortons remain physically in the same place while the z'-axis flips back and forth.

We now recall that Table I indicates that the (charged) ξ and η spinors may be electrically either positive or negative. Consultation with Fig. 4 and Table I and considering the placement of the vorton's electromagnetic charges into the body frame x'y'z' reveals that to obtain a function which satisfies Eq. (33) one must, with the application of the parity operator, at the same time reverse the sign of either electric or magnetic charge. A more consistent formalism obtains by doing the latter, using the magnetic charge conjugation operator M. Thus the appropriate form of the parity operation to be used on these eigenfunctions is

$$P' \equiv PM \quad . \tag{43}$$

The need to consider these extended operations was originally pointed out by Ramsey.[62] With this extension, and tacitly including the electromagnetic charges as indicated in Table I, one writes

$$P'\xi = i\xi \tag{41'}$$

and

$$P'\eta = -i\eta \quad . \tag{42'}$$

An analogous argument with the simple time reversal operator T leads to the extended time conjugation operator

$$T' \equiv TM \quad . \tag{44}$$

Applied to the $\mathscr{D}^{\ell}_{m'm}$ functions, T' effects $m \rightarrow -m$ and $m' \rightarrow -m'$, but maintains electromagnetic properties compatible with Table I.

Similarly, the restrictions on the electromagnetic properties of the $\mathscr{D}^{\ell}_{m'm}$ given by Eq. (29) lead to the requirement for an extended charge conjugation operator,

$$C' \equiv CM \tag{45}$$

where C conjugates only the electric charge. C' has no effect on the rotational properties of the various eigenstates.

The effect of the operators P', C', T' on the vorton helicity states R, L, r, and ℓ are straightforward:

$$P'R = PR = R \quad ,$$
$$P'L = PL = L \quad , \qquad (46)$$
$$P'r = Pr = \ell \quad ,$$
$$P'\ell = P\ell = r \quad ,$$

$$T'R = TR = L \quad ,$$
$$T'L = TL = R \quad , \qquad (47)$$
$$T'r = Tr = \ell \quad ,$$
$$T'\ell = T\ell = r \quad ;$$

C' makes no changes.

D. State Identifications

We are now in a position to combine the eight surviving ($Q_H \neq 0$) vorton state labels given in column five of Table II with the four (electrically charged) spinor functions of Eqs. (39) and (40) to obtain a total of 32 possible two-component (charged) fermion spinors.[18] (Each of these spinors will, of course, have an electrically neutral isospin companion formed by setting n = 0.) These 32 spinors are collected in

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[18] These two-component spinors comprise what are known as the large components. In the free particle rest frame, these are the only ones which are required. In more general situations, however, particularly when relativistic considerations are important, admixtures of the spinors of opposite parity enter the picture (proportional to v/c) as the so-called small components.[(63)] (In some sense the spinor comprising these small components is "borrowed" from the anti-fermion.) Thus, the Dirac fermion actually requires both types of spinor for a complete description, and these two spinors are used jointly to describe both the fermion and the anti-fermion.

Table III in two columns, where the superscripts on η and ξ indicate the sign of the electric charge.

As remarked above, these state labels were chosen in such a way that, for example,

$$P'C'T'\ell_i = \bar{\ell}_i \qquad , \tag{48}$$

With this choice, the complete functions automatically satisfy

$$\ell_i\xi^-\uparrow \xrightarrow{P'C'T'} \bar{\ell}_i\eta^+\downarrow \qquad , \tag{49}$$

as one expects.(64) The arrows denote the spin direction (or helicity) of the pair state.

The collection of two-component spinor functions given in Table III (along with their electrically neutral isospin companions) are the basis functions of this model which are to be associated with the observed point-like fermions. One sees that the set in the first column is in accord with observation: negative leptons and positive baryons. The set in the second column is merely an image of the first column related by a 180° dyality angle rotation. By interpolation we see that there is actually an infinite number of such (degenerate) sets of basis states, labelled by the dyality angle Θ. A feature of this model, then, is that (at an early stage of the universe) Nature, by a spontaneous breaking of the dyality symmetry, has selected the set in column one. The set in column two is merely one of the infinite, but unrealized, possible sets available by symmetry;[19] the discovery of the Rainon would bear witness to their (conceptual) existence.

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[19] This symmetry breaking process has defined the reference angle of Θ, and consequently, what we call electric and what we call magnetic charge. But, of course, no matter what reference angle would be selected, we would by definition end up calling electric charges electric.

The structure of the vorton helicities along with the electric charge and spinor type has enabled us to construct particle eigenstates which are physically distinguishable from one another. By this construction, the number of different particle types is finite and specified. What this model offers, then, that others do not, is a physical description of an underlying structure for the point-like fermions and a physical explanation of the quantum numbers which label the types of particles.

Collecting together the 16 particle and antiparticle states from column one of Table III, we make the identifications as shown in Table IV. By setting $n = 0$, one neutral isospin companion is obtainable from each of the state functions on the left-hand side of Table IV. The particles so obtained are indicated in parentheses. We see that there are eigenfunctions enough to accommodate the three known lepton generations (e, μ and τ) and predict one more which we call T.[20] The eigenfunctions also accommodate the quark flavor isodoublets (n,p), (λ,λ_c), (b,t), and a fourth doublet which we shall label (h,o). Thus, this model predicts one new heavy lepton the T with its neutrino and three new quark flavors t, h, and o.

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[20] The large predicted mass,(33) $\sim 380\ \mathrm{GeV}/c^2$, motivated the choice of the letter T (capital τ) for the final charged lepton. Following the suggestion of Sato,(65) who draws upon the Greek numbers, we have used the letters h and o to denote the seventh (hepta) and eighth (oktō) quark flavors.

V. INTERACTIONS

At this point it might be tempting to seek to incoporate the various particle interactions into this model along the lines of the Grand Unified Theories[66] (GUTS) in which electromagnetism is just one facet or component of a single basic, but more general interaction. However, the basic tenet of this model, that electromagnetism is the one and only fundamental interaction, is incompatible with such an approach; consistency requires that the explanation of the other interactions must be sought in features already existing as part of the model. It follows, of course, that these nonelectromagnetic interactions are in a second echelon of a hierarchy of interactions, and as such will be effective theories most easily amenable to phenomenological description. This entails the unhappy consequence that their complete understanding requires detailed analysis of extremely difficult (underlying) dynamical processes. As compensation for this difficulty, this approach offers the possibility of achieving a maximum of economy at the fundamental level: one fundamental interaction and one fundamental particle, which is a (static) solution to that interaction.

