

QUARK GEOMETRY ACCOMMODATING SIX FLAVORS

AS WELL AS *CBT*-FLAVOR MIXTURES (MARIONS)*

PART I: *CBT* - DIAGRAMS COUNTERCLOCKWISE

A. E. Schinn
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

The $SU(4)$ weight diagrams for mesons and baryons ($J^P = 1/2^+$ and $J^P = 3/2^+$) are extended to include b - and t -quark flavors as well as mixed *cbt*-flavors yielding hypothetical particles for which the name marions is proposed.

Submitted to Physical Review D

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

I. INTRODUCTION

The purpose of this paper is to show all possible mathematical combinations of quark flavors and to outline these combinations with hypothetical particles geometrically. Mesons of the $q\bar{q}$ - and baryons of the qqq -type are described and it is assumed the top-flavor is meaningful.

The number of mathematically derivable quark combinations of the $q\bar{q}$ -type is 36. The $SU(4)$ geometry covers 16 of these, the remaining 20 fall into areas of b - and t -flavored combinations and of cbt -mixed flavored marions. The qqq -baryons are represented mathematically by 50 or 56 combinations for baryons based on the $J^P = 1/2^+$ octet and for baryons based on the $J^P = 3/2^+$ decuplet, respectively. The $SU(4)$ weight diagram for the $J^P = 1/2^+$ baryons houses 16 combinations, the remaining 34 belong to b - and t -flavored and to the marion diagrams. $SU(4)$ of the $J^P = 3/2^+$ contains 20 combinations, the missing 36 labels are again accounted for by bottom- and top-flavored combinations and by marions. Figure 1 is a histogram of the above quark configurations.

Particle physics is a rather young science. The first baryon was introduced with the identification of the proton in 1919, while the first meson, the π , has been brought to light in 1947.¹ The "eightfold way" appeared in 1961, and the success of the $SU(3)$ concept soon led to the hypothesis of quarks, of three quarks. The need for another quark, an additional quantum number and an enlargement of the symmetry group from $SU(3)$ to $SU(4)$ became apparent soon. This paper describes a further development of the $SU(4)$ geometry to include the full set of six quark flavors. The "bottom" number is indicated here as B^b while the "top" number is written T^t . The flavor superscripts have been added to

distinguish the two new quantum numbers from the established baryon number, B, and from the T, occasionally found used in the literature to designate isospin, respectively.

There are two possible formats the b - and t -structures can be shown geometrically, one is having the bottom- and top-flavors layed out in an increasing weight sequence analogous to the layout of the up- and down-flavors (uds -parallel) which is denoted here by a "counterclockwise" sense. The other format (uds -antiparallel) shows the b - and t -flavors opposite to the sense of increasing weight of the u - and d -flavors.² The geometries are similar. The " cbt -diagrams counterclockwise" geometry is the subject of this paper, the " cbt -diagrams clockwise" geometry will be described in a separate paper. (See Fig. 2.)

II. COMBINATORIAL ANALYSIS

The number of permutations of n elements selected from m elements with repetition is

$$\text{PERMUTATION}^{(r)} \quad {}_m P_n^{(r)} = m^n, \quad (1)$$

while the number of permutations of n elements selected from m elements without repetition is

$$\text{PERMUTATION} \quad {}_m P_n = \frac{m!}{(m-n)!} \quad (2)$$

The number of combinations of n elements selected from m elements without repetition is

$$\text{COMBINATION} \quad {}_m C_n = \frac{m!}{(m-n)! n!} \quad (3)$$

With six quark flavors ($m=6$) the possible number of permutations with repetition of $q\bar{q}$ mesons is $m^n = 6^2 = 36$ [Eq. (1)]. The possible number of permutations with one flavor present only ($n=1$) is ${}_6P_1 = 720/120 = 6$ [Eq. (2)], the number of permutations using two flavors ($n=2$) is ${}_6P_2 = 720/24 = 30$ [Eq. (2)]. The possible $q\bar{q}$ permutations are listed in Table I. For the qqq baryons $m^n = 6^3 = 216$, a majority of 160 of which are repetitions. The valid combinations are "alphabetized" by increasing quark weight, double flavor listings having priority (ddu instead of udd, etc.). The possible number of permutations with one flavor present is again 6, as two flavors also yield 30 permutations. The number of combinations with three flavors ($n=3$) without repetition is ${}_6C_3 = 720/6 \times 6 = 20$ [Eq. (3)]. The possible qqq permutations and combinations are listed in Table II.

III. CBT - GEOMETRY

The transition from the $SU(4)$ geometry to the $SU(6)$ -plus-marion geometry is temptingly straightforward. New quantum numbers are called for, such as a B^b for bottom or beauty, and a T^t for top or truth. The old hypercharge equation has to tolerate another modification and continues to remind of something like a magic formula, permitting to hold on to, say, as to a well proven date reckoning.

Considering the following quantum numbers,

Quark	Q	I ₃	S	C	I	Y	B ^b	T ^t	Y _M
u	2/3	1/2	0	0	1/2	1/3	0	0	1/3
d	-1/3	-1/2	0	0	1/2	1/3	0	0	1/3
s	-1/3	0	-1	0	0	-2/3	0	0	-2/3
c	2/3	0	0	1	0	-2/3	0	0	1/3
t	2/3	0?	0	0	?	-2/3	0	1	-2/3
b	-1/3	0?	0	0	?	-2/3	-1	0	-2/3
\bar{u}	-2/3	-1/2	0	0	1/2	-1/3	0	0	-1/3
\bar{d}	1/3	1/2	0	0	1/2	-1/3	0	0	-1/3
\bar{s}	1/3	0	1	0	0	2/3	0	0	2/3
\bar{c}	-2/3	0	0	-1	0	2/3	0	0	-1/3
\bar{t}	-2/3	0?	0	0	?	2/3	0	-1	2/3
\bar{b}	1/3	0?	0	0	?	2/3	1	0	2/3

where the *b*- and *t*-flavored isospin values need experimental exploration, the new formulas for hypercharge appear as follows:

$$Y = B + S - C + B^b - T^t \quad (4)$$

or

$$Y = 2(Q - (I_3 + C + T^t)) \quad (5)$$

and

$$Q = I_3 + \frac{1}{2} (B + S + C + B^b + T^t) \quad (6)$$

The marions seem to obey their own hypercharge equation:

$$Y_M = B + S + B^b - T^t \quad (7)$$

A few explaining remarks about the weight diagrams should help clarify their seemingly confusing perspectives. Figure 3 shows the SU(4) weight diagram of the vector meson mixed 16-plet redesigned to encompass all mathematically possible flavor combinations. The layouts of the added flavors, *b* and *t*, follow the charm diagram layout. The F particles are differentiated by flavor subscripts, the charges for both the *c*- and *t*-flavored F particles are identical, while the F_b particle should be recognized as a neutral particle. The equivalents to the charmed D have

received the letters B and T for the b - and t -flavored particles, respectively.

Figure 4 shows the six possible marions which, while all carrying the letter M, differ by subscripts identifying their wave functions. Figure 5 illustrates the SU(3) slices of the full-flavor weight diagram. The marions are tentatively provided with a defined vertical scale, a Y_M scale, only. A possible I_3 scale is still undefined. The M numbers along the vertical scales serve no other purpose yet as to oblige the formula³

$$M = C + B^b + T^t \quad . \quad (8)$$

Figures 6 through 8 are extensions of the baryon octet with spin and parity $J^P = 1/2^+$. In Fig. 7 the front structure (charm) is omitted to allow a better view of the marion structure, whose particles have been labeled with the upper case greek M (mu) with quark content subscripts.

Figures 9 through 11 then are expansions of the baryon decuplet with $J^P = 3/2^+$, where Fig. 10 shows a detailed view of the marions by omitting the front, or charm, pyramid. The set of symbols used in Figs. 3 through 11 is, where applicable, that of Lichtenberg,⁴ but uses subscripts for charmed, bottom and top mesons and baryons.

