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MAJORANA NEUTRINOS AND THEIR ELECTROMAGNETIC PROPERTIES*

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ABSTRACT

To help develop a picture of Majorana neutrinos, we study their electromagnetic properties. We show that CPT-invariance forbids a Majorana neutrino from having a magnetic or electric dipole moment. Then, by considering the process $\gamma\to\nu\bar{\nu}$, we find the most general expression for the matrix element of the electromagnetic current of a Majorana neutrino. The result is verified in a way which leads us to explore the behavior under parity of such a particle. Next, we see how electromagnetic properties which follow from one-loop diagrams conform to our general results. Finally, we show how the striking electromagnetic differences between Majorana and Dirac neutrinos can become invisible as the neutrino mass goes to zero.

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I. INTRODUCTION

A number of widely-discussed recent theoretical models[1] suggest that neutrinos are massive Majorana particles, identical to their antiparticles. Thus, it is of interest to develop a picture of the characteristics of a Majorana neutrino. Here we study its electromagnetic properties, and contrast them with those of a Dirac neutrino, which is distinct from its antiparticle. We begin by showing that CPT-invariance forbids a Majorana neutrino from having either a magnetic or an electric dipole moment. Next, we question whether a physical Majorana neutrino state is indeed an eigenstate of charge conjugation C. Without assuming that it is, we derive in two ways the most general form for the matrix element $\langle\nu^M(p_f,s_f)|J_\mu^{EM}|\nu^M(p_i,s_i)\rangle$, where J_μ^{EM} is the electromagnetic current operator, and $\nu^M(p,s)$ is a Majorana neutrino of momentum p and spin projection s. This matrix element contains only one form factor. We show that this fact follows very simply from the requirement that the final state in the cross-channel process $\gamma\to\nu^M\nu^M$ be antisymmetric. The derivations of the electromagnetic matrix element reveal that a Majorana neutrino has very interesting parity properties, which we discuss. Next, noting that a Dirac neutrino has three more form factors than a Majorana neutrino, we examine how the extra form factors manage to vanish when the electromagnetic properties of a Majorana neutrino are calculated in $SU(2)_L \times U(1)$ to one-loop order. Lastly, we compare the electromagnetic interactions of a Majorana and a Dirac neutrino in the massless limit. We find that they conform to what seems to be a general rule: If all weak currents are left-handed, then the difference between a Majorana and a Dirac neutrino becomes invisible as the mass goes to zero. This occurs in spite of gross differences between these particles when the mass is not negligible.

II. STATIC ELECTROMAGNETIC PROPERTIES

It has been argued on various grounds, both in ancient papers and recent ones,[2] that a Majorana neutrino cannot have a magnetic or electric dipole moment. It seems not to have been noticed, however, that this conclusion already follows trivially from the relatively weak assumption of CPT-invariance. Suppose a Majorana neutrino has a magnetic dipole moment μ and electric dipole moment d. Then, when it is at rest, its interaction energy in a combination of static, uniform magnetic and electric fields is of the form $-\mu\langle\vec{s}\cdot\vec{B}\rangle - d\langle\vec{s}\cdot\vec{E}\rangle$. Here $\vec{s}$ is, of course, the neutrino spin operator. Now, in the CPT-reflected state, the fields $\vec{B}$ and $\vec{E}$ are unchanged. However, the effect of CPT on a Majorana neutrino at rest is simply to reverse its spin (apart from a phase factor). Thus, the dipole interaction energy changes sign when we go to the CPT-reflected state, so if CPT-invariance holds, μ and d must vanish.[3]

III. BEHAVIOR OF ν^M UNDER C

It might be thought that, alternatively, one could argue that μ and d must vanish because of the C properties of a Majorana neutrino. <u>If</u> such a neutrino is an eigenstate of C, then $\langle\nu^M|-\mu\vec{s}\cdot\vec{B} - d\vec{s}\cdot\vec{E}|\nu^M\rangle = \langle\nu^M|C[-\mu\vec{s}\cdot\vec{B} - d\vec{s}\cdot\vec{E}]C^{-1}|\nu^M\rangle = \langle\nu^M|\mu\vec{s}\cdot\vec{B} + d\vec{s}\cdot\vec{E}|\nu^M\rangle$, since $\vec{B}$ and $\vec{E}$ are C-odd. Thus, $\mu = d = 0$.

It seems, however, that this argument must be viewed with caution. The difficulty is that it is not obvious that a physical, dressed Majorana neutrino is indeed an eigenstate of C. To be sure, in free field theory a Majorana neutrino is <u>defined</u> to be an eigenstate of C. What is not obvious is that this property of C self-conjugacy can be maintained once the C-violating weak interactions are turned on.

Consider, for example, the diagrams in Fig. 1, where the external Majorana neutrino is massive but relativistic, and has negative helicity. Due to its Majorana character, the incoming neutrino can produce both $\ell^- W^+$ and $\ell^+ W^-$ virtual states. However, if the charged-current weak interactions are purely left-handed, the vertices in the $\ell^+ W^-$ diagram are severely depressed by helicity, compared to those in the $\ell^- W^+$ diagram. Thus, the C mirror-image states $\ell^- W^+$ and $\ell^+ W^-$ occur with very unequal amplitudes in the dressed neutrino state. Consequently, that dressed state does not appear to be self-conjugate under C.

We shall assume that a photon can couple to a neutrino only by coupling, as in Figs. 2 and 3, to the charged particles in such virtual intermediate states as those in Fig. 1. Thus, if these same virtual states vitiate the C self-conjugacy of Majorana neutrinos, that fact can hardly be disregarded when one is trying to understand the electromagnetic properties of these particles. Consequently, we shall derive the most general expression for the electromagnetic matrix element $\langle \nu^M(p_f,s_f)|J_\mu^{EM}|\nu^M(p_i,s_i)\rangle$ without assuming that $|\nu^M\rangle$ is an eigenstate of C.

