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Production of Protons and Lambdas in e^+e^- Jets from Jet Calculus and the Recombination Model.*

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ABSTRACT

We compute the expected yields of protons and lambdas in e^+e^- annihilation, using the KUV jet calculus and the recombination model. Our results have many features in common with the data, including their approximate size. We derive a differential equation for baryon production, and show that the terms we have calculated are one of three physically different contributions.

I. INTRODUCTION

Recently there has been a great deal of attention given to baryons in jets, from both the experimental and theoretical points of view. Data currently available on quark jets in e^+e^- annihilation show that the baryon distributions are within an order of magnitude of the meson distributions (an unexpectedly large cross-section for the production of baryons)[1-5], and that they have similar shape in x at large x (Ref.1). Similar features can be seen in quark jet data in deep inelastic muon scattering, as shown by the European Muon Collaboration[6]. At the same time, even in the absence of data, the creation of baryons in parton models offers enough differences from the meson case to present a theoretical challenge.

Several theoretical papers have appeared discussing rather different physical pictures for baryon creation. We may divide the more recent ones into:

a) "string" pictures, such as the Lund model[7,8] and the Israeli model[9], in which baryons are produced by diquarks formed from breaking strings of colour force (see also references 10 and 11) and

b) "recombination" pictures, in which quark triplets found in a parton jet are recombined with an explicit "gluing together" function which makes them into a baryon[12,13],

although there exist other papers with less explicit models (for example, reference 14). In this paper we present another calculation in the recombination picture.

Our basic scheme is the production of quark jets using the jet calculus of Konishi, Ukawa and Veneziano[15] (KUV), followed by "gluon conversion" into quark-antiquark pairs[16], and the gluing together of all triplets of quarks into baryons. In this our work resembles that of Eilam and Zahir[13], but as we shall see, our answers differ from theirs both qualitatively and quantitatively. This is due to the facts (i) that we evaluate the full jet calculus expression for the three quark distribution, rather than cutting off the integrals and throwing out small "unfavoured" propagators; (ii) we have a different functional form for the recombination function and (iii) we prefer different values for the parameters Λ and Q_o in the jet calculus.

In Section II we discuss in detail the evaluation of our formula for the recombination contribution to baryon production. Certain features of the computation which simplify the evaluation are explained.

In Section III we show the results of the calculation and their dependence on the parameters in the theory, Q_o^2 and Λ. The dependence on the jet value of Q^2 is also shown and compared with the evolution more commonly assumed.

In Section IV we put this contribution to the baryon fragmentation function into focus by writing a set of differential equations describing the evolution in Q^2. The solution to these equations has three terms:

a) a term of the "ordinary" Uematsu-Owens type[17], whose size is governed by the "intrinsic" baryon content of the partons at Q_o^2;

b) the recombination term, whose size is governed by the recombination function assumed at Q_o^2, and

c) a term whose size is governed by the "intrinsic" diquark content of the parton at Q_o^2 and a recombination parameter for recombining a diquark and a quark into a baryon (this parameter is not independent of the "triquark" recombination size).

These terms have rather different Q^2 evolution properties.

Section V is a summary.

II. EQUATIONS

Our basic formula for the recombination contribution to the parton fragmentation into baryons is (see Fig. 1)

$$D_i^B(x,Q^2) = \int D_{a_1a_2a_3,i}(x_1,x_2,x_3;Q^2)R^B_{a_1a_2a_3}(x_1,x_2,x_3;x)dx_1dx_2dx_3 \qquad (2.1)$$

where $D_{a_1\,a_2\,a_3,i}(x_1,x_2,x_3;Q^2)$ is the probability for finding the three partons a_1,a_2,a_3 with momentum fractions x_1,x_2,x_3 when they originate from parton i of four momentum squared Q^2. In Eq.(2.1), the sum over the indices a_1,a_2 and a_3 is implied. We will usually suppress the summation sign and assume the summation convention for repeated indices.

The KUV jet calculus formula for this distribution $D_{a_1a_2a_3,i}(x_1,x_2,x_3;Q^2)$ is (see Fig. 2)

$$D_{a_1a_2a_3,i}(x_1,x_2,x_3;Q^2) = \sum_{jb_1b_2b_2'c_1c_2} \int_{Y_o}^{Y} dy \int_{Y_o}^{y} dy' \int dxdzdz'dx''dx'dw_1dw_2dw_3$$

$$D_{a_1b_1}(w_1,y-Y_o)\ D_{a_2c_1}(w_2,y'-Y_o)\ D_{a_3c_2}(w_3,y'-Y_o)\hat{P}_{b_2'\to c_1c_2}(z').$$

$$D_{b_2'b_2}(x'',y-y')\hat{P}_{j\to b_1b_2}(z)\ D_{ji}(x,Y-y).$$

$$\delta(x_1-w_1zx)\ \delta(x_2-z'x'w_2)\ \delta[x_3-(1-z')x'w_3]\ \delta[x'-x''x(1-z)] \qquad (2.2)$$

The variable Y is related to Q^2 by

$$Y=(2\pi\ b)^{-1}\ \ell n[1+\alpha_o b\ell n(Q^2/\Lambda^2)] \tag{2.3}$$

where $12\pi b = 11\ N_c-2N_f$, with $N_c = 3$ colors and $N_f = 3$ flavors. The partons j,b_1,b_2,b_2',c_1 and c_2 have momentum fractions $x,z,\ 1-z,\ x'',w_2$ and w_3, respectively, with intermediate momenta x' and z' integrated. The KUV propagator $D_{\ell k}(v;y(Q^2))$ gives the probability for finding parton ℓ with momentum fraction v in the QCD generated cloud of parton k characterized by four momentum squared Q^2. The $\hat{P}$'s are the Altarelli-Parisi[18] branching functions for virtual partons (with the delta functions at x=1 removed). We use the notation given by Willen[19] for QCD strength parameters

$$\Lambda'^2 = \Lambda^2 \quad e^{-1/(b\alpha_o)}$$

and

$$\alpha_s = \frac{1}{b\ell n(Q^2/\Lambda'^2)}$$

The original jet has "off shellness" Q^2 and the three partons are measured at off shellness Q_o^2.

Our choice for the three quark recombination function

$$R^B_{qqq}(x_1,x_2,x_3;x) = \frac{27x_1x_2x_3}{x^3} \quad \delta(x_1+x_2+x_3-x) \tag{2.4}$$

is discussed in the next Section. Since it has only integer powers of x_1,x_2,x_3 we can compute moments of the three quark contribution to $D^B_i(x,Q^2)$ in terms of moments of the individual parton propagators

$D_{ij}(n,Q^2)$ and the moments of the vertex functions,

$$P^{m,n}_{i\to cd} = \int_o^1 z^m \, (1-z)^n \, \hat{P}_{i\to cd} \, (z) \, dz$$

thus giving (from the three-quark recombination only)

$$D^B_i(n,Y) = 27(n-3)! \sum_{jb_1b_2b'_2c_1c_2q_1q_2q_3} \int_{Y_o}^{Y} dy \int_{Y_o}^{y} dy'$$

$$\sum_{m=o}^{n-3} \sum_{r=o}^{m} \frac{1}{(n-3-m)!} \frac{1}{r!(m-r)!} D^{n-m-2}_{q_1b_1} (y-Y_o).$$

$$D^{m-r+1}_{q_2c_1} (y'-Y_o) \, D^{r+1}_{q_3c_2} (y'-Y_o) \, D^{m+2}_{b'_2b_2} (y-y').$$

$$D^n_{ji} (Y-y) \; P^{n-m-2,m+2}_{j\to b_1b_2} \; P^{m-r+1,r+1}_{b'_2\to c_1c_2} \tag{2.5}$$

The integrals over intermediate values of Q^2 may be done analytically in the representation in which the propagators $D(n,Q^2) = \exp(A_n Y)$ are diagonal.

If we used only the $D_{q_1q_2q_3,i}(x_1,x_2,x_3;Q^2)$ for baryon production and the $D_{q_1q_2,i}(x_1,x_2;Q^2)$ for meson production, we would have an unphysical situation with many gluons "left over" at the end of the jet evolution. Clearly we must use an algorithm which somehow includes these gluons into the hadrons formed in the jet. However, due to the rather primitive state of the recombination phenomenology, we wish to create baryons by combining only three quarks rather than by looking at sets of (three quarks plus multiple gluons).

The ansatz used by Chang and Hwa[16], conversion of the gluons by fiat into quark-antiquark pairs in such a way that their momentum is conserved, was rather successful in the corresponding calculation of pion content for e^+e^- [16,20], so we shall use that here. Specifically, the probability that a gluon turns into a quark (antiquark) of momentum fraction z(1-z) is taken to be

$$\bar{P}_{g\to q\bar{q}}(z) = \frac{3}{2N_f}\left[z^2+(1-z)^2\right] \quad . \tag{2.6}$$

Note that no additional powers of the strong coupling are included. This formula is particularly simple to compute for our case, since the contributions from $D_{gqq,i}(x_1,x_2,x_3;Q^2)$ to the total take the simple form

$$\int D_{gqq,i}(x_1,x_2,x_3;Q^2)\,\bar{P}_{g\to q\bar{q}}(z)\,\delta.(\xi - zx_1).$$

$$R^p_{qqq}(\xi,x_2,x_3;Q^2)d\,\xi dx_1dx_2dx_3dz$$

$$= \int D_{gqq,i}(x_1,x_2,x_3;Q^2)\bar{P}_{g\to q\bar{q}}(z)\;R^p_{qqq}\;(zx_1,x_2,x_3;x)dx_1dx_2dx_3dz \tag{2.7}$$

and hence we see that all the moments of $D_{qqq,i}(x_1,x_2,x_3;Q^2)$ computed from x^m in the expression of Eq.(2.5) will just be multiplied by

$$\int_0^1 \bar{P}_{g\to q\bar{q}}(z)\;z^m\,dz.$$

Similar comments apply to the contributions from 2 and 3 gluons.

In order to organize the calculations in such a way that we can see the forest in spite of the trees, we utilize the counting scheme indicated in Appendices A and B. In calculating the contributions to proton and antiproton production from the three species of quark jets in e^+e^-, we realize that symmetry relates all the required terms to the "23 irreducible diagrams" shown in Appendix A. In these diagrams, all the intermediate states are summed. Similarly, for the lambdas, we need only the 18 irreducible diagrams shown in Appendix B.

The double integrals of all required combinations of parton propagators are evaluated once and stored. Then separate packages using the moments of the vertex functions assemble these for the irreducible graphs. Finally the irreducible graphs are multiplied by weights which indicate the number of times each occurs and the appropriate charge factor for the photon vertex, and the cross-section is obtained.

We have been able to evaluate nine moments ($3 \leq n \leq 11$) of the baryon fragmentation function in this way. Note that we keep all terms in the propagators; unlike Eilam and Zahir[13] we have not been forced to make approximations.

To invert the moments, we use Yndurain's method[21], which should work here. Due to the fact that we can only calculate moments n=3 and above (see Eq. (2.5)) our answers give the values of $D_i^B(x,Q^2)$ only for x above 0.3077. We could compute more moments if it were absolutely necessary, but the program already takes about 10 minutes on the SLAC computer, to produce the x distribution for each value of Q_o^2 and Λ, so we have therefore tried to be frugal in spite of our enthusiasm.

Since we use Yndurain's method, our answers should be reasonably accurate for all points except the two end points shown. In this we avoid the inaccuracies of a parametrization of the x dependence of the type used by Chang and Hwa[22] and Eilam and Zahir[13]. As discussed below in Section III, their technique has apparently resulted in incorrect evaluation of the Q^2 dependence of their expression at low x.

III. RESULTS

a. PARAMETERS

The size of the answers depends strongly on three parameters in the theory: Λ, the QCD constant that enters in α_s; Q_o^2, the off-shellness of the recombining partons; and R, the overall size of the recombination function (e.g. R=27 in Eq.(2.4)). The results also depend, both in magnitude and in qualitative features, on the way the gluons are treated. As far as this paper is concerned, only one method for handling the gluons has been considered - the one described in the previous section and Appendix C. We therefore consider this as fixed, and worry only about the dependence on the three parameters Λ, Q_o^2 and R.

Our value of Λ is the Q^2 at which $\alpha_s(Q^2) = 1.0$. The more usual parameter, Λ' - such that $\alpha_s(Q^2) = 1/b \ \ell n \ (Q^2/\Lambda'^2)$ - differs very little from it: $\Lambda' = 0.932\Lambda$. We therefore expect that values of Λ between 50 Mev and 200 Mev should be reasonable, in accord with recent fits to the scale-breaking behaviour of the nucleon structure functions[23].

The choice of Q_o^2 is much less clear. Our first concern, of course, must be that the strong coupling constant be small here. Since we are using a perturbative evolution, we should not have very large values of $\alpha_s(Q^2)$ at any point in the spray: this will be true if the values at the end of the evolution are still small.

Our second concern is that the value of Q_o^2 should be reasonable for the application of the recombination model in the simple (really primitive) form we are using here. This is currently more a matter of art than science. However, early applications of the recombination model involved partons coming forward in hadron-hadron interactions. These partons were not terribly far off their mass shell; probably they had Q_o^2 less than 5 Gev^2.

For this reason we will restrict the variation of Q_o^2 to values less than 5 Gev^2. The lower limit of variation should probably be that for which $\alpha_s(Q_o^2) = 1$, ie. $Q_o = 1.43\Lambda'$. In this paper we consider a rather more restricted range of variables, down to $Q_o^2 = 1.0$ Gev^2. We have no very good reason for this restriction of the range, other than a vague intuitive feel that the sum of the masses of the recombining partons ought to be quite a bit larger than the mass of the created hadron to allow for a lot of binding energy. Since the gluons split, their quark daughters will have smaller mass, so we want the Q_o^2 of the parent gluons to be fairly substantial.

In an earlier paper on pions[20], some of us tried to determine Q_o^2 by requiring that the Q^2 dependence of higher moments of the quark-antiquark contribution alone (without the gluon conversion terms) imitate that of a scale-breaking version of the Feynman-Field parametrization[24] at as low a Q^2 as possible. This led us to prefer small values of Q_o^2, near $Q_o^2 = 0.5$ Gev^2. (We should note that the Λ value used at that time was larger than the one now in vogue). It is not obvious to us now that this is a necessary or even a desirable condition. To require that the recombination model result imitate the Owens-Uematsu equations is to require that Q_o^2 be so close to Λ that α_s is large at Q_o^2 (see our discussion below of the Eilam-Zahir choice for these parameters).

It is, however, desirable to calculate the large Q^2 recombination predictions for pions, protons, kaons and lambdas using the same values of Λ and Q_o^2. This would result in a phenomenological "best fit" value for Q_o^2. This work is under way and will be reported in a later short note.

b. THE RECOMBINATION FUNCTIONS

Our choice of the form

$$R^{B}_{qqq}(x_1,x_2,x_3;x) = R\,\frac{x_1x_2x_3}{x^3}\quad\delta(x_1+x_2+x_3-x) \tag{3.1}$$

is motivated more by a desire for simplicity in the evaluation of the moments in Eq. (2.1) than by rigorous physical arguments. However, there are some physical arguments which would lead us to a formula of roughly this form.

As pointed out by van Hove [25], the recombination function is a type of rate, and must take the form (for mesons, m; v,s ≡ valence, sea quarks)

$$r(k_v,k_s)\quad\delta\;(k_m-k_v-k_s)$$

where $r(k_v,k_s)$ is a Lorentz invariant function. If we calculate such a rate in any simple model, it will come out as a polynomial in the appropriate dot products of vector momenta; hence, it would seem reasonable to use a polynomial in x_1, x_2 and x_3.

