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THE STATIC POTENTIAL IN QUANTUM CHROMODYNAMICS*

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ABSTRACT

The effective interaction between a static quark-antiquark pair is computed within the framework of $\left\langle G^a_{\mu\nu}\, G^{\mu\nu}_a \right\rangle \neq 0$. Due to the static approximation the interaction takes the form of a potential, which is in striking agreement with phenomenological potentials.

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A more fundamental understanding of the heavy quark-antiquark interaction in Quantum Chromodynamics (QCD) is needed to fully utilize the wealth of data being accumulated on the properties of heavy quarkonia. While it is widely known that a simple potential form for the interactions intrinsically neglects the important nonlocality that arises through retardation and quark motion effects [1], nonrelativistic potential models have met considerable success, particularly in spectroscopic calculations [2]. For this reason, it is an interesting problem to investigate the static quark potential in QCD as perhaps a crude model of the relevant physics, although an extension to the full dynamics appears problematic.

The static potential in QCD has a 1/R behavior at short distances, as mediated by perturbative one gluon exchange. It remains to satisfactorily include the nonperturbative contributions which will dominate outside this Coulombic regime. This can be done systematically using the techniques of Shifman, Vainshtein and Zakharov [3]. Their approach consists of parametrizing the present ignorance of the long wavelength nonperturbative structure of QCD with nonzero vacuum expectation values of certain gauge invariant field operators. Once incorporated into the theory, these parameters can be experimentally determined and used to make predictions for other processes. A set of remarkably successful charmonium sum rules [4] yields for the lowest dimensional purely gluonic operator

$$\left\langle 0 \left| \frac{g^2}{4\pi^2} G^a_{\mu\nu} G^{\mu\nu}_a \right| 0 \right\rangle \equiv M_0^4 = (330\ \text{MeV})^4 \tag{1}$$

with $G^a_{\mu\nu}$ the gluon field strength tensor.

Determining the effective static potential generated by this nonperturbative vacuum gluon condensate is equivalent to solving for the induced interaction between a static quark-antiquark pair immersed in constant uniform color electric and magnetic fields. The contributions of higher dimensional and possibly nonuniform vacuum fields must also be considered, and either included or shown to be negligible for this process. The effect of this gluon condensate on the static potential can be evaluated quite simply using multipole techniques. The procedure can be justified as long as the size of the vacuum gluon fluctuations is larger than the $Q\bar{Q}$ separation, and the expansion parameter $(\underset{\sim}{k}\cdot\underset{\sim}{r})$ is a small number. Since the vacuum condensate is effectively spatially homogeneous, a lowest order multipole expansion is sufficient out to relatively large distances. Higher order multipoles which couple to gradients of the vacuum fields could contribute when deviations from the homogeneous approximation are incorporated. However, as will be pointed out, these are negligible out to a distance of roughly a fermi.

The calculation consists of allowing a static $Q\bar{Q}$ pair in an asymptotic color singlet state and interacting via a perturbative one gluon exchange potential to couple an arbitrary number of times to the vacuum field. As known from the one gluon exchange potential, the intermediate color octet $Q\bar{Q}$ states are in a repulsive channel, and thus highly virtual with respect to the incident color singlet state. This forces the vacuum couplings to clump into short periods of octet propagation, separated by longer periods of on-shell color singlet propagation, as illustrated by fig. 1. What must be evaluated are these "octet clumps" of fig. 2, which when iterated on the singlet propagator give a corrected color singlet static potential

$$V_2(r) = -\frac{4\alpha_s}{3r} + h_1(r) \quad . \tag{2}$$

The expression for $h_1(r)$ is

$$h_1(r) = \sum_{n=0}^{\infty} (-i)^{2n+1} \int_0^{\infty} dt_{2n+2} \int_0^{t_{2n+2}} dt_{2n+1} \cdots \int_0^{t_3} dt_2$$

