BEAM LOADING AND EMITTANCE GROWTH FOR A
DISK-LOADED STRUCTURE SCALED TO 10 μm[*]

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ABSTRACT

Beam loading and transverse emittance growth are studied in a disk-loaded accelerating structure which has been scaled to a wavelength of 10 μm. The resulting limitations on the charge per bunch which can be accelerated in such a scaled structure should provide a crude estimate of the charge per bunch which can be accelerated in a laser driven grating accelerator operating at the same wavelength. For an accelerator 100 m in length delivering an energy of 500 GeV, it is found that the number of particles per bunch that can be accelerated is on the order of $10^5 - 10^6$.

INTRODUCTION

Laser driven grating accelerators have been proposed which are capable of achieving very high energy gradients at a wavelength on the order of 10 μm. The charge per bunch which can be accelerated in such a device, as determined by beam loading and transverse emittance growth considerations, is also of importance. Beam loading effects and transverse emittance growth are readily calculated if the longitudinal and transverse wake potentials for an accelerating structure are known. These wake potentials have been computed[1] for the SLAC disk-loaded structure, and experimental measurements have verified that these computed wake potentials are at least approximately correct. In the absence of similar detailed calculations for the grating geometries proposed for a near-field laser driven accelerator, we can obtain a rough estimate of the charge per bunch that can be accelerated by studying beam loading and emittance growth in a disk-loaded structure scaled to the same wavelength.

BEAM LOADING

Associated with each mode in an accelerating structure is a quantity k_n, called the loss parameter, defined by

$$k_n \equiv \frac{E_a^2}{4w} \tag{1}$$

where E_a is the accelerating gradient and w is the energy stored per unit length for that mode. Note that k_n scales as ω^2. The above quantity is called the loss parameter because the energy loss per unit length of structure to a point charge q is given by $\Delta U_n = k_n q^2$. Also, the average beam loading gradient per particle is $\bar{E}_{bn} = k_n q$. Let k_o be the loss parameter for the accelerating mode. The energy loss and average beam loading gradient per particle due to higher-order modes can be taken into account by the beam loading enhancement factor B, such that the total loss to all modes is given by

$$\Delta U = B(\sigma) k_o q^2$$

$$\bar{E}_b = B(\sigma) k_o q \quad .$$

Note that B is a function of the bunch length σ. For the SLAC disk-loaded structure ($\lambda = 10.5$ cm), B = 3.0 for a bunch length $\sigma = 1$ mm. The value of k_o is 2×10^{13} V/C-m.

In addition to an average energy loss per particle, there will be an energy spread within the bunch due to beam loading. The particles at the front of the bunch will experience very little energy loss, while the particles at the back of the bunch will see a beam loading gradient on the order of $2\bar{E}_b$. If the accelerating field is E_a, the total energy spectrum width will be on the order $2\bar{E}_b/E_a$. This energy spread can be reduced by letting the bunch ride ahead (in space) of the accelerating wave crest so that the more heavily loaded particles at the rear of the bunch pick up more energy from the accelerating wave. Let us ignore this complication and compute the average energy loss per particle normalized to the peak accelerating field,

$$\frac{\bar{E}_b}{E_a} = \frac{B k_o q}{E_a} \quad .$$

Rewriting this expression in term of the number of particles that can be accelerated for a given value of $\bar{E}_b/E_a$, using also Eq. (1),

$$N = \frac{E_a}{e B k_o} \left(\frac{\bar{E}_b}{E_a} \right) \quad . \tag{2}$$

If the SLAC structure is scaled to $\lambda = 10$ μm, k_o increases by the ratio $(10 \text{ cm}/10 \text{ μm})^2 = 10^8$ to $k_o = 2 \times 10^{21}$ V/C-m. Using also $E_a = 5$ GeV/m, $\bar{E}_b/E_a = 0.1$ and B = 3, we obtain $N = 5 \times 10^5$. The total relative energy spread in this example would be on the order of 20%. This could be reduced to a few per cent by moving the bunch ahead of the accelerating wave crest, but at the expense of an additional loss in average particle energy.