In order to give a physical rationale to this approach, it is proposed that the sources of the interactions are associated with various electromagnetic features of the point-like bound vorton pair spin 1/2 states. The point-like fermion states which have a certain feature, then, will participate in the associated interaction.

The most obvious such feature is the electric charge. Thus, once the value of the bound state electric charge is (properly) specified by Eq. (16), the conventional form of electrodynamics can automatically be incorporated into the model.

The assignment of the sources for the other well established interactions (strong, weak, and gravitation) is a reasonably straightforward matter. Since half of the point-like states have the (strong[21]) electric moments μ_E and μ_t, they (one or both) are designated as the source for the strong interactions; since all the point-like fermions have a μ_U, this will be considered as the source of the weak interaction (historically known as the Universal Fermi Interaction); and, motivated by general relativity, the energy-momentum tensor $T^{\mu\nu}$ will be taken as the source for gravitation. These source assignments are listed in Table V.

In looking to already existing features of the model, it is natural to identify magnetically bound vorton pairs with integral ℓ as the bosons which carry the nonelectromagnetic forces. One then assigns $\ell = 0$, 1, and 2 to the strong, weak, and gravitational interactions respectively.[22] These ℓ assignments correlate logically with the above specified sources. Since the individual vorton helicities have no preferred orientation in space or relationship to the fermion spin, their associated μ_E and μ_t are natural candidates for a scalar interaction;[23] since the μ_U is always

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[21] These moments are "strong" because they are associated with the rotations of the magnetic charge, which in the bound pair state satisfies the relationship $g \cong 137e$.

[22] It is interesting that the strength of the interactions diminish rapidly as ℓ becomes larger.

[23] The question of the nature of the basic strong interaction is a difficult one. The large coupling strength militates against clean calculations which may be subjected to conclusive experimental tests. At the present time, the major contender for the strong interaction is most certainly QCD carried by colored vector gluons. However, while the evidence for QCD appears favorable, it is by no means conclusive; confinement has not yet been theoretically proved, and colored objects have not yet been experimentally observed. Before the advent of QCD, some felt that the evidence for a basic scalar strong interaction was perhaps better than that for a vector interaction.(67) Furthermore, it is even conceivable that the basic scalar interaction suggested here could give rise to a (third echelon) vector interaction much along the lines of QCD.

along the fermion spin, it is a natural candidate for a "vector" interaction; and as shown by Weinberg,[68] if one has a massless spin 2 particle, given Lorentz invariance, it will naturally couple to the $T^{\mu\nu}$ in the manner specified by general relativity.

Feynman diagrams are a useful way to depict these interactions. For example, the photon, which carries the electromagentic force, couples to a fermion with a bare charge e_0, as shown in Fig. 5a. As is well known, the complete interaction vertex entails radiative corrections, and after renormalization the electromagnetic interaction is represented by a vertex as depicted in Fig. 5b. Similarly, the (composite) bosons as specified above would generate their respective forces through an elemental vertex as depicted in Fig. 6a, where it is seen that the vorton lines are continuous through the vertex. This feature maintains the conservation of the vortons' topological charge. The vortons comprising the force carrying bosons are obtained at the vertex by simply diverting into the boson states the constituent vortons of the fermion states.

When constructed in this manner, these boson states will have $Q_H = 0$, and will be amenable to treatment in an effective field theory. In such a field theory, since the bound vorton states have an intrinsic size $\sim \ell_L$, the interactions carried by these bosons will be subject to a physical cutoff at $\sim M_L$ (and hence will not be renormalizable). Radiative corrections to these "bare" vertices, then, will be in terms of quasi-divergent integrals multiplied by very small intrinsic coupling constants. (Recall that except for QED the postulated sources contain the length ℓ_L.) As with electromagnetism, these radiative corrections can in principle be functionally taken into account with a vertex function Γ_ℓ as depicted in Fig. 6b.

While these Γ_ℓ can be phenomenologically assigned, the challenge of this approach is to calculate them from first principles. It is unfortunate that this may be an arduous task, however, for one notes that even now the electromagnetic vertex function Γ_μ is only known in low order and only in terms of the renormalized charge.

On the other hand, certain advantages do accrue from this approach. For example, one would now be relieved from the requirement to find a renormalizable theory of gravitation, a task which to date has proved to be intractable.

Finally, it is appropriate to reiterate here that in this model it is assumed that the masses of physical particles are dynamically generated by the self-interactions of these particles, and their mass differences from a spontaneous symmetry breaking. This idea was first proposed by Nambu and his collaborators.(69)

From the assumption that interactions and mass (self-interactions) are described by one and the same theory, it follows that the binding energy causing the collapse of the magnetically bound vorton pair to a point-like state will be furnished by and limited to the mass (energy) of the original unbound, or free, vortons. This being the case, it has been argued(34) that in the point-like limit, the binding energy will be identically equal to the sum of the masses of the free vortons. One sees that this equality will obtain independent of the masses of the free vortons and of the coupling strength which binds them; the assumption that mass derives from self-interactions enables without contrivance the formation of massless composite particles of extremely small scale— a difficulty for some other particle models.

To the extent that this cancellation is not quite complete, one could expect the dynamical generation of a small magnetic self-energy or mass. This small mass, which we shall attribute to the pair and not to the individual vortons can be naturally accommodated in the Dirac equation (which would govern the spin 1/2 pair state) as the bare or "mechanical" mass m_0.[24] A finite m_0 is conceptually useful because it enables already at this stage the definition of a center-of-mass coordinate frame in which to describe the orbital motion of the bound pair.

The subsequent dressing of the pair by its own self-interactions and hence the acquisition of mass by the pair by other (self-)interactions (i.e., electric or strong) and the mass splittings by a spontaneous symmetry breaking are assumed not yet to have occurred. An analysis of how this might come about in the (charged) leptonic sector via the QED self-interaction has been published.(33) The hadronic sector which involves also the strong interactions is more complicated and will be left for the future.

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[24]This small mass would be associated with the presence of the μ_U and its self-interactions. Thus the recent evidence that ν_e may indeed have a small mass(70) is accommodated in a natural way in this model.

VI. SUMMARY AND DISCUSSION

This paper presents a qualitative description of a model for the structure of point-like fermions. In this model, it is assumed that there is only one fundamental interaction, electromagnetism, and one fundamental object, the vorton, which is a semiclassical monopole solution of Maxwell's equations. Vortons carry a generalized electromagnetic charge Q with a dyality angle Θ, one unit each of two different kinds of angular momenta, and a unit of topological charge Q_H.

