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A FINITE PARTICLE NUMBER APPROACH TO QUANTUM PHYSICS*

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ABSTRACT

Bridgman has contended that the "inside of an electron" cannot be given operational meaning. The basic reason for this is taken to be that when relativity is coupled to quantum mechanics the uncertainty principle in energy requires the existence of an indefinitely large number of particulate degrees of freedom corresponding to particles of finite mass when any system is examined at short distance, as was first pointed out by Wick. This principle is examined in the context of the nuclear force problem and shown to frustrate a precise theory of strong interactions using conventional approaches. However, once relativistic scattering theory is recast in the form of free particle wave functions and elementary scatterings, progress becomes possible. In particular, a unitary and covariant first approximation to the nuclear force problem using only two particles and one quantum can be formulated simply by postulating that particle (or anti-particle) can bind with the quantum to make a system of the same mass as the particle and physically indistinguishable from it. Extended to the two particle-two quantum sector this approach promises to unify the one-boson-exchange models and the dispersion-theoretic models for the nuclear force in a consistent way. Whether this approach can reproduce the quantitative successes of quantum electrodynamics and whether it can be extended to model the quantum chromodynamics of quarks and gluons remains an open question, but so far no barriers have been encountered.

Since this proposal suggests that relativistic quantum mechanics can be formulated in terms of free particle asymptotic wave functions, the question arises whether these constructs can be given an operational

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basis, using the paradigm of the double slit experiment viewed as either a measurement of mass via the Debroglie wavelength or as a quantum mechanical definition of the distance between two slits. It is found that if the detectors in the slits require particles of finite mass, this approach is frustrated and we conclude once again that the classical concept of the space-time continuum cannot be given even indirect operational meaning at short distance.

This leads us to the proposal that the underlying concepts of space, time and particle are inextricably linked and should be constructed together from discrete "elements of reality." In the absence of an a priori space-time framework, the concept of the indistinguishability of particles raises logical problems, which have been faced by Parker-Rhodes in his theory of indistinguishables. We accept his conclusion that in a theory of the type we seek, the "elements of reality" cannot be directly observed, but must be given specific content by indirect observations. For these we use the "yes-no" counter events of high energy particle physics as the paradigm. Following Kilmister we adopt a generation operation of these discrete elements starting from the empty set which thanks to an equivalence relation, a minimum labeling rule, and a principle of economy leads to the combinatorial hierarchy of Amson, Bastin, Kilmister and Parker-Rhodes, characterized by the terminating sequence of cardinals 3, 10, 137, 1.7×10^{38}. We note that this generation operation produces elements that can be represented by ordered strings of the existence symbols "0" and "1". Further, these strings can be grouped into a "label" of length 256 to which the hierarchy refers, and an "address" whose length increases as the construction proceeds. We interpret the label as a quantum number classification of the particles and the address as a random walk from which, following Stein, labeled "objects" of unique velocity relative to the limiting velocity c (a consequence of the construction) can be selected as statistical assemblages. Ensembles of objects can then be constructed and shown to have space-time "coordinates" relative to each other which satisfy a discrete version of the Lorentz transformation, and the Heisenberg uncertainty principle, thus defining $\hbar$ in our theory.

Returning to the combinatorial hierarchy, we introduce the concept of "interaction energy" dimensionally, and by a modification of an agrument due to Dyson identify 1/137 as a first approximation to $e^2/\hbar c$ and $1/1.7 \times 10^{38}$ as $Gm^2/\hbar c$ showing that the unit of mass implied by the theory is close to the proton mass. Thus the lack of scale invariance in nature, and the inadequacy of the concept of the space-time continuum receives a fundamental explanation. Following a calculation of Parker-Rhodes we show that there is reason to believe that the theory requires $m_p/m_e = 137\pi/[(3/14)(1+(2/7)+(2/7)^2)(4/5)] = 1836.1515$ in agreement with experiment. We further argue that the generation operation and labeling rule we have introduced are congruent with current ideas about the big bang cosmology and a baryon number for the universe somewhat less than 2^{256}, which is the right order of magnitude. The length of the strings at the current epoch is taken to be a measure of universal time, evidenced by the 3°K background radiation. The labels for the first three levels of the hierarchy can be interpreted, possibly, as representing conserved quantum numbers for which the obvious candidates are charge, spin, baryon and lepton number. A possible connection to the scattering theory discussed above is sketched.

In conclusion we argue that a finite model such as ours which contains conservation in the presence of a random background necessarily leads to the evolution of structures of increasing complexity, and forms an adequate physical basis for a materialist philosophy whose guiding principle is "fixed past-uncertain future."

> "The structure of our mathematics is such that we are almost forced, whether we want to or not, to talk about the inside of an electron, although physically we cannot assign any meaning to such statements."

The challenge posed by Bridgman[1] in the quotation I have chosen as the theme of my contribution has been with me ever since I first encountered it as a graduate student in physics. Thanks to the combinatorial hierarchy pioneered by Amson, Bastin, Kilmister and Parker-Rhodes[2,3] and recent work by them and others, I hope in this lecture to indicate how Bridgman might be answered today. Briefly what we attempt to do is to construct space, time and particles by discrete sequential processes that do not require taking the space-time continuum as an a priori foundation of natural philosophy. According to an analysis of Bastin's[4] which in large part I share, one major point on which Bohr went astray was in his assumption that there is no escape from describing physics in classical terms relying on the space-time continuum. Then quantum phenomena become paradoxical. Bohr met this problem by the profound idea of "complimentarity." Yet I have never been able to go beyond the qualitative insights this idea provides to the quantitative precision I demand of any natural philosophy.

Although I find the various versions of the "Copenhagen interpretation" which purportedly incorporate Bohr's approach inadequate at the fundamental level, I suspect that much of my thinking on this question parts company with many people here. In particular, I find nothing "idealistic," "irrational" or "anti-materialist" in accepting as a scientific hypothesis the existence of an intrisic randomness in every elementary scattering process. Indeed the distant correlation experiments of which I am aware prior to this Symposium, which were stimulated by Einstein, Podolsky and Rosen,[5] Bohm's gedanken experiment[6] and Bell's theorem[7] as reviewed by Clauser and Shimony[8] leave me little alternative. All I wish to say in advance is that I am comfortable with a natural philosophy that includes both randomness and conservation laws. I believe that the purpose of natural philosophy is to change the world and not just to understand it. As I have argued in the past, I find this purpose better served by the conjunction of these two principles than by nineteenth century determinism.

Since I am a physicist as well as a philosopher, I approach these fundamental questions from the point of view of specific problems, and not just in general terms. Much of my professional career has been concerned with the problem of nuclear forces, starting long enough ago so that this was still considered to be a branch of elementary particle physics. Until recently I must confess that even the problem of how to arrive at an unambiguous starting point for the theory of strong interactions has eluded me. That in regard to nuclear forces there is still no consensus as to where to start is small comfort. I would rather see the problem solved by anyone than to continue with my own frustrations in grapling with it. I share the view of Phipps[10] that the aim of a physicist is always to kill a problem and not to keep it alive for discussion. But thanks to recent work in collaboration with James Lindesay[11,12] I think I have made a start in the right direction. Since it ties in with ideas that come from the combinatorial hierarchy approach, I start with this concrete example before tackling the larger issues.

In a sense, of course, the fundamental starting point for a theory of strong interactions has already been provided by Wick's profound analysis[13] of Yukawa's meson theory.[14] As is illustrated in Fig. 1, if two systems are brought together within some distance r where they can interact coherently during the time δt when they are so localized, special relativity requires that $r \le c\delta t$. By Heisenberg's uncertainty principle $\delta t \approx \hbar/\delta E$. Assume that the interaction is in some sense due to the presence of some particle of mass μ and (from special relativity again) rest energy μc^2. This can only happen if the uncertainty in energy $\delta E \ge \mu c^2$. Hence

$$r \le c\delta t \approx \frac{c\hbar}{\delta E} \le \frac{c\hbar}{\mu c^2} = \frac{\hbar}{\mu c} \quad . \tag{1}$$

Following Newton we assume that the total momentum of the system must be conserved, but this does not define the relative momentum between the two systems before and after they enter the region of dimension r; we conclude that they will "scatter" in some manner that will be connected to the way they share momentum with μ during the time interval δt. Further, if the energy is high enough, the "hadronic quantum" of mass μ will appear, sometimes, in the final state.

This analysis has certainly served as an extraordinarily fruitful guide to experimental discovery and to the interpretation of the deluge of data which has poured out of the high energy particle accelerator laboratories. Yet it has proved to be extraordinarily difficult to reduce this semi-quantitative insight to a precise theory. Yukawa's theory was a modification of quantum electrodynamics to include finite mass quanta as the carriers of the nuclear force. Bohr and Rosenfeld[15] showed at about this time that the quantum communtation relations of the electromagnetic field components could indeed be derived from the uncertainty principle and a carefully chosen sequence of gedanken experiments, thus in a sense providing an operational underpinning to the mathematical procedure of second quantization. But they were careful to point out that their analysis was possible because QED contains only two independent dimensional constants, $\hbar$ and c, and not three. This allowed them to envisage the construction of arbitrarily complicated apparatus within the dimension of a single wavelength of the radiation field being explored. Then by the continuity of nature, or Occam's Razor, they felt justified in extrapolating the analysis to arbitrarily short distances. However, in any theory such as Yukawa's meson theory, or Dirac's theory of the electron, which contains a finite mass parameter, this scale invariance breaks down and the extrapolation is no longer justified. We can see why immediately from Wick's argument. Once we try to probe any system at short distance, the number of particles in the system becomes uncertain, and indeed increases without limit as we go to infinitessimal distances. This is, of course the major source of the infinities which plague quantum field theory.