Of course, if we knew the baryon wave function exactly at low Q^2, we could simply insert this into the formula for the rate and compute the function $r(k_v,k_s)$ explicitly. While we do not know the wave function at low Q^2, we do know it at high Q^2 [26]. Ordinarily this would be irrelevant, but it happens that the corresponding large Q^2 wave function for the pion [27]

$$\phi\ \sim x_1x_2$$

does give a commonly used form for the pion recombination function, $x_1x_2\quad\delta\,(x_1+x_2-x)$, when substituted into the formula for the rate. The similar Lepage-Brodsky large Q^2 wave function for the baryons [26], $x_1x_2x_3$, leads to a rate formula

$$\frac{x_1 x_2 x_3}{x^3} \quad \delta(x_1 + x_2 + x_3 - x) \ .$$

Finally, the form $x_1 x_2 x_3$ was used in some phenomenological fits by Ranft[28]. He claims that his results are not much affected by changes of the powers in the formula.

Having once decided on a functional form for the recombination function, we must normalize it using some reasonable convention. We believe that the only really correct normalization is the requirement that, after recombination occurs, the momentum into all hadrons in the jet must be bounded above by 1 (see Ref. 20 for a discussion of this in the meson case). As yet, we have been unable to implement this in a rigorous fashion for baryons and mesons together. For the time being, therefore, we use the normalization method of Teper[29]:

If

$$\xi(x_1, x_2, x_3; x) \quad \delta(x_1 + x_2 + x_3 - x)$$

is the recombination function, then

$$\xi(x_1, x_2, x_3; x_1 + x_2 + x_3)$$

must be the probability that the three quarks in cells $dx_1 dx_2 dx_3$ combine to make a baryon at some x. This probability should be bounded above by 1. For our case the maximum of the function $xy(1-x-y)$ lies at $x = y = \frac{1}{3}$; so the maximum value is $\frac{1}{27}$ and our normalization factor should be 27 or less.

If we take into account the fact that three quarks and three antiquarks in a cell in 6 dimensional phase space may go either into a baryon and an antibaryon or into three mesons, we see that the actual normalization should be less than 27 by some factor which is probably between 2 and 4. We compute the values of our function for R=27, but for proper agreement with the data these should be too high.

We believe this normalization argument, though crude, to be preferable to the sum rule used by Ranft[28]

$$R\int_0^1 d\,\xi_1 \int_0^{1-\xi_1} d\,\xi_2 \int_0^{1-\xi_1-\xi_2} d\,\xi_3 \qquad \xi_1\,\xi_2\,\xi_3\,(1-\xi_1-\xi_2-\xi_3) = 1$$

This is because in our basic formula, Eq. (2.1), the quantity which is differential in x_1 and x_2 is $D(x_1,x_2;Q^2)$, not $R(x_1,x_2,x)$. R plays the role of a quantity differential in x; therefore, any sum rule involving it should be an integral over x.

Other recent recombination model calculations involving baryons by Takasugi and Tata[30], and by Eilam and Zahir[13], use a different form for the recombination function suggested by the valon model of Hwa[31]:

$$R' \quad \left(\frac{x_1x_2x_3}{x^3}\right)^{3/2} \qquad \delta(x_1+x_2+x_3-x)$$

used by Takasugi and Tata and

$$(19.9)\ \frac{(x_1x_2)^{1.65}\quad x_3^{\ 1.35}}{x^{4.65}} \quad \delta(x_1+x_2+x_3-x)$$

used by Eilam and Zahir. Eilam and Zahir use not only this functional form but also this normalization, whereas Takasugi and Tata use the functional form but allow the normalization R' to run free. A priori, we do not expect that the essential features of the results should differ very much between this functional form and the one we have computed, although of course, details will change. However, we have not yet checked this expectation by evaluating functions with non-integer powers.

The valon model derivation of this function by Hwa and Zahir[32] is actually a fit to the valon distribution in the nucleon as measured in deep inelastic scattering. This should therefore be some average of the wave function squared over the small intrinsic transverse momenta (p_t) of the quarks in the nucleon (these are certainly less than one Gev^2). In the jet calculus generated spray, however, we are recombining quarks which are likely to be at rather large transverse momentum. We need a recombination function appropriate for picking baryons out of this sort of intrinsic momentum distribution; it would therefore be no more rigorous to apply the structure function - recombination function equation of Hwa (Eq. (2.38c) of Ref. 31) than the heuristic arguments given at the beginning of this section. Eilam and Zahir are aware of this problem, and they cut off the integrals in the jet calculus in order to be sure they are only combining sets of quarks with rather small transverse momentum. This has other consequences, as explained in the last subsection of this section.

c. GRAPHS OF RESULTS

i) Q^2 Dependence

Perhaps the most striking feature of results calculated from Eq. (2.1) is the fact that the results rise with Q^2 as shown in Figs. 3 and 4. It is not surprising that there is an initial rise with Q^2, since the formula vanishes at Q_o^2. What is less easily anticipated, however, is the fact that the rise continues for all values of Q^2 accessible with present and planned accelerators.

The exact details of the rise depend somewhat on the values of the parameters Q_o^2 and Λ. For example, if Q_o^2 is large the variation at small Q^2 is faster than if Q_o^2 is small (see Fig.5). This is because with large Q_o^2 and small Q^2 very few partons are produced in the spray and hence not much recombination can occur. Small Q_o^2 for the same Q^2 is closer to the "asymptotic" Uematsu-Owens[17] behaviour.

We see in Fig.4 that all the contributions are rising with Q^2, but that the smallest ones rise fastest. However, the decomposition of the total result, shown in Fig.6, remains similar in its main features.

Because of this strong Q^2 dependence through the region of current experimental interest, it should be possible to distinguish recombination model terms from the Uematsu-Owens type of behaviour. For reference we show in Fig.7 a typical Q^2 dependence predicted by the Altarelli-Parisi equations. As input we have taken the EMC data of Ref.33 to give D_u^p at Q^2=25 Gev^2, and we have assumed D_d^p=0.5 D_u^p (the down quark fragmentation is, of course, suppressed in all applications with photons because of the charge). The data shown are from e^+e^- at higher values of Q^2. We see that the Q^2 variation of the Uematsu-Owens equations is rather small compared to that needed to fit the annihilation data.

(ii) Leading Quark Effect

The next striking feature of the results, as stressed in our Letter[34], is that at large x most of the contribution comes from terms where the initial quark comes right through and recombines with two gluons. We shall term this the "leading quark" effect[35]. Various features of presently available data agree with this effect. For example, the data of Ref.33 show that, at large x, proton production from up quarks is very much larger than antiproton production, which, of course, contains no leading quark. The data in Ref.6 show that, in jets originating from quarks in protons, the production of lambdas is similar to the production of protons; a leading up quark can equally well pick (ud) or (sd) pairs up from the converting gluons.

At small x, on the other hand, most of the contribution comes from the three gluon terms. As a result we expect equal numbers of baryons and antibaryons in quark jets; this is seen by the EMC in deep inelastic muon scattering[6,33].

In Fig.8 we present the predictions of this model for the functions D_u^p, $D_u^{\bar{p}}$, D_u^{Λ} and $D_u^{\bar{\Lambda}}$. Whereas $D_u^{\bar{p}}$ and $D_u^{\bar{\Lambda}}$ exhibit a similar behaviour, D_u^{Λ} is more sharply peaked than D_u^p in the small x region.

Thus the results are almost totally dependent on the method used to treat the gluons in the spray, and it would behoove us to consider whether other techniques for handling them are possible. Certainly one can envision more general recombination functions which recombine any number of partons into a baryon (plus additional stuff which does not matter if we calculate the single particle inclusive distributions). We believe that further study of this sort should be done; to facilitate that we list in Appendix C the explicit forms of the recombination functions for two quarks and a gluon, for two gluons and one quark, and for three gluons, into a (baryon plus anything).

In Fig.9 we present results for lambda production and proton production, calculated with the same set of parameters. Note that the lambda production is a factor of 2 higher than the proton production at small x, where the triple gluon term dominates. We expect this, since there are only three different ways to assign the down quark from the proton to the three gluons in Fig.2, but there are 6 ways to assign the u,d and s quarks to the gluons for the lambda.

At large x, on the other hand, where the leading quark effect occurs, the ratio of lambda to proton production is determined by the number of ways that the leading quark can combine with quarks from gluon conversion to create the appropriate particles. The combinatorial counting is slightly more complicated than for the small x case where only one graph (the triple gluon graph) is involved. In the large x case, the graphs involved are (see Appendices A and B) 6,8 and 9 for the case of the Λ and 7, 10 and 11 for the case of the proton. The contribution of these graphs to the Λ/p ratio is 4/3.

(iii) Comparision with Data

Of course, the lambda production should be suppressed from the values we calculate for two reasons: in principle it is more difficult to make strange quarks than it is to make non-strange quarks and we should somehow include this in both the jet calculus evolution and in the gluon conversion term. We have done neither. Also, there may well be SU(3) symmetry breaking in the recombination function, and we lack this also.

When all this handwaving is over, we expect the number of lambdas predicted by the model to be no more than a factor of 3 smaller than the protons. At present there is no data on the large x distributions of protons; however, it is likely that the experimental lambda and proton cross-sections differ from each other at 1089 Gev^2 by less than a factor of 3.

We now compare with those high Q^2 data on lambda and proton production which are currently available. This is shown in Fig.10 for the parameters Q_o^2=5 Gev^2 and Λ=100 Mev. In Fig.11 we show how this comparision would change as the two parameters change.

d. COMPARISON WITH OTHERS

Earlier calculations of this general type were done by Chang and Hwa for pions [22] and by Eilam and Zahir for baryons[13]. The formula which they evaluate,

$$D_i^B(x,Q^2) = \int_{Y_o}^{T(Y)} dy \int_{Y_o}^{y} dy' [\text{same integrand as in Eq.2.2.}]\ R(x_1,x_2,x_3;x)dx_1dx_2dx_3$$

where $T(Y)=Y$ if $Q^2 < 30$ Gev^2 and $T(Y)=Y$ (30Gev^2) if $Q^2 > 30$ Gev^2, may be rewritten in the form

$$D_{j'i}\ (Y-T(Y))\, D_{j'}^B\ (x,T(Y))$$

From this we see immediately that for all values of Q^2 above 30 Gev^2 (the cut-off value they insert for the integrals) they should just have the Owens-Uematsu type behaviour. That is, they should have a slow fall at large x with increasing Q^2, and a slow rise at small x.

In fact, the results they display fall continuously with Q^2 at all x shown on their graphs. This is an artifact of the technique they use to invert the moments, which emphasizes the large x behaviour.

Eilam and Zahir claim their answers do not vary much with the cut-off. This is clearly a somewhat misleading statement - their answers will rise with Q^2 for Y below the cut-off, and then have Owens-Uematsu behaviour above this. While we sympathize with their attempt to limit the p_t of the recombining hadrons, we feel that this should be achieved by use of a modified jet calculus[36] which keeps track of p_t or which reorders the graphs rather than by truncating the KUV results.

Had Eilam and Zahir used the full jet calculus instead of their truncated version, with the same parameters Λ=650 Mev, Q_o^2=0.64 Gev^2 and with the recombination function

$$2 \quad \frac{x_1 x_2 x_3}{x^3} \quad \delta(x_1+x_2+x_3-x)$$

(this is the valon prescription normalization of our functional form according to Hwa[31]), they would have obtained the Q^2 dependence shown in Fig.12. We see that their truncation fo the integrals has led to a completely different form for the result.

In Fig.13 we compare this set of parameters with the data. The agreement is quite respectable. While we believe these parameters to be foreign to the spirit of the model (Λ is much larger than that used in other QCD phenomenology, and the ratio of Q_o^2 to Λ^2 is such that α_s is larger than 1 at the end of the spray), they produce a cross-section of reasonable size. This is larger (by two orders of magnitude) than the results obtained by Eilam and Zahir. At present we are unable to understand their results: experiments with our program lead us to believe that their truncation of the jet calculus should not produce a spectacular change in the size of the answers (see Fig.12). If the discrepancy is due to the different forms of the recombination function, then the sensitivity of these calculations to that form is remarkable and needs to be carefully investigated.

IV. ROLE OF THE RECOMBINATION TERM

The recombination term calculated above has a different dependence on Q^2 from that of the "normal" fragmentation functions obeying the Uematsu-Owens - Altarelli-Parisi Q^2 evolution. It is, therefore, natural to ask how the two might possibly be related. In this section we attempt an interpretation of the two terms (and of one other term) by writing a pair of differential equations for the probability that a baryon is found in a parton jet. This is similar to the equation derived by some of us in our recent paper on mesons in jets[9]; but, it has the additional feature that another construct, the diquark, must be introduced to make the derivation simple.

Let $D_i^p(n,y)$ be the n th moment of the $i\to p$ fragmentation function. To write a differential equation for the y (i.e., Q^2, see Eq. (2.3)) dependence of this function, we must study the ways in which the function is changed by a lengthening of the y interval. Our basic hypothesis is that the only allowed changes due to variations in y come from quantum chromodynamics - i.e., that the only additional vertices allowed when y is changed are QCD ones. Given this, a lengthening of the y interval leads to two ways in which the fragmentation functions can be changed:

a) The proton may already be present in the jet (prior to lengthening). In this case, lengthening of the interval can result in radiation from i (Fig.14);

b) Lengthening of the interval may create the vertex which ultimately adds the third quark. This might then recombine with a "two quark system "already present to form the baryon. We will refer to this "two quark system" as a "diquark" without prejudice about whether the two quarks are "semibound" or indeed whether they are even close to each other. Also, we will assume that the recombination in question occurs only at $Q^2=Q_o^2$; this is consistent with our desire to use only perturbative QCD down to Q_o^2 and the phenomenological recombination model at Q_o^2.

The fragmentation function then obeys a differential equation of the form

$$\frac{d}{dy} D_i^p\,(n,y) = A_{ji}(n)\, D_j^p\,(n,y) + f_{p,i}\,(n,y) \tag{4.1}$$

where

$$A_{ji}(n) = \sum_{c_3} A_i^{jc_3}\,(n) = \sum_{c_3} P^{n,0}_{i\to jc_3}$$

in the notation used in Eq. (2.5). The function $f_{p,i}(n,y)$ may be written in the form (see Fig. 15)

$$\begin{aligned} f_{p,i}(t,y) = &\int dz\, dw_1 dw_2\, \hat{P}_{i\to b_1 b_2}(z)\, D_{a\,b_1}(w_1,y-Y_o)\,\xi_{(bc)b_2}(w_2,y)\,\cdot \\ &\qquad\cdot\, R^p_{(bc)a}\,(zw_1,(1-z)w_2,t) \\ + &\int dz\, dw_1 dw_2\, \hat{P}_{i\to b_1 b_2}\,(z)\, D_{bb_1}\,(w_1,y-Y_o)\,\xi_{(ac)b_2}\,(w_2,y)\,\cdot \\ &\qquad\cdot\, R^p_{(ac)b}\,(zw_1,(1-z)w_2,t) \\ + &\int dz\, dw_1 dw_2\, \hat{P}_{i\to b_1 b_2}\,(z)\, D_{cb_1}\,(w_1,y-Y_o)\,\xi_{(ab)b_2}(w_2,y)\,\cdot \\ &\qquad\cdot\, R^p_{(ab)c}\,(zw_1,(1-z)w_2,t) \end{aligned} \tag{4.2}$$

We have introduced the new symbol $\xi_{(ab)c}(x,y)$ for the propagator of parton c at y to go to diquark (ab) at Y_o. Here x is the momentum fraction of c carried by the diquark.