Since Q is known to be large, it is argued that a pair of (magnetically) bound vortons will collapse to a point-like state, and that this point-like state will be essentially massless, the masses of the vortons being "consumed" by the binding force. Since the vorton mass and binding energy are both due to electromagnetism and both associated with the same charges, it is not unreasonable that the cancellation of these two energies should be (nearly) exact.

It is suggested that complete collapse to an actual mathematical point is prevented by physics associated with the Landau singularity of QED, and that as a consequence the size of the point-like state is characterized by the Landau length, which has roughly been shown to be in the vicinity of the Planck length.[33]

Since the ground state of the magnetically bound vorton pair is not, in fact, a mathematical point, but has a finite size, it is argued that such a state is describable by the generalized (symmetric top) eigenfunction $\mathscr{D}^{\ell}_{m'm}$ and that $\ell = 1/2$. It is shown that the $\mathscr{D}^{\ell}_{m'm}$ has four states of imaginary parity which may be mapped onto the usual Dirac four-component fermion spinor.

Using Schwinger's quantum condition for particles carrying both electric and magnetic charge, it is shown how the $\ell = 1/2$ leads to an isodoublet charge structure for these point-like fermions. Neutral fermions are formed when the two vortons have dyality angles differing by π and charged fermions when the dyality angles differ by slightly less than π.

It is argued that the vortons in the tightly bound pair state are fully relativistic and as such their intrinsic angular momenta are appropriately described in the helicity formalism. It is further argued that the structure of these helicities is decoupled from the orbital $\ell = 1/2$ and, consequently, has the character of a Lorentz scalar. The eigenstates of this structure are identified as the basis for a set of internal quantum numbers of the point-like pair states that distinguishes the various particle types from one another.

All possible pair states allowed by the model are enumerated. After certain of these states are dropped from consideration for cause (helicity structure of fully relativistic vortons, mutual annihilation allowed by $Q_H = 0$, and spontaneous symmetry breaking), sixteen four-component spinors remain. Using as a criterion, the cancellation or addition of the intrinsic vorton electric moments, these spinors partition naturally into eight leptonic and eight hadronic particle types. Eleven of these are identified with the quantum numbers of the experimentally known particles: e, ν_e, μ, μ_ν, τ, ν_τ; p, n, Λ, Λ_c and b. Thus one new heavy lepton the T with its neutrino and three new quark states are predicted by this model.

Since a basic premise of this model is that there is only one fundamental interaction, electromagnetism, the nonelectromagnetic interactions are placed in a second echelon in a hierarchy of interactions. Thus, in contrast to the GUTS theories which "horizontally" unify the interactions, this model unifies them "vertically." Unfortunately, the analysis of this aspect of the model is still at an early stage. While the approach in this model is expected to provide calculable particle masses and coupling constants (thus, in principle, enabling a theory unencumbered by a multitude of fundamental parameters), in fact, at present such calculations are too difficult. As a consequence, the masses and coupling constants, as well as the appropriate governing equations, must be introduced phenomenologically. On the other hand, we have seen that this model provides qualitative explanations for certain features of elementary particle spectra which are incorporated into other models by postulation. Specifically, it provides an enumeration of the quantum numbers (e.g., flavors) of the elementary particles, both hadrons and leptons, and the reason for the isodoublet structure of the fermion multiplets.

It also provides a rationale for some aspects of their interactions. It is suggested that the sources of these interactions are found in the various electromagnetic features which characterize the various point-like fermion states, and features are identified which are appropriate to associate with the four (established) basic interactions. It is further suggested that the bosons carrying the forces of the second echelon interactions (strong, weak, and graviation) may be identified respectively with the ℓ = 0, 1, and 2 bound vorton pair states. Thus, on the one hand while this model provides a consistent and satisfying qualitative picture of these basic interactions, on the other hand their quantitative treatment from a fundamental point of view remains to be done.

It is assumed that the masses of particles are dynamically generated by their self-interactions. While we are not able to predict the masses in the hadronic sector, a previous analysis of the QED self-interaction of the charged leptons[33] indicates that the mass of the T may be quite large, ~380 GeV/c^2. Consequently, from an experimental point of view, it appears that the most promising avenue for substantiating this model would be a search for its fundamental building block, i.e., the vorton.

Finally, it is interesting to point out that the fact that the baryon number of the universe appears to be a large positive number is not of serious consequence in this model. Baryon number and lepton number are just certain combinations of vorton helicities and as such are not absolutely conserved quantities; only Q_H is rigorously conserved. It is for this reason that leptons and hadrons have been assigned opposite Q_H. One would expect that $Q_H = 0$ for the universe as a whole which, of course, is possible with this assignment. A large anti-matter component of the universe is then neither required nor precluded by the restrictions of this model.

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APPENDIX A

Modification Due to QED of the Quantization Condition on Q

1. Generalities

This Appendix first recapitulates the salient points of the analysis in Ref. (1) that leads to the quantization condition on the magnitude of the vorton's electromagnetic charge Q, from which one obtains the dimensionless constant $Q_0^2/\hbar c = 4.87$. Modifications of the analysis by effects of QED are then discussed. These modifications lead to a dependence of the quantized value of Q^2 upon the vorton scale a; that is

$$Q^2 = Q^2(a) \quad , \tag{A-1}$$

where the a dependence derives from the scale introduced by (the masses of) the fermions of QED. Since this discussion is meant to illuminate the qualitative features of these modifications, only what are viewed as the leading contributions are considered.

2. Recapitulation

The total electromagnetic energy W_T of a vorton of electromagnetic charge Q is divided into two parts: 1) a static energy associated with the monopole field of the charge Q and 2) a dynamic energy associated with the "rotation" of that charge. For the purposes of convenient analysis it is assumed that the dyality angle $\Theta = \pi/2$, making Q an electric charge. Since Maxwell's equations are invariant with respect to Θ, the results of the analysis are valid for a general Θ. One can assert that this Θ invariance applies as well to the QED modifications (due to vacuum polarization loops) discussed below. For it has been argued[71] that the renormalization (also due to vacuum polarization loops) of electric and magnetic charges is the same. (Once a photon has been emitted from a charge, it doesn't "know" whether that charge is electric or magnetic.)