So far as quantum electrodynamics goes, these difficulties found a practical solution in the renormalization program of Tomonoga, Schwinger, Feynman and Dyson. Quantitative success in the calculation of some observed physical quantities with an accuracy of the order of one part in $(137)^4$ is indeed impressive. Further, very high energy scattering experiments conventionally interpreted reveal the existence of "pointlike" carriers of electric charge with no internal structure larger than an upper limit of about 10^{-16} cm. This experimental fact has been the starting point of much current elementary particle theory. Yet mathematical consistency still eludes us. As Dyson[16] pointed out long ago, the perturbation theory taken to N terms describes processes involving N charged particle-antiparticle pairs. Confined to a volume of linear dimension $\hbar/2mc$, these imply an electrostatic energy of order $Ne^2/(\hbar/2mc) = N(e^2/\hbar c)2mc^2$. Hence if we go beyond 137 terms in the perturbation series, there is enough electrostatic energy in the system to create additional pairs, and the interpretation of the theory becomes somewhat vague. In particular Dyson pointed out that in a theory in which like charges attract -- which amounts to changing e^2 to $-e^2$ in the perturbation series -- the system can gain energy from the vacuum fluctuations which produce additional pairs, and the system collapses with infinite binding energy. As a minimum this shows that the perturbation series is not uniformly convergent, and the usual assumption is that it is an asymptotic series which cannot be used beyond 137 terms. Some nonlinearity not included in conventional QED must be introduced if the theory is to survive. For purposes which will become clearer below, we summarize Dyson's result by saying that QED does not allow us to define what we mean by more than 137 charged particle-antiparticle pairs in a volume of linear dimension smaller than the Compton wavelength of the pair. This is, of course, yet another illustration of Wick's analysis.

Returning to strong interactions, the situation is much worse. Empirically the Yukawa particle -- the pion -- couples to nucleons with a coupling constant $g^2/\hbar c$ of about 14 in contrast to the 1/137 which could be exploited in QED. Thus perturbation theory cannot be used directly, and a general method of solution applicable to all problems has yet to be developed. Much ingenious work has been done, and much contact with experiment has proved to be possible, but typically it is hard to push the accuracy of prediction to an accuracy of better than 10-30% except in special circumstances. For the specific problem of constructing a "meson-theoretic potential" which could be used in a nonrelativistic Schroedinger equation for a system of A nucleons, and hence as a basis for nuclear physics, there has never been a consensus. Currently there is a candidate derived from dispersion theory, and another derived from the observed low energy spectrum of bosons, which both give good fits to nucleon-nucleon scattering data. Conceivably they are describing the same physics in different approximations, but this remains to be proved. Worse, it is difficult to connect up this approach in a clean way with quarks and quantum chromodynamics, which most high energy theorists believe to be the basic theory of strong interactions.

Faced with all these theoretical difficulties, one might ask why not determine the nuclear force law directly from experiment? The difficulty with this approach is that in quantum theory there is no way to go from the probability distributions that one measures in the laboratory directly to a force law, except in the special case that the forces can be derived from a potential which is only a function of the distance between the two particles. But we know from the Wick-Yukawa mechanism that there are bound to be hidden mesonic degrees of freedom, and in particular that at distances less than $\hbar/2m_\pi c$ where we encounter two pions, the large coupling constant implies that we are just as likely to encounter a nucleon-antinucleon pair. Thus the basic interaction must be nonlocal. And there are an infinite number of nonlocal interactions all of which predict identical nucleon-nucleon scattering amplitudes.

A new approach to the problem opened up when Faddeev[17] developed a rigorous scattering theory for three nonrelativistic particles. Potentials which differ in their mathematical details but still predict the same two particle scattering amplitudes can be shown to predict different three particle scattering amplitudes. So I, and a lot of other people, hoped that this fact might be exploited to gain new information about nuclear forces. However, as I finally realized in 1972, this does not work either.[18] In addition to the new information obtained in the three nucleon system, there is a new effect: the pion can now be exchanged among all three particles leading to what is called a "three body force," whose effects in the system are experimentally indistinguishable from the ambiguities in the "potential" we were seeking to resolve. This suggested to me that we should try to formulate the theory directly in terms of the two particle scatterings which we can observe without introducing these short range effects. Then at least we could compare such a theory with experiment and use the discrepancy to isolate the effect we needed to explain from what we already knew from two particle experiments. This led me to an analysis of scattering theory using only free particle wave functions and point scatterings, and hence into problems in the foundations of quantum mechanics.

One important stimulation for my approach came from a critique of quantum mechanics by Thomas Phipps[19] in which he points out that the conventional "derivation" of quantum mechanics from the Hamilton-Jacobi equations throws away half the degrees of freedom, namely those corresponding to the initial positions and momenta. By viewing the equations as operator equations this can be avoided and the theories unified. The classical equations have a constant state vector, the quantum mechanical equations require the action to be constant, and there is an intermediate class of solutions in which neither the state vector nor the action are constant -- which will not concern us here. In the quantum limit the only difference from conventional theory is that the Schroedinger wave function acquires a phase factor exp $-i\Sigma_k \underline{P}_k \cdot \underline{Q}_k$ containing the initial constants of the classical theory. Since this does not alter the probabilities computed from the theory, the usual results of quantum mechanics follow. But I realized that the sudden changes in these constants could be associated with stochastic scattering events and hence provide the breaking of the connection between the fixed past and the uncertain future required by quantum mechanics.[18,20] This then allowed me to develop scattering theory from free particle wave functions and stochastic processes without introducing the concept of "interaction energy" as I now demonstrate.

The contrast between the classical and the quantum situation is illustrated in Fig. 2. We assume that we start from three well separated beams of particles with momenta $\underline{P}_i$, which can be measured by a system of collimators and slits along the initial directions with the timing determined by counters; we will return below to a more precise discussion of these measurements. We are used to the factor $(\underline{q}_i - \underline{Q})$ in the phase which (usually with $\underline{Q} = 0$) defines the coordinate system in such a way as to guarantee translational invariance. What the Phipps' argument has done is to note that we should define this factor separately for each particle, the interpretation I give being that these $\underline{Q}_i$ are the coordinates of the last scattering before that particular beam enters the scattering region. Within the scattering region, the initial plane waves disappear and are replaced by outgoing plane waves with momenta $\underline{K}_i$. Since, observationally, we do not know where these scatterings occured within the scattering volume, we again must supply new and arbitrary coordinates $\underline{X}_i$ in the region feeding the collimators and counters which measure the $\underline{K}_i$ of the final state. Although we have illustrated the situation for a 3-3 scattering, the treatment can refer to any finite number of particles in and any finite number out. We see from the figure how the Phipps phase factor occurs naturally in this scattering problem.

Our next step is to assume that the outgoing wave function is proportional to the incoming wave function and to write down the wave function for the whole system in such a way that for distant past times we have only the noninteracting plane waves of the initial state. This is done using the usual Lippmann-Schwinger trick and leads to the energy denominator $(\Sigma_k \varepsilon_k - E - i0^+)$ with $\varepsilon_k = (m_k^2 + K_k^2)^{\frac{1}{2}}$ that in physical terms represents the uncertainty principle. Next we note that whenever the same particle occurs in the initial and the final state, there can be interference terms and that these would in general allow the measurement of the parameters $\underline{Q}_i$ and $\underline{X}_i$, which can therefore be thought of as "hidden variables." Relying on experiment, we <u>postulate</u> that in fact such terms cannot be observed, and require our sum to exclude them. The result is that the Phipps phase factor has to be the same in the initial and the final state, and that the Schroedinger wave function it multiplies is the conventional Goldberger-Watson scattering wave function. Since the details have been published,[21] I will not bother you with them here.

At first sight this exercise appears useless, since all we have done is to recover the conventional relativistic scattering theory. But this is not true. In the conventional theory, the scattering amplitude T is defined as the matrix element of an interaction, and the formula derived from a Hamiltonian that contains this interaction. But our derivation makes T descriptive rather than dynamical. Given the experimental results for any scattering process we can extract the T which describes them, or given a T we can calculate them. Thus we have separated kinematics from dynamics in quantum theory. In the conventional theory if the interaction Hamiltonian is hermitian, the prediction conserves flux, or as this is customarily referred to, it is unitary. But our more general formalism does not make this requirement. For this we must pay a price. There is good experimental evidence for flux conservation, and for time reversal invariance; this is much more powerful when we include the indirect evidence for detailed balance which comes from statistical equilibrium and thermodynamics. So the dynamical theory from which we compute T must directly guarantee these properties. This turned out to be the hardest technical problem to solve in my approach.

To cut a long story short, it turns out that a consistent zero range or on shell N-particle scattering theory is possible only when the input amplitudes continued to negative energies contain no singularities other than bound state poles.[22] Since the Wick-Yukawa mechanism does result in singularities at negative energies, this means that we are debarred from using the theory phenomenologically to describe nuclear forces, as originally envisaged. But what we gain instead is a theory in which mesons and nucleons can be treated on an equal footing, and thus have created a new approach to nuclear physics. Since the program is only just launched, and may not get far in its present form, my description will be brief. The basic idea is simply that a quantum of mass m_Q can bind to either a particle or an antiparticle of mass m to form a physical "bound state" also of mass m and physically indistinguishable from the particle. The resulting theory[12] for two particles and one quantum is illustrated in Fig. 3 and leads to a covariant, unitary theory for nuclear forces driven by single quantum exchange. In the nonrelativistic region this reduces to Yukawa "potential scattering," but since the kinematics are fully covariant, there is no problem with what are sometimes called "recoil corrections." Inclusion of spin leads to the Dirac equation in an appropriate $m_Q = 0$ limit, but the exact theory gives a fully covariant and unitary theory for the scattering of two spin 1/2 particles which in our opinion is superior to any such theory currently in the literature. For the nuclear force problem this clearly can provide a covariant "one boson exchange model" without going through the ambiguous step of constructing a "nonrelativistic potential." Thus I have taken the first step toward the theory of nuclear forces for which I have been searching for thirty years.