We note in passing that if one takes the electromagnetic interaction of a Majorana neutrino to be given by the diagrams of Fig. 3, plus similar diagrams with the photon attached to the W boson line, then one has taken J_μ^{EM} to be $-ie\bar{\ell}\gamma_\mu \ell$, which is C-odd, plus a C-odd term for the W-boson. Thus, if the dressed ν^M were a C eigenstate, these diagrams would give $\langle \nu^M|J_\mu^{EM}|\nu^M\rangle =$ $\langle \nu^M|CJ_\mu^{EM}C^{-1}|\nu^M\rangle = -\langle \nu^M|J_\mu^{EM}|\nu^M\rangle = 0$. However, explicit calculation shows that these diagrams do not yield a vanishing $\langle \nu^M|J_\mu^{EM}|\nu^M\rangle$.

IV. THE ELECTROMAGNETIC CURRENT OF A MAJORANA NEUTRINO

We shall obtain the most general form for $\langle \nu^M|J_\mu^{EM}|\nu^M\rangle$ in two ways. In the first of these we consider, not the s-channel process $\gamma+\nu\to\nu$ which this matrix element describes, but the related t-channel process $\gamma\to\nu+\bar{\nu}$. We do this because in the t-channel, the consequences of having neutrinos of Majorana character are particularly easy to see. (That the amplitudes for $\gamma+\nu\to\nu$ and $\gamma\to\nu+\bar{\nu}$ are related does not depend on C-conservation, or on any special C properties of ν.)

For $\gamma\to\nu\bar{\nu}$, the number of independent amplitudes is easy to determine. First, note that, even though the photon is off-shell, the conserved electromagnetic current to which it couples will only produce $\nu\bar{\nu}$ states with total angular momentum J=1. States with J=0 cannot be produced. For, if $|\Phi_{J=0}(q)\rangle$ represents any system with J=0 and momentum q, then $\langle\Phi_{J=0}(q)|J_\mu^{EM}|0\rangle = aq_\mu$, where a is a form factor. Current conservation then requires that $q_\mu\langle\Phi_{J=0}(q)|J_\mu^{EM}|0\rangle = aq^2 = 0$, so that a must vanish. Now, consider for a moment the Dirac case, in which the t-channel process is $\gamma\to\nu^D\bar{\nu}^D$. In the non-relativistic limit, the $\nu^D\bar{\nu}^D$ pair can be produced in any one of four J=1 states: $^3D_1$, $^3P_1$, $^3S_1$, and $^1P_1$. Thus, a Dirac neutrino has <u>four</u> independent electromagnetic form factors. In the Majorana case, the t-channel process is $\gamma\to\nu^M\nu^M$, with two identical fermions in the final state. Thus, this state must be antisymmetric. Of the four J=1 states just mentioned, only one, $^3P_1$, meets this requirement. Therefore, a Majorana neutrino has only <u>one</u> electromagnetic form factor.

Using standard techniques, one easily finds that for a Dirac neutrino, the electromagnetic matrix element may be written in the form

$$\langle\nu^D(p_f,s_f)|J_\mu^{EM}|\nu^D(p_i,s_i)\rangle = i\bar{u}_f[F_D(q^2)\gamma_\mu + G_D(q^2)(q^2\gamma_\mu - 2miq_\mu)\gamma_5 + M_D(q^2)\sigma_{\mu\nu}q_\nu + E_D(q^2)i\sigma_{\mu\nu}q_\nu\gamma_5]u_i \; . \tag{4.1}$$

Here F_D, G_D, M_D, and E_D are form factors, $q = p_f - p_i$, and m is the neutrino mass.[4] Before one imposes the constraint that the neutrinos in the t-channel are identical, the analogous matrix element for a Majorana neutrino has exactly the same form. To see this, note that this matrix element comes from perturbation theoretic diagrams in which the incoming ν^M is annihilated by a free Majorana field χ, and the outgoing one is created by the corresponding $\bar{\chi}$, or else the other way around. Now, the momentum-space expansion of χ is

$$\chi = \sum_{\vec{p},s} \sqrt{\frac{m}{E_{\vec{p}}V}}\,(a_{\vec{p},s}u_{\vec{p},s}e^{ipx} + a^{\dagger}_{\vec{p},s}v_{\vec{p},s}e^{-ipx}) \ , \tag{4.2}$$

in an obvious notation. Thus, $\langle\nu^M|J_\mu^{EM}|\nu^M\rangle$ must have the form

$$\begin{aligned}\langle\nu^M(p_f,s_f)|J_\mu^{EM}|\nu^M(p_i,s_i)\rangle &= \bar{u}_f\Gamma_\mu^A u_i - \bar{v}_i\Gamma_\mu^B v_f \\ &\equiv \bar{u}_f\Gamma_\mu u_i \ ,\end{aligned} \tag{4.3}$$

where Γ_μ^A, Γ_μ^B, and Γ_μ are some combinations of gamma matrices and momentum factors, and in the last step of Eq. (4.3) we have used the relation $v_{\vec{p},s} = \gamma_2 u^*_{\vec{p},s}$. If we then find the most general expression for Γ_μ consistent with conservation of J_μ^{EM}, we obtain for $\langle\nu^M(p_f,s_f)|J_\mu^{EM}|\nu^M(p_i,s_i)\rangle$ a form identical to that of Eq. (4.1). Arguing similarly for the t-channel process, we find that

$$\begin{aligned}\langle\nu^M(p_1,s_1)\nu^M(p_2,s_2)|J_\mu^{EM}|0\rangle = i\bar{u}_2[&F_M\gamma_\mu + G_M(q^2\gamma_\mu - 2miq_\mu)\gamma_5 \\ &+ M_M\sigma_{\mu\nu}q_\nu + E_M i\sigma_{\mu\nu}q_\nu\gamma_5]v_1 \ ,\end{aligned} \tag{4.4}$$

where F_M, G_M, M_M, and E_M are form factors, and here $q = p_1 + p_2$.