$$\times\, e^{-it_{2n+2}\left(V_8^o - V_1^o\right)} \ \frac{1}{3}\, \mathrm{Tr}\left(H_I(1) \ \ldots \ H_I(2n+2)\right) \tag{3}$$

where V_1^o (V_8^o) is the one gluon exchange color singlet (octet) potential, and the trace is reflective of the color singlet nature of the external state. The form for H_I, the vacuum gluon — $Q\bar{Q}$ coupling, is of a multipole form and has been derived by several authors [5]

$$H_I = -\sum_a \left[-Q_a A_a^o + \underset{\sim}{d}_a \cdot \underset{\sim}{E}_a + \underset{\sim}{m}_a \cdot \underset{\sim}{B}_a + \ldots\right] \tag{4}$$

where

$$\begin{aligned} Q_a &= g \int d^3r \ \ \bar{\Psi} \gamma_0 T_a \Psi \\ \underset{\sim}{d}_a &= g \int d^3r \ \ \underset{\sim}{r} \bar{\Psi} \gamma_0 T_a \Psi \\ \underset{\sim}{m}_a &= g \int d^3r \ \ \frac{1}{2}\left(\underset{\sim}{r} \times \bar{\Psi} \underset{\sim}{\gamma} T_a \Psi\right) \end{aligned} \tag{5}$$

with Ψ the $Q\bar{Q}$ "wave function", $\underset{\sim}{r}$ the $Q\bar{Q}$ separation, and T_a the generators of $SU_c(3)$. Restricting for the moment to color electric dipole coupling, rotating to Euclidean space, and doing the trivial time integrations, one finds

$$h_1(r) = -\frac{1}{3}\sum_{n=0}^{\infty}\int_0^{\infty} d\tau \, \frac{\tau^{2n}}{(2n)!} \, e^{-\left(V_8^o - V_1^o\right)\tau}$$

$$\times \frac{g\underset{\sim}{r}\cdot\underset{\sim}{E}^{a_1}}{2} \cdots \frac{g\underset{\sim}{r}\cdot\underset{\sim}{E}^{a_{2n+2}}}{2} \, 2\left(D^{a_2} \cdots D^{a_{2n+1}}\right)_{a_1 a_{2n+2}} \qquad (6)$$

where $(D^a)_{bc} \equiv d_{abc}$, which is defined in the standard way from the anti-commutators of the group generators. The origin of the product of D-matrices can be easily understood by evaluating an isolated coupling of a vacuum field to an octet projected $Q\bar{Q}$ state as in fig. 3

$$\left(\text{figure 3}\right) \sim \mathrm{Tr}\left(\left\{T_m, T_{a_i}\right\} T_n\right) \sim d_{a_i mn} \sim \left(D^{a_i}\right)_{mn} \quad . \qquad (7)$$

Equation (1) and Lorentz invariance of the vacuum [6] demand $\langle E_i^a E_j^b \rangle \to -\frac{1}{3}\delta_{ij} \frac{1}{8}\delta^{ab} |\underset{\sim}{E}_c^2|$, and thus the free indices of the product of D-matrices are pair-wise contracted. If the contractions are restricted to "planar" configurations the leading behavior in $(1/N_c)$ is obtained, and the product of matrices collapses to trivial form. The only nontrivial point is to determine the number of distinct planar pair-wise contractions of 2n vertices. The combinatoric factor is easily found to be $\frac{(2n)!}{(n+1)!n!}$ [7]. Equation (6) for $h_1(r)$ becomes

$$h_1(r) = \frac{g^2 r^2 |\underset{\sim}{E}_a^2|}{18} \int_0^{\infty} d\tau \, e^{-\left(V_8^o - V_1^o\right)\tau} \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!} \left(-\frac{5g^2 r^2 |\underset{\sim}{E}_a^2| \tau^2}{288}\right)^n \quad . \qquad (8)$$