The next step, to construct four particle equations along the same lines has been successfully taken,[23] and work on the covariant generalization and the generalization to N greater than four is proceeding smoothly. Where the theory differs from conventional field theory is that the analytic continuation connecting particle-particle to particle-antiparticle processes restricts the intermediate states to the same finite N degrees of freedom. Unitarity is guaranteed, as was proved in the three particle case by Freedman, Lovelace and Namyslowski[24] long ago, and the proof can be extended to our more general situation. What we lack from the field theory point of view is "crossing." This is obvious if by crossing we mean that the same function of the Mandelstam variables describe appropriately defined particle and antiparticle processes. Since our theory contains in any channel an infinite number of exchange processes if we make a multiple scattering series expansion, in the "crossed" channels we would necessarily obtain an infinite number of particles in the intermediate states, which violates our finite N restriction. Thus beyond some order fixed by N we do not reproduce the renormalized perturbation theory. We think this is a small price to pay for a theory which is unitary, unambiguous for any finite N, and retains the appropriate particle-antiparticle symmetries within that restriction. I will be happy to discuss the technical details informally with anyone who is interested.

What remains to be done is to show that we can reproduce in this finite theory the successful agreement with experiment achieved by quantum electrodynamics to order e^8. This can clearly always be done *ad hoc* by adding any terms our theory does not automatically generate and using our Faddeev-Yakubovsky dynamics to unitarize the result. We hope that the result will emerge automatically, but this remains to be proved. The test to order e^4, where the conventional theory generates infinities that have to be renormalized, but ours does not, should be complete by the end of this year, at least for the scalar case.

The four particle theory using two nucleons and two quanta is straightforward, and can be constructed to contain the boson-boson resonances phenomenologically. It also will be related to boson-nucleon scattering and hence contain the same physical content as the dispersion theory that makes use of an analytic continuation of the πN-πN amplitude to the $\pi\pi$-$N\bar{N}$ amplitude needed in the nuclear force problem. In this way we hope to unify the one-boson-exchange and the dispersion theoretic models. We also have done preliminary work on a more phenomenological approach to nuclear physics that puts the mesons and nucleons on an equal footing, but discussion here would take us too far afield.

So far our theory is analogous only to Abelian field theories with Yukawa type couplings. The extension to quantum chromodynamics is more speculative. There is no barrier to including the gluon-two gluon and two gluon-two gluon couplings required by QCD in our theory by appropriately defined scattering amplitudes. Also, our scattering theory technique appears to allow us to meet the problem of "confinement" by confining the asymptotic states to $q\bar{q}$ and qqq systems, which can be readily accomplished. As a minimum this will lead to a relativistic constituent quark model. Whether all the problems of QCD can be met in this way is still more dubious than our hope that we can recover the successes of renormalized QED, but we are optimistic.

Our purpose in presenting this sketch of a strictly finite elementary particle theory based on free particle wave functions has been provided to isolate the basic problem faced by a more fundamental theory -- namely how to construct these wave functions and the elementary scattering processes from first principles. All I can have hoped to do is to make it plausible that if we can do so, the route by which we can then connect up to laboratory experiment and conventional theoretical descriptions is straightforward, if tedious; I certainly have not proved my case.

The starting point of our more fundamental approach is an analysis of the double slit experiment using a beam of particles of mass M illustrated in Fig. 4. Since details have been published,[25] I will be brief. The arrangement differs from conventional discussions in that I have included the counters in the collimator in the discussion together with their time resolution, which allows the velocity of the particles in the beam to be defined to arbitrarily high precision. I have also introduced counters into the slits as well as into the detector array in order to accumulate the statistical data that exhibit the probability distributions which are predicted by quantum mechanics. The prediction is that the data sets $D_s D_c D_1 \bar{D}_2 D_3$ and $D_s D_c \bar{D}_1 D_2 D_3$ will exhibit single slit interference patterns appropriately connected geometrically to the positions D_1 and D_2 respectively, while the data set $D_s D_c \bar{D}_1 \bar{D}_2 D_3$ will exhibit a double slit pattern centered on the Z-axis whose envelope is the single slit pattern. Thus the apparatus collects all three types of data in the same experiment. The three probability distributions are classical and add without interference, as can be checked by varying the density of the material in counters D_1 and D_2. The intensity of the source can be cut down to the point where there is only one particle passing through the apparatus at a time, if we have patience enough, without altering the limit to which the distributions converge. Hence the predictions of quantum mechanics are in this sense objective and realistic within the limitations of a statistical theory. It is only when we ask a question that the apparatus cannot answer, namely in the third case "which slit did the particle pass through?", that we get into conceptual difficulty. The published analysis[25] attempts to demonstrate in detail that the experiment can be refined to check the predictions to arbitrarily high precision. Therefore from its own point of view, that is with respect to the questions it is allowed to answer, quantum measurement theory can be refined to any desired accuracy, just as can classical measurement theory. In a sense this experiment is a measurement of the mass M of the particles in the beam using their Debroglie wavelength.

However, when we analyze the action of the detectors in more detail we find a hidden assumption in the statement that the analysis can be arbitrarily refined. We assume that the detectors contain particles of mass m and a threshold detection energy below which they are not activated; we take account of the recoil correction when they are. The result is that we can make the check to arbitrary precision only when the mass m can be assumed to be arbitrarily small compared to M. For instance we could make the experiment with cannon balls, make the slits of armor plate, and use bird shot in the detectors. Of course the apparatus would then be of astronimical dimensions, and would exceed the resources of NASA, particularly when we realize that it must be enclosed in a light tight box so that we cannot see which slit the cannon balls go through! Of course the presence of photons is sufficient to destroy the phase coherence whether "we" see them or not. Perhaps some day some philosophically sophisticated type I civilization will perform the experiment.

Assume that the experiment has been performed and that the predictions of quantum mechanics are confirmed using macroscopic apparatus. The situation is then similar to the gedanken experiments constructed by Bohr and Rosenfeld[15] mentioned above, except that the dimensional constants are now $\hbar$ and M rather than $\hbar$ and c. The experiment could then be viewed as a measurement of the separation of the slits d and thus provide an operational definition of length within the framework of quantum mechanics. Because of the uniformity of nature, or Occam's Razor, we could then use scale invariance to extrapolate the measurement down to arbitrarily short distances and define operationally the concept of length as it occurs in quantum mechanics. But empirically there *is* a smallest mass in nature, namely m_e the mass of the electron. Even if some neutrinos should turn out to have finite masses, the situation does not change in principle. As Bastin has put it,[26] the basic quantization is the quantization of mass, and the task of a fundamental theory is to understand how this comes about. We conclude that the use of the space time continuum in quantum mechanics involves the hidden assumption that there are in nature an available collection of arbitrarily small masses at any scale, masses which have not been observed, or predicted by theory; otherwise quantum mechanics is inconsistent from an operational point of view. Such a hidden postulate we find too big a price to pay for the consistency of quantum mechanics, and look in another direction for our point of revision. So far, at least, all we have succeeded in doing is to confirm Bridgman's position that the space inside an electron is a meaningless concept from an operational point of view, whether we follow Bohr and Rosenfeld or our own modest coda to their grand performance.

The way in which I look at the successes of quantum theory and the elementary particle physics grounded in the experimental results achieved by the high energy particle accelerator laboratories, is that we have good evidence that there are discrete and indivisible processes in nature, and that these involve discrete masses. The type of analysis I have just given has also led me to the conclusion that it is beyond my competence to imbed these processes in a classical space-time continuum—which is forever unobservable, which for me has become an *ad hoc* introduction of a philosophical element into physics. Unfortunately I have not got away from the necessity of introducing unobservable elements into my fundamental theory. However, these elements are discrete, and although not directly observable, are introduced in such a way as to have observable consequences. For me this is still a point of view consistent with a materialist philosophy.

Once one rejects the *a priori* assumption of a space-time continuum, another problem comes to the fore, namely the concept of the identity of particles which plays such a crucial role in contemporary elementary particle physics. There it is met, more or less, by exploiting the duality created by the relation between probability amplitudes (wave functions) and their absolute square (observable probability densities) to interpret the fundamental difference between bosons and fermions in terms of symmetry and antisymmetry of wave functions and relate it to the exclusion principle. It has an older history starting at least as early as the Gibbs paradox, but was not at the forefront of the ideas which led to the combinatorial hierarchy model. It became an essential problem for Parker-Rhodes, which he tries to meet in his monograph, The Theory of Indistinguishables.[27] This volume gives a very brief history of the historical background of the combinatorial hierarchy in a prefatory paragraph, and the mathematical history of indistinguishables, such as it is, in the introduction. We start our discussion with his considerations here for logical rather than historical reasons.

