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NONLOCAL GAUGE THEORIES*

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ABSTRACT

Starting from a generalization of the covariant derivative, we develop nonlocal gauge theories. These theories enjoy local gauge invariance and associated Ward identities, a corresponding locally conserved current, and a locally conserved energy-momentum tensor, with the Ward identities implying the masslessness of the gauge field as in local theories. The ultraviolet behavior of these theories for non-vanishing fermion mass parameter is sufficiently convergent to remove the Adler-Bell-Jackiw anomaly.

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There exists at present an impressive body of evidence supporting the relevance of the gauge principle to the description of particle interactions. Currently, the greater part of the theoretical effort in extending the standard model to a more fundamental theory is concerned with the problems of mass hierarchy and family repetition.[1] The present work was motivated by the idea that a suitable generalization of gauge field theories which could enrich their structure while retaining the underlying gauge principle may lead to a resolution of these problems. Nonlocal gauge theories (NLGT) are promising candidates for this generalization insofar as (a) they involve form factors, so that the theory is intrinsically capable of accommodating any spectrum of mass scales, and (b) the fermion sector in a sense corresponds to the simultaneous description of a number of "sister" fermions of different masses in terms of a single spinor field.

Regardeless of how fruitful the above attributes turn out to be, we find NLGTs quite interesting and worthy of serious study in their own right.[2] Specifically, the theories developed here (a) possess an exact local gauge invariance and a locally conserved current, (b) have a locally conserved energy-momentum tensor, (c) are at least as well behaved as their local counterparts with regard to convergence properties, (d) naturally reduce to their local counterparts for distances well beyond the scale of nonlocality, and (e) they are in a sense complementary to lattice gauge theories[3] insofar as these may be considered special cases of the continuum NLGT corresponding to discrete form factors. Moreover, we observe that the traditional caveats of nonlocal theories, i.e., possible acausal behavior and discordance with the canonical formalism, are in fact inessential here since (a) one would presumably arrange for the scale of nonlocality to occur well beyond the observable domain, and (b) the quantization procedure is conveniently based on the path integral formalism. Finally, we mention the improved ultraviolet behavior of NLGTs which is owing to the high energy cut-off provided by the form factors. For a fermion propagating in an external field, for example, the short-distance behavior of the propagator is sufficiently benign to remove the Adler-Bell-Jackiw[4] (ABJ) anomaly for the chiral current, a fact which is obviously relevant to composite models based on chiral symmetry.[1]

We now proceed to outline the construction of NLGTs. This is most conveniently accomplished via a generalization of the covariant derivative by the following reasoning. The introduction of nonlocality entails the inclusion of derivatives of arbitrarily high order, symbolically representable by $i\partial^\mu{}_x \to i\partial^\mu{}_x\hat{f}(i\partial_x)$, where f is a suitable scalar function.[5] On the other hand, the local covariant derivative relative to gauge fields $A_a{}^\mu(x)$ and representation matrices λ_a is given by $\underset{\sim}{D}^\mu{}_x = \partial^\mu{}_x - ig\underset{\sim}{A}^\mu(x)$, where $\underset{\sim}{A}^\mu(x) = \underset{\sim}{\lambda}_a A_a{}^\mu(x)$.[6] The nonlocal generalization is accomplished by the substitution $\partial \to \underset{\sim}{D}$ and a symmetrization among the resultant noncommuting covariant derivatives by means of a Fourier integral. The result is

$$i\partial^\mu{}_x\hat{f}(i\partial_x) \to \int dz \, \exp[-z\cdot\underset{\sim}{D}_x] f_\mu(z), \tag{1}$$

where $f_\mu = \partial_\mu f$. An integral representation for (1) is obtained by means of the identity

$$\exp iz\cdot[i\partial_x + g\underset{\sim}{A}(x)]$$
$$= \exp(-\tfrac{1}{2}z\cdot\partial_x)\mathrm{Texp}\,[ig \int d\tau z\cdot A(x+z\tau)]\exp(-\tfrac{1}{2}z\cdot\partial x), \tag{2}$$

where "Texp" designates a τ-ordered exponential. Thus the kernel for the nonlocal covariant derivative is obtained in the form

$$\underset{\sim}{\Delta}_\mu(x,x'|f) = f_\mu(r)\mathrm{Texp}[ig \int_{-\frac{1}{2}}^{+\frac{1}{2}} d\tau\; r\cdot\underset{\sim}{A}(R+r\tau)],$$
$$r = x-x', \quad R = \tfrac{1}{2}(x+x'), \tag{3}$$

where the functional dependence of the covariant derivative on the form factor f has been explicitly denoted. Note that in the local limit $f(x) \to \delta(x)$, $\underset{\sim}{\Delta}^\mu(x,x'|f) \to \delta(x-x')\underset{\sim}{D}^\mu{}_x$. Moreover, under a gauge transformation effected by S, $\underset{\sim}{\Delta}$ transforms according to

$$\underset{\sim}{\Delta}_\mu(x,x'|f) \to \underset{\sim}{\Delta}'_\mu(x,x'|f) = \underset{\sim}{S}^{-1}(x)\underset{\sim}{\Delta}_\mu(x,x'|f)\underset{\sim}{S}(x'). \tag{4}$$

