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CAN SOFT GLUON EFFECTS BE MEASURED IN ELECTRON-POSITRON ANNIHILATION?*

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ABSTRACT

The energy-energy correlation at large angles in e^+e^- annihilation is calculated by resumming soft gluon contributions through two-loop level. The result is compared with experimental data. No agreement is obtained using a purely perturbative analysis. The relevance of nonperturbative effects at present energies is emphasized.

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There has been a great interest in the processes characterized by two large but different mass scales in perturbative Quantum Chromodynamics (QCD).[1] One well-known example is the production of two almost acollinear hadrons in e^+e^- annihilation. In this process the total energy Q and the relative (scaled) transverse momentum Q_T between two hadrons play the role of the two scales [$Q_T^2 \simeq Q^2 \sin^2(\theta/2)$; collinearity angle $\theta \sim 180^\circ$]. The facts that experimental data are already available (PETRA)[2] and that new data will be available in the near future (PEP) make this process particularly interesting also from an experimental point of view.

To analyze such a process, it is useful to introduce a well-defined and measurable cross section: the energy-energy correlation.[3] This quantity is expected to give a clean test of QCD since not only its energy dependence but also its absolute magnitude can be calculated. In the acollinear configuration the energy-energy correlation is governed by the effective quark form factor[4] built up by the resummation of large corrections due to soft gluon effects to all orders.[5] This problem has been analyzed at one loop level by several authors.[1]

In order to obtain a reliable answer in such a configuration (i) it is important[6,7] to perform a perturbative calculation through two loops[7] and (ii) the question about the sensitivity of the result to nonperturbative effects must be investigated.

Using the unintegrated parton densities[4,8] $D(Q^2,p_T,x)$ the energy-energy correlation is written as[9] (see Fig. 1)

$$\frac{1}{\sigma_{TOT}} \frac{d\Sigma}{d^2Q_T} = \frac{1}{\sigma_{TOT}} \frac{1}{2} \sum_{A,B} \int x_A dx_A \int x_B dx_B \sum_{q,\bar{q}} \int d^2p_T^A \, d^2p_T^B \, d^2p_T^S \times \delta^2\left(Q_T - \frac{p_T^A}{x_A} - \frac{p_T^B}{x_B} - p_T^S\right) D_q^A\left(Q^2,p_T^A,x_A\right) D_{\bar{q}}^B\left(Q^2,p_T^B,x_B\right) S\left(Q^2,p_T^S\right) \tag{1}$$

where the sums extend over all hadrons and all quark flavors respectively and $S(Q^2, p_T^S)$ represents the set of the two-particle irreducible contributions with external photon vertices included. Taking the Fourier transform into impact parameter (b_T) space the (nonsinglet) quark density[10] obeys the following evolution equation in the light-like gauge (gauge vector parallel to the antiquark momentum)[7]

$$Q^2 \frac{\partial}{\partial Q^2} D_q\left(Q^2, b_T, x\right) = \int \frac{dz}{z} \int dq_T^2 \left[\frac{\alpha_s(q_T^2)}{2\pi} + K\left(\frac{\alpha_s(q_T^2)}{2\pi}\right)^2 \right] \tag{2}$$
$$\times\ C_F\left(\frac{1+z^2}{1-z}\right)_+ \delta\left[z(1-z)Q^2 - q_T^2\right] J_0\left(\frac{bq}{z}\right) D_q\left(Q^2, \frac{b_T}{z}, \frac{x}{z}\right) ,$$

with J_0 the Bessel function of the first kind, $b = |b_T|$, $q = |q_T|$. The $(\ )_+$ notation indicates the usual regularization procedure. K has been given in Ref. 11 in the same gauge: $K = C_G[(67/18) - (\pi^2/6)] + N_F T_F(-10/9)$. We use the transverse momentum of the emitted gluon q_T^2 as the scale in the running coupling constant α_s which controls soft gluon effects.[1,12] In the soft gluon approximation ($z \sim 1$, $Q^2 >> q_T^2$), Eq. (2) can be solved and soft contributions resummed systematically. $S(Q^2, p_T^S)$ is calculated in the same way as the coefficient function in ordinary hard processes. Substituting the corresponding formulas[7] for D_q, $D_{\bar{q}}$ and S, Eq. (1) becomes

