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# IS THE STRONG COUPLING CONSTANT SMALL?*

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## ABSTRACT

It is important to examine the strong-coupling parameter $\Lambda$ in deep-inelastic scattering, since (e.g.) the proton lifetime in grand-unified theories is quite sensitive to $\Lambda$. We show that $\Lambda$ is not as small in $\mu N$ scattering as previously reported. Furthermore, $\Lambda$ extracted from $F_2(x,Q^2)$ is highly correlated with the parameterization of the gluon distribution. Other problems arise from $\sigma_L/\sigma_T$ assumptions.

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There has been considerable attention recently to the determination of the strong coupling constant $\alpha_s$ of Quantum Chromodynamics (or equivalently of the parameter $\Lambda$ where $\alpha_s \propto 1/\ln Q^2/\Lambda^2$). In particular, in the context of deep-inelastic scattering one frequently hears that $\Lambda$ is very small; around 100 MeV from muon experiments and around 200 MeV from neutrino experiments.[1] The magnitude of $\Lambda$ is relevant (among other things) to grand unified theories because the proton lifetime is very sensitive to it.[2] Earlier works have pointed out the importance of using $\Lambda$ determinations from second-order not leading-order calculations [3,4] and also the importance of considering the role of higher-twist corrections (which can lower $\Lambda$ significantly).[4]

In deep-inelastic lepton scattering, a value for $\Lambda$ can be obtained by measuring the $Q^2$ evolution of either the structure functions $F_2$ or the non-singlet structure functions, e.g., $F_2^p - F_2^n$ in $\mu N$ scattering or $xF_3$ in $\nu N$ scattering. These latter functions depend neither on the ratio $R = \sigma_L/\sigma_T$ nor on the gluon structure function as do the singlet functions $F_2$. However, in an experiment the structure function $F_2$ can be measured with much more precision than the non-singlet functions, which can only be extracted from differences of cross-sections. To overcome this limitation the more precise data for $F_2$ have sometimes been analyzed as if they were non-singlet distributions, by restriction to large values of x only. There, the contribution from the gluon function was hoped to be small.

In this paper we consider the impact of the gluon distribution and the ratio, $R = \sigma_L/\sigma_T$, on the analysis of the structure function $F_2(x,Q^2)$ and emphasize the difficulty of extracting a meaningful value of $\Lambda$ from $F_2$. Although we are concerned with the impact of these factors on $\Lambda$ determinations, we are not attempting to find a "best" value for $\Lambda$.

As a result, we have for simplicity used only the leading-order QCD evolution equations. However, previous work [5] on $xF_3$ indicates that the values of $\Lambda$ to either order are similar ($\Lambda^{(2)}$ in the $\overline{MS}$ scheme). We have examined the two lepton scattering data samples with the best statistics, the data from the European Muon Collaboration (EMC) [6] for $\mu N \to \mu X$ (iron target) and from the CERN-Dortmund-Heidelberg-Saclay (CDHS) collaboration [7] for $\overset{(-)}{\nu} N \to \mu X$, using the evolution equations for $F_2$ and $xF_3$ directly as described in ref. 4. In their analysis the EMC group has used only data above $x = 0.25$ and assumed that $\Lambda$ could then be extracted using the non-singlet evolution equations (which is equivalent to setting the glue contribution to zero in the evolution equation for $F_2^{singlet}$). As a result they got values for $\Lambda$ around 100 MeV. However, the use of the non-singlet equations for singlet data is not proper, and it leads to a much poorer fit to the data than does the use of the appropriate singlet equations ($\chi^2$ increases by 20).* In fact, we find that the singlet equations lead to $\Lambda \approx 300$ MeV not 100 MeV. We emphasize that even for $x \gtrsim 0.25$ these (and CDHS) data require a substantial gluon contribution. Furthermore, a substantial antiquark sea (~15% of all sea) has been measured [7] for $x > 0.25$, and this also implies, intuitively, that the glue distribution is non-negligible.