The static or monopole energy

$$W_m = K_m Q^2 \quad , \tag{A-2}$$

and the dynamic or dipole energy

$$W_d = K_d Q^2 \left(\beta_\phi^2 + \beta_\psi^2\right) \quad , \tag{A-3}$$

where $K_m = 5/4\pi a$, $K_d = 1/6\pi a$, and β_ϕ and β_ψ are rotational parameters. β_ϕ is associated with the usual angular rotation about the z-axis and β_ψ with a conformal rotation (see Sec. II), also associated with the z-axis; it was shown that the same K_d serves for both types of rotation. By then noting that if one uses the Bohr-Sommerfeld prescription to quantize the action S (specifically an integral of the $\vec{j}\cdot\vec{A}/c$ term in the Lagrangian density which yields Maxwell's equations) in units of Planck's constant h (or the associated angular momenta in terms of $\hbar$) one obtains the equations

$$S^{(\phi)} = K_s Q^2 \beta_\phi = m_\phi h \tag{A-4}$$

and

$$S^{(\psi)} = K_s Q^2 \beta_\psi = m_\psi h \quad , \tag{A-5}$$

where m_ϕ and m_ψ are quantum numbers, and both types of rotations are again served by the same parameter, $K_s = 2/3c$.

Defining

$$\bar{\beta} \equiv \frac{\beta_\phi}{m_\phi} = \frac{\beta_\psi}{m_\psi} \quad , \tag{A-6}$$

one obtains from Eqs. (A-4) and (A-5) the relation

$$Q^2 = \frac{h}{K_s \bar{\beta}} \tag{A-7}$$

In terms of $\bar{\beta}$, the total vorton energy

$$W_T = W_m + W_d = K'_m \bar{\beta}^{-1} + K'_d \bar{\beta} \quad , \tag{A-8}$$

where

$$K'_m = \frac{K_m h}{K_s} \tag{A-9}$$

and

$$K'_d = \frac{K_d h}{K_s} \left(m_\phi^2 + m_\psi^2 \right) \quad . \tag{A-10}$$

W_T has a minimum with respect to $\bar{\beta}$ when

$$\bar{\beta}^2 = \frac{K'_m}{K'_d} = \frac{K_m}{K_d \left(m_\phi^2 + m_\psi^2 \right)} \quad . \tag{A-11}$$

Choosing $\bar{\beta}$ to satisfy Eq. (A-11) yields

$$W_m = W_d = \frac{W_T}{2} = \sqrt{K'_m K'_d} \tag{A-12}$$

and the quantization condition

$$Q^2 = \frac{h}{K_s} \sqrt{\frac{K_d}{K_m}} \sqrt{m_\phi^2 + m_\psi^2} \quad . \tag{A-13}$$

Putting in the values for the constants gives

$$Q^2 = \pi \sqrt{\frac{6}{5}} \sqrt{m_\phi^2 + m_\psi^2} \, \hbar c \quad , \tag{A-14}$$

independent of the vorton scale a.

A "ground state" vorton is defined by $|m_\phi| = |m_\psi| = 1$, and Eq. (A-14) becomes

$$Q^2 = 4.87\ \hbar c \equiv Q_0^2 \quad . \tag{A-15}$$

This semiclassical result applies for $a >> \lambda\!\!\!^{-}_e$, where the effects of QED may be neglected.

3. Scale Dependence

One expects a modification in the quantization condition on Q as a becomes small and QED effects enter the picture. In reviewing the analysis which led to Eq. (A-13), we see that three electromagnetic quantities were calculated: the static (or monopole) energy W_m, the dynamic energy W_d, and the action S. Thus, it is appropriate to examine the scale dependence of these three quantities and the QED modifications to their evaluation as the scale becomes small.

W_m is the Coulomb self-interaction of the charge density given by an integral of $q_1 q_2/r$, where q_1 and q_2 are incremental portions of the charge density. Integration yields $W_m = 5Q^2/4\pi a$, an expected 1/a dependence, since a sets the scale of the vorton charge distribution. W_d is an integral of $\vec{j}_1 \cdot \vec{A}_2$, where the vorton potential $\vec{A}_2$, which is due to an incremental current $\vec{j}_2$, also goes like 1/r. Integration (when W_T is minimized) yields $W_d = 5Q^2/4\pi a$, again the expected 1/a dependence. With these results in mind, a glance at Eq. (A-11) shows that the $\bar{\beta}$ obtained from the minimization of W_T has no a dependence; the two 1/a factors cancel. This cancellation carries over into Eq. (A-13).

Now S is also an integral of $\vec{j} \cdot \vec{A}$. However, the 1/a dependence of this $\vec{j} \cdot \vec{A}$ integral is removed by the Bohr-Sommerfeld quantization condition, which specifies that the action integral is to be taken over one period of the relevant (cyclic) variable. Since $\bar{\beta}$ is a constant, the period goes like a as the vorton size diminishes, cancelling the 1/a dependence of the $\vec{A}$ and leaving S with no a dependence.

Using the above results in Eq. (A-13), one sees why there is no a dependence of Q^2 as specified by the Bohr-Sommerfeld quantization condition. Of course, one could have reached this conclusion on general grounds because electromagnetism has no intrinsic scale,[72] but the above discussion is a useful background for the consideration of QED effects. One presumes that the effects of QED on the quantization condition can be estimated by considering the QED modification of the 1/r potentials which figure directly in the calculation of the three electromagnetic quantities W_m, W_d, and S. One expects such a modification from vacuum polarization, the lowest order of which is depicted in Fig. 1.

As is the case with the renormalization of the electric charge, it is assumed here that vacuum polarization is the major relevant QED effect. For example, it is assumed that the vorton charge distribution is not modified by vacuum fluctuations in a way which significantly changes the W_m, W_d, and S. The rationale for this assumption is that the charge distribution is smooth and continuous; if a vacuum fluctuation causes a small region of the charge distribution to be displaced, an adjacent region will, by the same fluctuation, be caused to move over into the vacated location, leaving the energy of the configuration constant (to lowest order).

In atomic physics, the effects of vacuum polarization lead to what is known as the Uehling potential.[73] Its form is well known and for an electron in the field of a point charge Ze is given by[(74)]

$$V_{11}(r) = -\frac{Z\alpha}{r} R_{11}(r) \quad . \tag{A-16}$$

where

$$R_{11}(r) = \frac{2\alpha}{3\pi}\int_1^\infty \frac{dt}{t}\left(1+\frac{1}{2t^2}\right)\left(1-\frac{1}{t^2}\right)^{1/2} \exp\left\{-\frac{2rt}{\lambdabar_e}\right\} \tag{A-17}$$

is the ratio of the (second order approximation to the) Uehling potential to the Coulomb potential.