$$\frac{1}{\sigma_{TOT}} \frac{d\Sigma}{d\cos\theta} = \frac{Q^2}{8\pi} \int d^2 b_T\, e^{i b_T Q_T}\, e^{T(b)} \sum_A \int dx_A x_A\, D_q^A\left(\frac{1}{b^2}, b, x_A\right) \tag{3}$$
$$\times \sum_B \int dx_B x_B\, D_{\bar{q}}^B\left(\frac{1}{b^2}, b, x_B\right) ,$$

where $\exp[T(b)]$ is the effective form factor[7] and

$$T(b) = \frac{C_F}{\pi} \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \left[\ell n \frac{Q^2}{q^2}\, \alpha_s(q^2) + \ell n \frac{Q^2}{q^2} \frac{K}{2\pi} \alpha_s^2(q^2) + 2\ell n \frac{e^{\gamma_E}}{2}\, \alpha_s\left(\frac{1}{b^2}\right) - \frac{3}{2}\, \alpha_s(q^2) \right] \tag{4}$$

with γ_E Euler constant. We choose in Eqs. (3) and (4) the starting value of perturbative evolution to be[13] $1/b^2$ to eliminate logarithmic corrections other than[14] $\ln Q^2b^2$. Due to this fact the residual density $D(1/b^2,b,x)$ would be safely expanded in terms of $\alpha_s(1/b^2)[\alpha_s(1/b^2) << 1]$: $D(1/b^2,b,x) = D(1/b^2,x) + \mathcal{O}[\alpha_s(1/b^2)]$. $D(1/b^2,x)$ is the usual decay function which satisfies the sum rule: $\sum_A \int dx_A x_A\, D(1/b^2,x_A) = 1$. Equation (4) contains all the contributions of the form[15] $B(B/L)^n$ [first term in Eq. (4)] and $(B/L)^n$ $(n \geq 1)$ where $L \equiv \ln Q^2/\Lambda^2$ and $B \equiv \ln Q^2b^2$ with Λ the fundamental scale parameter. Collins and Soper, with a different formalism, have suggested a method[6] to compute such terms. The neglected terms are of order $\mathcal{O}[(1/L)^\gamma\ (B/L)^n]$ $(n \geq 0,\ \gamma \geq 1)$ and $\mathcal{O}(1/Q^2)$. Such a classification can be obtained by taking the limit $L \to \infty$ with B/L fixed and proves to be effective to pick up dominant contributions.[7]

In Fig. 2 we compare Eq. (3) (numerically integrated) with PLUTO data[2] at $Q \sim 30$ GeV using $N_F = 5$ and $\Lambda = 0.1, 0.2, 0.3$ GeV. The purely perturbative answer of Eq. (3) shows no agreement with the data.[16] To clarify the situation the cross section (Fig. 3) and the effective form factor $\exp[T(b)]$ (Fig. 4) are plotted also at $Q = 100$ GeV. In Figs. 3 and 4 shown by dashed lines are the curves obtained keeping only the terms $B(B/L)^n$ [first term in Eq. (4)] which correspond to the result derived in Ref. 17. A detailed analysis of the contributions of these terms has been done in Ref. 19. Figures 3 and 4 indicate that the contributions of terms $(B/L)^n$ cannot be neglected compared to the terms $B(B/L)^n$. For this reason the summation of the $B(B/L)^n$ terms gives only a partial result. (Note that when Q is larger $B(B/L)^n$ terms tend to be more important.) Keeping both these terms and neglecting $\mathcal{O}[(1/L)^\gamma\ (B/L)^n]$ and $\mathcal{O}(1/Q^2)$ terms in the exponent $T(b)$ in Eq. (3) make a sensible approximation. This fact can be explained by noticing that the region of small values of b is the relevant one in the integration of Eq. (3) due to the Sudakov-type effects in b_T space: the strong suppression of large b when Q^2 is large[18] (see Fig. 4). Since the perturbative calculations are unreliable in the large b region, this observation supports the above evaluation of the perturbative contributions. In the case that the large b suppression is not sufficiently strong, the final result should receive some corrections which come from not only the uncertainties in the perturbative estimations but also the nonperturbative effects (important at large b).

