SLAC-PUB-2877
January 1982
(T)

A STATISTICAL MECHANICAL CONSTRAINT ON FIELD THEORY*

Subhash Gupta and L. C. Biedenharn†
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We point out that statistical mechanics put a stronger constraint than asymptotic freedom on the number of flavors in confining gauge theories.

Submitted to Physical Review Letters

* Work supported in part by the Department of Energy, contract DE-AC03-76SF00515 and by the National Science Foundation.

† On sabbatical leave from Duke University, Durham, NC 27706.

The existence of asymptotic freedom in non-Abelian gauge theories implies, as is well known, a constraint on the possible number of flavors, n_f: for $SU(3^c)$, $n_f \leq 16$. We shall demonstrate below that if thermodynamics is applicable to a non-Abelian confining gauge theory then there exists a stronger constraint on the number of flavors. The constraint occurs for any number of colors, and for $n_c = 3$, we find $n_f < 12$.

The argument is quite straightforward, even elementary. Let us consider the thermodynamic properties of the vacuum at finite temperature in an imaginary world where the fundamental Lagrangian of the universe is a Lagrangian of a confining theory[1] such as quantum chromodynamics (QCD). We must state clearly our assumptions:

(i) The Hamiltonian is bounded below and is temperature independent. This assumption is necessary and sufficient to prove that the specific heat c_v is positive.[2]

(ii) Next let us assume a specific theory such as $SU(3^c)$ with n_f flavors of quarks which are massless. Such theories are assumed[3] to have a chiral realization with $n_f^2 - 1$ Goldstone bosons of zero mass and in addition a bound state spectrum of massive particles.

At very low temperatures only the degrees of freedom corresponding to the massless modes would be effective, the massive bound state spectrum being essentially frozen-out. Thus the effective number of degrees of freedom, $N_<$, is at least the number of Goldstrone modes: $N_< > (n_f^2 - 1)$, and the corresponding energy is $E \geq c(T)\ kT(n_f^2 - 1)$.

For the confining theories we are considering, there is a phase transition[4] at a critical temperature T_c, and the system becomes deconfined. Physically this is just a consequence of asymptotic freedom and the fact that at finite temperature the particles start overlapping.

Now let us consider the number of degrees of freedom that exist just above the critical temperature $T = T_{c+}$. Massless gluons contribute $8 \cdot 2$ degrees of freedom for $SU(3^c)$ and the fermion modes contribute $12 \cdot n_f \cdot \frac{7}{8}$ degrees of freedom (particle/antiparticle spin color; the factor $\frac{7}{8}$ comes from Fermi-Dirac vs Bose-Einstein statistics). We find:
$N_> = 16 + \frac{21}{2} n_f$.

From this we can obtain the energy $E_<$ below and $E_>$ just above the critical temperature by multiplying the number of degrees of freedom by kT, and in turn the latent heat of transition:

$$\Delta E = E_> - E_< = c(T_c)\; kT_c (N_> - N_<)$$

$$\leq c(T_c)\; kT_c \left(16 + \frac{21}{2}(n_f) - (n_f^2 - 1)\right) \quad .$$

This result shows that for n_f sufficiently large, ΔE becomes <u>negative</u>, which is physically <u>impossible</u>.[2] This provides the sought-for constraint: $n_f < 12$.

<u>Remark</u>:

(1) The possible existence of condensates like the $\bar{\psi}\psi$ and FF does not affect the argument since they could only strengthen the inequality, as these are present only below T_c.

(2) The possible existence of monopoles also does not affect the argument since monopoles are believed to be massive and would accordingly be frozen-out at $T \approx T_c$. Also, the number of degrees of freedom for $SU(3^c)$ monopoles is only 2 and independent of n_f; hence does not change the limit.

(3) If the chiral symmetry realization is by means of massless spin ½ baryons, then the constraint on n_f is even stronger, as in that case the number of spin ½ baryons varies as n_f^3, for large n_f. For $SU(3^c)$ we find the number to be around 7.

(4) Of course the above considerations are applicable to all confining gauge theories.

ACKNOWLEDGEMENTS

We would like to thank Tom Weiler and Christine Di Lieto for discussions. Especially we would like to thank Sid Drell and Helen Quinn for critical remarks and helpful suggestions. This work was supported in part by the Department of Energy, contract DE-AC03-76SF00515 and by the National Science Foundation.

REFERENCES

1. M. Creutz, BNL 30432 (1981); see references therein.

2. L. D. Landau and E. M. Lifshitz, Statistical Physics, Pergamon, London (1958), Chapter 2, Sections 20 and 21.

3. B. Svetitsky, S. D. Drell, H. R. Quinn and M. Weinstein, Phys. Rev. D22, 490 (1980). There the chiral realization is proven on a lattice version of QCD. t'Hooft, Cargese Institute Lectures (1979).

4. A. M. Polyokov, Phys. Lett. 72B, 477 (1978). The proof here uses a lattice version of the theory with no fermions. However, it is a widely held belief that the result should be true even in the presence of any number fermions and in the continuum.