The parameter of integration t is the energy of the virtual Coulomb photon in units of $m_e c^2/2$. Thus, as a result of the exp $\{-2rt/\lambdabar_e\}$, photons of energy $> \hbar c/r$ effectively do not contribute to the Uehling potential at the distance r. This is in accord with one's intuition and, as will be shown below, is also true for the Coulomb potential itself.

One may use Eq. (A-17) (in the region $a > \lambdabar_\mu$) to estimate the modification to the value of Q^2 specified by the Bohr-Sommerfeld quantization condition; the quantities K_m, K_d, and K_s each should be increased by a factor of approximately[25] $1 + R_{11}(a)$. (Whereas in Eq. (A-16), $V_{11}(r) < 0$, increasing the binding energy in atoms, the modification to W_m, W_d, and S will be positive, increasing the self-energy of a vorton charge configuration.) In the calculation of $\bar{\beta}$, the augmentation in W_m and W_d will (essentially) cancel, leaving the correction to K_s as the dominant factor. Thus, for the ground state vorton, one uses Eq. (A-13)

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[25] This approximation makes use of the fact that $\langle 1/r\rangle \cong 1/a$ for the distributions in the integrals of W_m, W_d, and S. Such an approximation is adequate for the purposes of this analysis, which is to illuminate qualitative behavior.

to write

$$Q^2(a) \cong \frac{Q_0^2}{1 + R_{11}(a)} \quad . \qquad \text{(A-18)}$$

To get a feel for the size of this correction, we note that $R_{11}(\lambda\!\!\!^{-}_e) = 5.6 \times 10^{-5}$, $R_{11}(0.1\ \lambda\!\!\!^{-}_e) = 1.7 \times 10^{-3}$, and $R_{11}(0.01\ \lambda\!\!\!^{-}_e) = 5 \times 10^{-3}$; the effect increases (slowly) as a decreases, and is small because the coupling α of the polarization loop to the propagating photon is small.

As one considers smaller and smaller scales,[26] one sees from Eq. (A-17) that some simplifications may be made in the calculation. In particular, one notes that the product $\left[1 + (1/2t^2)\right]\left[1 - (1/t^2)\right]^{1/2}$ differs from unity over only a small fraction of the range of integration. Thus, omitting these factors and noting that the upper limit to the integration is effectively set by the exponential, one can write

$$R_{11}(r) \rightarrow \frac{2\alpha}{3\pi} \int_1^{\lambda\!\!\!^{-}_e/2r} \frac{dt}{t} = \frac{2\alpha}{3\pi}\ \ell n\ \frac{\lambda\!\!\!^{-}_e}{2r} \quad , \qquad \text{(A-19)}$$

showing a logarithmic dependence of $R_{11}(r)$ upon r.

This result (which is appropriate to the range $\lambda\!\!\!^{-}_\mu < r < \lambda\!\!\!^{-}_e$) is a manifestation of the fact that the (Feynman gauge) photon propagator

$$\frac{-i}{q^2} \rightarrow \frac{-i}{q^2}\left(1 + \frac{\alpha}{3\pi}\ \ell n\ \frac{-q^2}{m_e^2\ c^2}\right) \quad , \qquad \text{(A-20)}$$

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[26] It is appropriate to remark here that it has been shown that at small distance the Uehling potential dominates other corrections to the Coulomb potential.(75)

when one makes the correction for the possibility of one loop of vacuum polarization.[76] [For q^2 the four-vector notation of Bjorken and Drell[77] is used: $q^2 = q_0^2 - \vec{q}^{\,2}$.]

If one approximates the photon propagator by the summation of diagrams of concatenations of all possible numbers of photon loops, as depicted in Fig. 2, Eq. (A-20) becomes[78]

$$\frac{-i}{q^2} \rightarrow \frac{-i}{q^2} \frac{1}{1 - \frac{\alpha}{3\pi} \ell n \frac{-q^2}{m_e^2 c^2}} \quad . \tag{A-21}$$

The next refinement is to realize that there are (effectively) N_f point-like fermions which electromagnetically couple to the photon. Thus, Eq. (A-21) is rewritten:

$$\frac{-i}{q^2} \rightarrow \frac{-i}{q^2} \frac{1}{1 - \frac{N_f \alpha}{3\pi} \ell n \frac{-q^2}{\bar{m}^2 c^2}} \quad , \tag{A-22}$$

where $\bar{m}$ is the r.m.s. value of the masses of the N_f fermions.

Using Eq. (A-22) to rewrite Eq. (A-18) yields

$$Q^2(a) \cong Q_0^2 \left(1 - \frac{2N_f \alpha}{3} \ell n \frac{\Lambda}{2\bar{m}c} \right) \quad , \tag{A-23}$$

where $\Lambda = \hbar/a$ is the momentum associated with the scale a.

Since α is small and the variation with scale is logarithmic, the leading log approximation given in Eq. (A-23) does not get into obvious difficulty for many orders of magnitude. For example, using $N_f = 8$, the number expected from this model, $\Lambda = 1.22 \times 10^{19}$ GeV/c, the Planck momentum, and $\bar{m} = 500$ MeV/c^2, a guess, yields $Q^2(\ell_p) = Q_0^2$ (1 - 0.544).

Whether Eq. (A-23) applies over this range and, in particular, how to extend it even further depends upon one's assumptions about the short scale behaviour of the photon propagator function. The assumptions which are employed here are those which have been detailed and discussed in Ref. (33).

In brief, these assumptions are: 1) that only QED bears upon the photon propagator (Grand Unification schemes are not adopted here), 2) that the form of photon propagator is suitably approximated by the right-hand side of Eq. (A-22), and 3) that this form may be extended by analytic continuation beyond the point where

$$\frac{N_f \alpha}{3\pi} \ell n \frac{P_L^2}{m^2 c^2} = 1 \ , \qquad \text{(A-24)}$$

which (in leading log approximation)[27] defines the Landau momentum P_L as the location of the Landau singularity.[28]

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[27]There are higher order Feynman diagrams which have been omitted, but it is argued in Ref. (33) that these are quantitatively too small in effect to change the qualitative behavior of Eq. (A-22). That is, their inclusion will move, but not eliminate the Landau singularity.