At present energies exp[T(b)] has still a rather long tail into large b region (Fig. 4a). This situation may be considered to be the case mentioned above. We have checked that the integration of Eq. (3) with an artificial cut off $b_{max} = M_0$ [$M_0 \sim \mathcal{O}(1\ \mathrm{GeV}^{-1})$] to roughly isolate purely perturbative contributions does not change the result and no better agreement with the data can be obtained. This suggests that at present energies the region of large b is still important. In fact the details of the answer (e.g., the slope of the cross section) are rather sensitive also to the precise behavior of exp[T(b)] in the damping regions.[19] If this region is also within the perturbative domain [$\alpha_s(1/b^2) \ll 1$], exp[T(b)] can be calculated perturbatively and a reliable answer will be obtained. At present energies however, this region cannot be treated only by the perturbative approach. For this reason we believe that it seems to be quite difficult to find a characteristic feature of perturbative QCD at present energies without a convincing way to handle nonperturbative effects.[16] In this specific kinematic region the usual treatment of nonperturbative dynamics[20] would decrease the cross section in Eq. (3). This is expected as well by intuitive pictures about hadronizations. A better agreement with the data is likely. However, the crucial role played by the behavior of the form factor exp[T(b)] in the damping region will make such treatment too strongly model dependent and probably too naive.[21] As the energy Q increases (e.g., Q = 100 GeV) on the other hand, the shrinking into smaller b region[22] (Fig. 4b) makes the result of the perturbative approach more promising.

In conclusion we would like to stress that at present energies in order to explain the data it is essential to understand nonperturbative effects. Experiments at higher energies (LEP and/or COLLIDER) could answer the question whether the dynamics of soft gluons can be described perturbatively.

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9. Our normalization is different by a factor 2 from the original definition in Ref. 3 and we assume the factorized form of the cross section.
10. Within our approximation it is sufficient to analyze only the nonsinglet density.
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13. We calculate in the modified minimal subtraction ($\overline{MS}$) renormalization scheme.

14. We restrict our analysis to the perturbative region of b [$\alpha_s(1/b^2) << 1$]. When $1/b^2$ is small one should use another scale M_0^2 [$\alpha_s(M_0^2) < 1$] as the starting value of the evolution.

15. The two-loop correction to the running coupling constant introduces contributions of the type $(\ell n\ L)(B/L)^n$.

16. For previous analyses, see Ref. 1 and references therein.

17. See, e.g., G. Parisi and R. Petronzio, Nucl. Phys. B154, 427 (1979).

18. This corresponds to the usual Sudakov suppression in momentum space at small transverse momentum. Interesting arguments on related problems can be found in Ref. 19.

19. S. D. Ellis, N. Fleishon and W. J. Stirling, Phys. Rev. D24, 1386 (1981).

20. Nonperturbative effects are usually parametrized to be relevant only in the large b (small Q_T) region (see Ref. 17).

21. In the intermediate angles region, the inclusion of nonperturbative effects seems to be possible. See Ref. 3 and L. S. Brown and S. D. Ellis, Phys. Rev. D24, 2383 (1981).

22. It may be interesting to point out that as Λ^2 decreases at fixed Q^2, exp[T(b)] develops a longer tail.

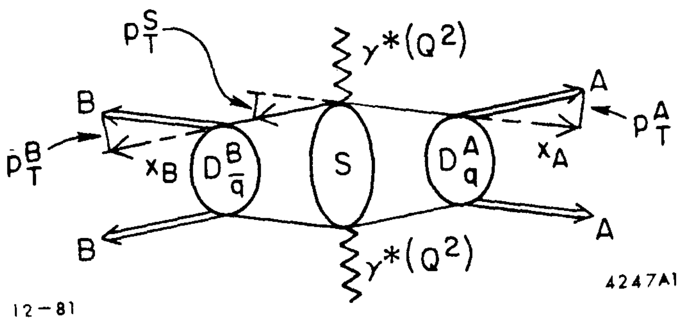
Figure Captions

Fig. 1. The process $e^+e^- \to \gamma^* \to A + B + X$ with kinematics. Single (double) line represents quark or antiquark (hadron).

Fig. 2. The comparison between the theory Eq. (3) and the PLUTO data for the energy-energy correlation at large angles ($Q \sim 30$ GeV).

Fig. 3. The energy-energy correlation at the two different energies. (a) $Q = 30$ GeV, $\Lambda = 0.2$. (b) $Q = 100$ GeV, $\Lambda = 0.2$. The solid (dashed) line is for the full result (first term) of Eq. (4).