Equation (4) is the cornerstone of local gauge invariance for NLGTs.

The field strength tensor is constructed as a generalized commutator of covariant derivatives:

$$\underset{\sim}{F}_{\mu\nu}(x,y|f) = \int dx'[\underset{\sim}{\Delta}_\mu(x,x'|f)\underset{\sim}{\Delta}_\nu(x',y|f) - \underset{\sim}{\Delta}_\nu(x,x'|f)\underset{\sim}{\Delta}_\mu(x',y|f)], \tag{5}$$

with the local limit $F_{\mu\nu}(x,y|f) \to \delta(x-y)F_{\mu\nu}(x)$, and a gauge transformation property identical to (4). Clearly $\underset{\sim}{\Delta}$ and $\underset{\sim}{F}$ are string-like objects.

Finally, the above elements may be assembled to form the action for a NLGT of the Yang-Mills type as follows[6]

$$
\begin{aligned}
W &= \int dx dx' \bar{\psi}(x)[i \underset{\sim}{\not{\Delta}}(x,x'|f) - m\delta(x-x')]\psi(x') \\
&\quad -(N/4)\int dx dx' \operatorname{tr}[\underset{\sim}{F}_{\mu\nu}(x,x'|h)\underset{\sim}{F}^{\mu\nu}(x',x|h)] \ , \\
N^{-1} &= \int dx [h(x)]^2 ,
\end{aligned}
\tag{6}
$$

where the normalization factor N ensures a correct local limit. Note that the form factors f and h, appearing in the fermion and gauge sectors respectively, are entirely independent. Henceforth, we shall take the local limit in the gauge sector, retaining only f.

The equations of motion are obtained from (6) by funcational differentiation with respect to ψ and A (while considering a field and its derivatives functionally dependent):

$$
\frac{\delta W}{\delta \bar{\psi}(\xi)} = 0, \ \int dx' [i \underset{\sim}{\not{\Delta}}(\xi,x'|f) - m\delta(\xi - x')]\psi(x') = 0,
\tag{7}
$$

$$
\frac{\delta W}{\delta \underset{\sim}{A}(\xi)} = 0, \ [\underset{\sim}{\Delta}^{\mu}, \underset{\sim}{F}_{\mu}{}^{\nu}(\xi)] = \underset{\sim}{j}^{\nu}(\xi) = \underset{\sim}{\lambda}_a j_a{}^{\nu}(\xi),
\tag{8}
$$

$$
j_{a\nu}(\xi) = \int dx dx' \bar{\psi}(x)[-\delta/\delta A_a{}^{\nu}(\xi)] i \underset{\sim}{\not{\Delta}}(x,x'|f)\psi(x'),
\tag{9}
$$

with another equation adjoint to (7). As expected, the current given by (9) is that which would be obtained as a consequence of gauge invariance. To verify its local conservation, i.e., $[\underset{\sim}{\Delta}^{\mu}, \underset{\sim}{j}_{\mu}(\xi)] = 0$, we use the formula [derivable from Eq. (4)]

$$
\begin{aligned}
&[\delta_{ac}\partial_{\xi}^{\nu} - ig f_{abc} A_b{}^{\nu}(\xi)]\delta/\delta A_c{}^{\nu}(\xi) \ [\underset{\sim}{\Delta}_{\mu}(x,x'|f)] \\
&= ig[\delta(x'-\xi)\underset{\sim}{\Delta}_{\mu}(x,x'|f)\underset{\sim}{\lambda}_a - \delta(x-\xi) \ \underset{\sim}{\lambda}_a \underset{\sim}{\Delta}_{\mu}(x,x'|f)],
\end{aligned}
\tag{10}
$$

which is a primitive form of the Ward-Takahashi identify (f_{abc} are the structure constants of the gauge group). Upon inserting (10) in (9), and using the adjoint of (7), current conservation follows. We note here that Eq. (9) for the current reduces in the local limit to the point-split definition (including the all-important line integral factor) first given by Schwinger for the Abelian case.[7]