Values of $F_2$ can only be extracted from the cross-section data by first measuring a value for R. CDHS assumed a constant value of $R = 0.1$

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*In making this comparison we first reproduced the EMC results and then used the singlet equations under exactly analogous conditions (but with one less free parameter than EMC used). We applied the same cuts as in ref. 6 for this comparison; these are different from the cuts and changes described later in this paper. This result is, however, independent of cuts.

whereas the EMC showed $F_2$ for both R = 0 and R = 0.2 assumptions.* [8] For both data sets we required $Q^2 \geq 10\ \text{GeV}^2$ and $W^2 \geq 10\ \text{GeV}^2$ in order to minimize higher-twist effects. The $W^2$ cut (applied after the $Q^2$ cut) eliminates only 3 of 138 EMC data points but reduces $\Lambda$ by 90 MeV. We also considered the effects of cutting out small x. These effects were, in general, small (for $\Lambda$, the value would be increased slightly by eliminating small x).† Let us consider the parameterization of the gluon distribution. First note [4] that one does not obtain adequate fits to $F_2$ (or $xF_3$) data without allowing for a term (as expected theoretically) such as ax in

$$F_2\left(x,Q_0^2\right) = C(1-x)^{\alpha}\ (1+ax) \quad . \tag{1}$$

For this reason and because it is expected theoretically, we considered a similar parameterization for the glue distribution (with $C_g$ determined from the momentum sum rule)

$$G\left(x,Q_0^2\right) = C_g(1-x)^{\beta}\ (1+gx) \tag{2}$$

and also an equivalent distribution

$$G\left(x,Q_0^2\right) = C_g(1-x)^{\beta}\left[1 + g'(1-x)^2\right] \quad . \tag{3}$$

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*We found that we could not reproduce several of the EMC points for $F_2$ (R = 0.2) calculated from their R = 0 points. This suggests [8] that these points were not in the center of their energy bins. For consistency we eliminated these few points.

†The EMC data were at E = 120, 250 and 280 GeV. The latter two energy sets overlapped in $Q^2$ and x so we normalized them. We tested the E = 120 GeV normalization by allowing it to be a free parameter, but it had little impact on our results. Since these data had $\chi^2$/degree of freedom significantly above 1.0, we felt it was essential for a meaningful comparison with the two data sets to include small percentage systematic errors (1.5% for EMC, 0.5% for CDHS) which were considerably smaller than the systematic errors referred to in refs. 6 and 7.

One finds that $\beta$ and g are highly correlated. Clearly any sensitivity to G is found at low x where it is difficult to isolate the $(1-x)^{\beta}$ behavior. Figure 1 shows the one standard deviation contour for this correlation. Figure 2, shows how different the resulting G can be for different choices of $\beta$ and g. A meaningful determination of $\beta$ is <u>not</u> possible with these data nor do we expect it to be possible using other deep-inelastic data for $F_2$. We should emphasize that the (perturbative) QCD evolution equations break down for G at small x. So one must check that no results depend on the small x region. For fig. 1 no visible difference occurs if $x \leq 0.1$ data are excluded.

In examining the data for $F_2$ one finds that there is a significant correlation between the values of $\Lambda$ and $\beta$ (whether g is fixed or free). The same is, of course, true of $\Lambda$ and g. Figure 3 shows the $\Lambda$-$\beta$ correlation. From the EMC data (assuming R = 0) we find at the 1 S.D. level that $\Lambda$ can be anywhere between 0.23 and 0.62 GeV (centered at 0.34). From the CDHS data (assuming R = 0.1) $\Lambda$ is between 0.25 and 0.85 GeV (centered at 0.42). Results are very similar using eq. (3). Again, no visible difference occurs if $x \leq 0.1$ data are excluded.