Fig. 4. The effective form factor $\exp[T(b)]$ in b_T space; (a) $Q = 30$ GeV, $\Lambda = 0.2$; (b) $Q = 100$ GeV, $\Lambda = 0.2$. The solid (dashed) line is defined as in Fig. 3.



p_T^S
$\gamma^*(Q^2)$
B
A
p_T^B
x_B
$D_{\bar{q}}^B$
S
D_q^A
x_A
p_T^A
B
A
$\gamma^*(Q^2)$
4247A1
12-81

Fig. 1

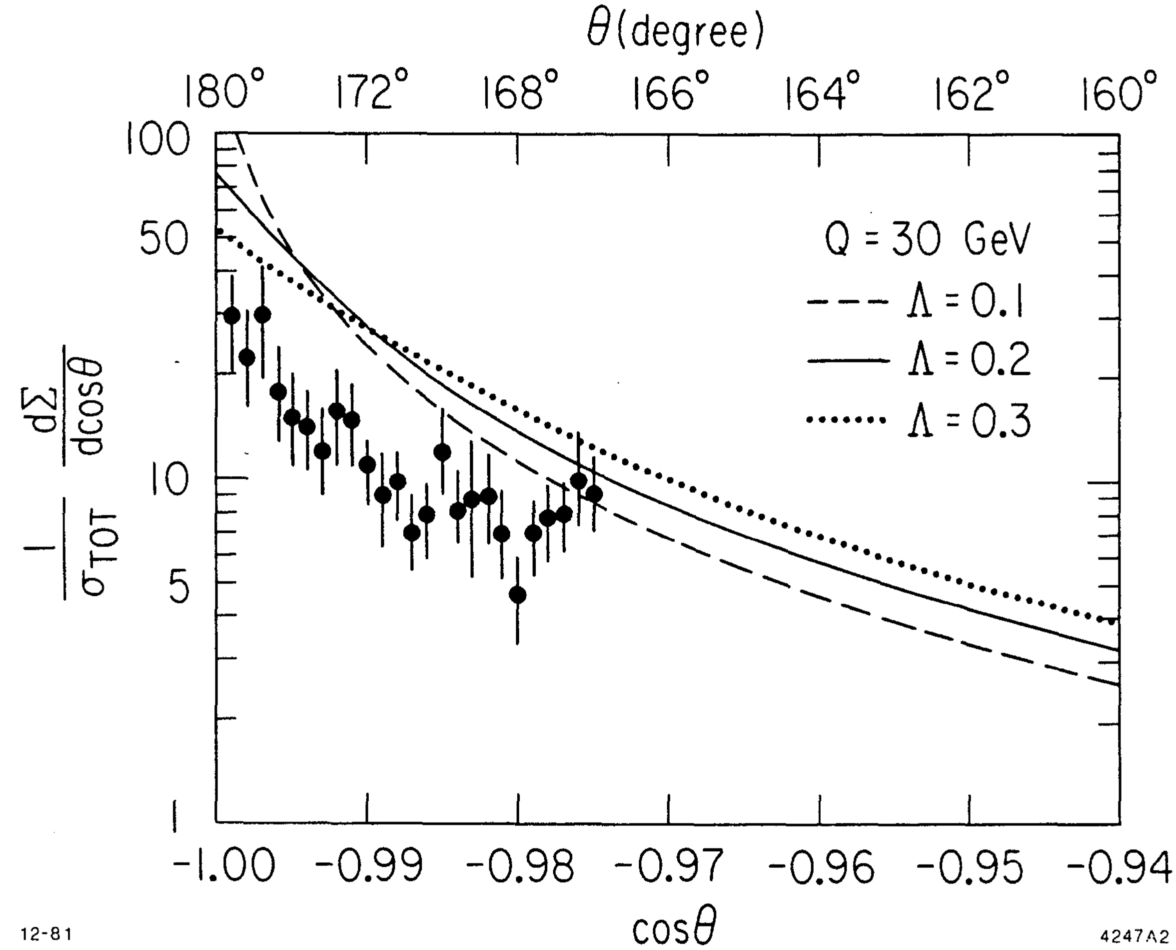
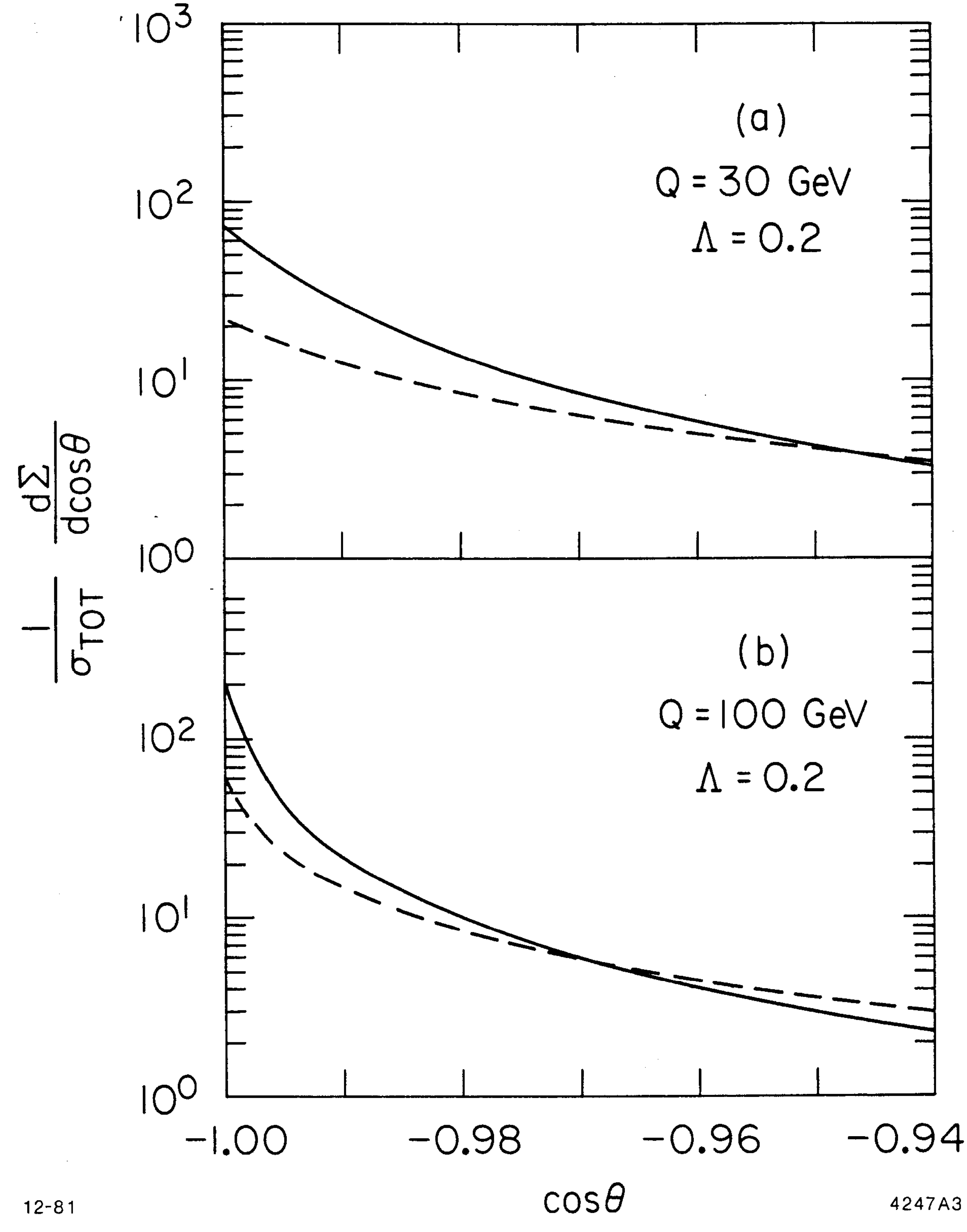