Next we consider translational invariance. Imposing this invariance on W leads to the condition

$$[\partial^\mu{}_x\bar{\psi}(x)]\delta W/\delta\bar{\psi}(x) + \delta W/\delta\psi(x)\ [\partial^\mu{}_x\psi(x)] + \delta W/\delta A_a{}^\nu[\partial^\mu{}_x A_a{}^\nu(x)] = 0, \tag{11}$$

which is satisfied by virtue of the equations of motion. On the other hand, this equation is to imply the local conservation of an energy-momentum tensor $\Theta_{\mu\nu}$. After some search, one finds (for the Abelian case)

$$\begin{aligned}\Theta^{\mu\nu}(\xi) = -\tfrac{1}{2}\int dxdx' \int_{-\frac{1}{2}}^{+\frac{1}{2}} d\tau \\ &\{2r^\mu\delta(R+r\tau-\xi)\bar{\psi}(x)[\partial^\nu{}_r L(x,x')]\psi(x') \\ &-ig\partial^\mu_\xi[D^{(0)}(x-\xi)-D^{(0)}(x'-\xi)]\bar{\psi}(x) \\ &i\not{A}(x,x'|f)\psi(x')r^\alpha\partial^\nu_R A_\alpha(R+r\tau)\}\ , \\ L(x,x') &= i\not{\partial}(x,x')-m\delta(x-x'), \\ \partial^2{}_x D^{(0)}(x) &= \delta(x).\end{aligned} \tag{12}$$

This complicated expression (which is neither symmetric nor manifestly gauge invariant) in fact reduces in the local limit to the appropriate energy-momentum tensor (apart from a total divergence).

Are the gauge fields massless in NLGTs? Just as in the local theory, the answer depends upon the details of the dynamics.[8] However, given the assumptions needed to supplement the Ward-Takahashi identity for the usual "proof" of masslessness, the same result is true there as a consequence of Eq. (10). Since this is not an entirely expected result, we shall outline a proof for the Abelian case.

Consider the generating functional in the presence of an external current J^{μ}, with the spinor fields integrted out and some gauge condition imposed:

$$\begin{aligned} Z(J) &= \int[dA]\, \exp[\mathrm{Tr}(\ell nL) + iW_{G}], \\ W &= \int dx dx'[\tfrac{1}{2}G_{\mu\mu'}{}^{(0)-1}(x-x')A^{\mu}(x)A^{\mu'}(x')] \\ &\quad -\int dx A^{\mu}(x)J_{\mu}(x) \ , \end{aligned} \tag{13}$$

where $G^{(0)}(\bar{G})$ is the free (fully interacting) vector propagator, and where "Tr" denotes a generalized trace. Using partial functional integration, we obtrain from (13)

$$\begin{aligned} &\int d\xi G_{\alpha\beta}{}^{(0)-1}(\xi-\xi')\,\langle A^{\beta}(\xi')\rangle = J_{\alpha}(\xi) + \langle j_{\alpha}(\xi)\rangle, \\ &\langle A_{\mu}(\xi)\rangle = [i\delta/\delta J^{\mu}(\xi)]\ell nZ(J), \\ &\langle j_{\alpha}(\xi)\rangle = i\int d[A]\exp[\mathrm{Tr}(\ell nL)+iW_{G}][\delta/\delta A^{\alpha}(\xi)]\ \mathrm{Tr}(\ell nL)/Z(J), \end{aligned} \tag{14}$$

where the latter is the current induced by J through vacuum polarization.

To verify the transversality of the induced current, we use Eq. (10) to deduce that

$$\begin{aligned} &\partial^{\alpha}_{\xi}\ [\delta/\delta A^{\alpha})]\mathrm{Tr}(\ell nL) \\ &\qquad = \mathrm{tr}\Big\{\int dx dx' L^{-1}(x,x')ig[\delta(x'-\xi)-\delta(x-\xi)] \\ &\qquad\quad L(x',x)\Big\} = 0, \end{aligned} \tag{15}$$

where "tr" refers to the (discrete) indices only. Upon substituting (15) in (14), we arrive at the transversality of the induced current and therefore of the polarization tensor $\pi = G^{-1}-G^{(0)-1}$ as well. On the other hand, using (3) we may write

$$[\delta/\delta A_\alpha(\xi)]\mathrm{Tr}(\ell n L) = ig\int dx dx' \int_{-\frac{1}{2}}^{+\frac{1}{2}} d\tau\, r^\alpha \delta(R+r\tau-\xi)$$
$$\mathrm{tr}[i\not{A}(x,x')L^{-1}(x',x)], \tag{16}$$