IV. CONCLUSIONS

This paper apparently has a two-fold purpose. It serves to demonstrate the feasibility of combining existing quark models such as the three dimensional SU(4) weight diagrams both for mesons and baryons with two additional dimensions. The required geometry follows symmetrical patterns and readily accepts all mathematically derivable combinations of quarks into logical locations which hopefully will prove agreeable to

both the mathematician and the physicist. The second point of the paper is to serve as yet another support for the quark concept.

V. ACKNOWLEDGEMENTS

I would like to thank the Head of the Electronics Division at SLAC, Ray Larsen, whose understanding helped make writing this paper possible. I am indebted to Dave Fryberger for informative discussions and comments. Eugene Cisneros supplied helpful criticism and encouragement. My special thanks go to Marion Schinn for giving me incentive and the permission to borrow her name. This work was supported by the Department of Energy, contract DE-AC03-76SF00515.

REFERENCES

1. For a review see P. Marmier and E. Sheldon, "Physics of Nuclei and Particles," Academic Press, New York and London (1969).
2. See D. Carydas and D. B. Lichtenberg, "Is the d quark heavier than the u quark?" J. Phys. G: Nucl. Phys. 7, 1345-1348 (1981).
3. An anticipated usefulness of the M quantity, probably, when defining the horizontal scale for the marions, could be called a second reason for recording Eq. (8). A third justification, admittedly, is a motivation to extoll Marion, the author's daughter, herself a pleasant combination of charm, beauty and truth.
4. See D. B. Lichtenberg, "Unitary Symmetry and Elementary Particles," Academic Press, New York, San Francisco and London (1978).

TABLE I
Possible $q\bar{q}$ Permutations

Quark Flavors	6^P_1	6^P_2	Quark Flavors	6^P_1	6^P_2
$u\bar{u}$	1	-	$c\bar{u}$	-	16
$u\bar{d}$	-	1	$c\bar{d}$	-	17
$u\bar{s}$	-	2	$c\bar{s}$	-	18
$u\bar{c}$	-	3	$c\bar{c}$	4	-
$u\bar{b}$	-	4	$c\bar{b}$	-	19
$u\bar{t}$	-	5	$c\bar{t}$	-	20
$d\bar{u}$	-	6	$b\bar{u}$	-	21
$d\bar{d}$	2	-	$b\bar{d}$	-	22
$d\bar{s}$	-	7	$b\bar{s}$	-	23
$d\bar{c}$	-	8	$b\bar{c}$	-	24
$d\bar{b}$	-	9	$b\bar{b}$	5	-
$d\bar{t}$	-	10	$b\bar{t}$	-	25
$s\bar{u}$	-	11	$t\bar{u}$	-	26
$s\bar{d}$	-	12	$t\bar{d}$	-	27
$s\bar{s}$	3	-	$t\bar{s}$	-	28
$s\bar{c}$	-	13	$t\bar{c}$	-	29
$s\bar{b}$	-	14	$t\bar{b}$	-	30
$s\bar{t}$	-	15	$t\bar{t}$	6	-

TABLE II

Possible qq̄q Permutations and Combinations (r: repetitions)

Quark Flavors	6^P_1	6^P_2	6^C_3	r
uuu	1	-	-	-
uud	-	1	-	-
uus	-	2	-	-
uuc	-	3	-	-
uub	-	4	-	-
uut	-	5	-	-
udu	-	-	-	P
udd	-	-	-	P
uds	-	-	1	-
udc	-	-	2	-
udb	-	-	3	-
udt	-	-	4	-
usu	-	-	-	P
usd	-	-	-	C
uss	-	-	-	P
usc	-	-	5	-
usb	-	-	6	-
ust	-	-	7	-

Quark Flavors	6^P_1	6^P_2	6^C_3	r
ucu	-	-	-	P
ucd	-	-	-	C
ucs	-	-	-	C
ucc	-	-	-	P
ucb	-	-	8	-
uct	-	-	9	-
ubu	-	-	-	P
ubd	-	-	-	C
ubs	-	-	-	C
ubc	-	-	-	C
ubb	-	-	-	P
ubt	-	-	10	-
utu	-	-	-	P
utd	-	-	-	C
uts	-	-	-	C
utc	-	-	-	C
utb	-	-	-	C
utt	-	-	-	P

Quark Flavors	6^P_1	6^P_2	6^C_3	r
duu	-	-	-	P
dud	-	-	-	P
dus	-	-	-	C
duc	-	-	-	C
dub	-	-	-	C
dut	-	-	-	C
ddu	-	6	-	-
ddd	2	-	-	-
dds	-	7	-	-
ddc	-	8	-	-
ddb	-	9	-	-
ddt	-	10	-	-
dsu	-	-	-	C
dsd	-	-	-	P
dss	-	-	-	P
dsc	-	-	11	-
dsb	-	-	12	-
dst	-	-	13	-

Quark Flavors	6^P_1	6^P_2	6^C_3	r
<i>dcu</i>	-	-	-	C
<i>dcd</i>	-	-	-	P
<i>dcs</i>	-	-	-	C
<i>dcc</i>	-	-	-	P
<i>dcb</i>	-	-	14	-
<i>dct</i>	-	-	15	-
<hr/>				
<i>dbu</i>	-	-	-	C
<i>dbd</i>	-	-	-	P
<i>dfs</i>	-	-	-	C
<i>dbc</i>	-	-	-	C
<i>dbb</i>	-	-	-	P
<i>dbt</i>	-	-	16	-
<hr/>				
<i>dtu</i>	-	-	-	C
<i>dtb</i>	-	-	-	P
<i>dfs</i>	-	-	-	C
<i>dte</i>	-	-	-	C
<i>dtb</i>	-	-	-	C
<i>dtb</i>	-	-	-	C
<i>dtb</i>	-	-	-	P

Quark Flavors	6^P_1	6^P_2	6^C_3	r
<i>suu</i>	-	-	-	P
<i>sud</i>	-	-	-	C
<i>sus</i>	-	-	-	P
<i>suc</i>	-	-	-	C
<i>sub</i>	-	-	-	C
<i>sut</i>	-	-	-	C
<hr/>				
<i>sdu</i>	-	-	-	C
<i>sdd</i>	-	-	-	P
<i>sds</i>	-	-	-	P
<i>sdc</i>	-	-	-	C
<i>sdb</i>	-	-	-	C
<i>sdt</i>	-	-	-	C
<hr/>				
<i>ssu</i>	-	11	-	-
<i>ssd</i>	-	12	-	-
<i>sss</i>	3	-	-	-
<i>ssc</i>	-	13	-	-
<i>ssb</i>	-	14	-	-
<i>sst</i>	-	15	-	-

Quark Flavors	6^P_1	6^P_2	6^C_3	r
<i>scu</i>	-	-	-	C
<i>scd</i>	-	-	-	C
<i>scs</i>	-	-	-	P
<i>scc</i>	-	-	-	P
<i>scb</i>	-	-	17	-
<i>sct</i>	-	-	18	-
<hr/>				
<i>sbu</i>	-	-	-	C
<i>sbd</i>	-	-	-	C
<i>sbs</i>	-	-	-	P
<i>sbc</i>	-	-	-	C
<i>sbb</i>	-	-	-	P
<i>sbt</i>	-	-	19	-
<hr/>				
<i>stu</i>	-	-	-	C
<i>std</i>	-	-	-	C
<i>sts</i>	-	-	-	P
<i>stc</i>	-	-	-	C
<i>stb</i>	-	-	-	C
<i>stt</i>	-	-	-	P