Once one has entertained the possibility that there are things which are indistinguishable except for spacial location, and has abstracted from that to a logical world in which there is no such thing as spacial location, one way of formalizing the situation is to introduce three rather than two parity relations. In Parker-Rhodes' notation

$=$ ab means that 'a and b are identical'

$\hat{=}$ ab means that 'a and b are <u>twins</u>'

$\dot{-}$ ab means that 'a and b are distinct'

The negation of these statements creates in a sense an asymmetric situation:

$\neq$ ab means that 'a and b are nonidentical'

which in this context implies that they can be either distinct or twins

$\hat{+}$ ab means that 'a and b are <u>bipar</u>'

which in this context means that they can be either identical or distinct

$+$ ab means that 'a and b are indistinct'

which in this context means that they can be either identical or twins. We should see immediately that this is not a "three valued logic," and also that the ambiguity produced by negation makes all statements in the theory context dependent. Thus one has to work out not only the object language of the theory but a context-dependent semantics and a meta-language in terms of which theorems can be asserted and proved. Both use standard two-valued logic. Another consequence is that collections of twins can be assigned a finite cardinal number but cannot be ordered within the collection. Thus one has to go outside the theory of finite sets to what Parker-Rhodes calls a "sort theory" which encompases set theory in the <u>biparitous</u> situation.

Our own interest in this theory arises because it tackles a fundamental problem in the theory of elementary particles at the logical level. More specifically the theory gives mathematical form to the problem of how to intorudce discrete "indistinguishable" elements into physics. This is done by the basic semantic postulate of the theory, which is that all "observation" must be biparitous. Our own basic operational paradigm drawn from the experience of high energy particle physics is to start from whether a counter fires or not; this is a "yes-no" event and is clearly biparitous. If one accepts Parker-Rhodes fundamental semantic postulate, "twins" can never be directly observed. They can only be indirectly inferred from biparitous observations. Thus for us the "elements of reality" are discrete "indistinguishables" which in another line of development are related to what von Weizsacher[28] calls "urs."

The subsequent development of Parker-Rhodes' theory gives flesh to his basic semantic postulate by considering only those types of "indistinguishables" which can be mapped onto a biparitous theory, and hence given indirect observational consequences. This turns out to be extremely restictive from a mathematical point of view, and allows him to enumerate and exaust the "sorts" of indistinguishables which he will admit. From this he goes on to construct space, time, particles and cosmology in ways that depart widely from our own approach. So I will not deal further with the development of his theory here, except to return later on to a remarkable calculation which we are trying to absorb into our own framework.

The first publication on the combinatorial hierarchy[2] made use of a less fundamental starting point. The basic elements of the theory were taken to be ordered strings of the existence symbols "0" and "1", $S_n(x) = S_n(x_i) = (x_1, x_2, \ldots, x_i, \ldots, x_n)$ with $x_i \in 0,1$. The basic operation in the theory was taken to be the combination of two strings to form a third string by adding the ordered elements pairwise using addition mod 2. Since $+_2$ is defined by $0 +_2 0 = 0$, $1 +_2 0 = 1 = 0 +_2 1$, $1 +_2 1 = 0$, and hence $S_n(x) \oplus S_n(y) = S_n(x +_2 y) = (\ldots, x_i +_2 y_i, \ldots)$, the combination of two identical strings gives the null string, while the combination of two nonidentical strings with $n = 2$ or greater yields a third string which is different from either. Thus the operation discriminates between strings which are the same (null result) or different (novel result) and is called "discrimination." Clearly this simple starting point has some of the elements of quantum mechanics — in particular discreteness — and if considered in some sense a candidate for the elementary material interaction from which the world is build up has the important characteristic of producing novelty, and hence the possibility of evolution. Further, as was proved in the original publication,[2] a second operation imposed on this basis using the somewhat vague idea of "information preservation" in a natural way leads to a unique hierarchy of four levels with the cumulative cardinals 3, 10, 137, $2^{127} - 1 + 137 \approx 1.7 \times 10^{38}$ which cannot be further extended. Thus a logical construction led rather directly to the fundamental scale constants of physics. To get from this initial insight, which obviously many people would take to be a coincidence, to a more articulated fundamental theory has taken, and is taking, a long time.

Before showing how this remarkable result can be achieved, we will first discuss more recent developments. In particular, Kilmister was bothered by the somewhat *ad hoc* way in which the hierarchy was constructed, and by the fact that the individual levels are characterized by the cardinals 3, 7, 127, $2^{127} - 1$ rather than by the cumulative sums of these cardinals which have the intriguing connection to the scale constants of physics. He therefore looked for a more fundamental way to generate the hierarchy, and found it in a construction of the integers given by Conway[29] which could be adapted to a quite different purpose. The generating process G yields new elements to adjoin to a set S of previously constructed elements. The generating operation chosen is: if L,R are disjoint subsets of S, adjoin $\{L/R\}$ to S. This choice has the great advantage that in a sense no starting point is needed, since from the empty set $\emptyset$ one obtains immediately $\{\emptyset/\emptyset\}$ which one calls 0. The next two elements are $\{\emptyset/0\}$ and $\{0/\emptyset\}$ and so on. Kilmister[30] then goes on to define an equivalence relation on S which turns out to be the same discrimination operation D or + that we defined more concretely above, once it has been fleshed out. He then goes on to call (in our terminology) the equivalence classes under D "labels." By then defining "discriminate closure" in terms of subsets which are singletons or which have the characterization that discrimination between any two labels in the set leads to a third label in the set, he proceeds to construct the combinatorial hierarchy. For example, since

$$(10) \oplus (01) = (11) \qquad (10) \oplus (11) = (01) \qquad (11) \oplus (01) = (10)$$

the three strings (10), (01), and (11) form a discriminately closed subset.

I must confess that I find Kilmister's development hard to folow, and hence will not try to justify it for you here. I therefore find the original construction of the hierarchy more appropriate for my current purpose of exposition. For the example at hand we first note that only two of the strings are linearly independent, since (10)+(01)+(11) = 0, where we have dropped the circle around + for simplicity in further presentation. Since a+a = 0 from our basic definition, if we have two linearly independent elements a,b we can always form the discriminately closed subset {a,b,a+b} as well as the two singletons {a} and {b}. Similarly, if we have three linearly independent elements a,b,c we can form the seven discriminately closed subsets {a}, {b}, {c}, {a,b,a+b}, {b,c,b+c}, {c,a,c+a}, {a,b,c,a+b,b+c,c+a,a+b+c}, and so on. We see that from n linearly independent strings we can always construct $2^n - 1$ discriminately closed subsets because this is the number of ways we can take n things 1,2,...,n at a time.

We can now briefly explain one way to get the hierarchy. Start from strings of length 2. Then we get $2^2 - 1 = 3$ DCsS (discriminately closed subsets). Find 2×2 matrices which are nonsingular (i.e., do not map onto zero), have these DCsS as their only eigenvectors, and are linearly independent. Rearrange these as strings of length 4. Since we now have three linearly independent strings, these can be used, as just proved, to form $2^3 - 1 = 7$ DCsS. Map these by seven linearly independent 4×4 matrices to obtain $2^7 - 1 = 127$ DCsS. Map these by 127 linearly independent 16×16 matrices to give 127 strings of length 256 from which we can construct $2^{127} - 1$ DCsS. Note that at each step we have in a sense preserved the information about discriminate closure contained in the next lower level. It has been proved by Kilmister[31] that the mapping matrices always exist, and by Noyes[32] that there are explicit exemplifications at each level. But if we try to repeat the process once more with 256×256 matrices, we cannot do so because at most 256^2 of them are linearly independent, and 256^2 is much less than $2^{127} - 1$. Thus the combinatorial hierarchy has four and only four levels, and is characterized by the cumulative cardinals 3, 10, 137, and 1.7×10^{38}.

We emphasize that this explicit construction in terms of strings of existence symbols and mapping matrices is not required for the mathematical development. John Amson has derived the hierarchy and proved its uniqueness using only set theory,[33] and the abstract definition of discrimination. As we have just seen, Kilmister has recently reduced the required postulates to a still more abstract and simple structure. But I have found the representation in terms of strings easier to think about when it comes to physical interpretation -- although this reification of an abstract structure has also at times led me into error. It was tempting for me to try to interpret the existence symbols in the strings as representing the presence or absence of conserved quantum numbers. This led to much tentative physical interpretation in our 1979 paper,[3] some of which may ultimately prove to be correct, but which in general was probably prematurely concrete. I will try to be more cautious here, but my optimism may once again lead me astray.

The new development which for me arises out of Kilmister's generation operation is that it goes on chugging out longer and longer strings even after we have exausted the hierarchical scheme for labeling them. The cutoff point which seems natural from the hierarchy development is when we have reached labels of length 256. Of course only 256 of these are linearly independent. From these we can choose any 127 which are linearly independent as a basis from which to construct the $2^{127}-1$ DCsS of the fourth level of the hierarchy. Since there are $2^{256}-1$ distinct non-null ordered strings of length 256, these DCsS can still be labeled by some scheme with labels of this length, even though a still larger level of the hierarchy has been proved not to exist. So what do we do with the ever increasing {L/R} elements, which as already proved by Conway can go on increasing for as long as we want to talk about integers? My proposal is simply to label them by some scheme which assigns existence symbols in strings long enough to accommodate them. For instance every time we generate an element which is novel and which cannot be accommodated within the scheme of strings of length n already established (which can, for instance be shown by finding that on discrimination with one of them is the result is null) we add to that representation either (01) or (10) thus increasing the length from n to n+2; which is added is to be random in the sense that each choice occurs with equal frequency and there are no other correlations. We also assume that discriminations are going on at random between existing elements of the set. When these produce a null result, we adjoin two new elements to the set, one obtained by adding (10) to the common (2n) string of existence symbols, and the other by adding (01) to the set. If the discrimination produces a novel string, this is also to be added to the set. It is clear that in this way we will generate strings with an even number of existence symbols whose length will eventually exceed the 256 which the hierarchy scheme can accommodate. For reasons developed below we call (as before) the first 256 entries in the string the label and the remainder of the string, however long, the address.