[28]One can perform a Wick rotation(79) to go from the p of Minkowski space to a P of Euclidean space: $p_0 = iP_4$, $p_j = P_j$, and $p^2 = -P^2$. In the latter space the Landau singularity lies on the surface of a four-sphere. Using this concept and the above numerical quantities i.e., $N_f = 8$, $\alpha = 1/137$, and $\bar{m} = 500$ MeV/c^2, yields the Landau momentum $P_L = 5.6 \times 10^{34}$ GeV/c and the Landau length $\ell_L = \hbar/P_L = 3.6 \times 10^{-49}$ cm. The important feature of this model is that it is assumed that there is a physical Landau singularity at some very small scale; its location is not critical. Even given that this model is correct, these estimates are, of course, subject to computational errors of some orders of magnitude because of exponentiation operations involved in their calculation.

The last assumption (when one uses the principal value prescription) "controls" the computational problems associated with the Landau singularity, and the logarithmically divergent renormalization integrals of QED become well-defined and finite and may (in principle) be evaluated[33]; in effect, they are cut off in a natural way at the Landau singularity.

The a dependence of the quantization condition on Q^2 beyond the Landau singularity derives (mainly) from the effect of the momentum cut-off on the Coulomb potential itself. To investigate this effect, one may write the three-dimensional Fourier expansion for the Coulomb potential of a charge e,

$$\phi(r) = \frac{e}{r} = \iiint \psi(\vec{k})\, e^{i\vec{k}\cdot\vec{r}}\, d\vec{k} \quad , \qquad \text{(A-25)}$$

where $\psi(\vec{k})$ is the Coulomb potential in k-space;[80]

$$\psi(\vec{k}) = \frac{e}{2\pi^2}\,\frac{1}{\vec{k}^2} \quad . \qquad \text{(A-26)}$$

Putting Eq. (A-26) back into Eq. (A-25) yields

$$\phi(r) = \frac{e}{2\pi^2}\iiint \frac{e^{ik\cdot r}}{\vec{k}^2}\, d\vec{k} = \frac{2e}{\pi r}\int_0^{\infty} \frac{\sin k}{k}\, d\vec{k} \quad . \qquad \text{(A-27)}$$

Since $\int_0^{\infty} (\sin x / x)\, dx = \pi/2$, one regains $\phi(r) = e/r$, checking Eq. (A-26) as the solution for $\psi(\vec{k})$.

The $1/\vec{k}^2$ as a factor in $\psi(\vec{k})$, of course, derives from the usual momentum space form of the photon propagator. One can consider the corrections to the photon propagator to be equivalent to a k-space weighting function w(k) and simply include it in the integral expressions of Eq. (A-27). That is, one may write

$$\phi(r) = \frac{e}{2\pi^2} \iiint w(k) \frac{e^{i\vec{k}\cdot\vec{r}}}{\vec{k}^2} \, d k = \frac{2e}{\pi r} \int_0^\infty w(k) \frac{\sin kr}{k} \, d k \quad , \qquad \text{(A-28)}$$

where it is assumed that w(k) is isotropic.

The cutoff, which in this model is found at the Landau singularity, may be represented by

$$w(k) = 1, \quad k < k_L \quad ,$$

and

$$w(k) = 0, \quad k \geq k_L \quad , \qquad \text{(A-29)}$$

where $k_L \equiv P_L/\hbar$. For simplicity, the slowly varying logarithmic Uehling potential has been omitted from Eq. (A-29)

Putting Eq. (A-29) into the right-hand side of Eq. (A-28) yields

$$\phi(r) = \frac{2e}{r} \int_0^{k_L r} \frac{\sin x}{x} \, d x \quad . \qquad \text{(A-30)}$$

This done, one notes that the major contribution to $\phi(r)$ comes in the range $0 \leq x \leq 2$; $\int_0^2 (\sin x/x)\, dx = 1.6054 \approx \pi/2$. Beyond $x = 2$, the value of the integral oscillates about the final value of $\pi/2$ by diminishing amounts.

Qualitatively speaking, this means that the existence of an effective maximum on the photon momentum leads to a transition at

$$r \sim \frac{1}{k_L} \qquad \text{(A-31)}$$

between the usual $1/r$ Coulomb dependence and a constant potential, independent of r. For $r k_L \ll 1$, one may replace $\sin kr$ with kr and perform the final integration on the right-hand side of Eq. (A-28) obtaining

$$\phi(r)\Big|_{r \ll \frac{1}{k_L}} = \frac{2ek_L}{\pi} \quad , \tag{A-32}$$

the "zero range" value of the potential.

When this result is taken into account, one notes that there will again be a cancellation in the ratio K_d/K_m, again leaving the value of $\bar{\beta}$ (essentially) invariant. In the calculation of S, however, for $r < \hbar/P_L$ the $1/r$ variation of $\vec{A}$ goes over to a constant, and the Bohr-Sommerfeld prescription discussed above introduces an uncompensated linear r dependence into K_s. Thus for $a < \hbar/P_L$, one has the approximation

$$Q^2(a) \sim \frac{Q_0^2\hbar}{aP_L} \quad ; \tag{A-33}$$

from its minimum at the Landau length, the value of $Q^2(a)$ rises rapidly going like $1/a$ as a diminishes.

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Table I. Electromagnetic configuration of the charged bound vorton pair states.

State	Allowed s_n	s_g	s_e
$\mathscr{D}^{\frac{1}{2}}_{\frac{1}{2}\,m}$	−1	+1	−1
$\mathscr{D}^{\frac{1}{2}}_{\frac{1}{2}\,m}$	−1	−1	+1
$\mathscr{D}^{\frac{1}{2}}_{-\frac{1}{2}\,m}$	+1	+1	+1
$\mathscr{D}^{\frac{1}{2}}_{-\frac{1}{2}\,m}$	+1	−1	−1

Table II. Structure of bound vorton pairs states.

Helicities	Q_H	μ_E	μ_t	State Label
Rr Rr	2	yes	yes	h_1
Rr Rℓ	0	yes	no	--
Rr Lr	0	no	yes	--
Rr Lℓ	2	no	no	$\bar{\ell}_2$
Rℓ Rr	0	yes	no	--
Rℓ Rℓ	-2	yes	yes	$\bar{h}_2$
Rℓ Lr	-2	no	no	ℓ_1
Rℓ Lℓ	0	no	yes	--
Lr Rr	0	no	yes	--
Lr Rℓ	-2	no	no	ℓ_2
Lr Lr	-2	yes	yes	$\bar{h}_1$
Lr Lℓ	0	yes	no	--
Lℓ Rr	2	no	no	$\bar{\ell}_1$
Lℓ Rℓ	0	no	yes	--
Lℓ Lr	0	yes	no	--
Lℓ Lℓ	2	yes	yes	h_2

Table III. The 32 surviving ($Q_H \neq 0$) point-like two-component (charged) fermion spinors.