Quark Flavors	6^P_1	6^P_2	6^C_3	r
<i>cuu</i>	-	-	-	P
<i>cud</i>	-	-	-	C
<i>cus</i>	-	-	-	C
<i>cuc</i>	-	-	-	P
<i>cub</i>	-	-	-	C
<i>cut</i>	-	-	-	C
<i>cdu</i>	-	-	-	C
<i>cdd</i>	-	-	-	P
<i>cds</i>	-	-	-	C
<i>cdc</i>	-	-	-	P
<i>cdb</i>	-	-	-	C
<i>cdt</i>	-	-	-	C
<i>csu</i>	-	-	-	C
<i>csd</i>	-	-	-	C
<i>css</i>	-	-	-	P
<i>csc</i>	-	-	-	P
<i>csb</i>	-	-	-	C
<i>cst</i>	-	-	-	C

Quark Flavors	6^P_1	6^P_2	6^C_3	r
<i>ccu</i>	-	16	-	-
<i>ccd</i>	-	17	-	-
<i>ccs</i>	-	18	-	-
<i>ccc</i>	4	-	-	-
<i>ccb</i>	-	19	-	-
<i>cct</i>	-	20	-	-
<i>cbu</i>	-	-	-	C
<i>cbd</i>	-	-	-	C
<i>cbs</i>	-	-	-	C
<i>cbc</i>	-	-	-	P
<i>cbb</i>	-	-	-	P
<i>cbt</i>	-	-	20	-
<i>ctu</i>	-	-	-	C
<i>ctd</i>	-	-	-	C
<i>dts</i>	-	-	-	C
<i>ctc</i>	-	-	-	P
<i>ctb</i>	-	-	-	C
<i>ctt</i>	-	-	-	P

Quark Flavors	6^P_1	6^P_2	6^C_3	r
<i>buu</i>	-	-	-	P
<i>bud</i>	-	-	-	C
<i>bus</i>	-	-	-	C
<i>buc</i>	-	-	-	C
<i>bub</i>	-	-	-	P
<i>but</i>	-	-	-	C
<i>bdu</i>	-	-	-	C
<i>bdd</i>	-	-	-	P
<i>bds</i>	-	-	-	C
<i>bdc</i>	-	-	-	C
<i>bdb</i>	-	-	-	P
<i>bdt</i>	-	-	-	C
<i>bsu</i>	-	-	-	C
<i>bsd</i>	-	-	-	C
<i>bss</i>	-	-	-	P
<i>bsc</i>	-	-	-	C
<i>bsb</i>	-	-	-	P
<i>bst</i>	-	-	-	C

Quark Flavors	6^P_1	6^P_2	6^C_3	r
<i>bcu</i>	-	-	-	C
<i>bcd</i>	-	-	-	C
<i>bcs</i>	-	-	-	C
<i>bcc</i>	-	-	-	P
<i>bcb</i>	-	-	-	P
<i>bct</i>	-	-	-	C
<hr/>				
<i>bbu</i>	-	21	-	-
<i>bbd</i>	-	22	-	-
<i>bbs</i>	-	23	-	-
<i>bbc</i>	-	24	-	-
<i>bbb</i>	5	-	-	-
<i>bbt</i>	-	25	-	-
<hr/>				
<i>btu</i>	-	-	-	C
<i>bt\bar{d}</i>	-	-	-	C
<i>bts</i>	-	-	-	C
<i>btc</i>	-	-	-	C
<i>btb</i>	-	-	-	P
<i>btt</i>	-	-	-	P

Quark Flavors	6^P_1	6^P_2	6^C_3	r
<i>tuu</i>	-	-	-	P
<i>tud</i>	-	-	-	C
<i>tus</i>	-	-	-	C
<i>tuc</i>	-	-	-	C
<i>tub</i>	-	-	-	C
<i>tut</i>	-	-	-	P
<hr/>				
<i>tdu</i>	-	-	-	C
<i>tdd</i>	-	-	-	P
<i>tds</i>	-	-	-	C
<i>tdc</i>	-	-	-	C
<i>tdb</i>	-	-	-	C
<i>tdt</i>	-	-	-	P
<hr/>				
<i>tsu</i>	-	-	-	C
<i>tsd</i>	-	-	-	C
<i>tss</i>	-	-	-	P
<i>tsc</i>	-	-	-	C
<i>tsb</i>	-	-	-	C
<i>tst</i>	-	-	-	P

Quark Flavors	6^P_1	6^P_2	6^C_3	r
<i>tcu</i>	-	-	-	C
<i>tcd</i>	-	-	-	C
<i>tcs</i>	-	-	-	C
<i>tcc</i>	-	-	-	P
<i>tcb</i>	-	-	-	C
<i>tct</i>	-	-	-	P
<hr/>				
<i>tbu</i>	-	-	-	C
<i>tbd</i>	-	-	-	C
<i>tbs</i>	-	-	-	C
<i>tbc</i>	-	-	-	C
<i>tbb</i>	-	-	-	P
<i>tbt</i>	-	-	-	P
<hr/>				
<i>ttu</i>	-	26	-	-
<i>ttd</i>	-	27	-	-
<i>tts</i>	-	28	-	-
<i>ttc</i>	-	29	-	-
<i>ttb</i>	-	30	-	-
<i>ttt</i>	6	-	-	-

FIGURE CAPTIONS

- Fig. 1. Histogram of full flavored quark configurations. (The assignment of the b^- and t^- , and of the marion-flavored combinations to spin and parity groups is tentative.)
- Fig. 2. cbt -diagrams counterclockwise and clockwise.
- Fig. 3. Full-flavor weight diagram based on the vector meson nonet with spin and parity $J^P = 1^-$.
- Fig. 4. Weight diagram of the cbt -mixed flavor mesons (marion mesons).
- Fig. 5. $SU(3)$ slices of the full flavor weight diagram based on the pseudoscalar meson nonet with $J^P = 0^-$.
- Fig. 6. Full-flavor weight diagram based on the baryon octet with $J^P = 1/2^+$.
- Fig. 7. Weight diagram of marion baryons based on $J^P = 1/2^+$.
- Fig. 8. $SU(3)$ slices of the full-flavor weight diagram based on the baryon octet with $J^P = 1/2^+$.
- Fig. 9. Full-flavor weight diagram based on the baryon decuplet with $J^P = 3/2^+$.
- Fig. 10. Weight diagram of marion baryons based on $J^P = 3/2^+$.
- Fig. 11. $SU(3)$ slices of full-flavor weight diagram based on the baryon decuplet with $J^P = 3/2^+$.