Our choice of computer terminology here is deliberate. Early thinking about the hierarchy grew out of this background, and it is still providing fruitful insights. To anticipate where we are going, the labels which we have so far treated abstractly are candidates to be identified at some stage with a classification scheme for elementary particles, and the addresses will be the basis from which a discrete version of space-time coordinates will be constructed. Those of you who are still with me will realize that the whole scheme presupposes in some sense a "memory," which would seem out of place in a scheme whose avowed aim is to generate the whole universe from the sequential action of elementary processes. Our brief answer to this problem is that it can only be met retrodictively just as in any discussion by human beings about times before there were human beings.

The whole approach at this stage may appear to be bizarre, and in particular the address -- label scheme completely unmotivated. Actually it arose from attempts to incorporate within the scheme work by Irving Stein[34] which was initially completely independent of the work on the hierarchy. Briefly what he did, starting from a random walk model with unequal probabilities for taking discrete steps left or right was to construct both the Lorentz transformation and the uncertainty principle referring to ensembles of "objects" which were themselves collections of initially indistinguishable material elements. Thus he had in some sense already achieved our objective of constructing space, time and particles in such a way that all three concepts came together, and could not be separated. The specific representation of Kilmister's generation operation I proposed above was specifically created to provide strings of random numbers in binary form from which Stein's construction could proceed. Whether this is the right way to go is currently very much a matter of controversy within the combinatorial hierarchy community, so this presentation is for the moment strictly a personal point of view. But now it is time to give a brief presentation of Stein's work as I see it.

We have seen that our construction leads to labeled "addresses" of length 2N and since the strings are ordered we can always think of them as containing ordered pairs which we symbolize by $(\ell_1, r_1, \ldots, \ell_i, r_i, \ldots, \ell_N, r_N)$ where $\ell_i, r_i \in 0,1$. From these we select an assemblage characterized by two numbers $n = n_\ell + n_r$ and $v = \langle(n_r - n_\ell)\rangle/n$ where $n_\ell = \sum_{i=1}^{N} \ell_i$ and $n_r = \sum_{i=1}^{N} r_i$ and all have the same label. Since the process by which these strings are generated is random, this defines a binomial distribution with a probability of $\langle n_\ell\rangle/n$ and $\langle n_r\rangle/n$ of finding any particular value of n_ℓ and n_r for any one string. Clearly the standard deviation of this distribution has the value

$$\sigma = \left[\frac{\langle n_\ell\rangle\langle n_r\rangle}{n}\right]^{\frac{1}{2}} = \left\{\frac{1}{4n}\left[\left(\langle n_r\rangle + \langle n_\ell\rangle\right)^2 - \left(\langle n_r\rangle - \langle n_\ell\rangle\right)^2\right]\right\}^{\frac{1}{2}} = n^{\frac{1}{2}}(1-v^2)^{\frac{1}{2}} . \tag{2}$$

We now introduce dimensional units by assuming that the label corresponds to a parameter m of the dimension of a mass and hence a characteristic length $L = \hbar/mc$ where $\hbar$ and c have the dimensions of action and velocity but are at this point otherwise undefined. Clearly we can think of this model if we wish as a biased random walk on a line with a step length L in which n steps have been taken, and hence can define a "time" for this process to take place by $t = nL/c$. With this interpretation we see that we automatically have a limiting speed for the peak of the distribution which is c and which is attained only in the case when n_ℓ or n_r is zero. Further, the peak of the distribution has from this point of view "moved" with an average velocity $V = vc$ during the time t. We now assume that the position of the object is to be measured by some elementary scattering event, ultimately to be modeled by our discrimination operation, and which defines the position x of the object to be at one standard deviation from the peak. Since the standard deviation depends on our choice of n and v, or in dimensional units on t and V we have, referred to some "absolute"— but so far arbitrary— system, that

$$x - Vt = L\sigma(t,V) = (cLt)^{\frac{1}{2}}(1 - V^2/c^2)^{\frac{1}{2}} . \tag{3}$$

To remove this arbitrariness we refer this position to the position of a similar object "at rest," i.e., with $V=0$, and find that since $x'=\sigma(t,0) = (cLt)^{\frac{1}{2}}$

$$x' = (x - Vt)/(1 - V^2/c^2)^{\frac{1}{2}} . \qquad (4)$$

But if we now concentrate on an object whose position is x' in the coordinate system with $V=0$, its position in a second coordinate system moving with velocity V with respect to the initial frame must be

$$x = x'(1 - V^2/c^2)^{\frac{1}{2}} + Vt . \qquad (5)$$

To make our description fully relativistic we refer this motion not to a coordinate system but to a second object with a different label L' which is at rest. Since it will in general be generated by a different number of steps we must assign it a different time t' as well as the position x'. By the principle of relativity [in this context the "principle of relativity" simply means that since the dichotomous choice between "ℓ" and "r" is arbitrary the choice between $+V$ and $-V$ must also be arbitrary]. it must have velocity $-V$ with respect to the object with Label L, and hence

$$x' = x(1 - V^2/c^2)^{\frac{1}{2}} - Vt' . \qquad (6)$$

Now, it is a simple matter of algebra to solve (5) and (6) for t and t' and obtain

$$t = (t' - xV/c^2)/(1 - V^2/c^2)^{\frac{1}{2}} \quad ; \quad t' = (t + xV/c^2)/(1 - V^2/c^2)^{\frac{1}{2}} \qquad (7)$$

which completes Stein's derivation of the Lorentz transformation for labeled objects which are assemblages of our basic strings. There are subtle objections which might be raised to this derivation, but we believe it is basically correct. Clearly we are now fully justified in identifying our dimensional constant c with the limiting speed of special relativity.

The next step is to define "particles" as ensembles of objects with a distribution of velocities characterized by a standard deviation δV. We then ask how far two such distributions which differ by δV in their velocities must move in order that the two distributions can be distinguished by a Raleigh-type criterion. Calling this distance δx we find that by equating it to $\sigma(t,0)$ it is

$$(\delta V)t = \delta x = \sigma(t,0) = (cLt)^{\frac{1}{2}} \quad \text{or} \quad t = cL/\delta V^2 . \qquad (8)$$

Hence

$$\delta x = (\delta V)t = cL/\delta V \quad \text{or} \quad \delta x\ \delta V = cL \quad . \tag{9}$$

Defining $\delta p = m\ \delta x$ under the supposition that both distributions carry the same label and recalling that the step length $L = \hbar/mc$ we find that

$$\delta p\ \delta x \ \geq \ \hbar \tag{10}$$

independent of the mass or step length assigned to the two particles with different velocity distributions. Thus, like c, our dimensional constant $\hbar$ is independent of the choice of either L or m, and hence can be identified with Planck's constant. This derivation is a little sloppy in that we have apparently used a nonrelativistic definition of p, but Stein[35] assures me that a more careful treatment shows that δx should carry the factor $(1 - V^2/c^2)^{\frac{1}{2}}$ in an arbitrary coordinate system and $m\ \delta V$ its inverse, which suffices to maintain the Lorentz invariance of the result.

Since we have already established the Lorentz invariance of $x^2 - c^2t^2$ for "particles" defined as ensembles of "objects" with a specified velocity V with a standard deviation δV characterized by a mass m assumed (from our construction) invariant since it refers to a binary string label, we can immediately define another dimensional constant E via the relation $E^2 = p^2c^2 + m^2c^4$ and from the already proved invariance of m, c and $\hbar$ arrive at both the Lorentz invariance of $E^2 - p^2c^2$ and the uncertainty principle in energy

$$\delta E\ \delta t \ \geq \ \hbar \tag{11}$$

on which our whole preliminary analysis drawn from Wick rests. Thus we have come to standard relativistic kinematics "on a line" starting from our discrete model.

To go from this derivation to the Lorentz transformations in $(3+1)$ space time requires an argument which has not been refined to the point of presentation here, and for all we know at present may turn out to be fatally flawed. It requires reliance on the assumption that level 3 of the hierarchy is the last step within which we can define exactly (or approximately) good additive conservation laws for quantum numbers. The three levels allow us to apply the Stein argument to three different types of labels, leading to three different momenta, while discriminate closure within the levels does not allow us, without a more specified sequential history, to select within the levels. Again, without more sequential information, we cannot say which level is which, establishing the isotropy of the 3-space we can construct in a static sense. Self-consistency then establishes the complete relativistic kinematics, and the uncertainty principle on which the finite particle number scattering theory discussed earlier rests.

To go from this particulate quantum mechanics (or rather quantum kinematics, since we have still not defined "interactions") to wave mechanics is more difficult. Some thought has been devoted to relating the step-length periodicity contained within the Stein model to the operational conditions under which a "particle" (which necessarily in this approach has a distribution in the velocities of the "objects" from which it is constructed) can subsequently be "detected." This line of thought seems to lead to what can be identified formally as both a "wave" and a "group" velocity, which was the starting point of Debroglie's first great contribution to physics, but whether it can lead to a precise definition of a "wave function" remains to be seen.