We have also used the recombination function

$$R^{p}_{(bc)a}(x_1,x_2,x)$$

which tells how the diquark recombines with the third quark to form a baryon. To take moments of Eq.(4.2) for use in Eq.(4.1), we need to know the functional form of this recombination function R. We will address this problem shortly.

CASE OF "FRAGILE" DIQUARKS

Next we consider the functions $\xi_{(ac)b}(w,y)$. As the interval is lengthened, there are in general three possibilities:

1. The diquark is already formed and radiation occurs from the initial quark.
2. The lengthening of the interval creates the basic vertex for forming the diquark.
3. The diquark is already formed, and radiation occurs from the diquark. This last possibility does not arise for the proton fragmentation function; the proton is a color singlet whereas the diquark carries color. In this subsection we assume that all recombination, including the recombination into diquarks, occurs only at Q_o^2; hence this term will not contribute. This is equivalent to assuming that the diquarks are rather tender objects which cannot sustain themselves under the impulse of any hard process, including radiation.

In the next subsection we discuss what happens if recombination into diquarks at larger Q^2 is allowed, and radiation from the diquark is inserted.

Since our present philosophy excludes possibility (3), the equation for the diquark propagator ξ becomes (see Fig. 16)

$$\frac{d}{dy}\,\xi_{(ab)b_2}(t,y) = \sum_{c_2c_3}\int_t^1 \frac{dx}{x}\,P_{b_2\to c_2c_3}(t/x)\,\xi_{(ab)c_2}(x,y)$$

$$+\sum_{jk}\int P_{b_2\to jk}(z)D_{aj}(w_1,y-Y_o)D_{bk}(w_2,y-Y_o)K(x_1,x_2,x).$$

$$\cdot\quad \delta(x_1-w_1z)\quad \delta(x_2-w_2(1-z))\;dx_1dx_2dw_1dw_2dz \qquad (4.3)$$

where the function $K(x_1,x_2,x)$ somehow describes the combination of two quarks at Q_o^2 to make a diquark.

The recombination functions $K(x_1,x_2,x)$ and $R^p_{(ab)c}(x_1,x_2,x)$ may be determined by the following requirements:

a) All the diquark clumpings (ab)c, (bc)a and (ac)b must be treated in the same way.

b) The overall recombination function of 3 quarks into a proton (which we will achieve by first combining two quarks to make a diquark and then combining the diquark with another quark to make a proton) must take the form

$$R^p_{abc}(x_1,x_2,x_3,x) = R\,\frac{x_1x_2x_3}{x^3}\qquad \delta(x_1+x_2+x_3-x)$$

c) The recombination of two quarks into a diquark (achieved by K) and of the diquark and bachelor quark into a proton (achieved by R) must preserve "moment" at the vertices. This is a property of the jet calculus vertices; we will concoct our theory to also have this form. While this is not strictly necessary it allows for much simpler interpretation.

By implementing these requirements we deduce that the recombination function to make a diquark from two quarks is

$$K_{ab}^{(ab)}(x_1,x_2,x) = E_{ab}^{(ab)} \frac{x_1 x_2}{x^2} \delta(x_1+x_2-x) .$$

This has the same form as the recombination function which makes a pion from a quark and an antiquark. One should not be surprised by this. In the discussion by Brodsky and Lepage of the pion wave function[27], the form $(x_1 x_2)$ at $Q^2 \to \infty$ is largely determined from the Born graph, single gluon exchange. This will also be the Born graph for the diquark system; hence the wave function of the diquark system will be similar to that of the pion in the limit $Q^2 \to \infty$. As we explained in the previous section, we might hope that known similarities in the large Q^2 behaviour reflect corresponding known similarities in the small Q^2 behaviour.

Of course, there are infrared difficulties associated with the diquark which are not present for the pion, due to the color of the diquark system. These would make exact replication of the Brodsky-Lepage calculation complicated. For our purposes here, however, singularities due to radiation from the diquark's color are being ignored (since, in the net hadronic amplitude, singularities in the various components of the calculation would presumably cancel). We only mention the Brodsky-Lepage arguments as hints that the diquark recombination function might conceivably be the same as that for the pion; within our framework, at any rate, we need the particular form x_1x_2/x^2.

With this recombination function, the solution to Eq.(4.3) takes the form

$$\xi_{(ab)b_2}(n,y) = D_{c_2b_2}(n,y-Y_o)\,\xi_{(ab)c_2}(n,Y_o) + \int_{Y_o}^{y} dy'\, D_{c_2b_2}(n,y-y') \sum_{r=o}^{n-2} \binom{n-2}{r} E_{ab}^{(ab)} P_{c_2\to jk}^{r+1,n-r+1} \cdot D_{aj}(r+1,y'-Y_o)\, D_{bk}(n-r-1,\, y'-Y_o) \qquad (4.4)$$

When we insert the recombination which makes the diquark with momentum fraction w and the bachelor quark with momentum fraction x_3 into a proton,

$$R^p_{(bc)a}(x_3,w,x) = R^p_{(bc)a}\,\frac{x_3w^2}{x^3}\,\delta(x_3+w-x)$$

we can then reduce Eq.(4.2) to moments

$$f_{p,i}(N,y) = \sum_m \binom{N-3}{m} P^{m+1,N-1-m}_{i\to b_1b_2} \cdot$$

$$\Big\{ D_{ab_1}(m+1,y-Y_o)\,\xi_{(bc)b_2}(N-1-m,y)\,R^p_{(bc)a} + D_{bb_1}(m+1,y-Y_o)\,\xi_{(ac)b_2}(N-1-m,y)\,R^p_{(ac)b} + D_{cb_1}(m+1,y-Y_o)\,\xi_{(ab)b_2}(N-1-m,y)\,R^p_{(ab)c}\Big\} \qquad (4.5)$$

The solution to Eq. (4.1) is then

$$D_i^p\,(N,y) = D_{ki}\,(N,y-Y_o)\,D_k^p\,(N,Y_o) + \int_{Y_o}^{y} D_{ji}\,(N,y-y')\,f_{p,j}\,(N,y')dy'$$

or in other words

$$D_i^p\,(N,Y) = A + B + C$$

with

$$A = D_{ki}\,(N,y-Y_o)\,D_k^p\,(N,Y_o) \tag{4.6a}$$

$$B = \int_{Y_o}^{y} dy'\,D_{ji}\,(N,y-y')\sum_m \binom{N-3}{m} P^{m+1,N-1-m}_{j\to b_1 b_2}\,D_{c_2 b_2}\,(N-1-m,y'-Y_o)\,.$$

$$\cdot\left\{ R^p_{(bc)a}\,D_{ab_1}\,(m+1,y'-Y_o)\,\xi_{(bc)c_2}\,(N-1-m,Y_o) + \right.$$

$$+ R^p_{(ac)b}\,D_{bb_1}(m+1,y'-Y_o)\,\xi_{(ac)c_2}\,(N-1-m,Y_o) +$$

$$\left. + R^p_{(ab)c}\,D_{cb_1}\,(m+1,y'-Y_o)\,\xi_{(ab)c_2}\,(N-1-m,Y_o) \right\} \tag{4.6b}$$

(see Fig. 17)

$$C = \int_{Y_o}^{y} dy'\,D_{ji}\,(N,y-y')\int_{Y_o}^{y'} dy''\sum_m \binom{N-3}{m} P^{m+1,N-1-m}_{j\to b_1 b_2}\,.$$

$$\cdot D_{c_2 b_2}\,(N-1-m,y'-y'')\sum_r \binom{N-3-m}{r} P^{r+1,N-2-m-r}_{c_2\to \ell k}\,.$$

$$\cdot \left\{ R^p_{(bc)a} \; E^{(bc)}_{bc} \; D_{ab_1} \; (m+1, y'-Y_o) \; D_{b\ell} \; (r+1, y''-Y_o) \; D_{ck} \; (N-2-m-r, y''-Y_o) \right.$$

$$+ R^p_{(ac)b} \; E^{(ac)}_{ac} \; D_{bb_1} \; (m+1, y'-Y_o) \; D_{a\ell}(r+1, y''-Y_o) \; D_{ck} \; (N-2-m-r, y''-Y_o)$$

$$\left. + R^p_{(ab)c} \; E^{(ab)}_{ab} \; D_{cb_1} \; (m+1, y'-Y_o) \; D_{a\ell} \; (r+1, y''-Y_o) \; D_{bk} \; (N-2-m-r, y''-Y_o) \right\} \qquad (4.6c)$$

Term A is the Owens-Uematsu term, corresponding to the process in Fig.14. This needs no discussion. Term B corresponds to the process shown in Fig.17. Here $\xi_{(\alpha\beta)b_2}(N-1-M, Y_o)$ is some "intrinsic" amount of diquark in the parton at Q_o^2. Term C corresponds to the process of Fig.2: this is the recombination term calculated in the earlier sections[37].

CASE OF "TOUGH" DIQUARKS

In the discussion so far, we have assumed that all the recombinations occur at Y_o. If one believes that the diquark is a relatively fragile object, which can exist only at small Q^2, this is a reasonable point of view. However, there is some feeling in the literature that many of the observed violations of scaling in deep inelastic electron scattering come in fact from scattering off diquarks. While these contributions are higher twist, and therefore, dominate at small Q^2, it is not clear that a Q^2 which is small for those purposes is necessarily our small Q_o^2. We should therefore investigate the possibility that diquarks exist also at large Q^2.

More explicitly, our study above uses

$$\frac{d}{dy} \; D^p_i \; (n,y) = A_{ji} \; D^p_j \; (n,y) + f_{p,i} \; (n,y)$$

where

$$f_{p,i}(x,y) = \int dz dx_2 dx_3 \hat{P}_{i\to k\ell}(z)\, D_{ak}(x_3,y-Y_o)\, \xi_{(bc)\ell}(x_2,y,Y_o)\, R^{p}_{(bc)a}(x_3,x_2,x)$$

$$+ \text{ cyclic permutations}$$

with $\xi_{(bc)\ell}(x_2,y,Y_o)$ being the probability of finding a diquark (bc) at Y_o from a parton ℓ at y. We wrote this in terms of a recombination function K in Eq.(4.3)

Now let us imagine that the recombination into diquarks can occur at any y' (we will continue to assume that the recombination into protons occurs only at Y_o, due to the known extended structure of hadrons). What we are exploring here is the possibility that the diquarks might be treated as pseudo-fields, analogous to the quarks and gluons and with some reasonable existence at larger Q^2 (i.e., presumably a less extended structure than the proton). We generalize the recombination function $K(x_1,x_2,x)$ to the new function $E^{(ab)}_{ab}(x_1,x_2,x,y')$ which indicates that the recombination takes place at y' and we explore the dependence of the diquark propagator $\xi(x,y,y_d)$ on both its initial (y) and final (y_d) variables.

As we change y (the mass label of the initial quark) we again have the two possible contributions from radiation off the quark and recombination. Now, however, the recombination may occur at any of the y's available in the range. The differential equation analogous to Eq. (4.3) is therefore

$$\frac{\partial}{\partial y} \xi_{(bc)\ell} (x,y,y_d) = \sum_j \int \frac{dz}{z} \hat{P}_{\ell\to ij} (z) \xi_{(bc)i} \left(\frac{x}{z}, y, y_d \right)$$

$$+ \int_{y_d}^{y} dy' \int dz\, dw_1 dw_2\, dx_1 dx_2\, \frac{dx'}{x'} \hat{P}_{\ell\to ij}(z)\, D_{ai} (w_1, y-y')\, D_{bj}(w_2, y-y').$$

$$\cdot E_{ab}^{(ab)} (x_1, x_2, x', y')\, \delta(x_1 - w_1 z)\, \delta[x_2 - w_2(1-z)]\, D_{qq} \left(\frac{x}{x'}, y'-y_d\right) . \qquad (4.7)$$

Similarly, as we increase the observation point of the diquark, two possible things can happen:

(a) We may lose a possible radiation vertex from the diquark (note that the diquark is assumed to radiate like an antiquark, so we know the vertex for this).

(b) We may lose a possible recombination into a diquark from all possible "creation" vertices.

These thus give us the differential equation for the variation of the diquark propagator ξ in its other variable

$$\frac{\partial}{\partial y_d} \xi_{(bc)\ell} (x,y,y_d) = - \int_x^1 \frac{dz}{z} P_{q\to qg} (z)\, \xi_{(bc)\ell}\left(\frac{x}{z}, y, y_d\right)$$

$$- \int_{y_d}^{y} dy'' \int \hat{P}_{k\to ij} (z)\, D_{ai} (w_1, y''-y_d)\, D_{bj} (w_2, y''-y_d).$$

$$\cdot E_{ab}^{(ab)} (x_1, x_2, x, y_d)\, \delta(x_1 - w_1 z \xi)\quad \delta[x_2 - w_2(1-z)\xi].$$

$$\cdot D_{k\ell} (\xi, y-y'')\, dz\, dw_1 dw_2 d\xi\, dx_1 dx_2 \qquad (4.8)$$

Since we will not study the behaviour of these equations in detail, we do not give here the most general solution; however, it is clear that the recombination contribution to the solution is

$$\xi_{(ab)\ell}(x,y,Y_o) = \int_{Y_o}^{y} dy'' \int_{Y_o}^{y''} dy' \int D_{k\ell}(\xi,y-y'')\,\hat{P}_{k\to ij}(z)\,D_{ai}(w_1,y''-y')\,.$$

$$\cdot D_{bj}(w_2,y''-y')\,E_{ab}^{(ab)}(x_1,x_2,x',y')\,D_{qq}\left(\frac{x}{x'},y'-Y_o\right).$$

$$\cdot\ \delta(x_1-\xi z w_1)\quad \delta[x_2-\xi(1-z)w_2]\,dz\,dx_1\,dx_2\,d\xi\,dw_1\,dw_2\,\frac{dx'}{x'} \qquad (4.9)$$

We see that allowance for the diquark radiation leads to a more complicated expression than the one we have evaluated, and it is not so easily related to the concept of a jet calculus evolution followed by the recombination function $x_1x_2x_3/x^3 \quad \delta(x_1+x_2+x_3-x)$.

Thus our approach fits neatly with the diquark concept only if one assumes that the diquarks are interesting objects for the Q_o^2 at which proton formation is accomplished, but not for arbitrarily large Q^2.

V. SUMMARY AND CONCLUSION

We have shown that there are general reasons for expecting the jet fragmentation into baryons to have three major contributions, with differing Q^2 dependence. One of these is the usual Owens-Uematsu term; the others involve recombination of a quark with a diquark, or of three quarks.

We have evaluated the three quark recombination into protons and lambdas for a particular choice of the recombination function, and shown that (for currently accepted values of the QCD constant Λ and reasonable values of Q_o^2 and size R of the recombination function) we can obtain rates for e^+e^- annihilation into these baryons which are in the neighbourhood of presently observed rates.

It is thus possible that the recombination mechanism will account for the observed rise of the amount of quark jet momentum going into baryons. Detailed fits to data require more work: (a) The parameter Q_o^2 must be chosen to fit both meson and baryon data; (b) The contributions of the other terms need to be estimated as carefully as possible and added to the term evaluated here. This work is underway.