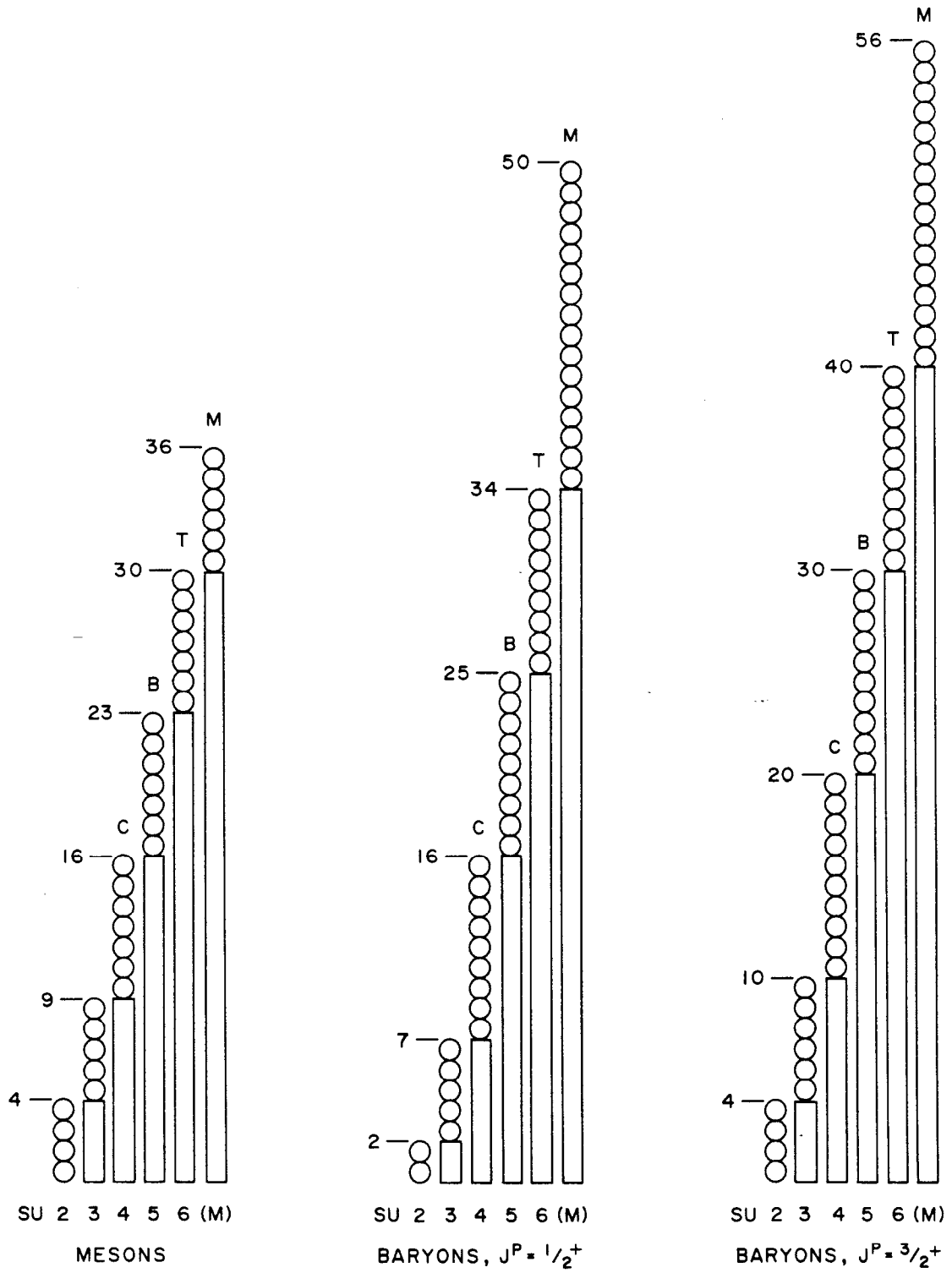


Fig. 1

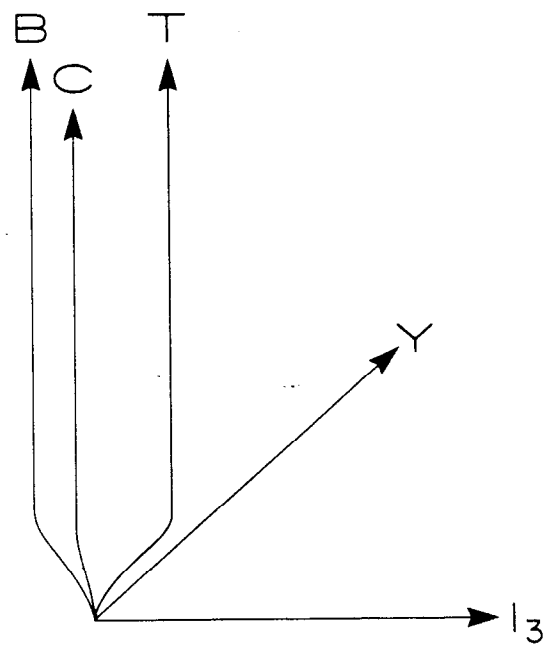
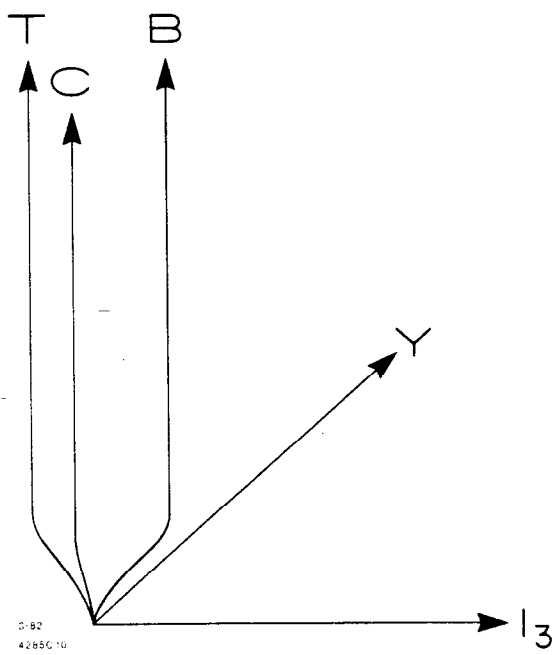