Defining

$$\xi \equiv \sqrt{\frac{5g^2 r^2 |\underset{\sim}{E}_a^2|}{288}} \quad ,$$

it is noted that the sum over n in eq. (8) can be explicitly evaluated as $J_1(2\xi\tau)/\xi\tau$, where J_1 is the first Bessel function. The integral over τ can then be analytically done giving

$$h_1(r) = \frac{8}{5}\left(\sqrt{\left(V_8^o - V_1^o\right)^2 + \frac{5g^2 r^2 |\underset{\sim}{E}_a^2|}{72}} - \left(V_8^o - V_1^o\right)\right) . \qquad (9a)$$

Putting in one gluon exchange expressions for $V^o_{1,8}$, and using $|\underset{\sim}{E}_a^2| = \pi^2 M_0^4$ from eq. (1), yields

$$V_1(r) = -\frac{4\alpha_s}{3r} + \frac{8}{5}\left(\sqrt{\left(\frac{3\alpha_s}{2r}\right)^2 + \frac{5\pi^2 M_0^4}{72} r^2} - \frac{3\alpha_s}{2r}\right) \qquad (9b)$$

for the effective color singlet static potential in the presence of non-zero $\langle G^2\rangle$. Note that for small r, $V \sim -(4\alpha_s/3r) + O(r^3)$, as shown previously [8], and for $r \geq 0.5$ f

$$V_1(r) \longrightarrow \frac{2\pi M_0^2}{3}\sqrt{\frac{2}{5}}\; r \longrightarrow \left(0.144 \text{ GeV}^2\right) r \quad .$$

This compares very favorably with static potentials used to fit the upsilon spectrum [5] which have $V(r) \to (0.155 \text{ GeV}^2)r$, and those used to give a Regge slope of 0.9 GeV^{-2} which have $V(r) \to (0.14 \text{ GeV}^2)r$. In fig. 4, the potential of eq. (9b) is compared with conjectured phenomenologically successful static potentials.

The effect of the vacuum magnetic field on the static potential can also be computed to leading order in $(1/M_Q)^2$ giving an effective "hyperfine interaction"

$$V^{HF}(r) = -\frac{1}{6}\left(1 - \frac{S(S+1)}{3}\right)\frac{\pi^2 M_0^4}{M_Q^2} \frac{1}{\sqrt{\left(\frac{3\alpha_s}{2r}\right)^2 + \frac{5\pi^2 M_0^4 r^2}{18}}} \qquad (10)$$

where S is the spin of the $Q\bar{Q}$ pair.

Finally, the physical relevance of these results must be addressed both philosophically and numerically. As stated previously, we calculated a static potential with complete neglect of quark motion, which explicitly ignores the unavoidable retardation effects well known in bound state problems. Thus, the applicability of the derived potential to bound state problems is neither direct nor assured, but is hopefully an approximation to the relevant physics. Also, it is straightforward to estimate the contributions from neglected higher order multipoles to our process using one simple assumption. It is conjectured that because the scale of the vacuum fluctuations of $G^a_{\mu\nu}G^{\mu\nu}_a$ is M_0, higher dimensional operators that occur in the expansion by increasing the number of derivatives or fields differ only by their power of M_0. Using the next order term of the interaction Hamiltonian, eq. (4), the first nonleading term is suppressed by a factor of roughly $(1/3!)(rM_0/2)^2$. For $r \lesssim 1$ fermi, this is less than a 10% correction. Therefore, the calculation is technically limited to a distance of less than a fermi. This should be adequate for attempted applications to heavy quarkonia.

REFERENCES

[1] L. S. Brown and W. I. Weisberger, Phys. Rev. D 20 (1979) 3239; T. Appelquist and M. Dine, Phys. Lett. 69B (1977) 231; M. B. Voloshin, Nucl. Phys. B154 (1979) 365; ITEP-30-1981.

[2] W. Celmaster, H. Georgi and M. Machacek, Phys. Rev. D 17 (1978) 886, 879; E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane and T.-M. Yan, Phys. Rev. Lett. 34 (1975) 369; W. Buchmuller and S. Tye, Phys. Rev. D 24 (1981) 132.

[3] M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. B147 (1979) 385.