Various other features of the model also need further investigation. In particular, most of the results depend on the method used to convert the gluons present in the jet spray into quark-antiquark pairs, thus producing effective recombination functions which recombine both gluons and quarks. Thus far, only one method has been investigated. The theoretical significance of this is not at all clear.

The form assumed for the three quark-baryon recombination function also deserves further study. Estimates by another group using a different form have obtained quite different results. The significance of this form is not independent of the gluon conversion; in the end only the net functions which effect the recombination of the partons into the baryons and mesons have any physical meaning. A great deal more thought needs to be devoted to this subject also. We feel, however, that the results reported here are encouraging enough to make this further study worthwhile.

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APPENDIX A

INDEPENDENT TERMS FOR $p + \bar{p}$

We list in Fig. A1 the 23 irreducible diagrams for $e^+e^- \to pX$ within the KUV jet calculus formalism (see also Fig. 2).

APPENDIX B

INDEPENDENT TERMS FOR $\Lambda + \bar{\Lambda}$

We list in Fig. B1 the 18 irreducible diagrams for $e^+e^- \to \Lambda X$ within the KUV jet calculus formalism (see also Fig. 2).

APPENDIX C

GENERALIZED RECOMBINATION FUNCTIONS

If we consider the gluon conversion and the quark recombination to form some new effective recombination functions, we find (the overall normalization factor R has been omitted):

a. GLUON(x_1) - QUARK(x_2) - QUARK(x_3)

$$\theta(x - x_2 - x_3)\theta(x_1 + x_2 + x_3 - x) \frac{x_2 x_3}{x^3} \left(\frac{x - x_2 - x_3}{x_1}\right) \cdot$$

$$\cdot \frac{3}{2N_f}\left[\left(\frac{x - x_2 - x_3}{x_1}\right)^2 + \left(\frac{x - x_2 - x_3 - x_1}{x_1}\right)^2\right]$$

This is shown in Fig. C1 as a function of the gluon momentum fraction x_1, for various values of the quark momenta. Note that the function is zero unless $x_2 + x_3 < x < x_1 + x_2 + x_3$, so in general "ultra-wee" gluons do not contribute.

b. GLUON(x_1) - GLUON(x_2) - QUARK(x_3)

$$\left(\frac{3}{2N_f}\right)^2 \frac{x_1 x_2 x_3}{x^3} \int_0^1 d\xi_1 \int_0^1 d\xi_2 \; \xi_1[\xi_1^2 + (1 - \xi_1)^2]\xi_2[\xi_2^2 + (1 - \xi_2)^2] \cdot$$

$$\cdot \; \delta(x_1\xi_1 + x_2\xi_2 + x_3 - x)$$

This is shown in Fig. C2. Now we only have contributions if $x_3 < x < x_1 + x_2 + x_3$. For fixed x, x_3 and x_2, this means that either $x_1 = 0.0$ is not reached at all (as in the gluon-quark-quark case) or the function goes to zero there.

c. GLUON(x_1) - GLUON(x_2) - GLUON(x_3)

$$\left(\frac{3}{2N_f}\right)^3 \frac{x_1 x_2 x_3}{x^3} \int_0^1 d\xi_1 \int_0^1 d\xi_2 \int_0^1 d\xi_3 \; \xi_1[\xi_1^2 + (1 - \xi_1)^2] \cdot$$

$$\cdot\ \xi_2[\xi_2^2 + (1 - \xi_2)^2]\xi_3[\xi_3^2 + (1 - \xi_3)^2]\delta(x_1\xi_1 + x_2\xi_2 + x_3\xi_3 - x)$$

This is shown in Fig. C3. Here there are contributions only if $x < x_1 + x_2 + x_3$. Again, for fixed x and x_2, x_3 this means either that $x_1 = 0.0$ is not used or that the function has a zero there.

DISCUSSION

We have seen above that despite the "minimal" form of our three quark recombination function $x_1x_2x_3/x^3$, the resulting recombination functions still vanish when the gluon momentum is "wee". A three quark recombination function with a higher power of x_1 will of course provide a more rapidly vanishing gluon recombination function.

REFERENCES

1. Tasso Collab., R. Brandelik et al., Phys. Lett. 105B, 75 (1981).

2. Tasso Collab., R. Brandelik et al., Phys. Lett. 94B, 444 (1980); Jade Collab., Phys. Lett. 104B, 325 (1981).

3. D. Haidt, Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies (published by Physikalisches Institut, Bonn, editor W. Pfeil), p. 558.

4. R. Hollebeek, ibid., p. 1.

5. R. Felst, ibid., p. 52.

6. H. Montgomery, ibid., p. 508.

7. B. Andersson, G. Gustafson, G. Ingelman and T. Sjöstrand, "Baryon Production in Lepton-Nucleon Scattering and Diquark Fragmentation", Lund preprint LU-TP 81-6.

8. B. Andersson, G. Gustafson and T. Sjöstrand, "A Model for Baryon Production in Quark and Gluon Jets", Lund preprint LU-TP 81-3.

9. A Casher, H. Neuberger and S. Nussinov, Phys. Rev. D20, 179 (1979).

10. T. Meyer, "A Monte Carlo Model to Produce Baryons in e^+e^- Annihilation", DESY preprint DESY B1-046.

11. G. Schierholz and M.J. Teper, "Baryon Production in QCD Jets", DESY preprint DESY 81-041.

12. W. Hoffmann, "Inclusive Production of Baryons in Decays of the Υ (9.46) and in Quark Jets", DESY preprint DESY 81-019.

13. G. Eilam and M.S. Zahir, "Forward Production of Baryons in Quark and Gluon Jets", preprint OITS-169(1981).

14. S. Pokorski, "How Do Quark-like Jets Look Like?", Warsaw preprint IFT/16/77. This paper predicts that the shape of the proton inclusive distribution for $.3 < x < .9$ should be very similar to that of the pion distribution.

15. K. Konishi, A. Ukawa and G. Veneziano, Phys. Lett. 78B, 243 (1978); Nucl. Phys. B157, 45 (1979).

16. V. Chang and R.C. Hwa, Phys. Rev. Lett. 44, 439 (1980).

17. T. Uematsu, Phys. Lett. 79B, 97 (1978); J.F. Owens, Phys. Lett. 76B, 85 (1978).

18. G.A. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977).

19. D. Willen, Phys. Rev. D21, 1781 (1980).

20. L.M. Jones, K.E. Lassila, U. Sukhatme and D. Willen, Phys. Rev. D23, 717 (1981).

21. F.J. Yndurain, Phys. Lett. 74B, 68 (1978).

22. V. Chang and R.C. Hwa, Phys. Rev. D23, 728 (1981).

23. F. Eisele, "Experimental Results on Structure Functions", Talk at DESY QCD Workshop, Fall 1981.

24. This was prepared by taking the values of Feynman and Field (Nucl. Phys. B136, 1 (1978)) as input at $Q_o^2 = 4.0$ and applying the Uematsu-Altarelli-Parisi equations.

25. L. van Hove, Schladming Lectures, p. 621 (1979), (Springer-Verlag, Wien).

26. G.P. Lepage and S.J. Brodsky, Phys. Rev. Lett. 43, 545 (1979); ib. 1625.

27. G.P. Lepage and S.J. Brodsky, Phys. Lett. 87B, 359 (1979); Phys. Rev. D22, 2157 (1980).

28. J. Ranft, Phys. Rev. D18, 1491 (1978).

29. M.J. Teper, "Applying the Parton Model to the Fast Hadrons at low p_t", Rutherford preprint RL-78-022/A, 1978 (unpublished).

30. E. Takasugi and X. Tata: a) "Particle Correlations in the Recombination Model Associated with Modified Kuti-Weisskopf Functions", U. of Texas Preprint; b) Phys. Rev. D23, 2573 (1981); see also E. Takasugi, Talk presented at XI International Symposium on Multiparticle Dynamics, Bruges, Belgium (1980).

31. R.C. Hwa, Phys. Rev. D22, 1593 (1980).

32. R.C. Hwa and M.S. Zahir, Phys. Rev. D23, 2539 (1981).

33. European Muon Collaboration, Phys. Lett. 103B, 388 (1981).

34. R. Migneron, L.M. Jones and K.E. Lassila, "Recombination Mechanism for Baryon Production in Jets", SLAC-PUB-2850, Fermilab PUB 81/75-THY and ILL-(TH)-81-54.

35. F. Gutbrod, "Review of Baryon Production in Hard Processes", Talk at the IV International Warsaw Symposium on Elementary Particle, 1981.

36. A. Bassetto, M. Ciafaloni and G. Marchesini, Nucl. Phys. B163, 447 (1979); Phys. Lett. 83B, 207 (1979); Phys. Lett. 86B, 366 (1979); L.M. Jones and K.E. Lassila, "On the Modified Jet Calculus of BCM", ILL-(TH)-81-18.

37. For convenience and simplicity we have chosen Q_o^2, the recombination point, to be the boundary value point for the differential equation. This is not necessary. As in Ref. 20, we can choose any Q_1^2 to be the boundary in the regions of integration in Eq. (4.6b) and Eq. (4.6c). Then start at Y_1, and Eq. (4.6a) is $D_{ki}(N,Y-Y_1)D_k^p(N,Y_1)$. Other parts of Eq. (4.6) are unchanged.

Figure Captions

Fig. 1. The basic components of our calculation. QCD evolution of the jet produces the three partons to be recombined plus other partons. The recombination schematically indicated here includes conversion of the gluons to $q\,\bar{q}$ pairs, and recombination of quark triplets.

Fig. 2. Diagramatic version of Eq. (2.2) for the 3 parton inclusive distribution.

Fig. 3. Dependence of the $(p + \bar{p})$ inclusive distribution on Q^2 ($Q_o^2 = 2$ Gev^2, $\Lambda = 200$ Mev.).

Fig. 4a. Dependence on Q^2 of the most important terms, 3g and 2g + q for $(\Lambda + \bar{\Lambda})$ production ($Q_o^2 = 1.5$ Gev^2, $\Lambda = 100$ Mev.).

Fig. 4b. Dependence on Q^2 of the less important terms, g + 2q and 3q, for $(\Lambda + \bar{\Lambda})$ production ($Q_o^2 = 1.5$ Gev^2, $\Lambda = 100$ Mev.).

Fig. 5. At low Q^2, evaluation of Eq. (2.1) for larger Q_o^2 leads to more rapid Q^2 variation for $e^+e^- \to (p + \bar{p})X$: (a) $Q_o^2 = 1$ Gev^2, $\Lambda = 50$ Mev; (b) $Q_o^2 = 5$ Gev^2, $\Lambda = 50$ Mev.

Fig. 6. The decomposition of total $(\Lambda + \bar{\Lambda})$ production is similar at (a) $Q^2 = 1089$ (Gev)^2 and (b) $Q^2 = 10^7$ Gev^2, despite different variation of the contributions shown in Fig. 4.

Fig. 7. The behaviour with Q^2 expected from the more usual Uematsu-Owens term; the input is at $Q_o^2 = 25$ Gev^2 from the EMC data[33]. The data are from ref. 2.

Fig. 8. As samples of the functions measured in inelastic muon scattering, we show D_u^p, D_u^{Λ}, $D_u^{\bar{p}}$ and $D_u^{\bar{\Lambda}}$ at $Q^2 = 25$ Gev^2, $Q_o^2 = 2$ Gev^2, $\Lambda = 200$ Mev.

Fig. 9. Predictions for the relative size of $(\Lambda + \bar{\Lambda})$ and $(p + \bar{p})$ cross

Fig. 9. (continued)

sections. At $x = .31$ the ratio $R = (\Lambda + \overline{\Lambda})/(p + \overline{p})$ is 1.87 (the dominant 3g terms have a ratio of 2); at $x = .92$ $R = 1.33$ (the dominant 2g terms have a ratio of 1.34); $Q^2 = 1089$ Gev^2, $Q_o^2 = 2$ Gev^2, $\Lambda = 200$ Mev. All SU(3) breaking effects have been neglec

Fig. 10. Comparison with available data, using $Q_o^2 = 5$ Gev^2, $\Lambda = 100$ Mev:

a) $e^+e^- \to (p + \overline{p})X$ at $Q^2 = 144$ Gev^2, data from ref. 2.

b) $e^+e^- \to (p + \overline{p})X$ at $Q^2 = 900$ Gev^2, data from ref. 2.

c) $e^+e^- \to (\Lambda + \overline{\Lambda})X$ at $Q^2 = 1089$ Gev^2, data from ref. 1.

Fig. 11. Variation of the predicted cross sections for $e^+e^- \to (\Lambda + \overline{\Lambda})X$ with Q_o^2 and Λ at $Q^2 = 1089$ Gev^2.

Fig. 12. Evaluation of our formula for $e^+e^- \to (p + \overline{p})X$ for $Q_o^2 = 0.64$ Gev^2, $\Lambda = 650$ Mev (Eilam and Zahir parameters[13]). The valon normalization of our functional form was used.

Fig. 13. Comparison of the Eilam-Zahir normalization with data.

Fig. 14. The first term in Eq. (4.1): Owens-Uematsu evolution. (The blob indicates the intrinsic proton at Q_o^2).

Fig. 15. Quark-diquark recombination, the second term in Eq. (4.1). The black box represents the diquark propagator ξ.

Fig. 16. Pictorial representation of the terms contributing to the "diquark propagator" in the case of "fragile" diquarks.

Fig. 17. Pictorial representation of the term B in Eq. (4.6b). The blob represents intrinsic diquark in the parton at Q_o^2.

Fig. A1. The 23 KUV irreducible diagrams for $e^+e^- \to pX$.

Fig. B1. The 18 KUV irreducible diagrams for $e^+e^- \to \Lambda X$.

Fig. C1. Behavior of the gluon-quark-quark recombination function.

Fig. C2. Behavior of the gluon-gluon-quark recombination function.

Fig. C3. Behavior of the three-gluon recombination function.