Fig. 2

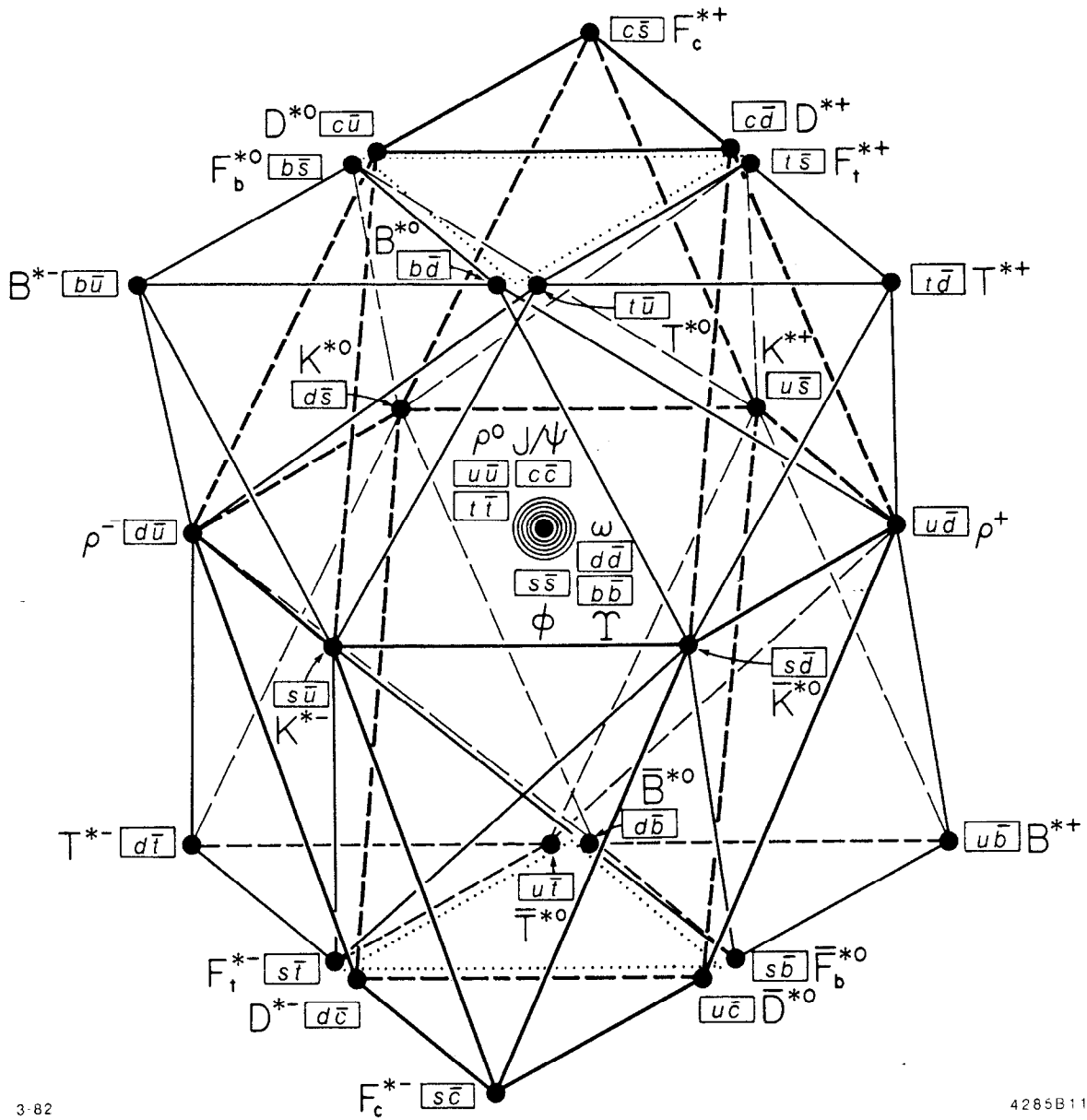


Fig. 3

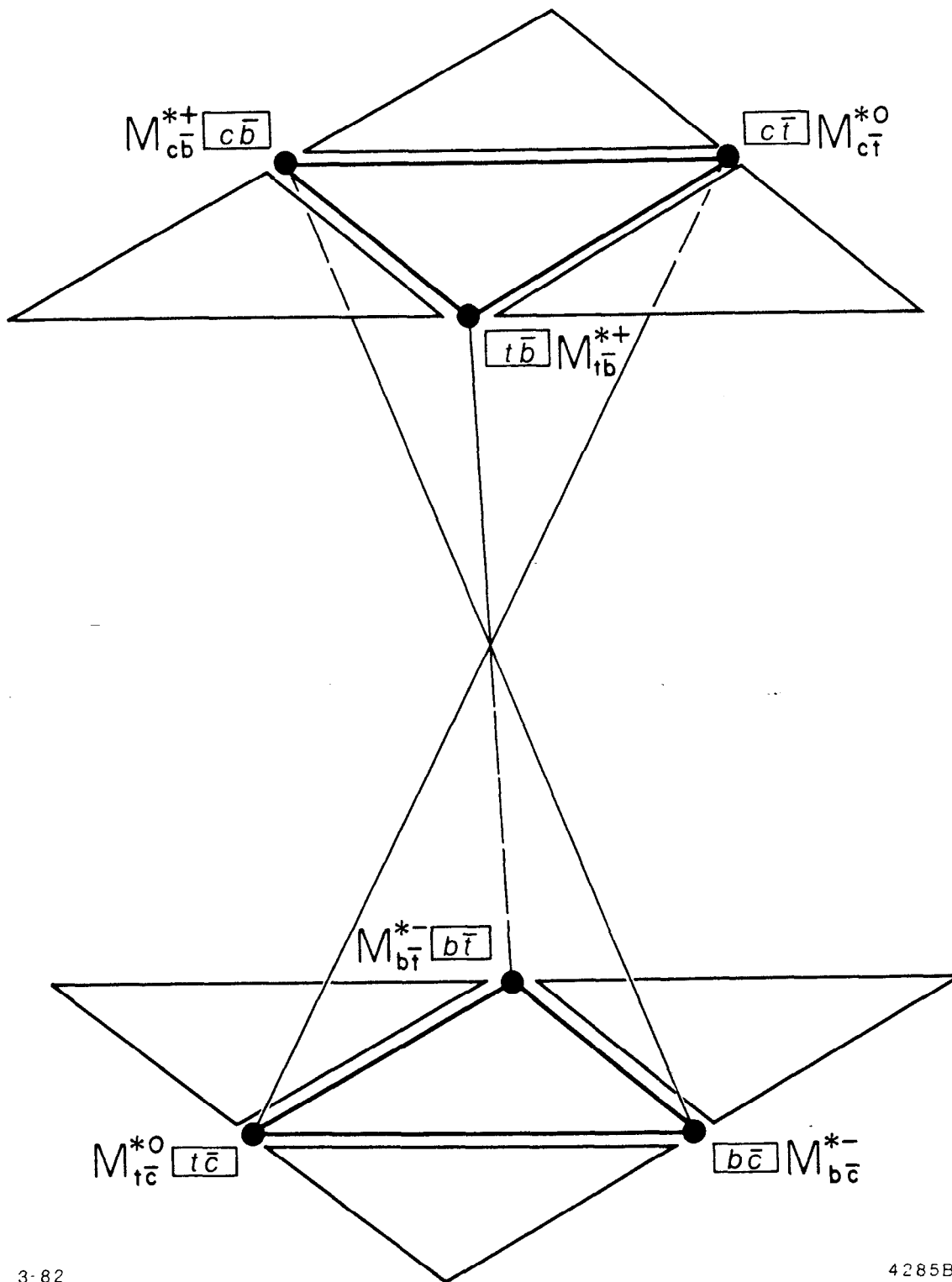


Fig. 4