[4] M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. B147 (1979) 448.

[5] T.-M. Yan, Phys. Rev. D 22 (1978) 1652; M. Peskin, Nucl. Phys. B156 (1979) 365; K. Shizuya, Phys. Rev. D 23 (1981) 1180.

[6] M. B. Voloshin, Nucl. Phys. B154 (1979) 365.

[7] J. Koplik, A. Neveu and S. Nussinov, Nucl. Phys. B123 (1977) 109.

[8] C. Flory, Phys. Lett. 101B (1981) 98.

FIGURE CAPTIONS

Fig. 1. $Q\bar{Q}$ pair interacting via one gluon exchange and coupling to the vacuum fields.

Fig. 2. "Octet clump" of $Q\bar{Q}$-gluon condensate coupling.

Fig. 3. Vacuum gluon coupling to a $Q\bar{Q}$ pair in a projected color octet state. $T_m/\sqrt{2}$ is the normalized color octet projection operator.

Fig. 4. Derived potential compared to two successful phenomenological potentials. The numbers refer to the following references: (1) this work; (2) A. Martin, Phys. Lett. 93B (1980) 338; and (3) Cornell group, Ref. [2]. We use $\alpha_s = 0.4$.

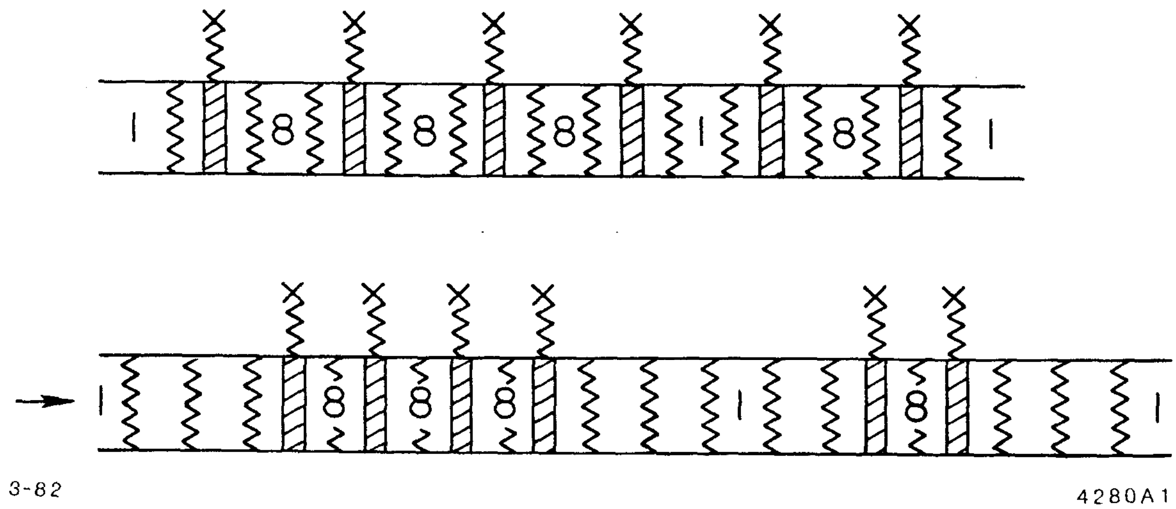

Fig. 1

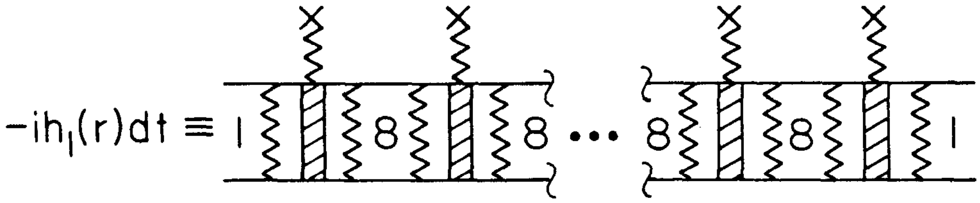

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Fig. 2

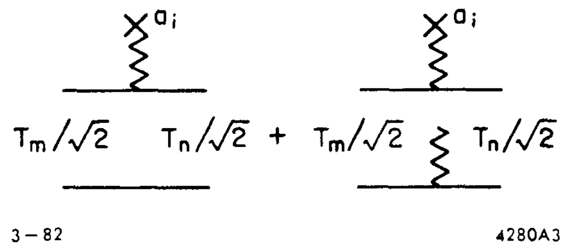

Fig. 3

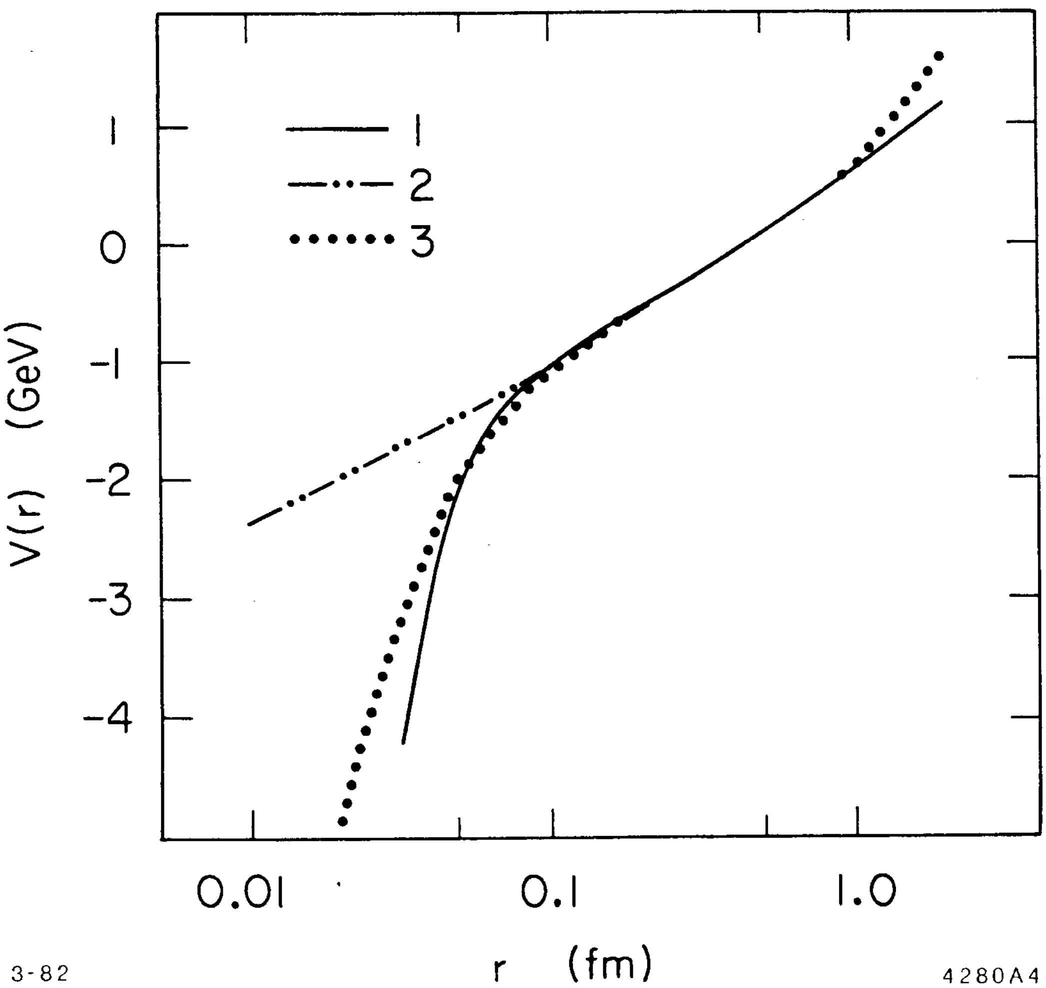

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Fig. 4