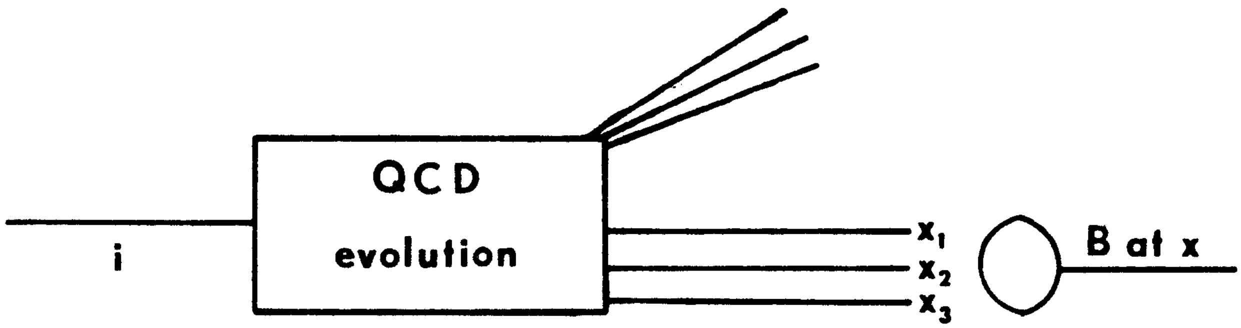


Figure 1

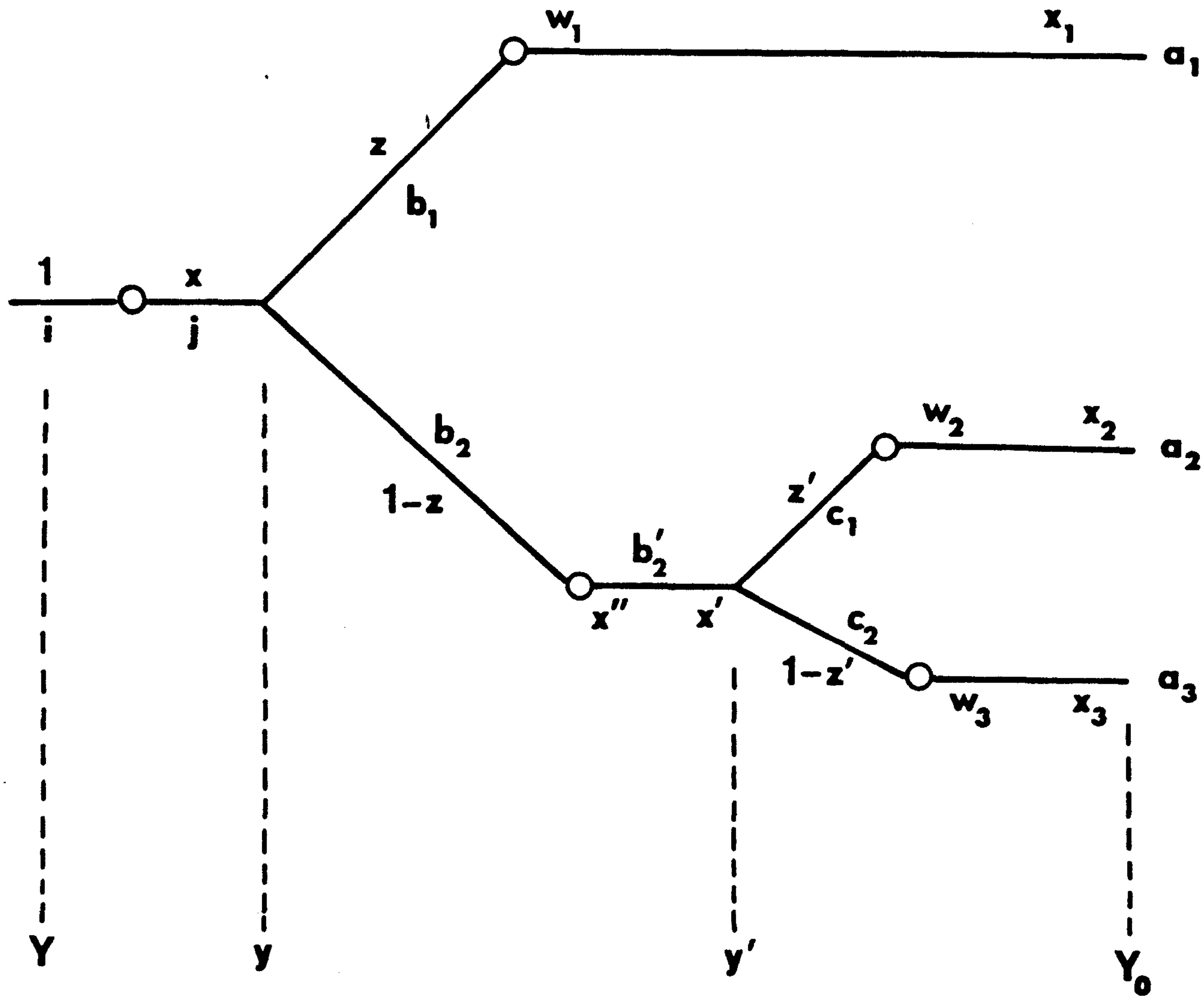


Figure 2

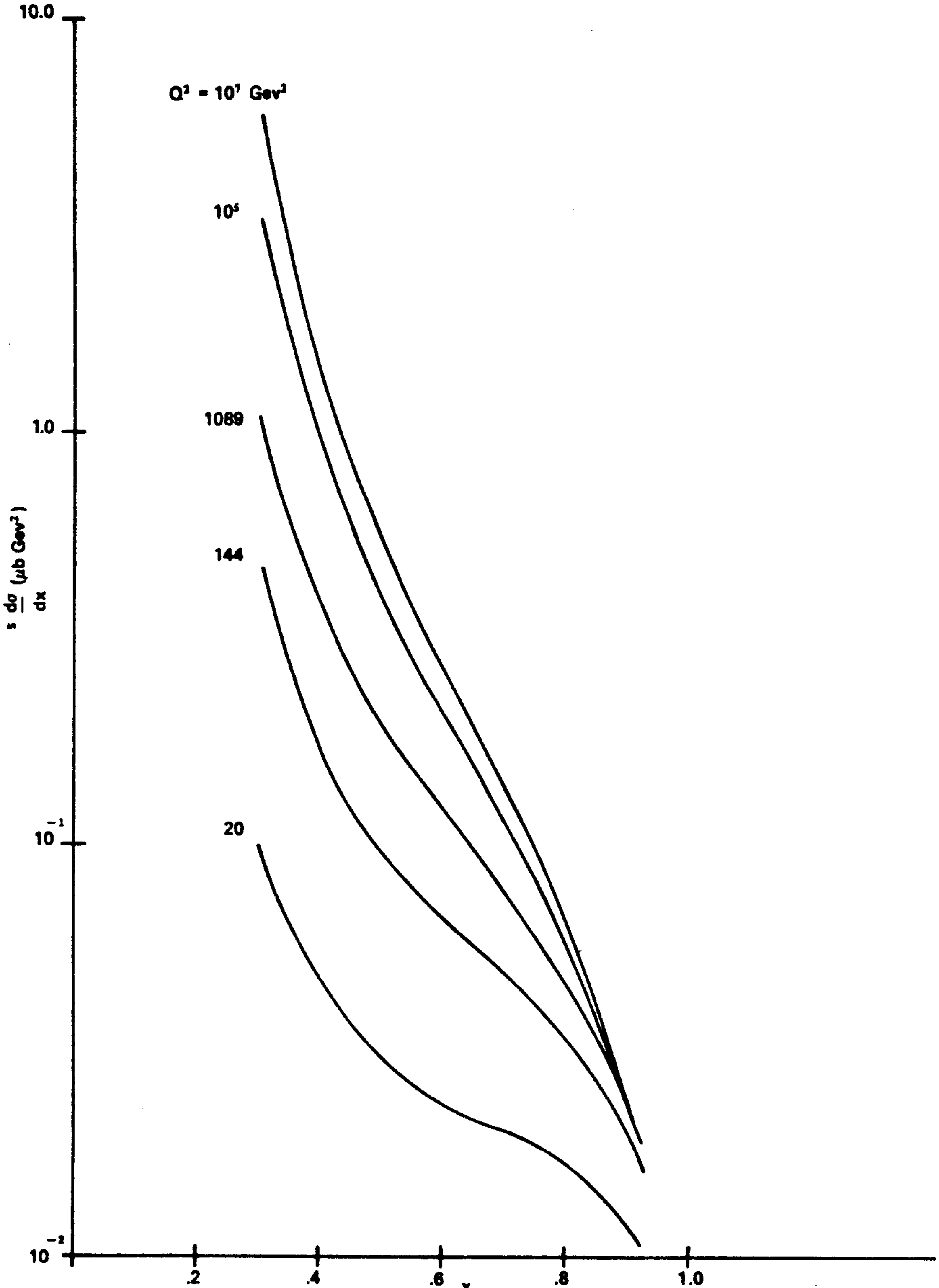
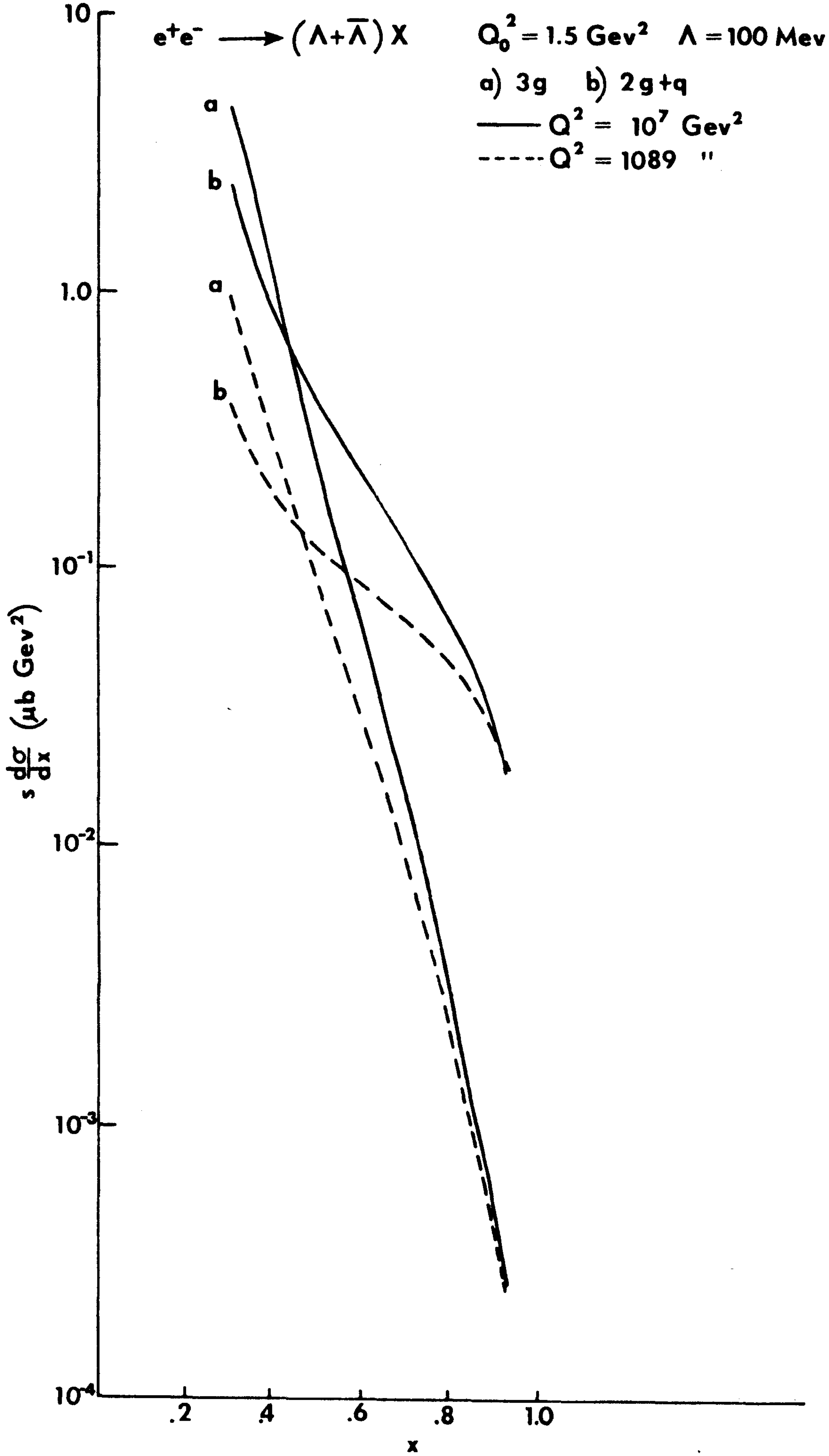
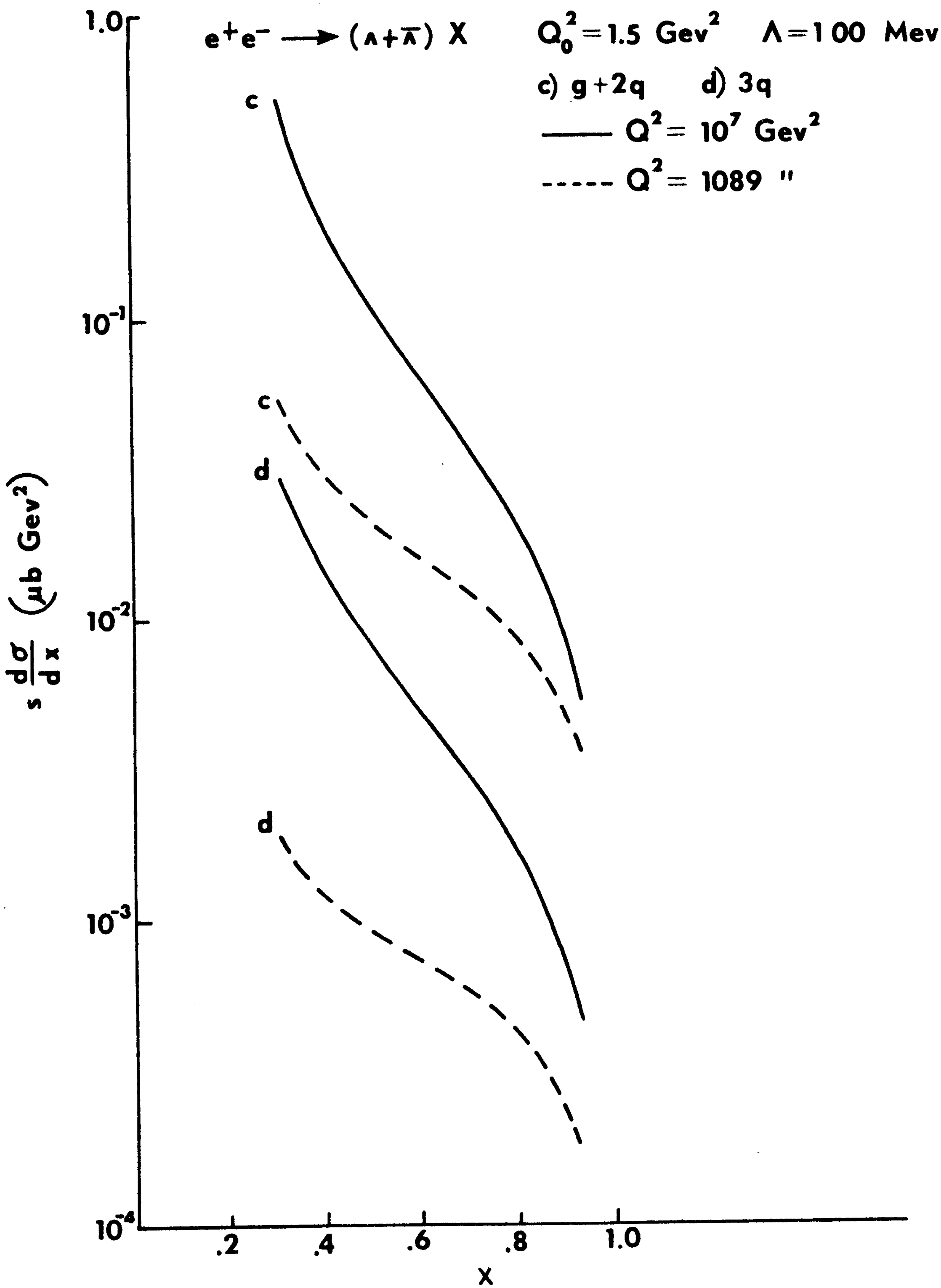

10.0
1.0
10^{-1}
10^{-2}
$s\,\frac{d\sigma}{dx}$ (μb Gev^2)
$Q^2 = 10^7$ Gev^2
10^5
1089
144
20
.2
.4
.6
.8
1.0
x