I	II
$\ell_1\ \xi^-$	$\ell_1\ \xi^+$
$\ell_1\ \eta^-$	$\ell_1\ \eta^+$
$\ell_2\ \xi^-$	$\ell_2\ \xi^+$
$\ell_2\ \eta^-$	$\ell_2\ \eta^+$
$h_1\ \xi^+$	$h_1\ \xi^-$
$h_1\ \eta^+$	$h_1\ \eta^-$
$h_2\ \xi^+$	$h_2\ \xi^-$
$h_2\ \eta^+$	$h_2\ \eta^-$
$\bar{\ell}_1\ \eta^+$	$\bar{\ell}_1\ \eta^-$
$\bar{\ell}_1\ \xi^+$	$\bar{\ell}_1\ \xi^-$
$\bar{\ell}_2\ \eta^+$	$\bar{\ell}_2\ \eta^-$
$\bar{\ell}_2\ \xi^+$	$\bar{\ell}_2\ \xi^-$
$\bar{h}_1\ \eta^-$	$\bar{h}_1\ \eta^+$
$\bar{h}_1\ \xi^-$	$\bar{h}_1\ \xi^+$
$\bar{h}_2\ \eta^-$	$\bar{h}_2\ \eta^+$
$\bar{h}_2\ \xi^-$	$\bar{h}_2\ \xi^+$

Table IV. Particle identifications of four-component Dirac spinors.

$\ell_1\ \xi^-$, $\bar{\ell}_1\ \eta^+$	e^-, e^+	$(\nu_e, \bar{\nu}_e)$
$\ell_1\ \eta^-$, $\bar{\ell}_1\ \xi^+$	μ^-, μ^+	$(\nu_\mu, \bar{\nu}_\mu)$
$\ell_2\ \xi^-$, $\bar{\ell}_2\ \eta^+$	τ^-, τ^+	$(\nu_\tau, \bar{\nu}_\tau)$
$\ell_2\ \eta^-$, $\bar{\ell}_2\ \xi^+$	T^-, T^+	$(\nu_T, \bar{\nu}_T)$
$h_1\ \xi^+$, $\bar{h}_1\ \eta^-$	p, $\bar{p}$	$(n, \bar{n})$
$h_1\ \eta^+$, $\bar{h}_1\ \xi^-$	λ_c, $\bar{\lambda}_c$	$(\lambda, \bar{\lambda})$
$h_2\ \xi^+$, $\bar{h}_2\ \eta^-$	t, $\bar{t}$	$(b, \bar{b})$
$h_2\ \eta^+$, $\bar{h}_2\ \xi^-$	o, $\bar{o}$	$(h, \bar{h})$

Table V. Particle interactions.

Type	Source	Particles	Carrier
Electromagnetism	e	electrically charged	photon
Strong	μ_E, μ_t	hadrons	ℓ = 0 bound vorton pair
Weak	μ_U	all	ℓ = 1 bound vorton pair
Gravitation	$T^{\mu\nu}$	all	ℓ = 2 bound vorton pair

FIGURE CAPTIONS

1. Vacuum polarization loop which yields the lowest order approximation to the Uehling potential.

2. Sum of vacuum polarization loops.

3. Estimated variation of the magnitude of the vorton charge Q as a function of the vorton scale a (in cm). For a >> $\lambda\!\!\!^{-}_e$, Q (a) = Q_0 of Eq. (9). As a becomes less than $\lambda\!\!\!^{-}_e$, Q is expected to drop logarithmically due to quantum mechanical corrections. After a minimum value at a scale on the order of the Landau length ℓ_L, Q (a) rises, quickly reaching a value compatible with the Schwinger quantization condition, Eq. (2). The actual location of ℓ_L is subject to considerable computational uncertainty.

4. The four symmetric top eigenfunctions $\mathscr{D}^{\frac{1}{2}}_{m'm}$. The x,y,z frame (heavy lines) represents the lab frame, and the x'y',z' frame (light lines) represents the (most probable orientation of the) rotating body frame. The arrows indicate the direction of rotation.

5. a) Bare electromagnetic interaction vertex.

   b) Renormalized electromagnet interaction vertex.

6. a) Elemental vertex of second echelon interactions.

   b) Renormalized vertex of second echelon interactions. Lorentz indices are omitted from the function Γ_ℓ; g_ℓ is the coupling constant.

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Fig. 1

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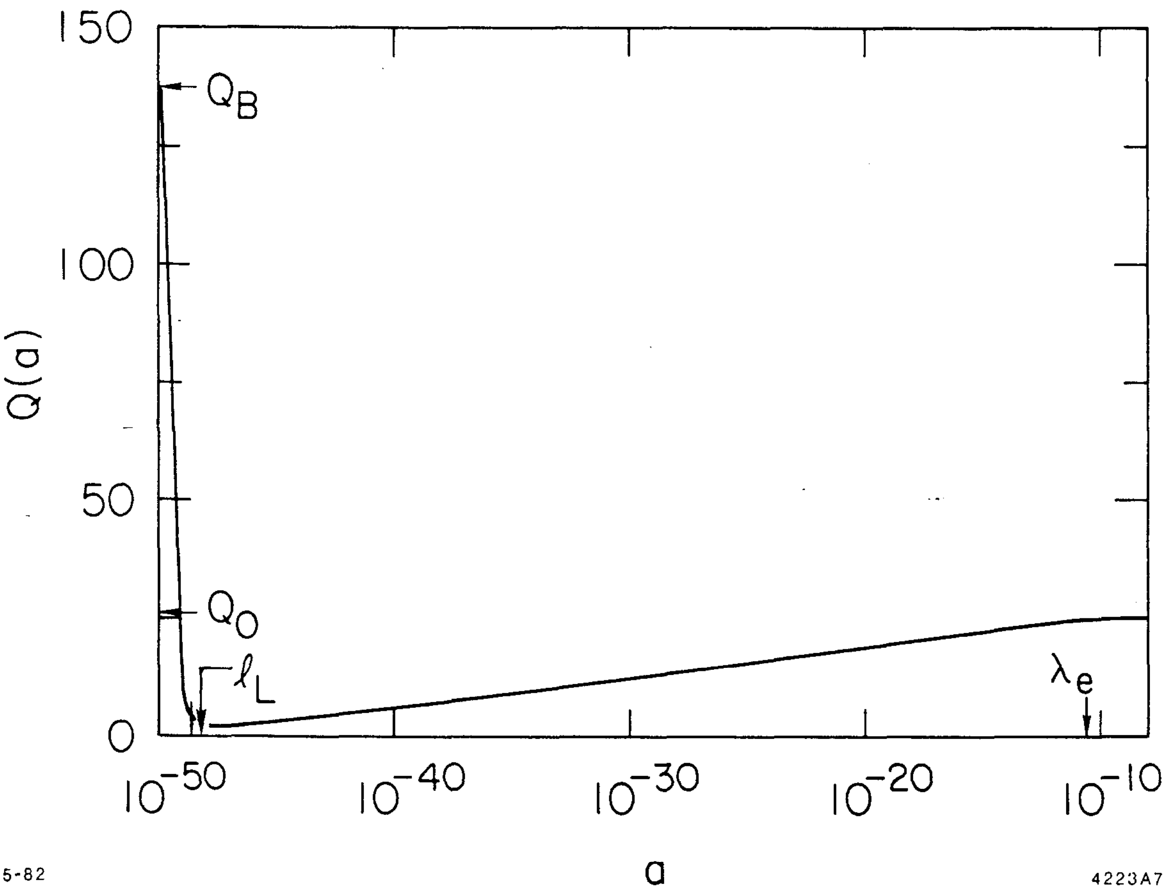
Fig. 2