Now, as we saw, Fermi statistics has the result that, of the four terms in Eq. (4.4), only the linear combination which produces the $^3P_1$ state in the non-relativistic limit is non-vanishing. To see which combination this is, let us examine the non-relativistic limits of the four terms. More precisely, let us look at the non-relativistic limit of the amplitude $\eta_\mu\langle\nu^M\nu^M|J_\mu^{EM}|0\rangle$ for an off-shell photon of momentum q and polarization η to decay into $\nu^M(p_1,s_1)$ + $\nu^M(p_2,s_2)$. We work in the rest frame of the photon, so that $\vec{p}_2 = -\vec{p}_1 \equiv \vec{p}$, and choose a gauge where $q_\mu \eta_\mu = 0$. Now, in terms of the Pauli spinors φ_s and charge-conjugate Pauli spinors φ_s^c, the singlet and triplet amplitudes are, respectively, $S = \varphi^\dagger_{s_2} \varphi^c_{s_1}$, and $\vec{T} = \varphi^\dagger_{s_2} \vec{\sigma}\, \varphi^c_{s_1}$. In terms of these quantities, the amplitudes corresponding to production of the $^1P_1$, $^3S_1$, $^3P_1$, and $^3D_1$ final states are, respectively, $\vec{\eta}\cdot\hat{p}S$, $\vec{\eta}\cdot\vec{T}$, $\vec{\eta}\cdot\vec{T}\times\hat{p}$, and $(3\vec{\eta}\cdot\hat{p}\ \vec{T}\cdot\hat{p} - \vec{\eta}\cdot\vec{T})$. Table I shows the non-relativistic limits of the various terms in Eq. (4.4) in terms of these amplitudes. From Table I, we see that only the $(q^2\gamma_\mu - 2miq_\mu)\gamma_5$ term corresponds to production of $^3P_1$. Thus, converting the matrix element of Eq. (4.4) into one for the s-channel by appealing to the general form of the fields χ and $\bar{\chi}$, we conclude that

$$\langle \nu^M(p_f,s_f)| J_\mu^{EM} |\nu^M(p_i,s_i)\rangle = i\bar{u}_f G_M(q^2)(q^2\gamma_\mu - 2miq_\mu)\gamma_5 u_i \ . \tag{4.5}$$

This is the most general expression for the matrix element of the electromagnetic current of a Majorana neutrino.

Note that Table I shows explicitly that in the t-channel matrix element, the $(q^2\gamma_\mu - 2miq_\mu)\gamma_5$ term is antisymmetic in the non-relativistic limit, while the others are not (the quantities $\vec{T}$ and S are symmetric and antisymmetic, respectively). Of course, we need not have gone to the non-relativistic limit to discover the symmetry properties of the different terms. The bracketed quantity in Eq. (4.4), Γ_μ, is symmetric under $(p_1,s_1) \leftrightarrow (p_2,s_2)$, so this interchange simply corresponds to $i\bar{u}_2\Gamma_\mu v_1 \leftrightarrow i\bar{u}_1\Gamma_\mu v_2$. But $i\bar{u}_1\Gamma_\mu v_2 = -i\bar{u}_2(C^{-1}\Gamma_\mu C)^T v_1$, where $C = \gamma_4\gamma_2$. Hence, from the charge conjugation properties of the various combinations of gamma matrices, it follows immediately that under $(p_1,s_1) \leftrightarrow (p_2,s_2)$ all the candidate terms in Eq. (4.4) remain unchanged, except for the $(q^2\gamma_\mu - 2miq_\mu)\gamma_5$ term, which undergoes the required change of sign.

The electromagnetic matrix element of Eq. (4.5) is confirmed by finding the most general effective electromagnetic current, $J_\mu^{eff}(\bar{\chi},\chi)$, which can be constructed out of free Majorana fields χ and $\bar{\chi}$ and their derivatives up through second order, and taking the matrix element of this effective current between free Majorana neutrino states. The construction of J_μ^{eff} does not assume C self-conjugacy of <u>dressed</u> Majorana neutrinos. Indeed, J_μ^{eff} incorporates C-violating weak effects such as those in the diagrams of Fig. 3, replacing all but the external legs of such diagrams by a "black box". Since χ^c, the charge-conjugate of the free Majorana field χ, is χ itself, $J_\mu^{eff}(\bar{\chi},\chi)$ will necessarily be C-even, in contrast to the bare electromagnetic current, which is C-odd before it receives weak corrections.

Eliminating from J_μ^{eff} such candidate terms as $\bar{\chi}(\partial_\mu\chi) - (\partial_\mu\bar{\chi})\chi$, which vanish because $\chi^c \equiv C\bar{\chi}^T = \chi$, we are left with

$$J_\mu^{eff} = \partial_\mu[\bar{\chi}(a+b\gamma_5)\chi] + (c+d\,\Box^2)(\bar{\chi}\gamma_\mu\gamma_5\chi) + [\bar{\chi}\sigma_{\mu\nu}(e+f\gamma_5)(\partial_\nu\chi) - (\partial_\nu\bar{\chi})\sigma_{\mu\nu}(e+f\gamma_5)\chi]\ , \tag{4.6}$$

where a, . . ., f are constants. Current conservation then requires that

$$\begin{aligned}\partial_\mu J_\mu^{eff} &= \Box^2[\bar{\chi}(a+b\gamma_5)\chi] + (c+d\,\Box^2)2m(\bar{\chi}\gamma_5\chi) \\ &\quad + [(\partial_\mu\bar{\chi})\sigma_{\mu\nu}(e+f\gamma_5)(\partial_\nu\chi) - (\partial_\nu\bar{\chi})\sigma_{\mu\nu}(e+f\gamma_5)(\partial_\mu\chi)] \\ &= 0.\end{aligned} \tag{4.7}$$

This constraint demands that

$$a = b + 2md = c = e = f = 0\ . \tag{4.8}$$

Thus,

$$J_\mu^{\text{eff}} = \frac{g}{m^2}[-\Box^2(\bar{\chi}\gamma_\mu\gamma_5\chi) + 2m\partial_\mu(\bar{\chi}\gamma_5\chi)] \ , \tag{4.9}$$

where g is a dimensionless constant. Taking the matrix element of $J_\mu^{\text{eff}}(0)$ between free ν^M states, we obtain once again the result of Eq. (4.5), apart from the form factor. The latter corresponds to permissible additional powers of $\Box^2$ operating on the entire current of Eq. (4.9).[5]