FIGURE 3



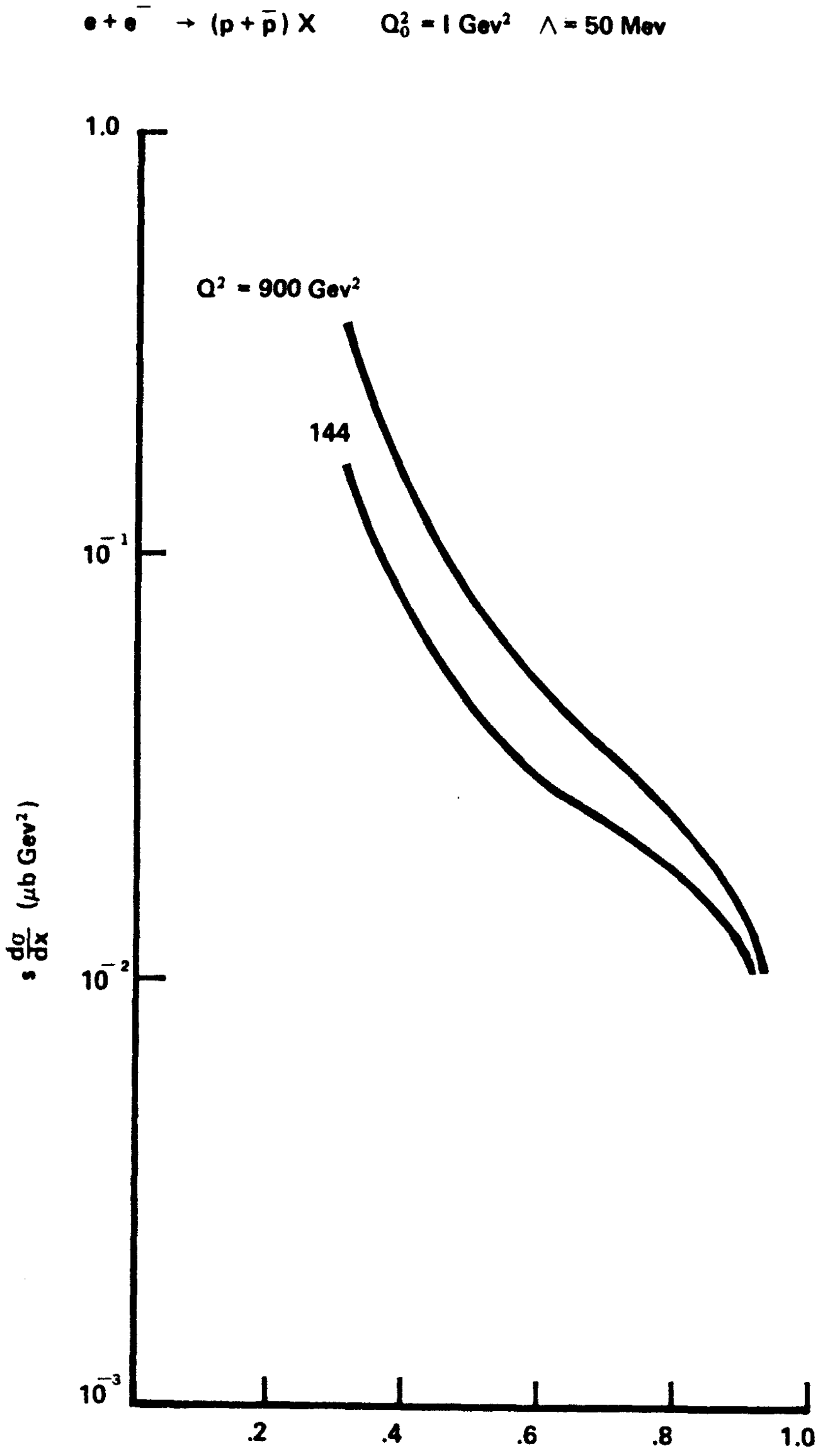
$e^+e^- \longrightarrow (\Lambda+\bar{\Lambda})X$
$Q_0^2 = 1.5$ Gev^2 $\Lambda = 100$ Mev
a) 3g b) 2g+q
—— $Q^2 = 10^7$ Gev^2
----- $Q^2 = 1089$ "
a
b
a
b
10
1.0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$s\frac{d\sigma}{dx}$ (μb Gev^2)
.2
.4
.6
.8
1.0
x

Figure 4a



$e^+e^- \longrightarrow (\Lambda+\bar{\Lambda})\ X$
$Q_0^2 = 1.5\ \text{Gev}^2$ $\Lambda = 100$ Mev
c) g+2q d) 3q
—— $Q^2 = 10^7\ \text{Gev}^2$
----- $Q^2 = 1089$ "
c
c
d
d
1.0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$s\,\frac{d\sigma}{dx}\ (\mu b\ \text{Gev}^2)$
.2
.4
.6
.8
1.0
X

Figure 4b



e + e⁻ → (p + p̄) X Q_0^2 = 1 Gev² Λ = 50 Mev
1.0
10^{-1}
10^{-2}
10^{-3}
$s\frac{d\sigma}{dx}$ (μb Gev²)
Q² = 900 Gev²
144
.2
.4
.6
.8
1.0
x

FIGURE 5A

$e^+e^- \longrightarrow (p+\bar{p})X \qquad Q_0^2 = 5\ \mathrm{Gev}^2 \qquad \Lambda = 50\ \mathrm{Mev}$

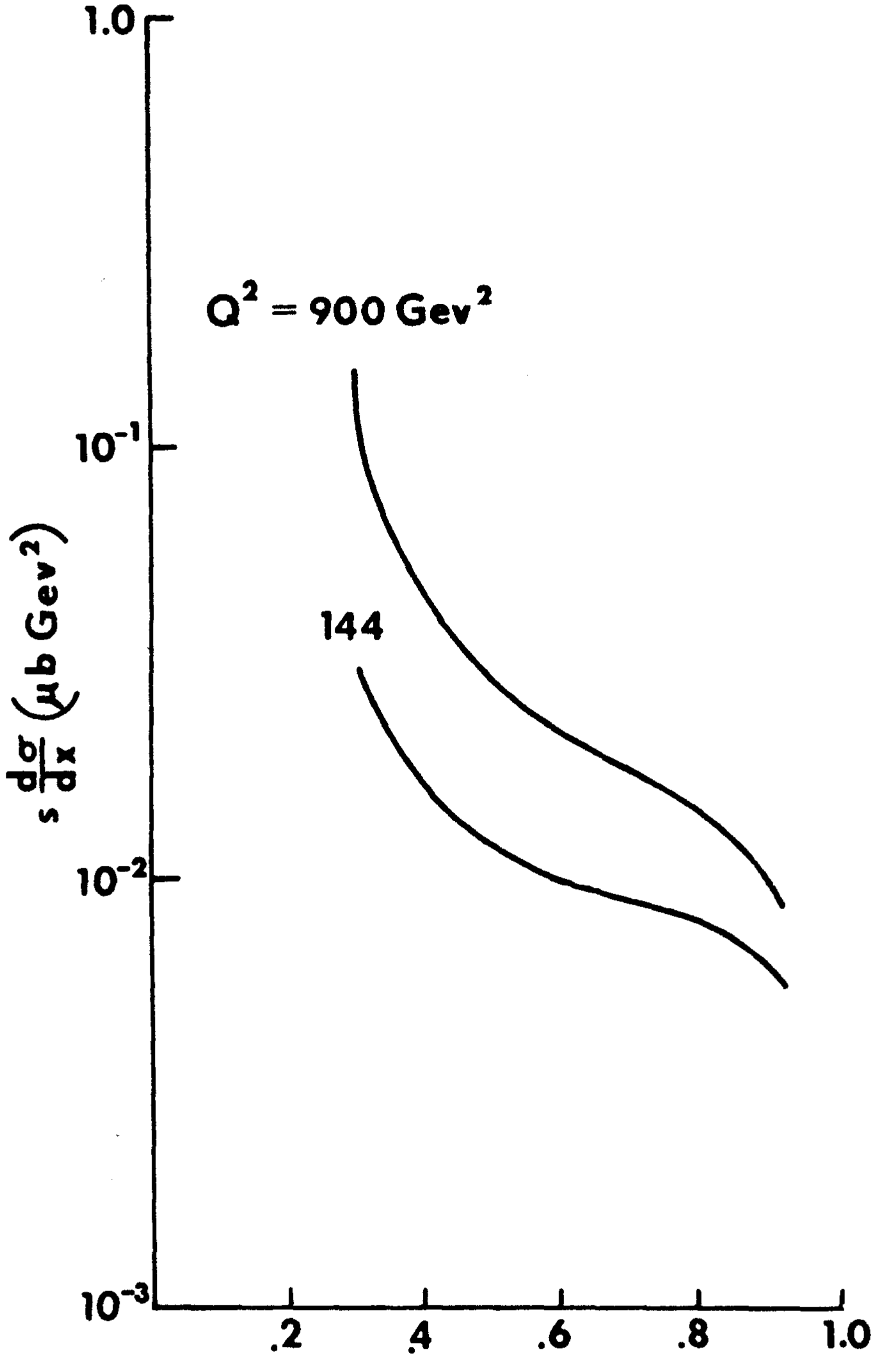
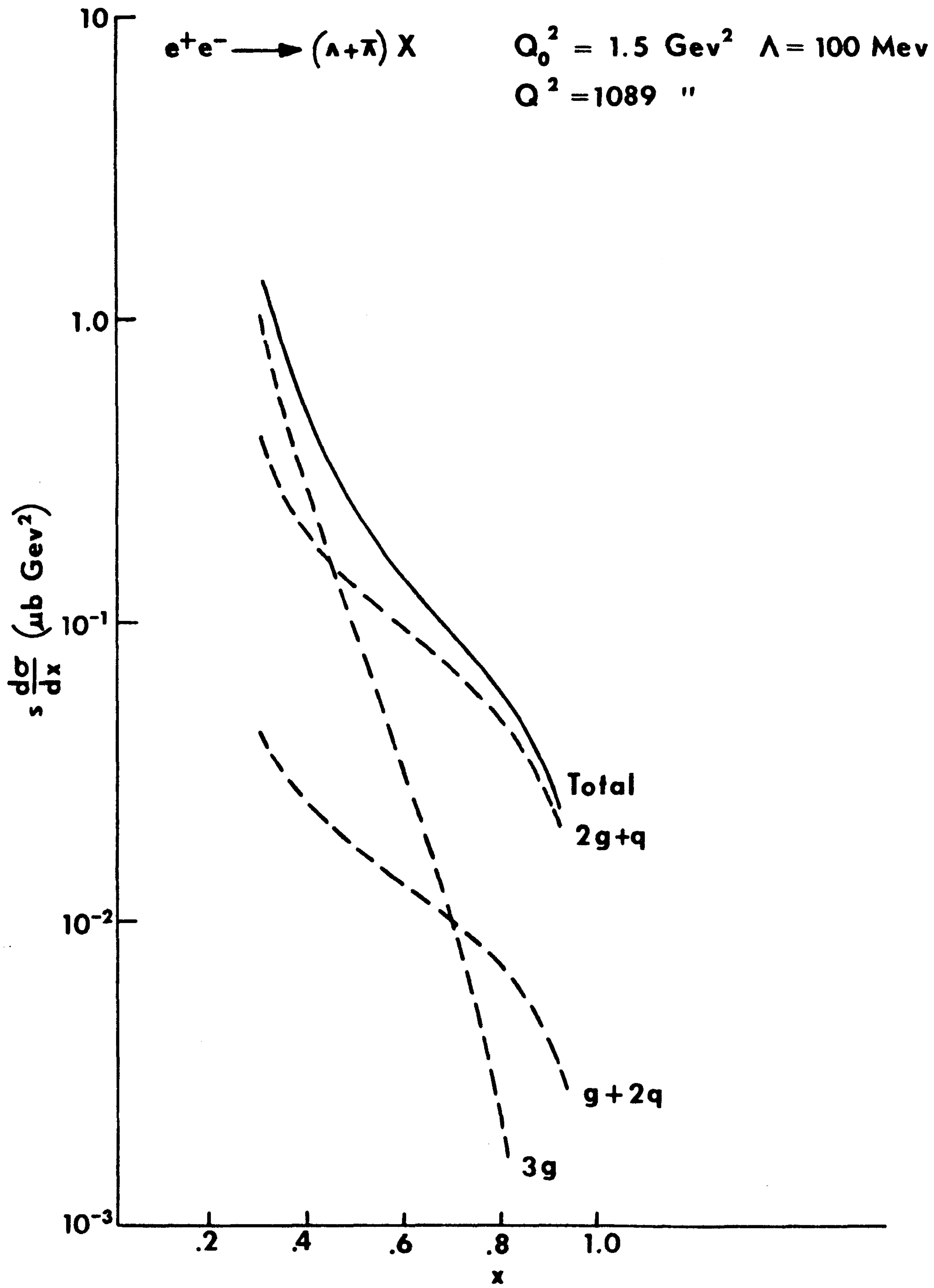
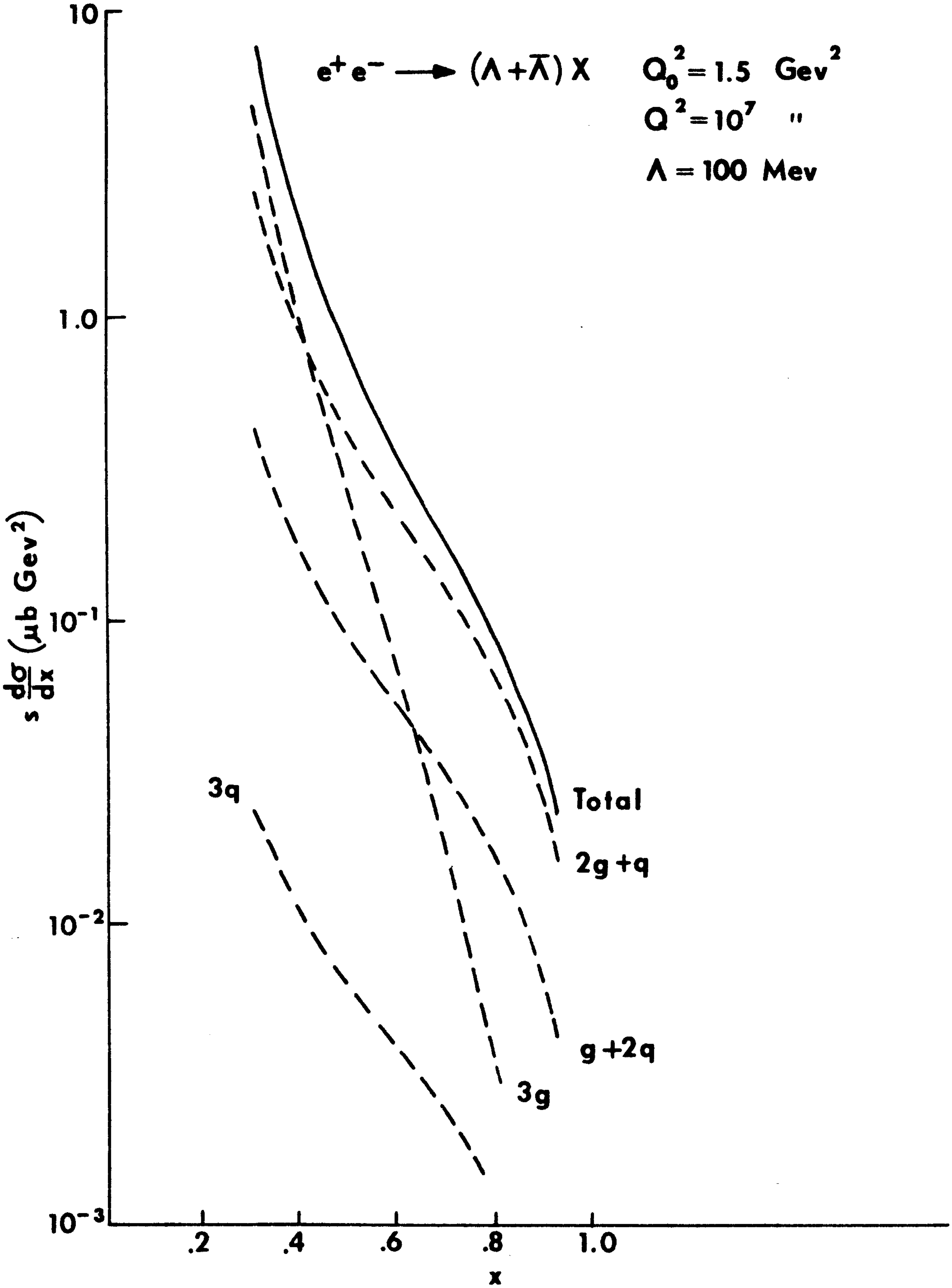