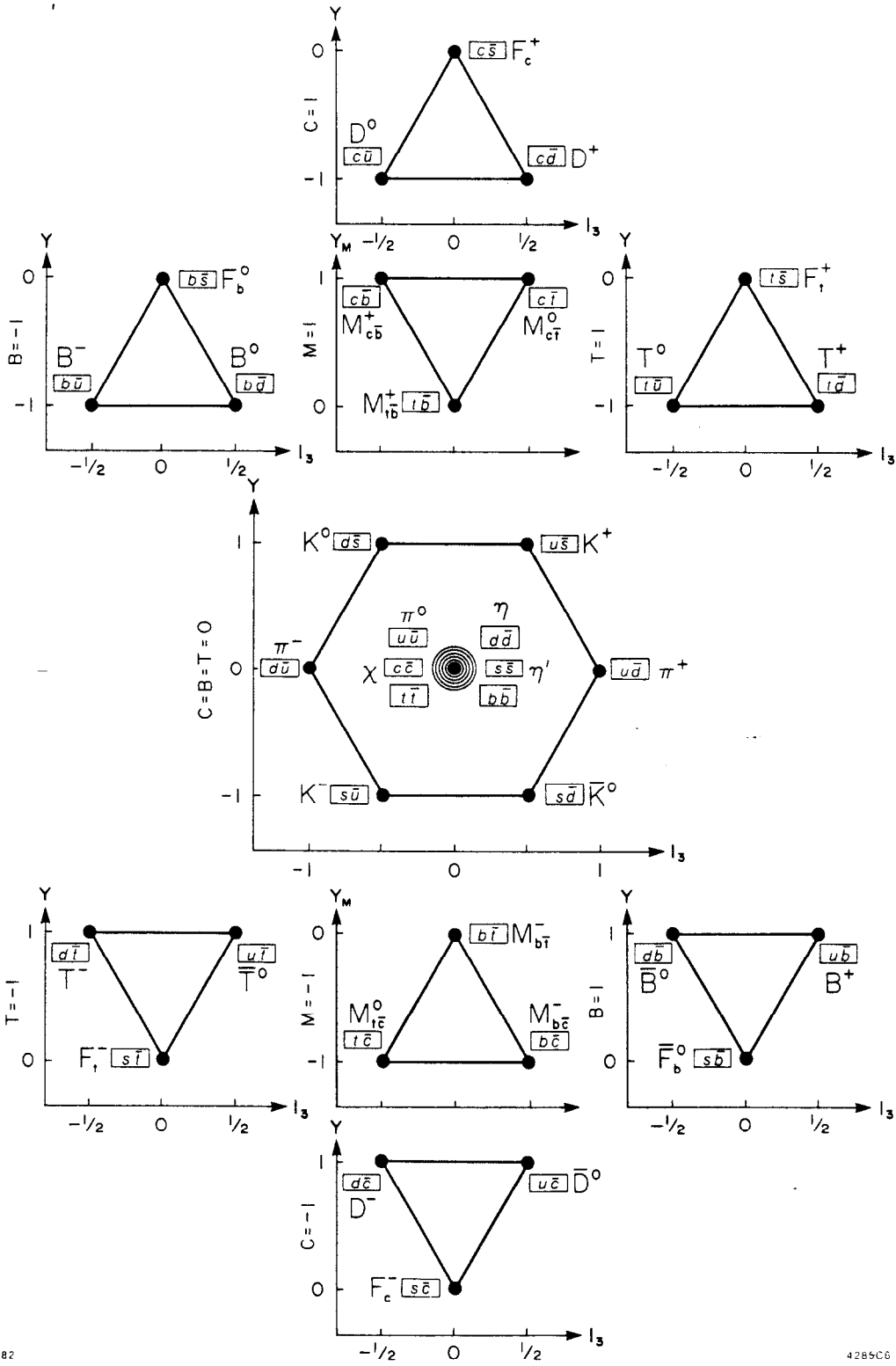


Fig. 5

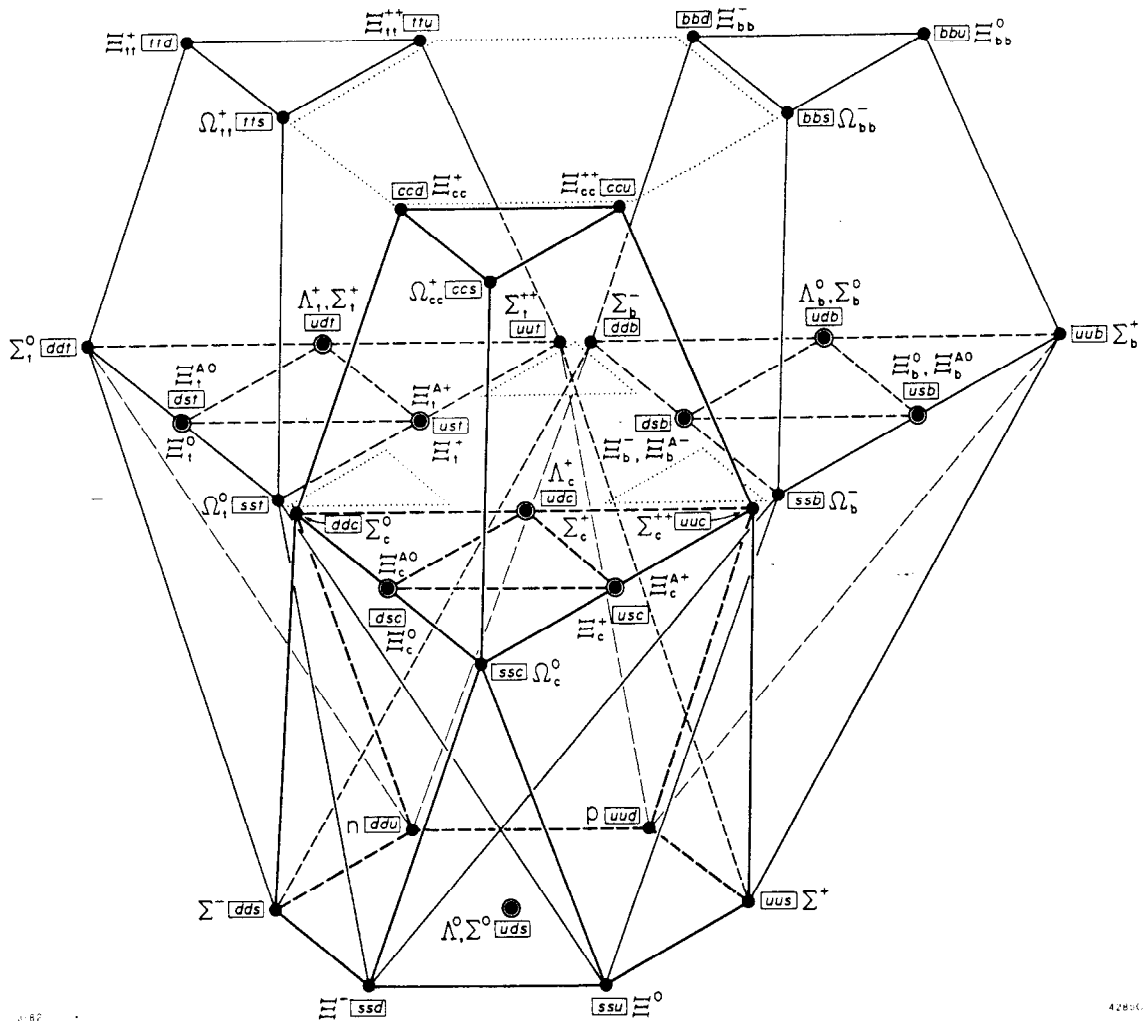


Fig. 6

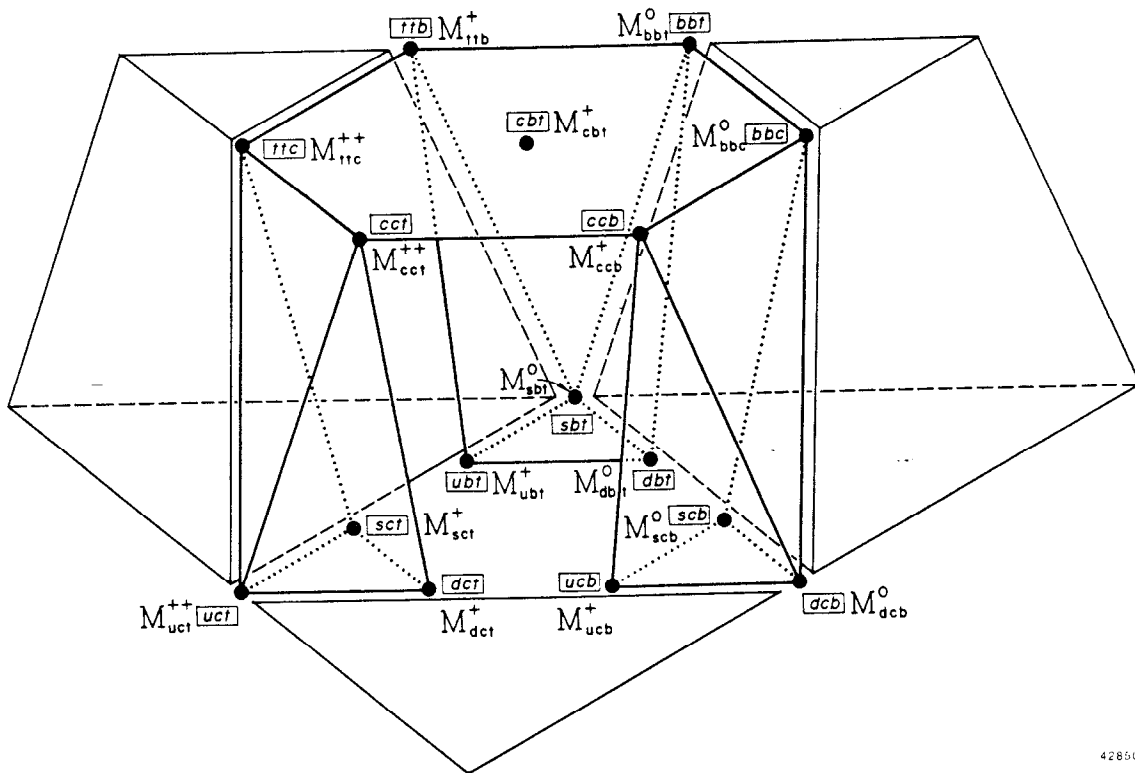