V. PARITY OF MAJORANA NEUTRINOS

The structure of J_μ^{eff}, Eq. (4.9), calls attention to the interesting parity properties of Majorana neutrinos. It is not obvious, but can be shown, that under parity P, a free Majorana field χ behaves just as does a Dirac field:

$$\chi(\vec{x},t) \xrightarrow{P} \eta_P\gamma_4\chi(-\vec{x},t) \ , \tag{5.1}$$

where η_P is a phase factor.[6] Thus, we can determine the parity of J_μ^{eff} by inspection, disregarding the fact that χ is not a Dirac field. We see that $\vec{J}^{\text{eff}}$ has even parity. However, from the t-channel analysis, we know that this operator, acting on the vacuum, produces a $\nu^M\nu^M$ state which in the non-relativistic limit is $^3P_1$. Hence, we learn that

$$P|\nu^M\nu^M;{}^3P_1\rangle = +|\nu^M\nu^M;{}^3P_1\rangle \ . \tag{5.2}$$

That is, a state consisting of two identical Majorana neutrinos in a P-wave has positive parity! This result may seem somewhat strange, but it is true. It reflects the fact that the "intrinsic" parity of a single Majorana neutrino at rest is $\pm i$:[7]

$$P|\nu^M(\vec{p}=0,s)\rangle = \pm i|\nu^M(\vec{p}=0,s)\rangle \ . \tag{5.3}$$

Thus, the intrinsic parity of two identical Majorana neutrinos is $(\pm i)^2$. This has measurable consequences, at least in principle. For example, in a world where parity is conserved, the decay $\Psi(2^+) \to \nu^M\nu^M$ of a particle Ψ with $J^P=2^+$ would yield final states with <u>odd</u> values of orbital angular momentum. This fact would influence the angular distribution of the neutrinos.

To see why the intrinsic parity of a free Majorana neutrino is ±i, consider the transformation law (5.1) for the free field. This law induces a related one for the charge-conjugate field χ^c; namely,

$$\chi^c(\vec{x},t) \equiv \gamma_4\gamma_2\bar{\chi}^T(\vec{x},t) \xrightarrow{P} -\eta_P^*\gamma_4\chi^c(-\vec{x},t) \ . \tag{5.4}$$

Since $\chi^c(\vec{x},t) = \chi(\vec{x},t)$, we must require that the right-hand sides of (5.1) and (5.4) be equal. Thus, we must have $\eta_P = \pm i$. Insertion of the plane-wave expansion of χ in (5.1) then leads to the conclusion that $P|\nu^M(\vec{p},s)\rangle = \pm i|\nu^M(-\vec{p},s)\rangle$. (One finds that it also shows again that the properties of Majorana neutrinos under P are not consistent unless $\eta_P = \pm i$.) The fixed value of the square of the intrinsic parity of a Majorana neutrino is in sharp contrast to the intrinsic parity of a Dirac particle, which is completely arbitrary.

VI. HOW LOOP DIAGRAMS YIELD ONLY ONE MAJORANA FORM FACTOR

The general expression for $\langle\nu^M|J_\mu^{EM}|\nu^M\rangle$, Eq. (4.5), shows that not only do the magnetic and electric dipole moments of a Majorana neutrino vanish, but the entire magnetic and electric dipole form factors, $M_M(q^2)$ and $E_M(q^2)$, vanish as well. The "electric charge distribution" form factor, $F_M(q^2)$, vanishes also.

Now, for Dirac neutrinos, the electromagnetic form factors have been calculated in $SU(2)_L \times U(1)$ in terms of one-loop diagrams such as that in Fig.

2.[8] (In this and subsequent Figures in which a photon is attached to a charged lepton line, the additional diagram in which the photon is attached to the W boson line will always be understood. Also, we imagine that the calculations we shall discuss are carried out in a gauge in which diagrams such as that of Fig. 2, but with the W replaced by an unphysical charged Higgs particle, or by W and charged Higgs lines meeting at a photon-W-Higgs vertex, do not contribute.[8]) It is well known that the loop diagrams yield a non-vanishing magnetic moment for a Dirac neutrino.[8,9] But suppose the external neutrino is replaced by a Majorana one. How is it that these diagrams will now give vanishing values, as they must, for the magnetic moment and, indeed, for all the form factors which are forbidden to a Majorana neutrino? The answer follows from Fig. 3. This reminds us that when the incoming neutrino is a ν^M, there will be an extra diagram in which the neutrino produces an ℓ^+W^- intermediate state, rather than an ℓ^-W^+ one. The term in the weak Lagrangian which was active at the initial (final) vertex of the original diagram will be active at the final (initial) vertex of the new one. Figure 3 indicates at each vertex the interaction term that is active there, but in the extra diagram peculiar to ν^M, this interaction is written in terms of charge-conjugate lepton fields, with ν^c identified as ν. Comparing the extra diagram with the original one, we see that they are the same except that ℓ is replaced by ℓ^c and W^+ by W^-, so that the sign of the coupling to the photon is reversed, and, in addition, γ_5 is replaced by $-\gamma_5$. Comparing the expansion of the Majorana field χ, Eq. (4.2), to that of the analogous Dirac field, we see that the diagram common to Dirac and Majorana neutrinos gives the same amplitude in both cases. Now, suppose that this diagram leads to

$$\langle \nu_f^D | J_\mu^{EM} | \nu_i^D \rangle = \bar{u}_f \Gamma_\mu(\gamma_5) u_i \ , \tag{6.1}$$

with some specific $\Gamma_\mu(\gamma_5)$ of the form indicated by Eq. (4.1), involving γ_5. Then the sum of diagrams in Fig. 3 will lead to

$$\langle \nu_f^M | J_\mu^{EM} | \nu_i^M \rangle = \bar{u}_f[\Gamma_\mu(\gamma_5) - \Gamma_\mu(-\gamma_5)]u_i \ . \tag{6.2}$$

That is, in $\langle \nu^M | J^{EM} | \nu^M \rangle$, only terms proportional to γ_5, which are parity--violating and come from terms in the diagrams involving a γ_5 at one weak vertex and unity at the other, survive. Thus, the Majorana neutrino will have no γ_μ or $\sigma_{\mu\nu}q_\nu$ electromagnetic form factors. It will develop no $\sigma_{\mu\nu}q_\nu\gamma_5$ form factor either, because such a form factor would be CP-violating, and the original one-loop diagram common to ν^D and ν^M does not contain any CP violation to begin with. Thus, the one-loop diagrams conform in a very simple way to the requirement that three out of the four form factors of a ν^D must vanish for a ν^M.