FIGURE 5B



$e^+e^- \to (\Lambda+\bar{\Lambda})\,X$
$Q_0^2 = 1.5\ \mathrm{Gev}^2\quad \Lambda = 100\ \mathrm{Mev}$
$Q^2 = 1089$ "
10
1.0
10^{-1}
10^{-2}
10^{-3}
$s\frac{d\sigma}{dx}\ (\mu b\ \mathrm{Gev}^2)$
Total
2g+q
g+2q
3g
.2
.4
.6
.8
1.0
x

Figure 6a



$e^+e^- \longrightarrow (\Lambda+\bar{\Lambda})X$
$Q_0^2 = 1.5$ Gev^2
$Q^2 = 10^7$ "
$\Lambda = 100$ Mev
10
1.0
10^{-1}
10^{-2}
10^{-3}
$s\frac{d\sigma}{dx}(\mu b\ Gev^2)$
3q
Total
2g +q
g +2q
3g
.2
.4
.6
.8
1.0
x

Figure 6b

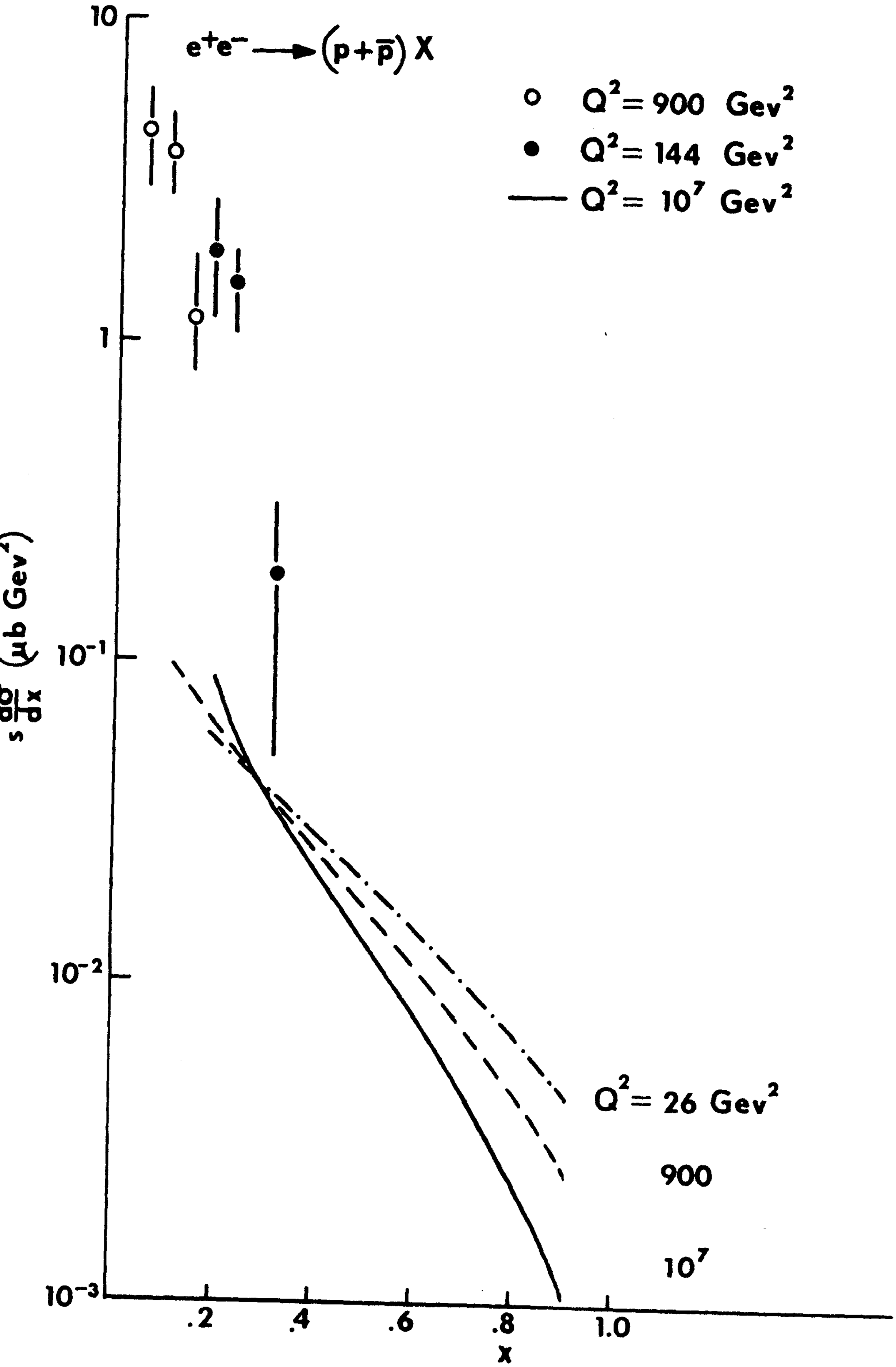

$e^+e^- \longrightarrow (p+\bar{p})\,X$
$Q^2 = 900$ Gev^2
$Q^2 = 144$ Gev^2
$Q^2 = 10^7$ Gev^2
$Q^2 = 26$ Gev^2
900
10^7
$s\frac{d\sigma}{dx}$ (μb Gev^2)
10
1
10^{-1}
10^{-2}
10^{-3}
.2
.4
.6
.8
1.0
x