Fig. 7

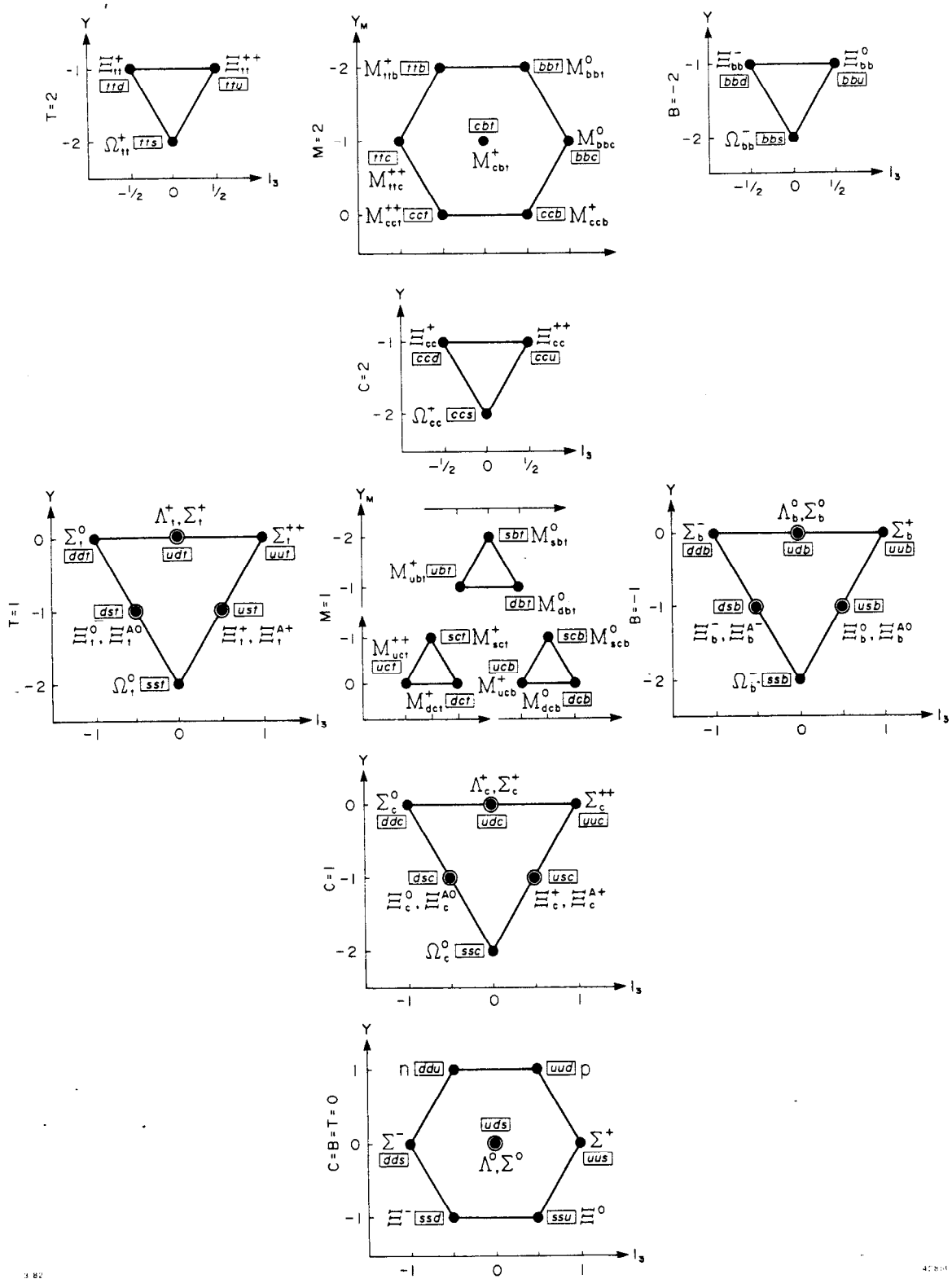


Fig. 8

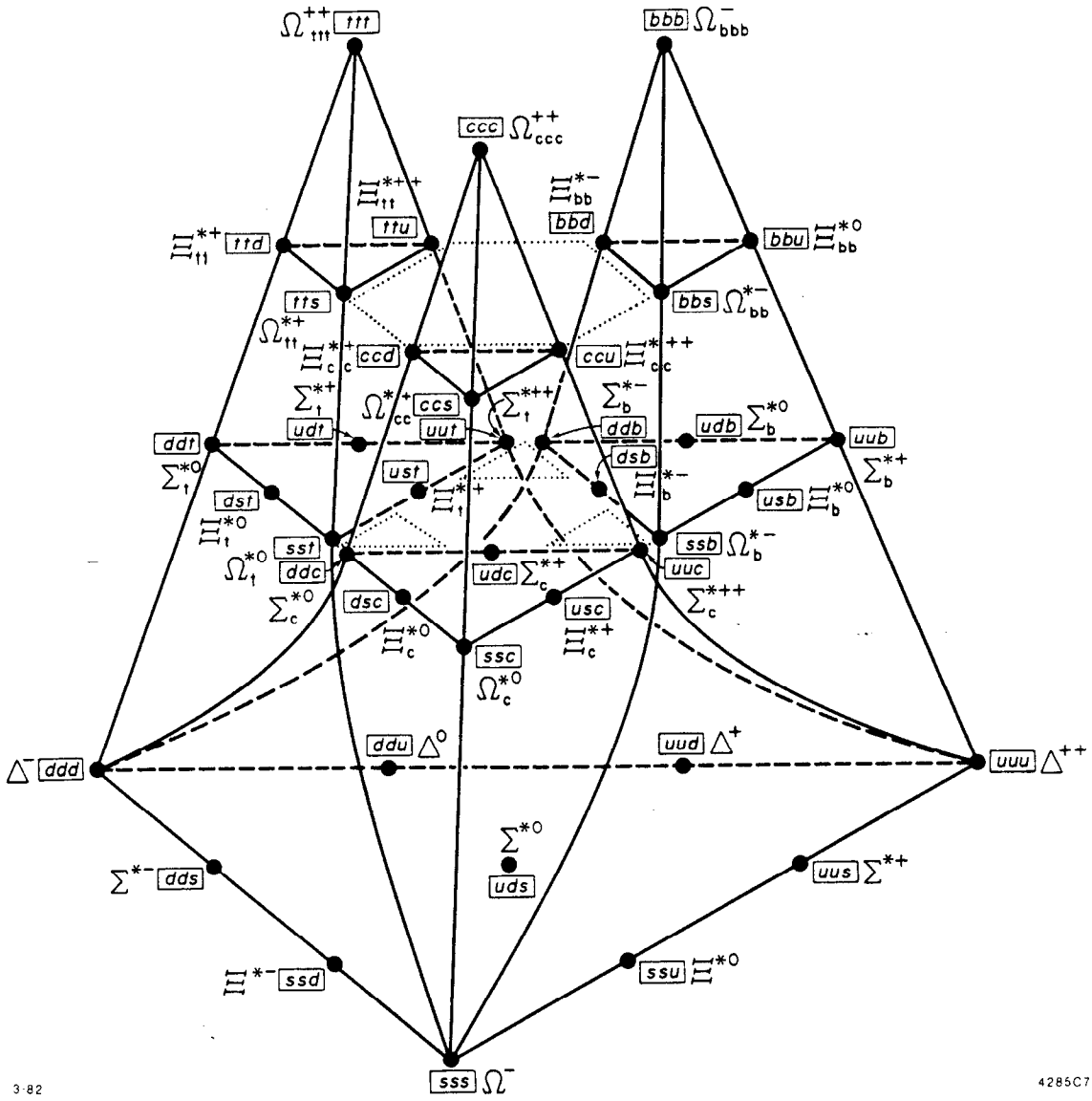