VII. THE MASSLESS LIMIT

For massless neutrinos in a world where all weak currents are left-handed, there is no distinction between a two-component Dirac (i.e., Weyl) neutrino and a Majorana neutrino. That is, the left-handed Dirac neutrino and its right-handed so-called antineutrino can equivalently be regarded as the left- and right-handed states of a Majorana neutrino. Once the mass is non-zero, however, then no matter how small it is, a Dirac and a Majorana neutrino are different: the former involves four distinct states with a common mass, and the latter only two. Nevertheless, it appears that for practical purposes the massless limit is still a smooth one. Indeed, it seems that this limit can be described by a "Practical Majorana-Dirac Confusion Theorem": Assume that all weak currents are left-handed. Assume further that experiments on a given neutrino are always done with one of two incoming states--a state of negative helicity, "ν_-", or its positive-helicity CPT-conjugate, "$\bar{\nu}_+$". Then, as the neutrino mass goes to zero, it gradually becomes impossible to tell experimentally whether "ν_-" and "$\bar{\nu}_+$" are actually ν_-^D and $\bar{\nu}_+^D$, two of the four states of a Dirac neutrino (ν_-^D, $\bar{\nu}_+^D$, ν_+^D, $\bar{\nu}_-^D$), or ν_-^M and ν_+^M, the two states of a Majorana neutrino (ν_-^M, ν_+^M).

This "theorem" applies, in particular, to the behavior which has been found[10] for neutrino neutral-current weak interactions. We shall show explicitly that it also applies to the electromagnetic interactions of neutrinos, even though, as we have seen, these interactions distinguish sharply between a Majorana particle and a Dirac one when the mass is not small. We shall also compare the electromagnetic interactions of Majorana and Dirac neutrinos in the massless limit when right-handed currents are present.

Assume first that there are no right-handed currents. Then the electromagnetic interactions of a Dirac neutrino are described by diagrams such as those of Fig. 4. The initial and final vertices are always weak vertices involving either a charged or neutral left-handed current. Now, suppose the neutrino mass m is very small compared to its momentum $|\vec{p}|$. Then, due to the handedness of the final vertex, the helicity-flipping transition $\nu_-^D \rightarrow \nu_+^D$ is highly suppressed compared to the helicity-preserving one $\nu_-^D \rightarrow \nu_-^D$, as Fig. 4 indicates. From Eq. (4.1), this means that the helicity-flipping form factors $M_D(q^2)$ and $E_D(q^2)$ must go to zero with m,[11] and that the helicity-flipping dipole terms in Eq. (4.1) may be neglected when $m \ll |\vec{p}|$. In addition, the quantity multiplying G_D in Eq. (4.1) obviously simplifies to $q^2\gamma_\mu\gamma_5$ when $m \ll |\vec{p}|$, so that in this limit, the surviving, helicity-conserving electromagnetic matrix element obeys

$$\langle \nu_-^D | J_\mu^{EM} | \nu_-^D \rangle \longrightarrow i\bar{u}_{f(-)}[F_D\gamma_\mu + G_D q^2 \gamma_\mu \gamma_5] u_{i(-)} \tag{7.1}$$

$$\simeq \ (F_D + G_D q^2) i\bar{u}_{f(-)}\gamma_\mu u_{i(-)} \ . \tag{7.2}$$

Here $u_{i(-)}$ and $u_{f(-)}$ are spinors for initial and final states of negative helicity, and in obtaining Eq. (7.2) we have used the relation $\gamma_5 u_{(-)} \simeq u_{(-)}$, which holds when $m << |\vec{p}|$.

For a Majorana neutrino, Eq. (4.5) shows that as $m \to 0$, the electromagnetic matrix element reduces to the helicity-conserving $\gamma_\mu\gamma_5$ term, regardless of the nature of the weak currents. In particular,

$$\langle \nu_-^M | J_\mu^{EM} | \nu_-^M \rangle \xrightarrow[m \to 0]{} G_M q^2 i\bar{u}_{f(-)}\gamma_\mu u_{i(-)} \ . \tag{7.3}$$

Thus, if

$$G_M q^2 = F_D + G_D q^2 \tag{7.4}$$

when $m \to 0$, then not only do the electromagnetic interactions of ν_-^D and ν_-^M both become helicity-preserving in this limit, but they become completely indistinguishable. (Of course, the interactions of $\bar{\nu}_+^D$ and ν_+^M also become indistinguishable if Eq. (7.4) holds.) Note that indistinguishability requires that the Majorana and Dirac form factors be related by Eq. (7.4) for any given set of (left-handed) weak interactions. Otherwise, we could determine the weak interactions through some experiments which do not settle the Majorana-Dirac issue, then calculate both the Majorana and Dirac form factors from the measured weak interactions, then measure $\langle \nu_- | J_\mu^{EM} | \nu_- \rangle$, and see whether it is described by $G_M q^2$, or $F_D + G_D q^2$.

At the one-loop level, it is easy to show that relation (7.4) is indeed satisfied. In fact, from Fig. 3 we see that in the extra diagram peculiar to ν^M, the weak currents are effectively right-handed. Hence, when $m \to 0$, the contribution of this diagram to the transition $\nu_-^M \to \nu_-^M$ becomes negligible compared to that of the diagram common to ν^M and ν^D, so that

$$\langle \nu_-^M | J_\mu^{EM} | \nu_-^M \rangle \xrightarrow[m \to 0]{} \langle \nu_-^D | J_\mu^{EM} | \nu_-^D \rangle \; . \qquad (7.5)$$

Going to a bit more detail, let us first note that when there are no right-handed currents,

$$F_D = G_D q^2 \qquad (7.6)$$

when $m \to 0$. To see this, observe from Fig. 4 that $\langle \nu_+^D | J_\mu^{EM} | \nu_+^D \rangle$ must vanish with m. Through the analogue of Eq. (7.1) for $\langle \nu_+^D | J_\mu^{EM} | \nu_+^D \rangle$, this implies Eq. (7.6). Now, suppose that when $m \ll |\vec{p}|$, the one-loop diagram of Fig. 2 yields a matrix element $\langle \nu_-^D | J_\mu^{EM} | \nu_-^D \rangle$ of the form required by Eq. (7.1). Then, according to Eq. (6.2), the sum of diagrams in Fig. 3 will yield

$$\begin{aligned} \langle \nu_-^M | J_\mu^{EM} | \nu_-^M \rangle &= i\bar{u}_{f(-)} [F_D \gamma_\mu + G_D q^2 \gamma_\mu \gamma_5) - (F_D \gamma_\mu - G_D q^2 \gamma_\mu \gamma_5] u_{i(-)} \\ &= 2 G_D q^2 i \bar{u}_{f(-)} \gamma_\mu u_{i(-)} \; . \end{aligned} \qquad (7.7)$$