Although the full development of a discrete theory that can claim to explain "wave-particle dualism" starting from this base is still a long way off, aspects of the theory as now developed are promising enough to deserve attention. It will be noted that the Stein theory does not, and indeed cannot, give us any guide to the values of the parameter m which are allowed, except that as developed they have necessarily been assumed finite. However, once we have reached, as we claim we have above, the generalization $E^2 - p^2c^2 = m^2c^4$ there seems no barrier to also considering distributions characterized by $m = 0$ and hence, inverting the procedure followed above, assigning them a coordinate-system dependent step length $L_0 = \hbar c/E$. As strings they will be represented by $\ell_i \in 1,0$, $r_i = 0$ or visa versa and their "distribution" travels with the limiting velocity $\pm c$ without dispersion, as already noted. We clearly have the correct parameterization of "wavelength" and "frequency" dimensionally speaking, but we hesitate to call these objects "photons" until we know how they interact. So we must turn to the question of extracting from our theory a physical meaning for the abstract parameters E, p and m which so far we have (in spite of the seductive notation) no justification for calling "energy," "momentum" and "mass."

Grasping at the remarkable fact that the electromagnetic and gravitational strength parameters seem to turn up with approximately the correct values in the hierarchy construction, we take the plunge by assuming on dimensional grounds that we can give content to the concept of electromagnetic "interaction energy" by assuming it proportional to e^2/L where e^2 is the universally observed smallest (and quantized!) unit of electric charge. The only length we have defined at this point is $\hbar/mc$ with m arbitrary. Clearly, if the scheme is to work we must relate this to the third level of the hierarchy construction where we have in some sense 137 degrees of freedom, and -- as mentioned above -- are first able to make connection with conventional 3-space. Since at this point we have no way to distinguish between the different degrees of freedom, we must assign 1/137 of the interaction energy to each of them. But if we try to include more than 137 of them in this volume characterized by a single step length, we exceed the possibility of a static description in the case of attraction, since there is more interaction energy than there is mass to attract. This is our version of the Dyson argument given above, and now in our context shows that we can associate the pure number 137 with counting the maximum number of electromagnetically interacting particles we are allowed to consider within a minimum length we have arrived at from other considerations. Similarly if we clump more than 1.7×10^{38} gravitating nucleons within their characteristic length, the gravitational force is sufficient to make them collapse into, in conventional theory, a "black hole." We see that the termination of the hierarchy at this level and our physical interpretation are congruent.

We are now in a position to discuss a remarkable calculation by Parker-Rhodes published in his Theory of Indistinguishables[27] and which we have also presented in an earlier pass at the physical interpretation of the combinatorial hierarchy.[3] Our line of reasoning differs considerably from his, because of our different approach to physical interpretation; we are grateful to him for allowing us to try to adapt this remarkable result to our own language. The basic idea is that, since our unit of mass is already established as m_p, all other masses must be computed as ratios to m_p; we add to this an often held view that the mass of the electron is a consequence of its electromagnetic interactions. On dimensional grounds we therefore equate $m_e c^2$ to $\langle q^2\rangle\langle 1/r\rangle$ where q^2 is the the charge and r some characteristic length. Clearly the focus of the argument is on what statistical argument we can use to calculate these expectation values. As to distance, our model does not allow us to define it on a scale set by a shortest length which, in our interpretation of electromagnetic effects will be that of the Compton wavelength of a proton-antiproton pair $L = h/2m_p c$. Hence we can calculate electrostatic energy only due to stochastic separations which separate the effective charge by amounts greater than this. Thus $\langle 1/r\rangle = (2m_p/h)\langle 1/y\rangle$ with $y > 1$. Since we are assuming that charge is conserved we can write the effective square of the charge produced by the fluctuations as $e^2\langle x(1-x)\rangle$. We have already seen that at this stage of the theory $e^2 = \hbar c/137$. Hence

$$m_p/m_e = 137\pi/\langle x(1-x)\rangle\langle 1/y\rangle \quad . \qquad (12)$$

So far all we have done is to use dimensional analysis.

As argued above the three basic types of charge separation we can consider are those corresponding to the three levels of the hierarchy, which we have also argued give us the 3 dimensions of "space." Each of these will carry its own weighting factor of 1/y, and hence $P(1/y) = (1/y)^3$ and

$$\langle 1/y\rangle = \int_0^1 d(1/y)(1/y)P(1/y) \Big/ \int_1^\infty (dy/y^2)P(1/y) = \frac{4}{5} \quad . \qquad (13)$$

Since we are dealing with a static system, the charge must not only come apart with a probability proportional to q but also come together again with the same probability, which requires that $P(x(1-x)) = x^2(1-x)^2$. Here we must be careful since the fluctuations can produce both positive and negative charges, and it appears that x can have any value. But then the positive weighting factor would lead to a divergence. However because of charge conservation any fluctuation producing negative charge comes in the form of particle-antiparticle pairs, and statistically any contribution to x will cancel outside the interval $0 \le x \le 1$. Thus if we had only one degree of freedom we would have

$$K_1 = \langle x(1-x)\rangle_1 = \int_0^1 x(1-x)P(x(1-x))\,dx \Big/ \int_0^1 P(x(1-x))\,dx = \frac{3}{14} \quad . \tag{14}$$

Once the first fluctuation has taken place we have effective squared charges x^2 and $(1-x)^2$ so we can take account of the two additional degrees of freedom by writing the recursion relation

$$K_n = \int_0^1 \left[x^3(1-x)^3 + K_{n-1}x^2(1-x)^4\right]dx \Big/ \int_0^1 x^2(1-x)^2 dx$$

$$= \int_0^1 \left[x^3(1-x)^3 + K_{n-1}x^4(1-x)^2\right]dx \Big/ \int_0^1 x^2(1-x)^2 dx$$

$$= \frac{3}{14} + \frac{2}{7}K_{n-1} = \frac{3}{14}\sum_{\ell=0}^{n-1}\left(\frac{2}{7}\right)^{\ell} \quad . \tag{15}$$

Thus our final result is that

$$m_p/m_e = 137\pi \Big/ \left\{\frac{3}{14}\left(1 + \left(\frac{2}{7}\right) + \left(\frac{2}{7}\right)^2\right)\frac{4}{5}\right\} = 1836.151497\ldots \tag{16}$$

in comparison with the latest empirical result $m_p/m_e = 1836.15152 \pm 0.00070$.[36]

One criticism of this result is that the mass scale of our theory given by $\hbar c/GM^2 = 2^{127} - 1 + 137$ does not give precisely the proton mass but rather $M = 936.16$ MeV/c^2. However since, of course, both m_p and m_e must be referred to the same mass scale, this cancels out in the ratio. A more serious criticism is that we have used 1/137 for the fine structure constant rather than the empirical value. We would argue that this is consistent, since the calculation is conducted at level three of the hierarchy, and our suspicion is that the correction to this number will have to come at level four, where in our current view the weak interactions come in and will produce corrections of the order $1/256^2$.

Having, we hope, given an indication that our theory might be capable of producing general results of quantitative significant even prior to the development of a detailed dynamics, we now sketch our current views of how we might connect up to the finite particle number relativistic quantum dynamics discussed earlier in our talk. The general idea is to view the discrimination operation $a+b \rightarrow c$ as a discrete version of a Yukawa vertex and find the appropriate interpretation of the label so that quantum numbers are conserved. The momentum conservation of the process is then to come from the address part of the string and must connect consistently to the Stein construction and hence to our discrete version of space-time. There are various ways to accomplish this, but so far no unique scheme has emerged. So we content ourselves here with more general considerations. In particular we note that if we are to connect either to physical observations which require two in two out processes to discuss momentum conservation or if we look at the finite particle number scattering theory where the basic process is again a two in two out scattering, we should focus on how this is described rather than on vertices. This suggests that we should consider $a+b \leftrightarrow c \leftrightarrow d+e$ where this is to be related to discrimination by requiring that $a+b = c$ <u>and</u> that $d+e = c$. Since discrimination is not in itself an ordering relation, the ordering is to come from the relation between the address part of the string and the Stein construction, which provides a "background." Within the label part all we can require is that c belong to a DCsS, and we can see in a general way how this will lead to time reversal invariance, the connection between particles and antiparticles and the CPT theorem. We are not ready as yet to commit ourselves to a specific scheme for representing this.

One link we can already make between scattering processes and the hierarchy is that Vanzani[37] has proved that the minimum number of independent amplitudes which must be considered in an N particle scattering process is $2^{N-1} - 1$ corresponding to the distinguishable two cluster decompositions of the initial or final state. To illustrate this at level two where we have three linearly independent strings a,b,c, we define a fourth string uniquely by making it linearly dependent, i.e., $a+b+c+d = 0$. The one-to-one correspondence between two cluster decompositions and DCsS is then

Two cluster decomposition		Discriminately closed subset
(3,1)	(a)(bcd)	{a}
	(b)(cda)	{b}
	(c)(dab)	{c}
	(d)(abc)	{a,b,c,a+b,b+c,c+a,a+b+c}
(2,2)	(ab)(cd)	{a,b,a+b}
	(bc)(da)	{b,c,b+c}
	(ca)(db)	{c,a,c+a}

On the left we recognize the (3,1) and (2,2) configurations from which the Faddeev-Yakubovsky discussion of the four body problem starts.

Our next step is to recall for level one that the basis can be chosen as (10), (01), which might be interpreted as a representation of ± charge. One basis for level 2 is (1100) = a, (1110) = b, and (1101) = c. If we take the last two entries to represent ± charge as before, we can make a tentative physical interpretation in terms of the charge states of the seven bosons which are of greatest significance in the nuclear force problem. Our specific suggestion is to take $\pi^o = a$, $\pi^+ = b$ and $\pi^- = c$. Then $\pi^o + \pi^+ = \rho^+ = (0010)$, $\pi^+ + \pi^- = \rho^o = (0011)$, $\pi^- + \pi^o = \rho^- = (0001)$, $\pi^+ + \pi^- + \pi^o = \omega^o = (1111)$.