Clearly, $\Lambda$ cannot be determined accurately by these data for $F_2$. More sensitive determinations require additional input such as from non-singlet data, which do not involve gluon distributions. In muon scattering $\left(F_2^{\text{proton}} - F_2^{\text{neutron}}\right)$ can be used although it is sensitive to systematics [4] and has poorer statistics. In neutrino scattering $xF_3$ can be used. The CDHS data for $xF_3$ lead to $\Lambda = 0.2 \pm 0.1$ GeV compared with $\Lambda \approx 0.4$ from $F_2$. A simultaneous fit to $F_2$ and $xF_3$ data leads to a quite different correlation plot for $\Lambda$ and $\beta$, see fig. 3. Substantially smaller values for $\Lambda$ and larger values for $\beta$ are found. It is important to point out the smaller allowed region (one standard deviation contour) for $\Lambda$ and the smaller correlation between $\Lambda$ and $\beta$. This is a reflection of the strong constraints on $\Lambda$ coming from the simpler $xF_3$ evolution. Let us emphasize that a simultaneous fitting to $F_2$ and $xF_3$ requires the assumption that higher-twist terms are small, since these terms may enter in very different ways.

If we knew $\beta$ and g accurately (say from theoretical prejudices) then a more accurate determination of $\Lambda$ is possible from $F_2$.

As we stated previously, $F_2(x,Q^2)$ can only be extracted from data with knowledge or assumption about R $(= \sigma_L/\sigma_T)$. This is because the cross section depends on both $F_1$ and $F_2$ and (for $\mu$N scattering) is:

$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\left(y^2 F_1(x,Q^2) + \frac{1}{x}\left(1 - y - \frac{mxy}{2E}\right)F_2(x,Q^2)\right) \qquad (4)$$

where $y = \nu/E = Q^2/2mEx$. Then $F_2$ can be extracted by using

$$F_1(x,Q^2) = \frac{1}{2x} F_2(x,Q^2)(1 + R)\Big/\left(1 + \frac{4m^2x^2}{Q^2}\right) \quad . \qquad (5)$$

If one is given $F_2(x,Q^2)$, R and y (or E), then clearly one can obtain $d^2\sigma/dQ^2dx$. From that one can obtain $F_2(x,Q^2)$ for other assumptions of R. For CDHS data the y values were not available so we could not proceed. For the EMC data the E values are given, and we have considered the impact of different assumptions of R.

We found the following $\chi^2$ and $\Lambda$ values while fitting EMC data with $\beta = 4$ (fixed) for 130 degrees of freedom (1% systematics):

| | |
|---|---|
| R = 0 | $\chi^2 = 129.$ $\Lambda = 0.31$ |
| R = 0.1 | $\chi^2 = 147.$ $\Lambda = 0.36$ (assumption used by CDHS for R) |
| R = QCD (leading twist) | $\chi^2 = 142.$ $\Lambda = 0.29$ |

What can one conclude from these results? The determination of $\Lambda$ is only mildly sensitive to the assumption of R. However, the quality of the fit is extremely sensitive to R. For example, R = 0 (which is not consistent with QCD) has a $\chi^2$ which is far better than R of leading twist QCD ($\Delta\chi^2 = 12$). And, if we were to allow the value of R to be chosen by fitting the data (with R independent of $Q^2$ and x), we find that the nonsensical value R = -0.06 is preferred with $\chi^2 = 117$ (note — this clearly is not how one measures R). One can only conclude that systematic errors must be included. In fact, if one does a combined fit to the CDHS and EMC data without systematic errors with R = 0.1 in both cases, one obtains $\chi^2 = 345$ for 200 degrees of freedom. The systematic errors described earlier (1.5% for EMC; 0.5% CDHS) reduce $\chi^2$ to 203. Whether the actual systematic errors in the determination of R feed back to the determination of $\Lambda$, $\beta$, etc., is not clear, but one would be happier to find $\Lambda$ in a more consistent context.

In conclusion, the non-negligible role of the glue distribution in $F_2$ evolution makes it very difficult to determine $\Lambda$ accurately. The shape of the glue distribution is difficult to extract and has a big impact on $\Lambda$ determinations. It is very useful to combine singlet and non-singlet fits if $Q^2$ and $W^2$ are large. The magnitude of $\Lambda$ is probably between 150 and 350 MeV.