Comparing this to Eq. (7.3) and using the relation (7.6), we see that the requirement that $G_M q^2 = F_D + G_D q^2$ has been met.[12]

Now, how does the character of the massless limit change if both left- and right-handed weak currents are present? The electromagnetic transitions of ν^M are still helicity-conserving in this limit. However, as illustrated in Fig. 5, for ν^D there are now diagrams which lead to helicity-flipping transitions $\nu_-^D \to \nu_+^D$ whose amplitudes remain finite when $m \to 0$. The dipole form factors M_D and E_D, and in particular the magnetic and electric dipole moments of ν^D, need no longer vanish with m.[13] Thus, even when $m = 0$, a Dirac neutrino is now quite distinct from a Majorana neutrino. Through helicity-flipping in an external $\vec{B}$ or $\vec{E}$ field, ν_-^D and $\bar{\nu}_+^D$ can be converted, respectively, into ν_+^D and $\bar{\nu}_-^D$, the other two particles of the Dirac foursome. The latter two particles cannot be identical to $\bar{\nu}_+^D$ and ν_-^D. If they were, we would have a neutrino involving only two states, and could prove through one of the approaches in Sec. IV that its electromagnetic matrix element is given by Eq. (4.5) and does not allow any helicity-flipping when m = 0.

In addition, when there are right-handed currents, the helicity-preserving matrix elements $\langle \nu_-^M | J_\mu^{EM} | \nu_-^M \rangle$ and $\langle \nu_-^D | J_\mu^{EM} | \nu_-^D \rangle$ do <u>not</u> become equal, for a given set of weak interactions, as $m \to 0$. When the currents acting in the first diagram of Fig. 3 include right-handed pieces, those acting in the second diagram include effectively left-handed ones. Thus, the latter diagram, which is unique to ν^M, makes a non-vanishing contribution to $\langle \nu_-^M | J_\mu^{EM} | \nu_-^M \rangle$ when m = 0. Hence, Eq. (7.5) fails, as does, of course, the relation $G_M q^2 = F_D + G_D q^2$.

VIII. SUMMARY

While a Dirac neutrino has four electromagnetic form factors, a Majorana neutrino has only one. This result is obtained very easily by considering the t-channel process $\gamma \to \nu^M \nu^M$, and requiring that the two-neutrino final state obey Fermi statistics. The same procedure shows that it is an essentially axial vector form factor which survives in the Majorana case. The electric charge distribution form factor, and the magnetic and electric dipole form factors, all vanish. We saw explicitly how these form factors vanish when they are calculated from one-loop diagrams. We also observed that the magnetic and electric dipole <u>moments</u> are already forbidden by CPT-invariance.

The electromagnetic matrix element $\langle \nu^M | J_\mu^{EM} | \nu^M \rangle$ derived by the t-channel analysis is confirmed by constructing an effective electromagnetic current out of free Majorana fields. Neither of these derivations assumes that a physical Majorana neutrino is an eigenstate of C. Taken together, they point to the surprising fact that the intrinsic parity of a pair of identical free Majorana neutrinos is -1.

In spite of the very different electromagnetic properties of Dirac and Majorana neutrinos, the electromagnetic interactions of $(\nu_-^D, \bar{\nu}_+^D)$ and (ν_-^M, ν_+^M) smoothly become helicity-preserving and indistinguishable as $m \to 0$ if there are no right-handed weak currents. This strengthens our expectation that all differences between Dirac and Majorana behavior disappear with the neutrino mass if right-handed currents are absent. On the other hand, if such currents are present, then the electromagnetic interactions of $(\nu_-^D, \bar{\nu}_+^D)$ and (ν_-^M, ν_+^M) are very different, even when $m = 0$.

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4. Throughout this paper we shall omit the normalization factor $[(m/E_f V)(m/E_i V)]^{1/2}$ from expressions such as the right-hand side of Eq. (4.1).
5. After this work was mostly completed, we received a preprint by J. Nieves, in which another derivation of the electromagnetic matrix element $\langle \nu^M | J^{EM} | \nu^M \rangle$ is presented. The result of that derivation agrees with our Eq. (4.5). Also, an effective photon-Majorana neutrino interaction, expressed in a two-component formalism and involving one form factor, has appeared in J. Schechter and J. Valle, Ref. 2. Now, our t-channel analysis shows that there can only be one form factor. Thus, while it is not easy to go from the Schechter-Valle formalism to our Eq. (4.5), if we assume

that their effective interaction is a correct one, then it must actually be one way of expressing the most general effective interaction possible.

6. The author thanks L.N. Chang for a very helpful conversation in this connection.

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11. The form factor $M_D(q^2)$ calculated to one-loop order in Ref. 8 explicitly exhibits this behavior.

12. Relations between the Majorana and Dirac form factors for the electromagnetic transition between two different neutrinos ν_1 and ν_2 have been given by J. Nieves, Ref. 5.

13. See Ref. 8, and, for a specific example, M.A.B. Bég et al., Ref. 9.

TABLE I. Non-relativistic limits of terms in the t-channel electromagnetic matrix element. The final column lists the states produced by each term.

Term	Non-Relativistic Limit	States Produced
γ_μ	$(1 + \frac{p^2}{12m^2})\vec{n}\cdot\vec{T} - \frac{p^2}{6m^2}(3\vec{n}\cdot\hat{p}\,\vec{T}\cdot\hat{p} - \vec{n}\cdot\vec{T})$	${}^3S_1$, ${}^3D_1$
$(q^2\gamma_\mu - 2miq_\mu)\gamma_5$	$\vec{n}\cdot\vec{T} \times \hat{p}$	${}^3P_1$
$\sigma_{\mu\nu}q_\nu$	$(1 - \frac{\vec{p}^2}{12m^2})\vec{n}\cdot\vec{T} + \frac{\vec{p}^2}{6m^2}(3\vec{n}\cdot\hat{p}\,\vec{T}\cdot\hat{p} - \vec{n}\cdot\vec{T})$	${}^3S_1$, ${}^3D_1$
$\sigma_{\mu\nu}q_\nu\gamma_5$	$\vec{n}\cdot\hat{p}\ S$	${}^1P_1$

FIGURE CAPTIONS

Fig. 1. Virtual components in the state of a Majorana neutrino ν_-^M, whose subscript denotes its helicity. In the figure, ℓ is a charged lepton, and W the charged weak boson.