This looks promising, but what about the first two entries in the strings? We tentatively take these to refer to a second dichotomous quantum number such as baryon number. Then the physical interpretation of some of the strings which lie outside the hierarchy would be $p = (1010)$, $\bar{p} = (0101)$, $n = (1000)$, $\bar{n} = (0100)$, and we would have a way to describe the charge and baryon number degrees of freedom which enter into the dynamical model we proposed earlier for the nuclear force in the two nucleon - two quantum sector. Going on to level three where we have strings of length 16, the tentative suggestion is that we have four dichotomous variables, namely baryon number, lepton number, charge, and spin. Then if we use a representation[3] in which the first four entries are (1111...) or (0000...) we could let these represent baryon number and lepton number, and construct an interpretation of the seven basis vectors making them identifiable with $\pi^+, \pi^-, \pi^o, W^+, W^-, W^o, \gamma$. At the particle level we now have $e^+ e^- \nu \bar{\nu}$ in addition to the nucleons. Thus each four particle system has the same basis of $2^{4-1} - 1 = 7$, while the eight particle system has a basis of $2^{8-1} - 1 = 127$. Again, this looks promising as the weak vector bosons would come in together with the electromagnetic quantum and we might have a chance of understanding weak-electromagnetic unification with a first approximation to the coupling constant as 1/137. We note in passing that 1/10 at level two might be an approximation to the (pseudovector) pion-nucleon coupling constant $f^2 = 0.08$. We take comfort from the fact that we expect large corrections at this level before we can compare to experiment and hence need not take the discrepancy between 0.08 and 0.10 too seriously. Going on to level four we expect to encounter the quark degrees of freedom, and have tentative schemes to interpret them in terms of the hierarchy classification scheme, but it would be premature to present them here. There is even the possibility, since the dichotomous numbers now go from four to sixteen that we might be able to connect up with Harari's "rishons," but this is obviously very speculative.

Although tentative, we hope these ideas show that there is at least a chance that we can connect up our fundamental theory with current ideas about elementary particle physics, and go on to gravitation and cosmology. We have already seen that the mass scale of the theory is defined by the gravitational constant, and hence in a sense we have the starting point for a "grand unified theory." We also note that since, via the Stein construction, our labels that are thus tied to gravitational mass serve in his dynamics as inertial mass and thus make the equivalence of the two a consequence of our scheme and not an additional postulate as in general relativity. As to cosmology, since we have 2^{256} labels, once our generation operation has reached that length of strings we cannot go any further in assigning quantum number interpretations. Thus the upper limit to the number of particles in the universe in our model is 2^{256}. Among these, if we do our dynamics right, there will be a large number of very massive and highly unstable systems, so we generate a "big bang" with enormous energy. How many of these persist depends on just how the particulate quantum number conservation works, but at worst they will settle down to the $(2^{127} - 1 + 137)^2$ discriminations which the hierarchy keeps on describing. How many of these are baryons and hence determine the mass of the universe again depends of details we cannot yet calculate, but it looks like we are within the rather broad limits currently assigned by observational cosmology.

From here on our cosmology is fairly conventional, assuming we can get our dynamics into reasonable agreement with the current empirical content of high energy particle physics. On a time scale that can be referred to[38] as "the first three minutes" the significant constituents of our universe will settle down to electrons, protons and neutrons plus massless radiation, and the neutrons will be effectively unimportant on a time scale only five times as long. Currently observable p/He^4 ratios in our locality put primary constraints on acceptable models and d/p, d/He^3 ratios more sensitive constraints on early time historical parameters which paleophysicists use to discriminate between models. The time scale for the significant formation of hydrogen and helium atoms, and the consequent breakaway of electromagnetic radiation from "matter" takes a million times longer, reflecting the ratio between atomic and nuclear dimensions. Retrodiction to these longer times takes only the postulates of the uniformity of nature (in more historical terms the "Copernican hypothesis") or Occam's Razor coupled to currently available empirical information.

From here on, until other types of historical data become important, we are involved only in questions of detail. So it is important to fix the framework of discussion within our own context. Conventional views about physics and cosmology allow us to retrodict earlier situations on the basis of current observation and laboratory experiments. That the rather startling "big bang" cosmology resulted was treated with skepticism until the predicted $3^{o}K$ background radiation was observed and increasingly tightly correlated with other types of evidence. More recently it has been shown that our solar system is moving with 600 km/sec with respect to this historical referent, and a little increase in the sensitivity and sophistication of current technology will soon demonstrate the rotation of the earth around the sun using what I can only call an "absolute" reference frame for velocities. This tight mesh allows me, I believe, to talk about cosmic time. My basic point here is that if we accept Kilmister's generation operation articulated in something like the way I have proposed, the lengths of the strings, which I take as the "elements of reality," measured by some large number N, are a candidate within this model for a discrete measure of cosmic time. Looking only at the "addresses, the two unique strings $\ell_i = 1$ and $r_i = 0$ for all $i \leq N$ and visa versa define an "event horizon" for the universe we can observe, and connect this radius to a finite cosmic time scale. Our model does not allow us to specify even a 3-dimensional space until we add more structure, so for us the universe is isotropic at large enough "distance" in accord with observation. Thus that there should be a locally measureable parameter (velocity relative to the 3^oK radiation) related to the cosmic time scale is no problem for us. Our model clearly entails a universe with different properties at different cosmic times, in accord with observation. Whether a particular "time slice" is also isotropic is easily answered in the negative for our neighborhodd, while the "homogeniety" of the universe for earlier time slices is ambiguous for very early times in terms of current data. So we get back to more mundane matters like galaxies, stellar systems, and planets.

Since I have discussed elsewhere[39] the 15 aeon (15×10^9 year) development of galaxies, type I and II stars, the 4.5 aeon development of the surface of our planet and its biological systems, the significance of the cooperative nature of hominid social systems for the emergence of humanity several millions years ago, the rupture of this system by food production a few thousand years ago, the development of the hierarchical military state, and hence the roots of our planetary crisis, I will stick here to general principles. So far the fundamental theory we have outlined contains sequential discrete processes, random at each step, but constrained by conservation laws. Looking backward, this is all we need to understand why current systems can be more complex than earlier ones -- they have more options with which they can meet unexpected (i.e., "random") events within the range to which they are adapted. As the number and types of such systems increase, the range of adaptation can often be more easily understood in terms of the complex systems themselves than in terms of the "physical" background we have been at pains to develop here -- in modern jargon "ecological" considerations become increasingly important. Thus we see no gap between our explanatory model and mathematical discussions of biological selection and evolution such as that of Manfred Eigen.[40] Unfortunately, the biological experience does not contain the dimension of cross-cultural transmission which is so important in social evolution, so satisfactory paradigms in this realm are still, in my opinion, lacking.

It remains to ask whether the physics and cosmology I have sketched here form a satisfactory basis for a materialist philosophy. What we hope to have achieved by this approach is a simple model for random processes with conservation laws. Then the sequential development of the system will select ever more complicated correlated structures for stability against this random background. Thus we use the same explanatory principles for the stabilization of particles, cosmic evolution and biological and social evolution. For me the most important philosophical insight which comes from this approach can be described by the phrase "fixed past — uncertain future." To quote an earlier statement[9]

> "If the past is indeed fixed, but determines the probabilities of future events, study of the past can provide a significant guide to present action. The increasing precision which historical, evolutionary, and cosmological study has given to our understanding of how we have arrived at the current planetary crisis lends hope to this view. Yet if all we can predict are probabilities, we are not forced to choose courses which are likely to lead to disaster. We can always, with some finite hope of success, choose a more humane course of action. It is a tribute to the inherent wisdom of the peoples of this world that they have mainly taken this attitude of responsible moral choice, in spite of the erudite teachings of their theologians, philosophers and scientists."

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FIGURE CAPTIONS

Fig. 1. The coupling of relativity to quantum mechanics leads to new particulate degrees of freedom at short distance.

Fig. 2. (a) Diagrammatic representation of the Faddeev equation.

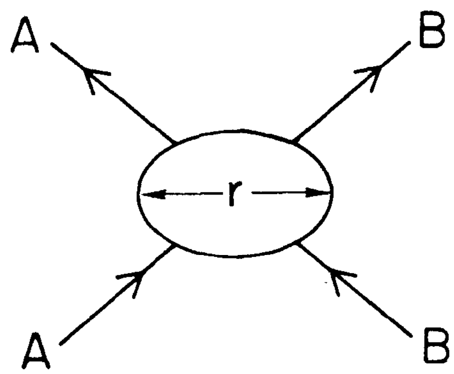
(b) The particle-particle-quantum sector when there is no direct particle-particle scattering.

(c) One way to obtain Thompson scattering in the quantum-quantum-particle sector.

Fig. 3. The Hamilton-Jacobi coordinates and initial states in classical and quantum mechanics, following Phipps.

Fig. 4. Double slit experiment with counters in the collimators to define time gates and counters in the slits to allow the same apparatus to give data exhibiting both the single and the double slit patterns.