θ(degree)
180°
172°
168°
166°
164°
162°
160°
100
50
10
5
1
Q = 30 GeV
Λ = 0.1
Λ = 0.2
Λ = 0.3
1/σTOT dΣ/dcosθ
-1.00
-0.99
-0.98
-0.97
-0.96
-0.95
-0.94
cosθ
12-81
4247A2

Fig. 2



(a)
Q = 30 GeV
Λ = 0.2
(b)
Q = 100 GeV
Λ = 0.2
10³
10²
10¹
10⁰
$\frac{1}{\sigma_{TOT}} \frac{d\Sigma}{d\cos\theta}$
-1.00
-0.98
-0.96
-0.94
cosθ
12-81
4247A3

Fig. 3

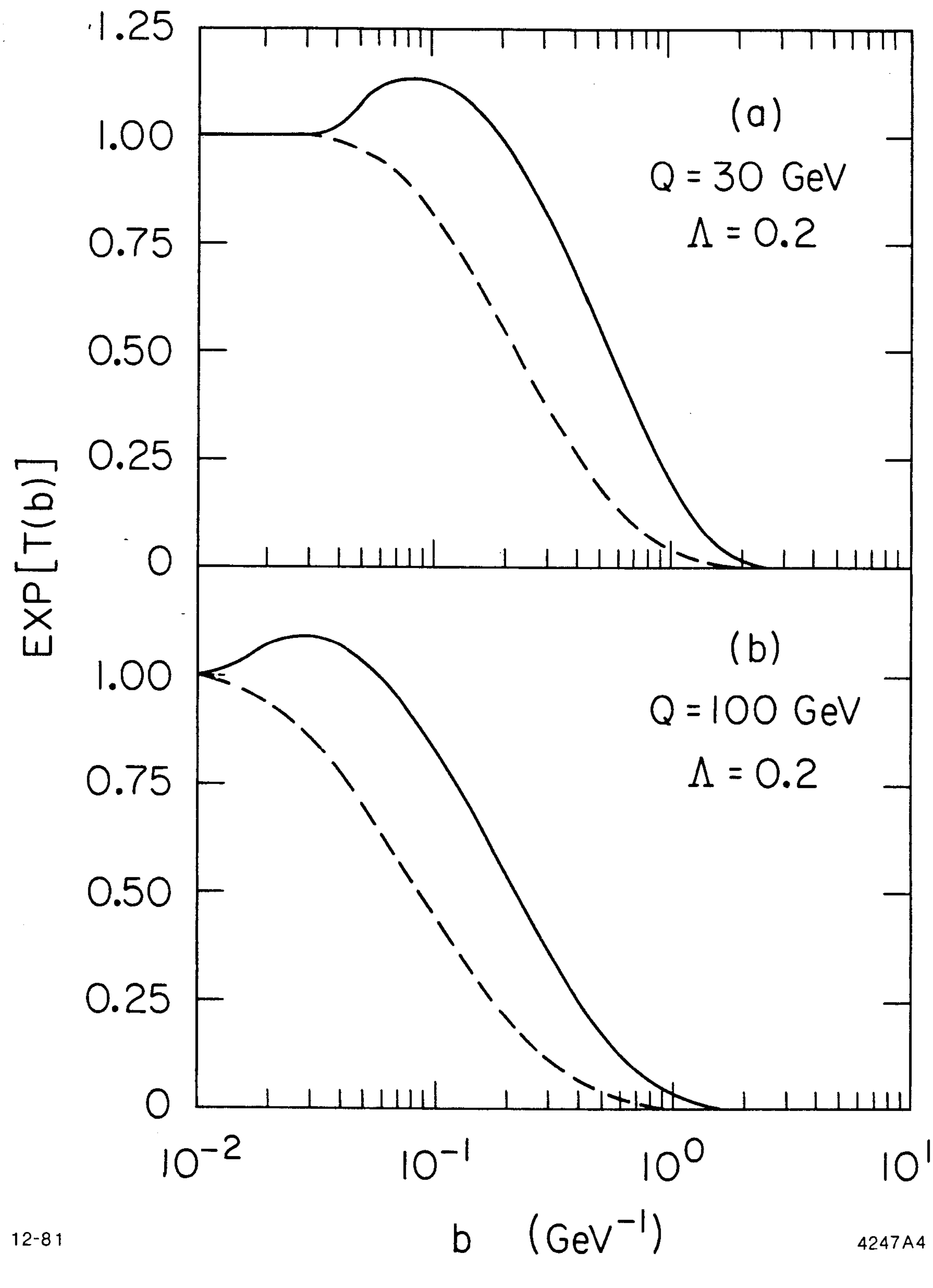

1.25
1.00
0.75
0.50
0.25
0
(a)
Q = 30 GeV
Λ = 0.2
1.00
0.75
0.50
0.25
0
(b)
Q = 100 GeV
Λ = 0.2
EXP[T(b)]
10⁻²
10⁻¹
10⁰
10¹
b (GeV⁻¹)
12-81
4247A4

Fig. 4