Fig. 2. One-loop diagram for the electromagnetic interaction of a Dirac neutrino ν^D with initial momentum and spin projection values i and final ones f. The term in the weak Lagrangian which is active at each vertex is written next to it.

Fig. 3. One-loop diagrams for the electromagnetic interaction of a Majorana neutrino ν^M. The symbol ℓ^c denotes the charge conjugate of the field ℓ.

Fig. 4. Electromagnetic interactions of a highly relativistic Dirac neutrino, when all weak currents are left-handed, as indicated at the initial and final vertices. The symbol Z^0 denotes the neutral weak boson, and the shaded area an arbitrary structure.

Fig. 5. A helicity-flipping transition involving the action of a right-handed current of strength r. The amplitude for this transition does not vanish when $m \to 0$.

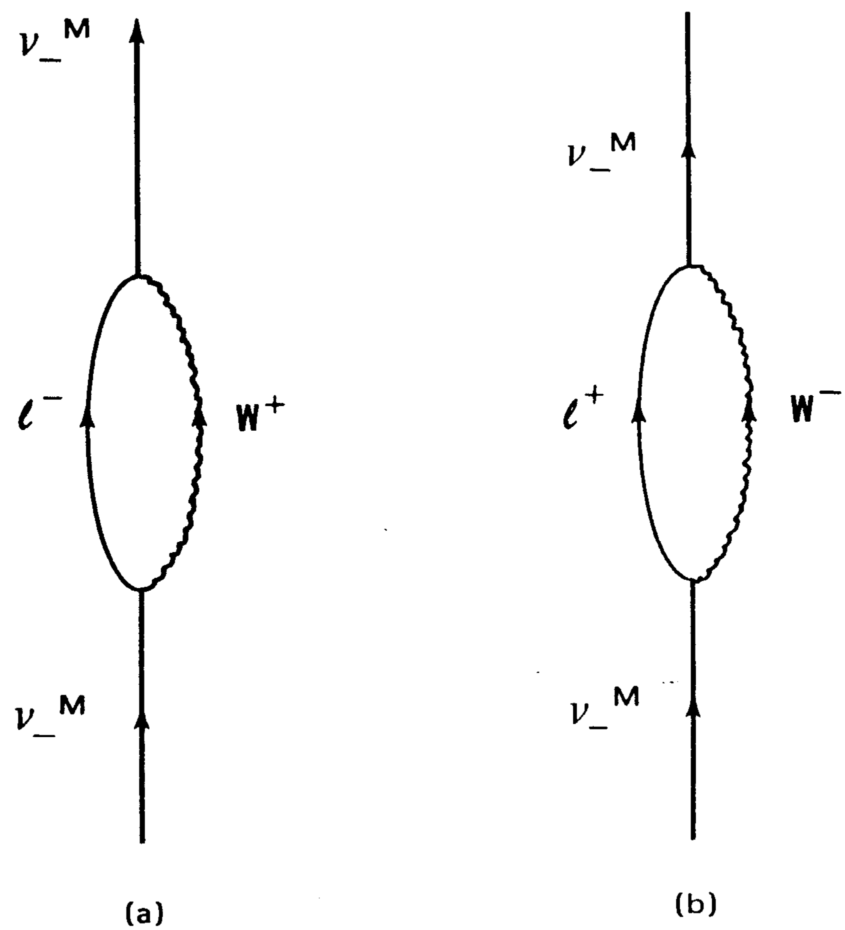


Fig. 1

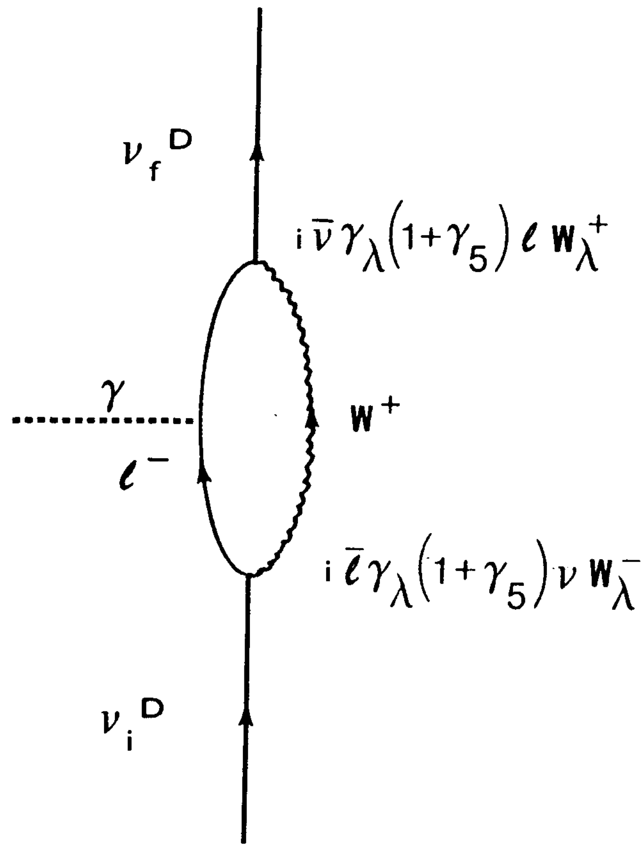

$\nu_f{}^D$
$i\bar{\nu}\gamma_\lambda(1+\gamma_5)\ell W_\lambda^+$
γ
W^+
ℓ^-
$i\bar{\ell}\gamma_\lambda(1+\gamma_5)\nu W_\lambda^-$
$\nu_i{}^D$