Following completion of this work we received two related papers.[9]

We would like to acknowledge useful conversations with J. Carr, F. Gilman and P. Zerwas.

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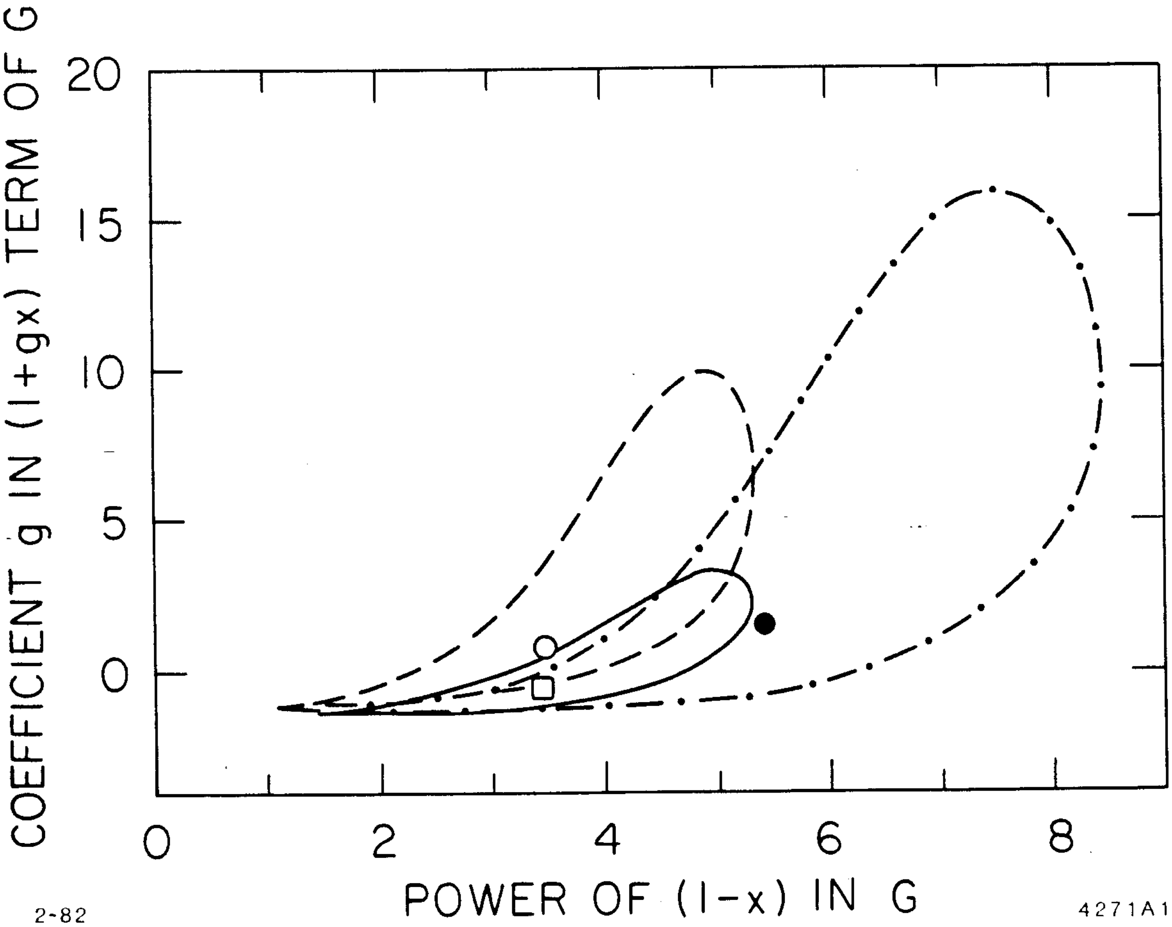
FIGURE CAPTIONS

Fig. 1. The one standard deviation contours for the correlation between the two glue distribution parameters g and β (the power of (1 - x)). The solid curve is for EMC μ data; the dashed curve is for $F_2$ from CDHS ν data; and the dot-dashed curve is for a combined fit to $xF_3$ and $F_2$ from CDHS ν data.