Fig. 9

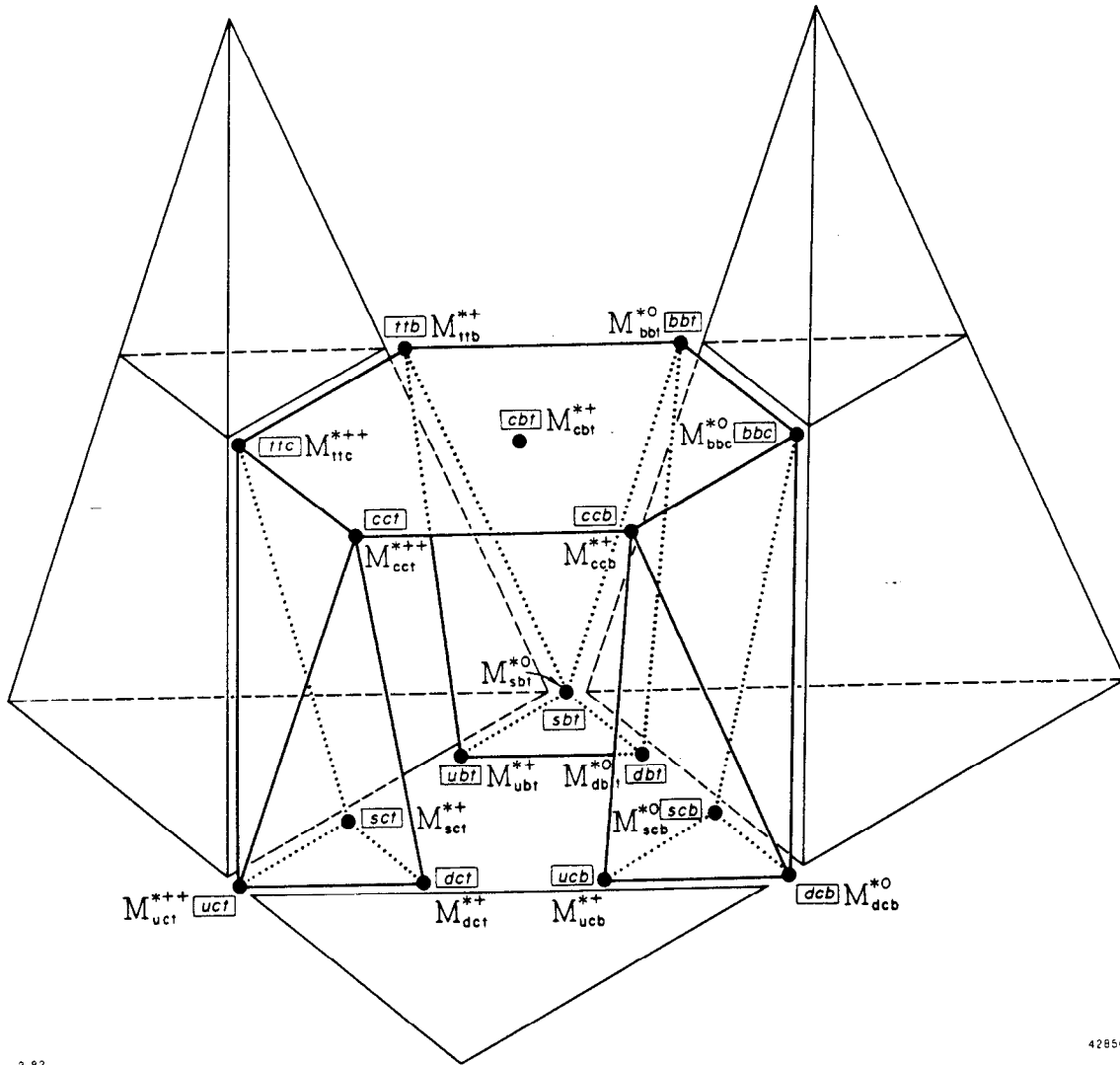


Fig. 10

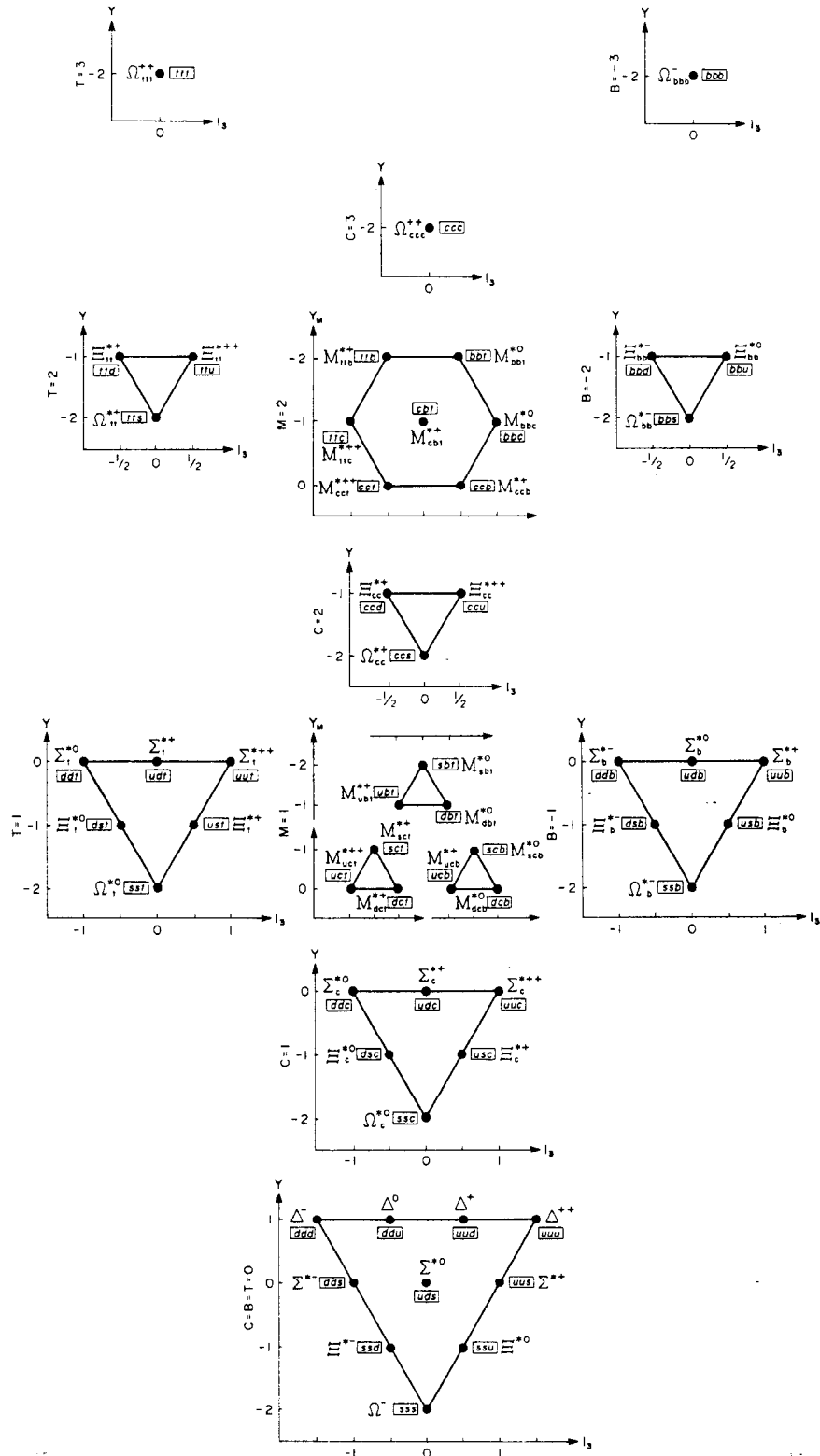


Fig. 11