The efficiency for transfer of energy stored in the structure to the bunch is also of interest. This efficiency is given by

$$\eta = \frac{q(E_a - \bar{E}_b)}{w} = \frac{4}{B}\left(\frac{\bar{E}_b}{E_a}\right)\left(1 - \frac{\bar{E}_b}{E_a}\right) \quad . \tag{3}$$

The maximum efficiency is seen to be $\eta = 1/B$ at $\bar{E}_b/E_a = 1/2$. For the preceding example with $\bar{E}_b/E_a = 0.1$ and $B = 3$, the efficiency is 12%.

EMITTANCE GROWTH

The presence of dipole (deflecting) modes in a disk-loaded accelerating structure can lead to single bunch emittance growth.[2] Assume that a bunch moves off axis at the beginning of an accelerator, executing betatron oscillations with a wavelength $\lambda_\beta = 2\pi/k_\beta$ and an initial amplitude x_0. The bunch can be modeled by a simple two-charge approximation, a head charge $q/2$ at $z' = +\sigma$ and a tail charge $q/2$ at $z' = -\sigma$, where z' is the coordinate with respect to the bunch center. The head charge excites a transverse deflection wake, which drives the betatron oscillations of the tail charge. After traveling length z, it is easy to show[3] that the amplitude of the oscillation of the tail charge is

$$x(z) = \frac{q\, x_0\, z\, w_\perp(2\sigma)}{4\, V_0\, k_\beta} \quad ,$$

where eV_0 is the beam energy and $w_\perp$ is the dipole wake potential. If $A \equiv x(z)/x_0$, then the number of particles that can be accelerated is

$$N = \frac{4\, A\, V_0\, k_\beta}{e\, z\, w_\perp(2\sigma)} \quad . \tag{4}$$

The transverse wake for the SLAC structure at $\lambda = 10.5$ cm and $2\sigma = 2$ mm is[1] $w_\perp = 3 \times 10^{15}$ V/C-m^2. The dipole wake scales[3] as ω^3; at $\lambda = 10$ m it therefore becomes $w_\perp = 3 \times 10^{27}$ V/C-m^2. Assuming $\lambda_\beta \approx 100$ m for SLAC, then $\lambda_\beta = 1$ cm at the shorter wavelength. As a numerical example, again assume a gradient of 5 GeV/m, a total energy of 500 GeV and a length of 100 m. Equation (4) is strictly valid only for V_0 constant, but take $V_0 = 250$ GeV as an approximation for uniform acceleration to 500 GeV. Let the amplitude of the oscillation of the tail particle grow by a factor of 10. Inserting these values in Eq. (4), we obtain $N = 1 \times 10^5$.

DISCUSSION

The luminosity in a linear collider is given by $\mathscr{L} = N^2 f_r/\left(4\pi\sigma_x^*\sigma_y^*\right)$, where f_r is the repetition rate and σ_x^* and σ_y^* are the transverse beam dimensions at the collision point. To be interesting as a machine for high energy physics, an accelerator must be capable of achieving a luminosity 10^{30} cm^{-2}s^{-1}, at the very least. Assuming

transverse beam dimensions of 0.1 μm and a repetition rate of 1 kHz, 10^9 particles per bunch are required to achieve this luminosity. The preceding analysis indicates that it will be very difficult to reach this high a charge per bunch in a grating accelerator for a beam with $\sigma_x^* \approx \sigma_y^*$. If the charge per bunch is lower, the repetition rate must increase above 1 kHz as N^{-2} to maintain a given luminosity with the above beam dimensions. It is difficult to believe that beams with a transverse dimension much less than 0.1 μm can be made to collide. Using a wide flat beam ($\sigma_x^* \gg \sigma_y^*$), or many beams in parallel, is then the only remaining possibility. The total beam width in this case would have to be on the order of 1 cm to achieve the above luminosity.

REFERENCES

[1] K. Bane and P. Wilson, 11th Int'l Conf. on High Energy Accelerators (Birkhäuser Verlag, Basel, 1980) p. 592.

[2] A. W. Chao, B. Richter and C. Y. Yao, 11th Int'l Conf. on High Energy Accelerators (Birkhäuser Verlag, Basel, 1980) p. 597.

[3] P. B. Wilson, 1981 Summer School on High Energy Particle Accelerators. To be published in AIP Conf. Proceedings.