WICK-YUKAWA MECHANISM



$$r \;<\; c\,\Delta t \;\approx\; c\frac{\hbar}{\Delta E} \;\leq\; \frac{c\hbar}{mc^2} \;=\; \frac{\hbar}{mc}$$

limiting velocity	uncertainty principle	mass -energy

Finite Mass
Momentum Conservation

3038A2

Fig. 1

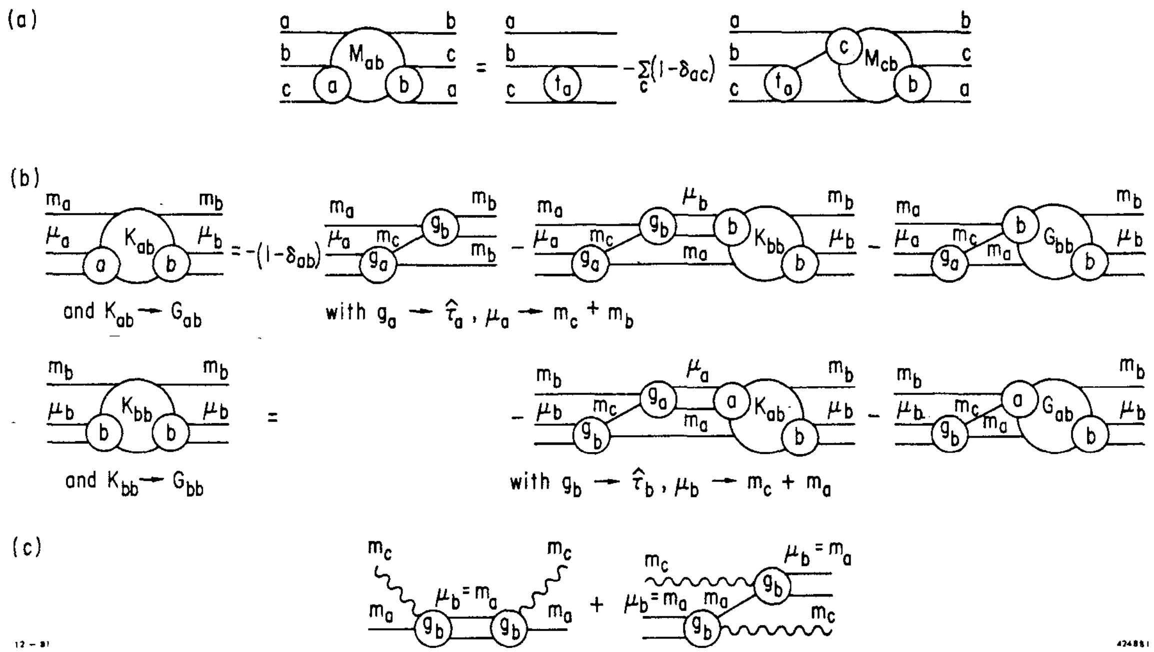
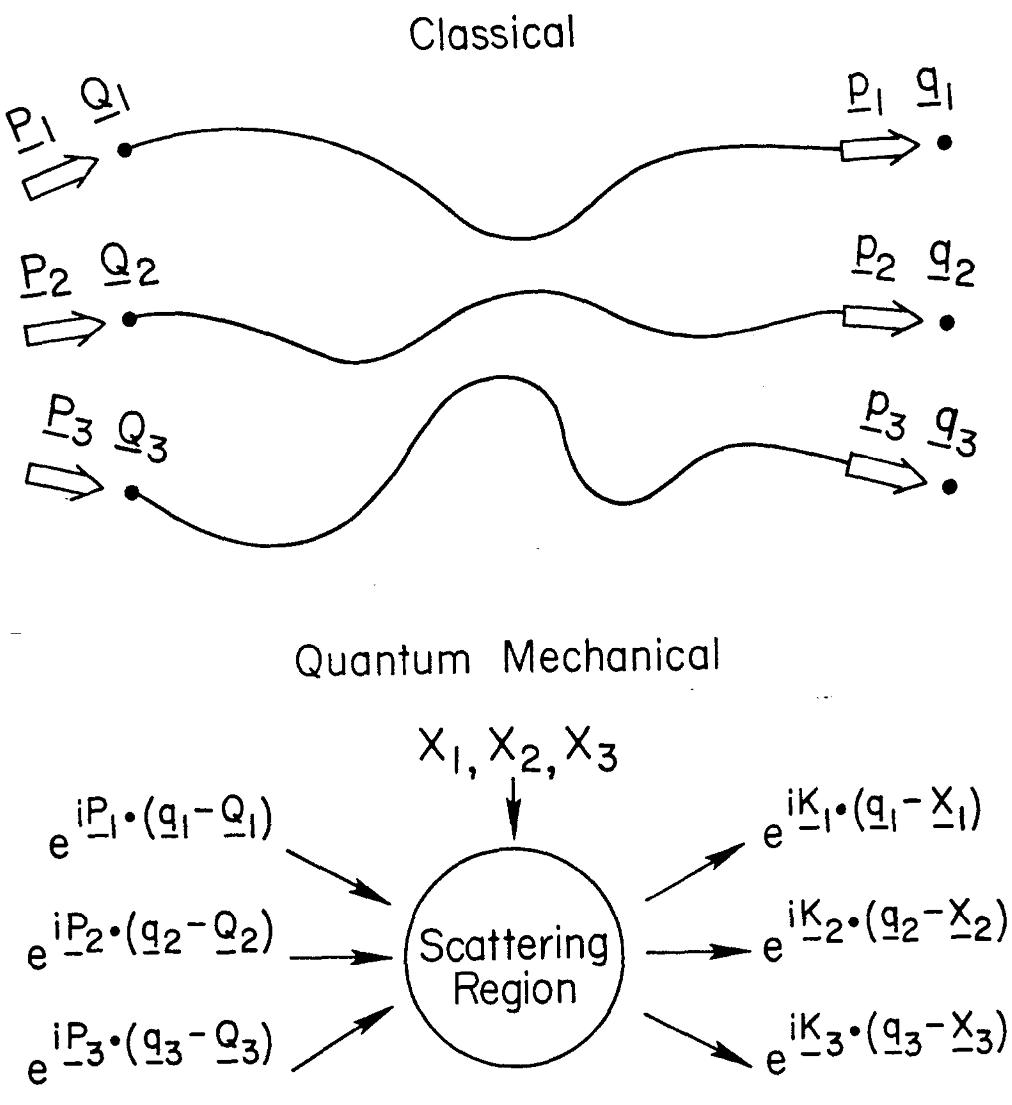

(a)
(b)
and $K_{ab} \rightarrow G_{ab}$
with $g_a \rightarrow \hat{\tau}_a$, $\mu_a \rightarrow m_c + m_b$
and $K_{bb} \rightarrow G_{bb}$
with $g_b \rightarrow \hat{\tau}_b$, $\mu_b \rightarrow m_c + m_a$
(c)
12 - 81
4248B1

Fig. 2

FIXED PAST $\Longrightarrow$ UNCERTAIN FUTURE

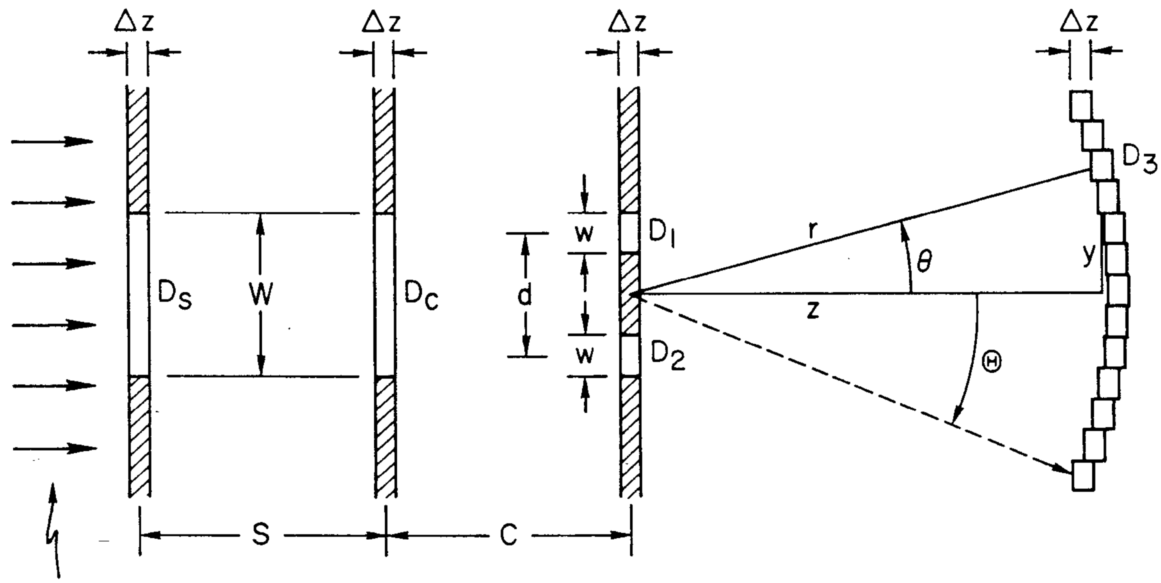


$$\Psi_{PHIPPS} = e^{-i\Sigma_k \underline{P}_k \cdot \underline{Q}_k} \Phi_{schroed}(q_k, t)$$

$$\rightarrow e^{-i\Sigma_k \underline{K}_k \cdot \underline{X}_k} \Phi_{schroed}(q_k, t)$$

3038A3

Fig. 3



D_S fires at $t = -(S+C)\,M/p_0 \pm \frac{\Delta t}{2}$

D_C fires at $t = -cM/p_0 \pm \frac{\Delta t}{2}$

(I_0): D_3 fires at $t = rM/p_0 \pm \frac{\Delta t_3}{2}$

(I_1): D_1 fires at $t = 0 \pm \frac{\Delta t_0}{2}$ D_3 fires at $t = rM/p_0 \pm \frac{\Delta t_3}{2}$

4-77 (I_2): D_2 fires at $t = 0 \pm \frac{\Delta t_0}{2}$ D_3 fires at $t = rM/p_0 \pm \frac{\Delta t_3}{2}$ 3155A1

Fig. 4