Fig. 2. The glue distribution [Eq. (2)] for three choices of β and g taken from the dot-dashed curve in fig. 1. The solid curve has β = 3.3 and g = -0.25; the dashed curve has β = 5.9 and g = -0.50; the dotted curve has β = 5.9 and g = 9.5.

Fig. 3. The one standard deviation contours for the correlation between Λ and the power of (1 - x) in the glue distribution. The solid curve is for EMC μ data; the dashed curve is for $F_2$ from CDHS ν data; and the dot-dashed curve is for a combined fit to $xF_3$ and $F_2$ from CDHS ν data.



COEFFICIENT g IN (1+gx) TERM OF G
20
15
10
5
0
0
2
4
6
8
POWER OF (1-x) IN G
2-82
4271A1


Fig. 1

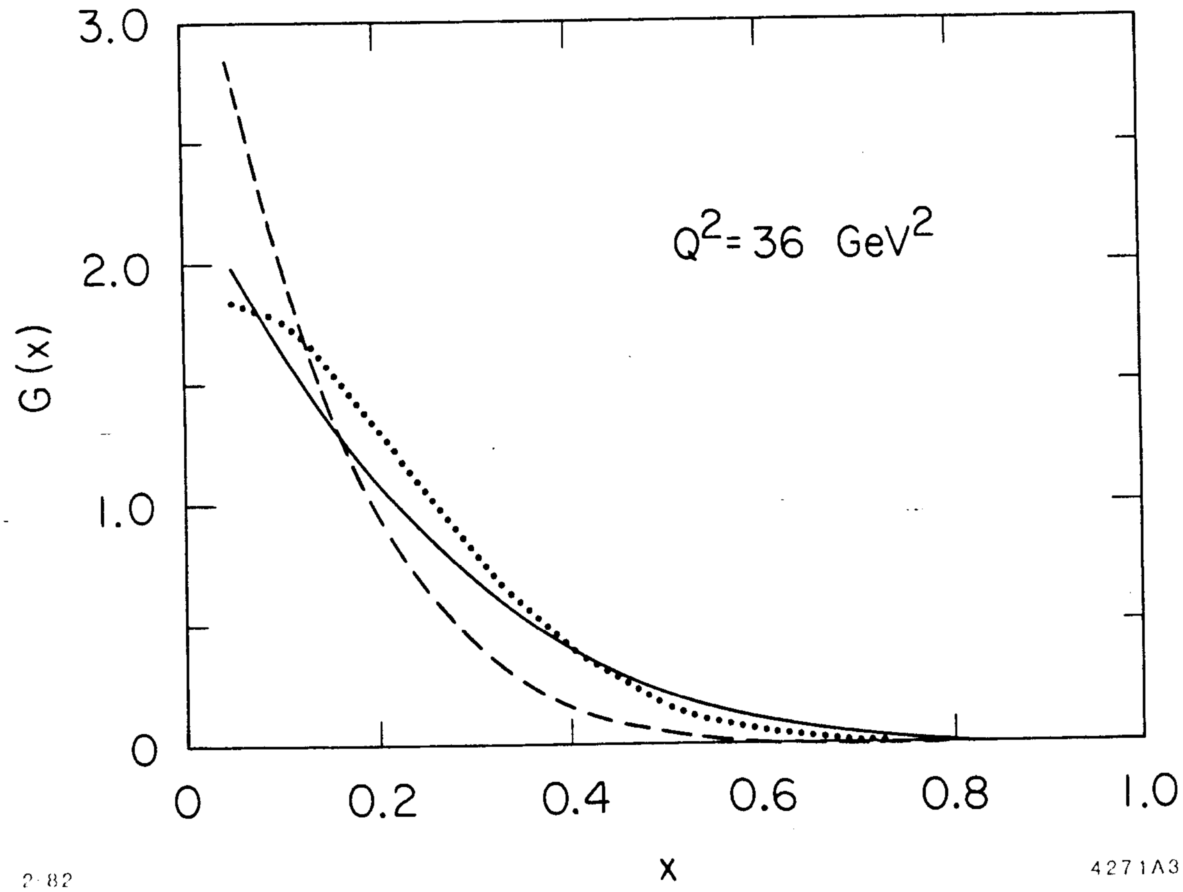



Fig. 2

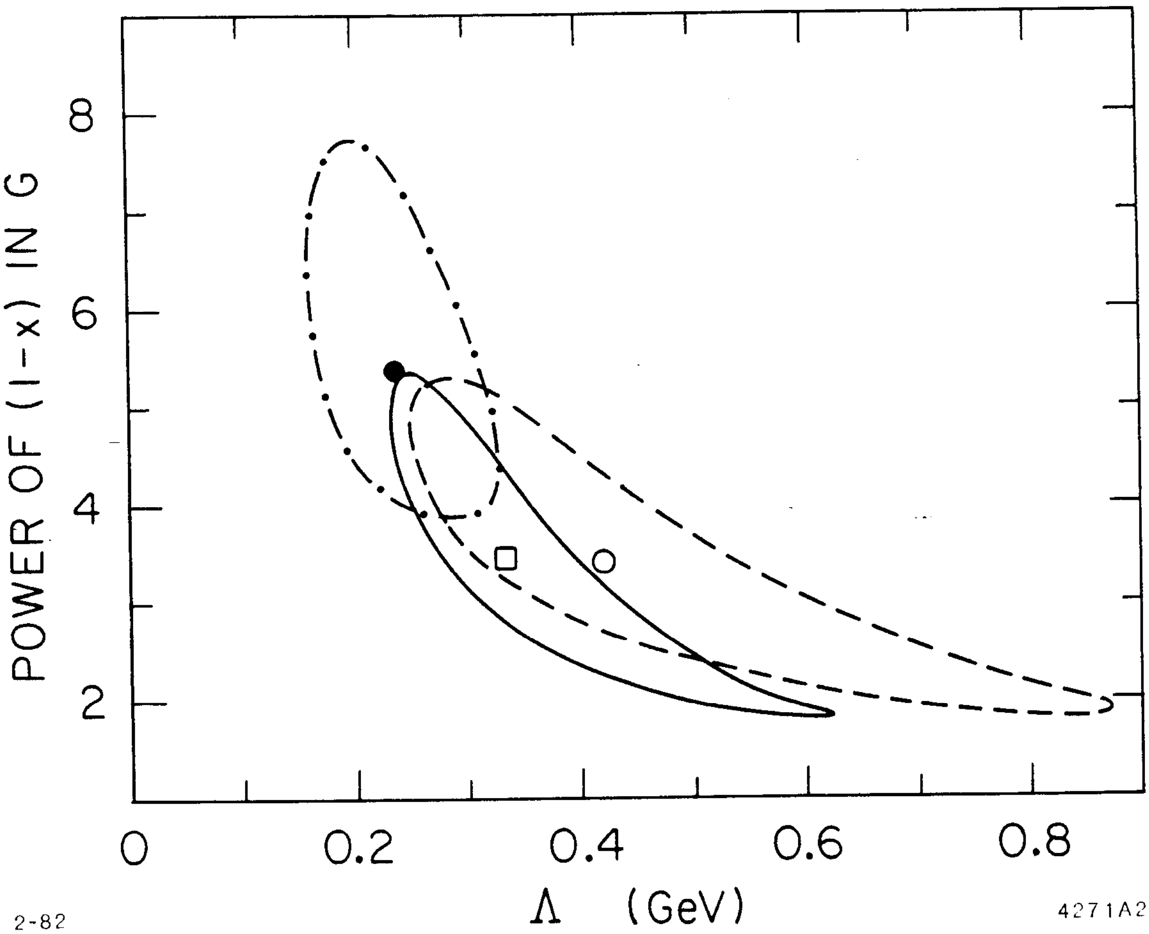



Fig. 3