and deduce therefrom the result

$$\int d\xi [\delta/\delta A_\alpha(\xi)]\mathrm{Tr}(\ell n L)$$
$$= ig\ \mathrm{tr}\left\{ \int dx dx' r^\alpha [L(x,x')+m\delta(x-x')]L^{-1}(x',x)\right\}, \tag{17}$$

which vanishes by virtue of the equations of motion. But then $\int d\xi \langle j\ (\xi)\rangle = 0$, hence the vanishing of the polarization tensor in the limit of zero momentum transfer. The latter in turn implies the masslessness of the gauge field. We repeat that just as in the local case, this argument is subject to the proviso that the above operations are not invalidated by "anomalies" (such as the Schwinger mechanism).

We now briefly consider perturbation theory and possible divergences (for the Abelian case). The new topological feature for NLGTs is the possibility of multiple emission, or the emergence of more than one gauge propagator, from a given vertex. Thus for an nth-order vertex, with the jth gauge propagator carrying the momentum q_j, the vertex function will be

$$\Gamma_{\mu_1\ldots\mu_n}(k',k|q_1\ldots q_n) = \int_{-\frac{1}{2}}^{+\frac{1}{2}} d\tau_1\ldots d\tau_n\, \partial_{\mu_1}\ldots\partial_{\mu_n} \not{s}\hat{f}(s),$$
$$s = k + \Sigma(\tfrac{1}{2}+\tau_j)q_j, \tag{18}$$

where k and k′ are the fermion momenta obeying the conservation condition $k'-k = \Sigma q_j$. The second modification in perturbation theory occurs in the fermion propagator, now given by $[\hat{f}(p)\not{p}-m]^{-1}$. A simple power counting then shows that multiple exchanges occurring in perturbation contributions do not change the degree of divergence relative to single ones, except when there is reabsorption at the emitting vertex. But the latter are easily shown to contribute constant terms to the phase of the fermion propagator and are therefore removable.[5] Restricting our attention to simple vertices then, we observe that the ultraviolet behavior of the perturbation expansion is determined by the behavior of $\hat{f}(p)$ for large (space-like) p.[10] Indeed if $p\hat{f}(p)$ dominates m for large p (which is always the case if m=0), then the worst ultraviolet behavior will differ from the local theory by factors of magnitude $[\partial/\partial\ell n|p|]\ell n\hat{f}(p)$, which for reasonably well-behaved $\hat{f}(p)$ will be O(1). Hence we expect an ultraviolet behavior similar to that of the local theory in this case. If the opposite case occurs (implying a nonvanishing m), say $|p|\hat{f}(p) = O(|p|^{-\mu})$, $\mu > 0$, then the resulting ultraviolet behavior will be modified by $O(|p|^{-\ \mu})$ relative to the local theory, where N is the number of (simple) vertices in the perturbation term under consideration. This is sufficient to remove all primitive logarithmic divergences,[11] hopefully rendering the perturbation expansion ultraviolet-finite. Although this argument does not constitute a rigorous proof, comparison with the local theory and sample calculations render it fairly convincing.

The improved ultraviolet behavior that occurs for $m \neq 0$ and f subject to the condition stated above obviously has profound consequences for applications to particle interactions. We have already mentioned the absence of the ABJ anomaly,[4] a fact which is verified by explicit calculation. Curiously, axial current conservation is nevertheless always spoiled, canonically when $m \neq 0$, and anomalously when $m=0$. The important difference remains, however, that in the case of NLGTs (for $m \neq 0$) the symmetry breaking term is proportional to m and presumably controlled by it.

Finally, we note that the criterion of the vanishing of $p\hat{f}(p)$ necessary for improved (probably finite) ultraviolet behavior of NLGTs also implies the (perturbative) existence of at least one fermion species of higher mass than the "low-energy" one of (perturbative) mass $\approx$ m. This linkage is obviously suggestive of a cancellation mechanism effected by the sister fermion of higher mass.

Clearly we have uncovered only the bare essentials of NLGTs here. To that extent, they appear promising and worthy of further study.

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9. A particularly transparent way to see this is to consider the fermion propagator interacting with an external potential, and expand not in A but in the exponential of A as occurs in the covariant derivative. All interaction terms then occur as the exponential of the sum of elementary current-current interactions, with the interaction of a given current with itself (a divergent constant) being the exponentiated form of the divergent terms mentioned in the text.
10. This behavior will be required to be no worse than O(1).

11. The usual quadratic divergence in the polarization tensor is automatically cured in the NLGT thanks to the presence of the line integral factor[7].