Fig. 2

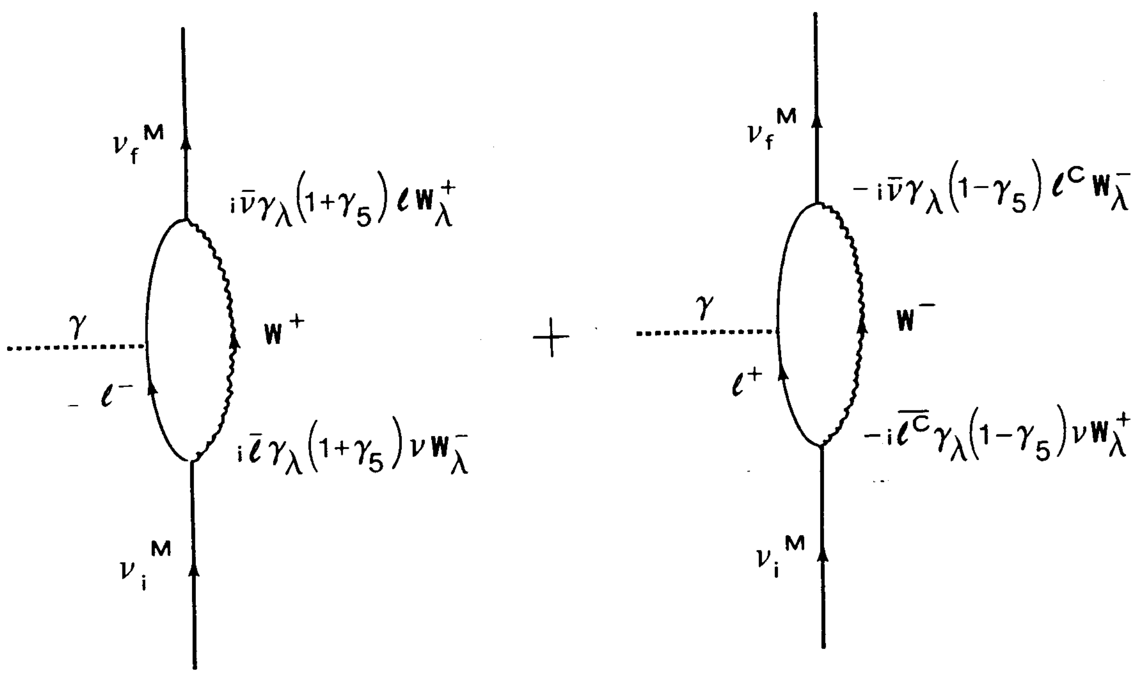
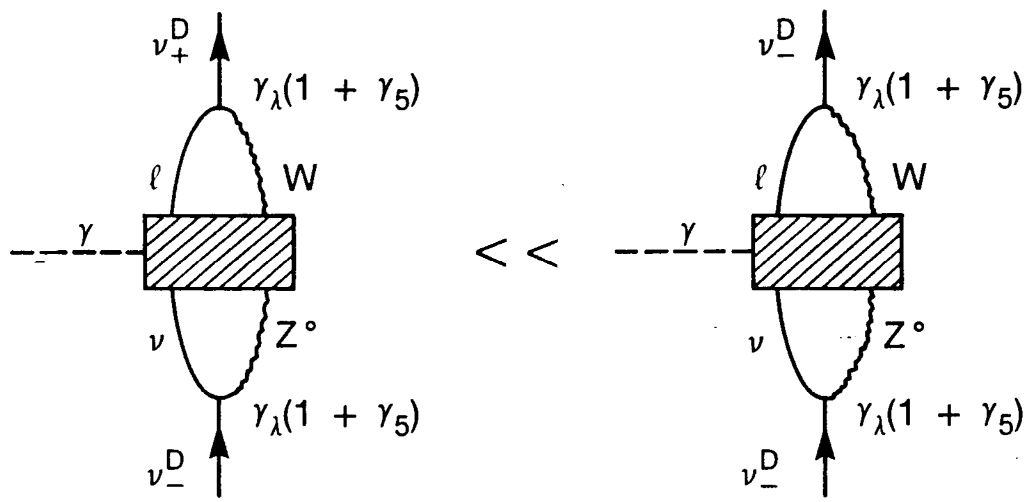
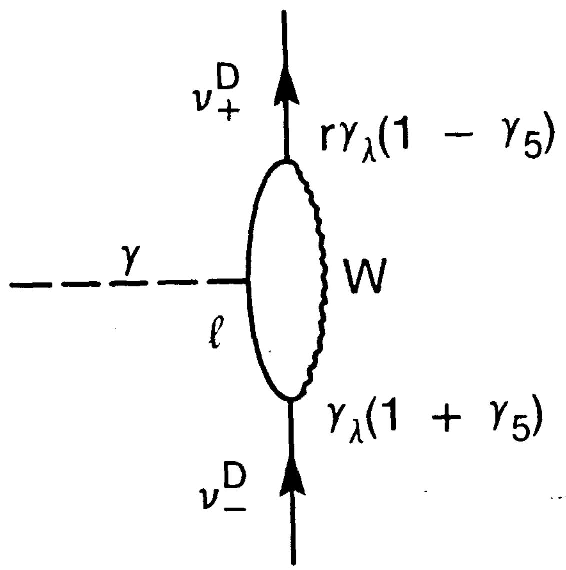

$\nu_f{}^M$
$i\bar{\nu}\gamma_\lambda(1+\gamma_5)\ell W_\lambda^+$
γ
W^+
ℓ^-
$i\bar{\ell}\gamma_\lambda(1+\gamma_5)\nu W_\lambda^-$
$\nu_i{}^M$
$+$
$\nu_f{}^M$
$-i\bar{\nu}\gamma_\lambda(1-\gamma_5)\ell^C W_\lambda^-$
γ
W^-
ℓ^+
$-i\overline{\ell^C}\gamma_\lambda(1-\gamma_5)\nu W_\lambda^+$
$\nu_i{}^M$

Fig. 3



ν_+^D
$\gamma_\lambda(1 + \gamma_5)$
ℓ
W
γ
ν
Z°
$\gamma_\lambda(1 + \gamma_5)$
ν_-^D
<<
ν_-^D
$\gamma_\lambda(1 + \gamma_5)$
ℓ
W
γ
ν
Z°
$\gamma_\lambda(1 + \gamma_5)$
ν_-^D

Fig. 4



ν_+^D
$r\gamma_\lambda(1 - \gamma_5)$
γ
W
ℓ
$\gamma_\lambda(1 + \gamma_5)$
ν_-^D

Fig. 5