150
100
50
0
Q(a)
Q_B
Q_0
ℓ_L
λ_e
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10^{-40}
10^{-30}
10^{-20}
10^{-10}
a
5-82
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Fig. 3

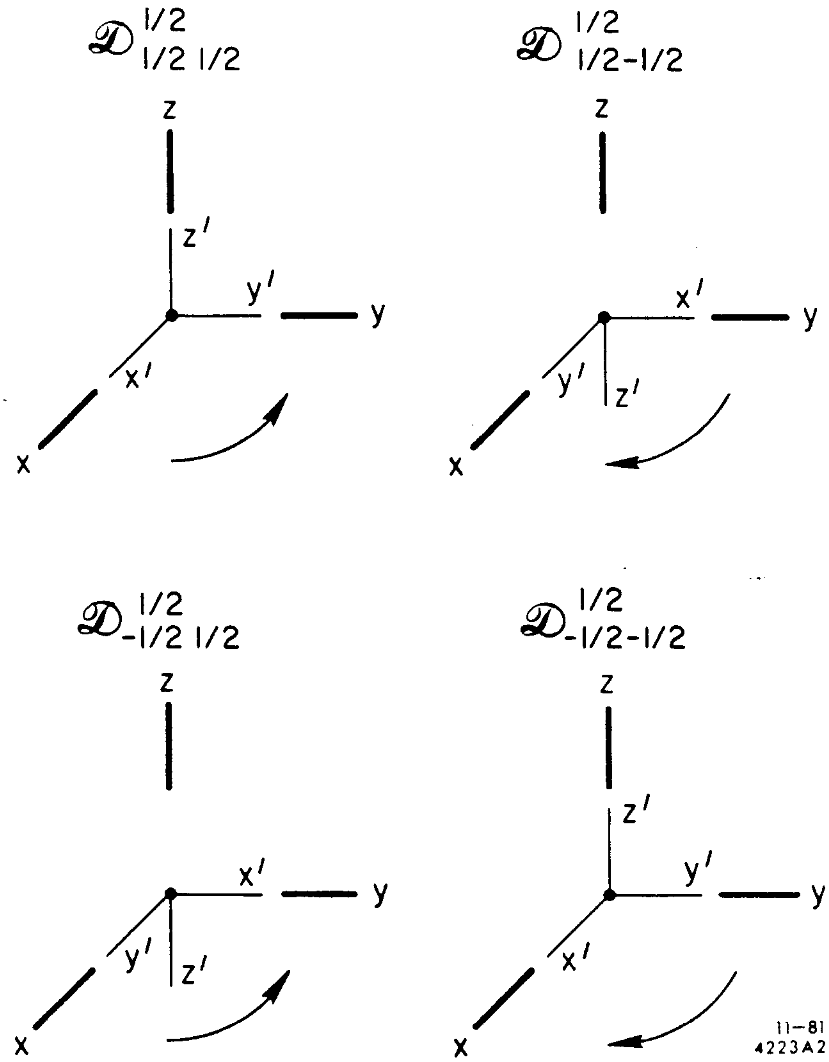
$\mathscr{D}^{1/2}_{1/2\,1/2}$
$\mathscr{D}^{1/2}_{1/2\,-1/2}$
$\mathscr{D}^{1/2}_{-1/2\,1/2}$
$\mathscr{D}^{1/2}_{-1/2\,-1/2}$
z
z'
y'
y
x'
x
11-81
4223A2

Fig. 4

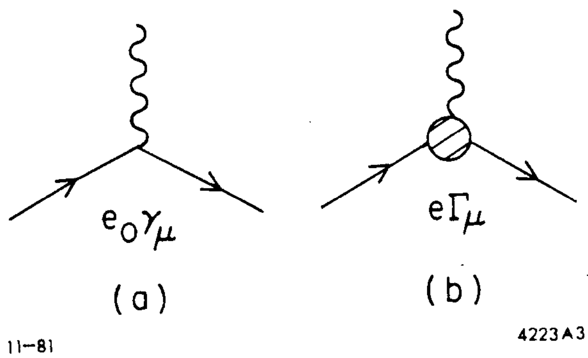

Fig. 5

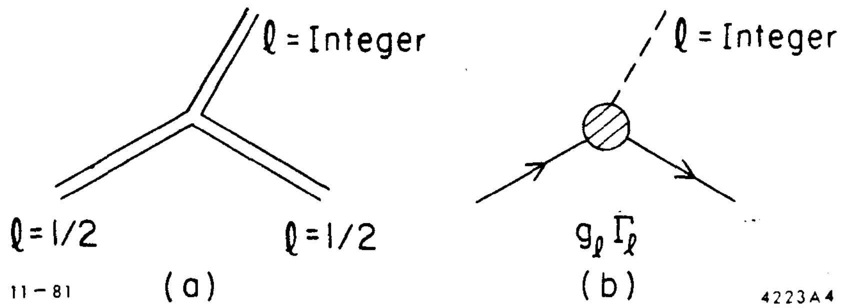

Fig. 6