Figure 7

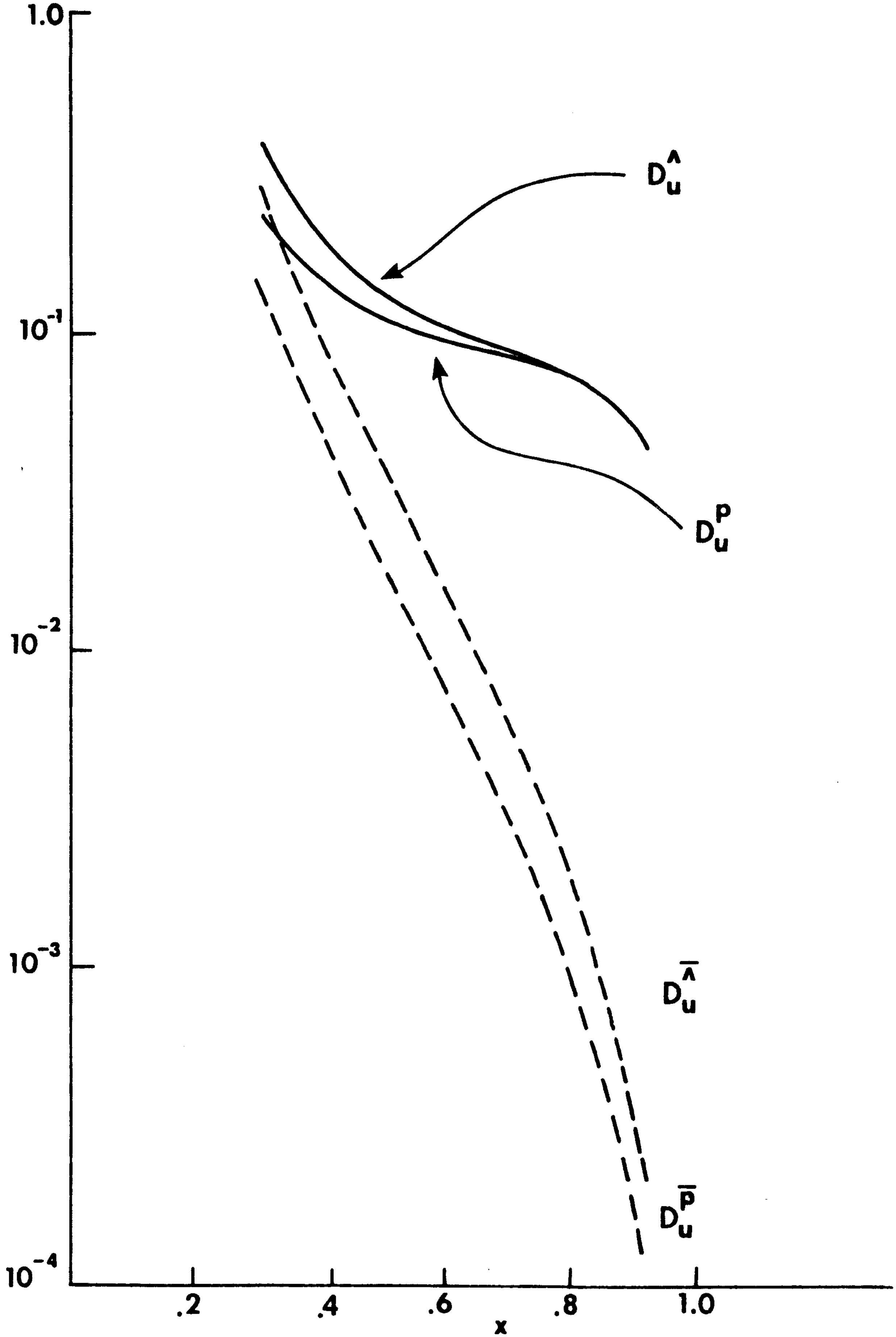


Figure 8

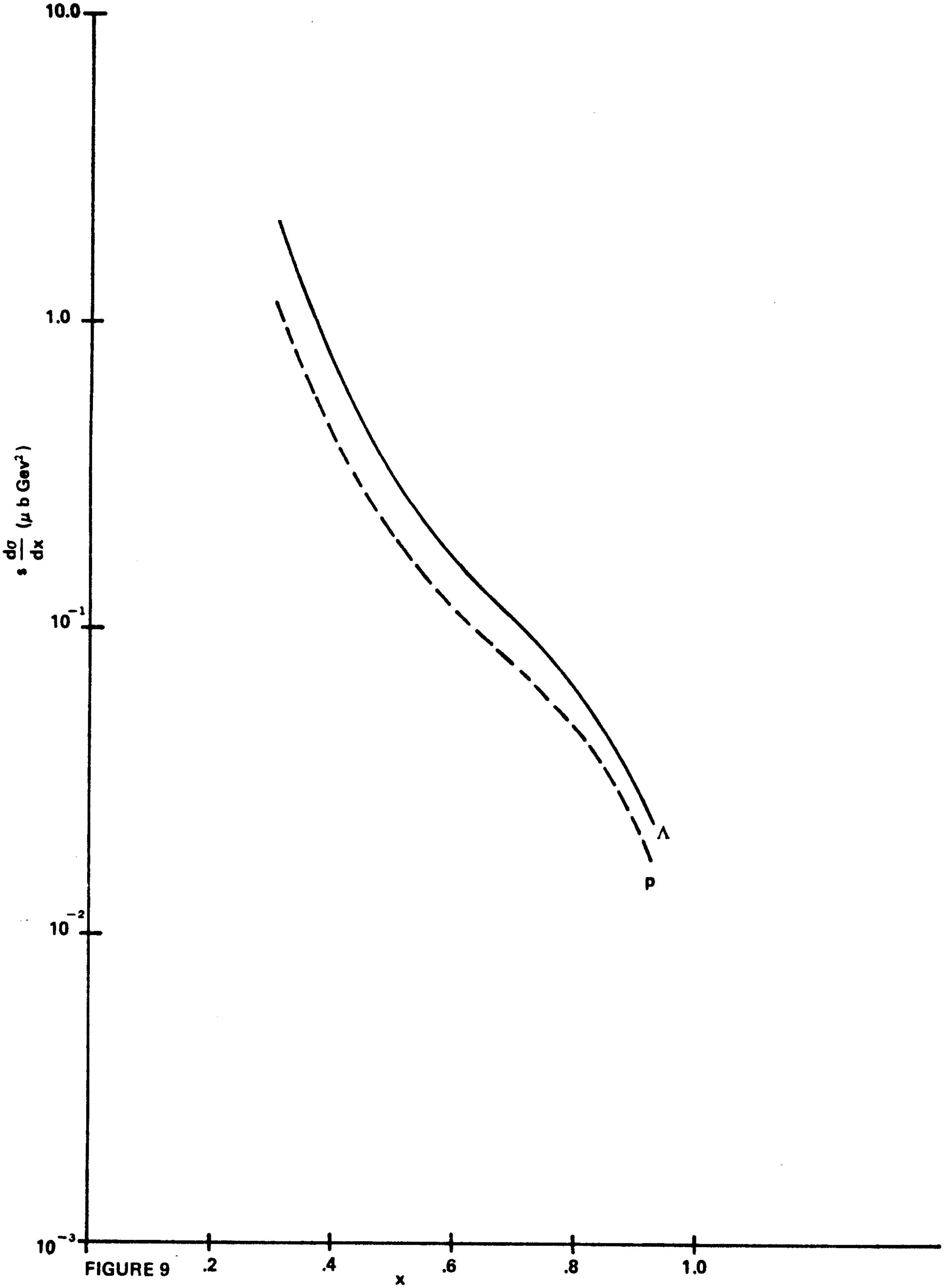
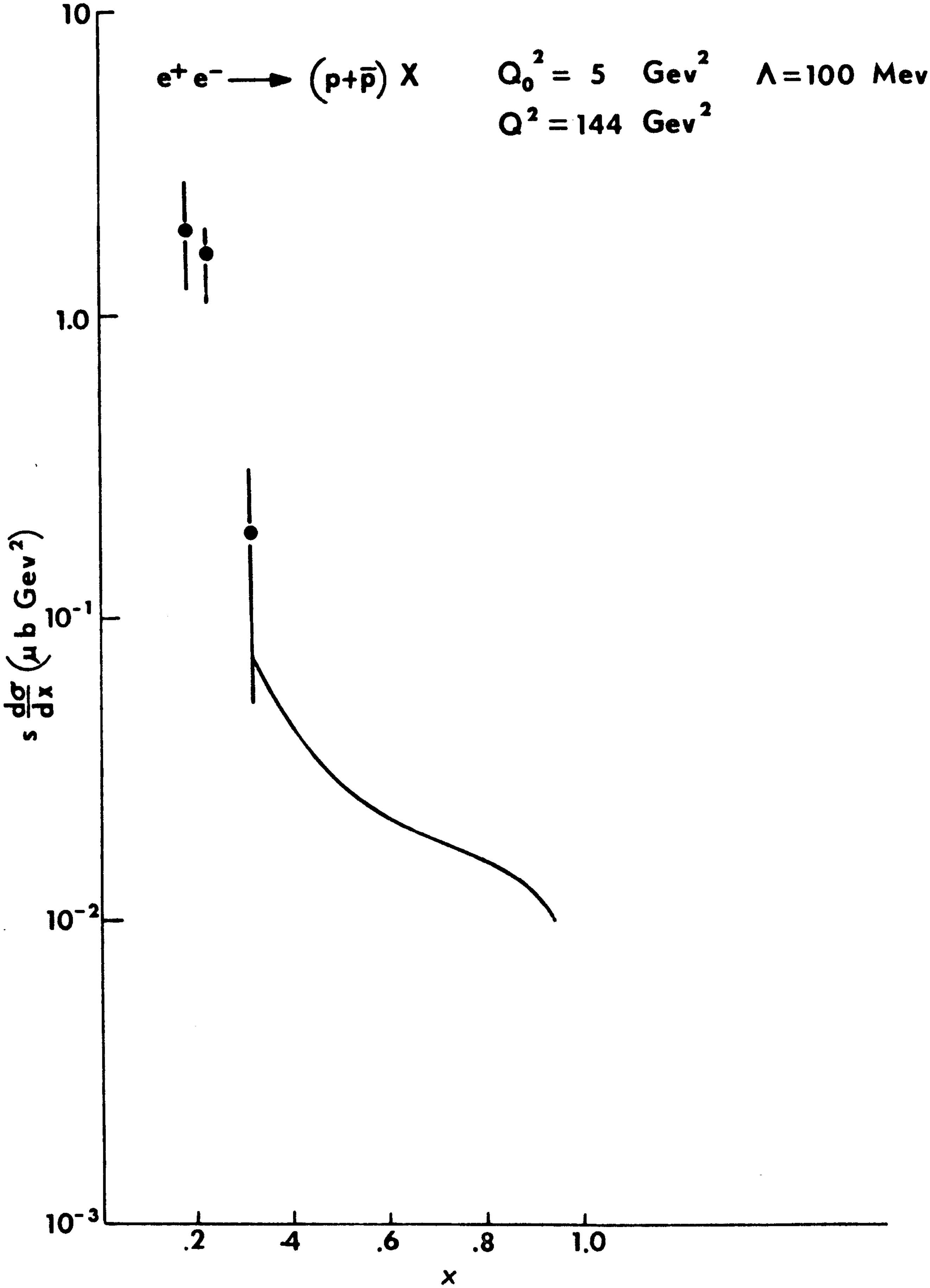
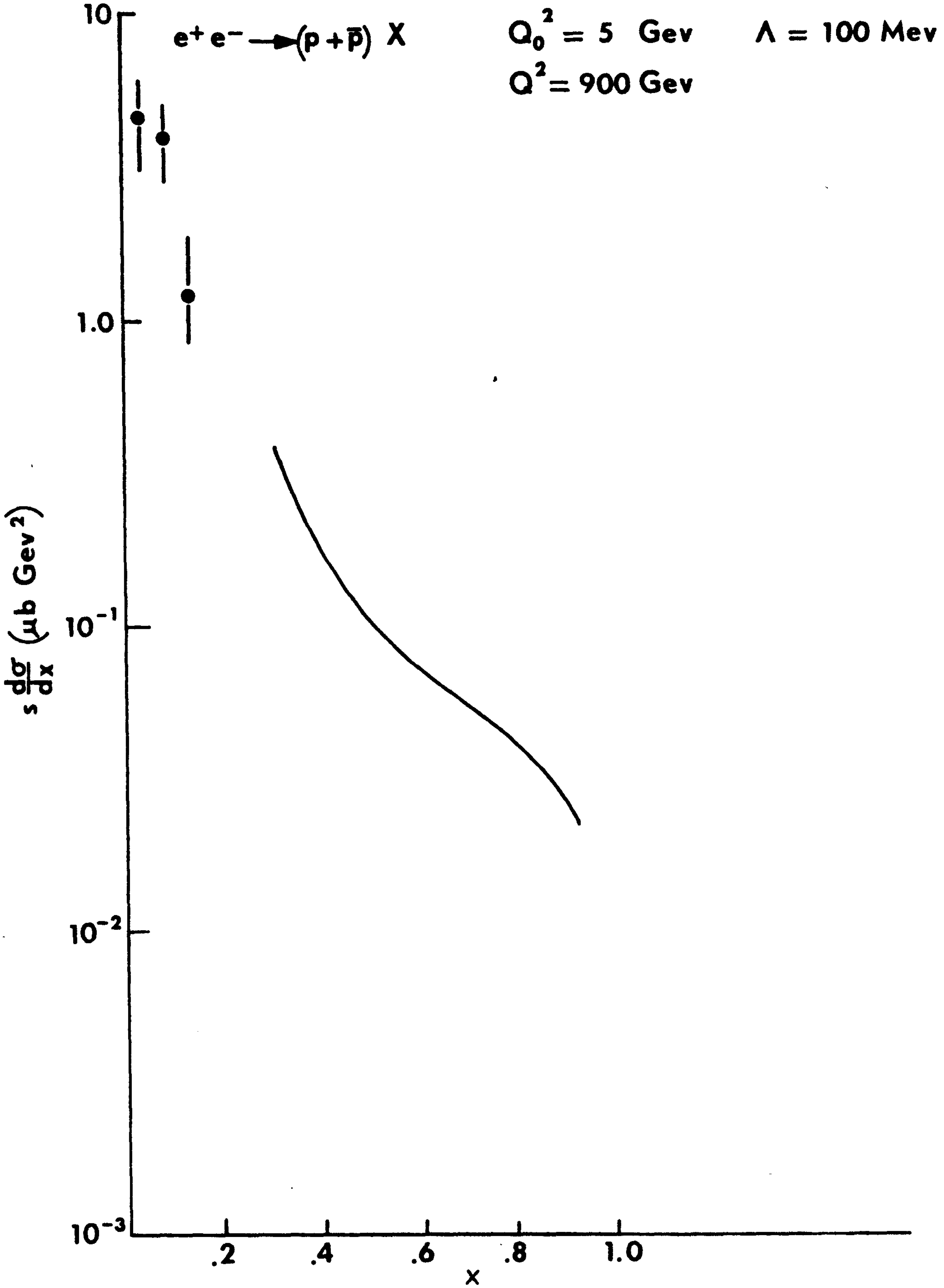


FIGURE 9



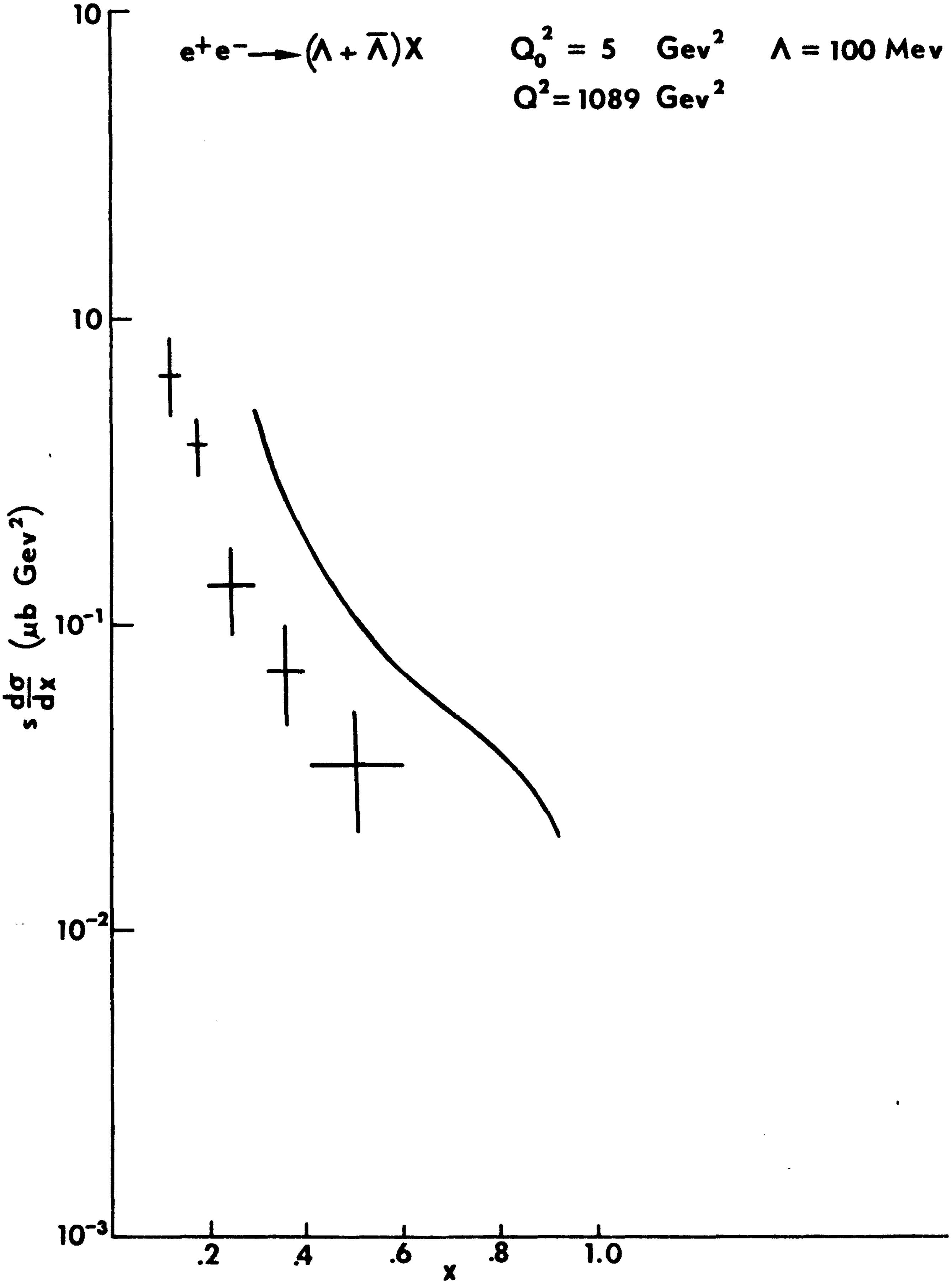
$e^+e^- \longrightarrow (p+\bar{p})\,X$
$Q_0^2 = 5$ Gev^2
$\Lambda = 100$ Mev
$Q^2 = 144$ Gev^2
10
1.0
10^{-1}
10^{-2}
10^{-3}
$s\,\frac{d\sigma}{dx}\,(\mu b\ \mathrm{Gev}^2)$
.2
.4
.6
.8
1.0
x

Figure 10a



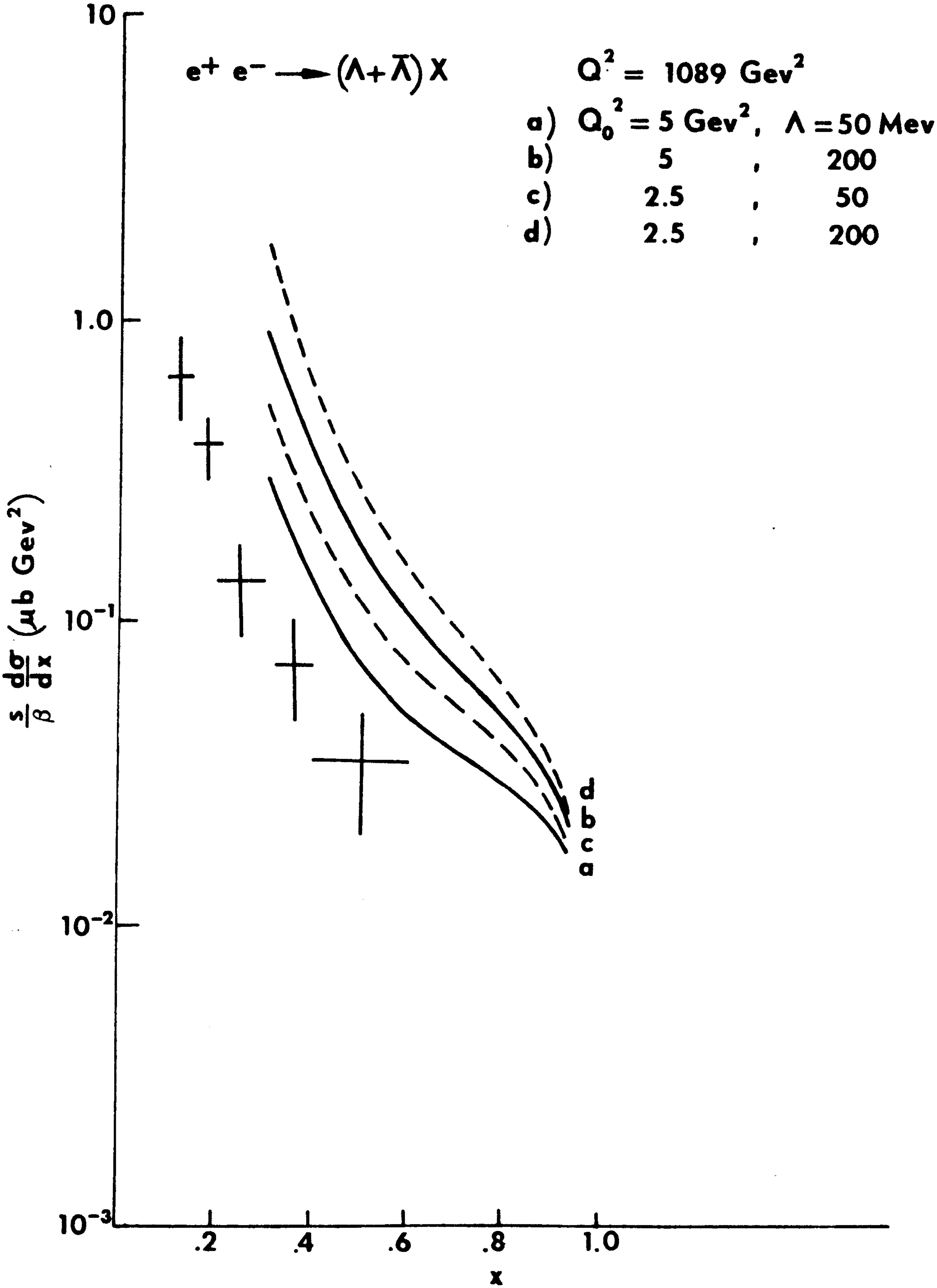
$e^+e^- \rightarrow (p+\bar{p})\ X$
$Q_0^2 = 5$ Gev
$Q^2 = 900$ Gev
$\Lambda = 100$ Mev
$s\frac{d\sigma}{dx}$ (μb Gev^2)
10
1.0
10^{-1}
10^{-2}
10^{-3}
.2
.4
.6
.8
1.0
x

Figure 10b



$e^+e^- \to (\Lambda + \bar{\Lambda})X$
$Q_0^2 = 5$ Gev^2
$\Lambda = 100$ Mev
$Q^2 = 1089$ Gev^2
10
10
10^{-1}
10^{-2}
10^{-3}
$s\frac{d\sigma}{dx}$ (μb Gev^2)
.2
.4
.6
.8
1.0
x

Figure 10c



10
1.0
10^{-1}
10^{-2}
10^{-3}
$e^+ e^- \rightarrow (\Lambda + \bar{\Lambda}) X$
$Q^2 = 1089$ Gev^2
a) $Q_0^2 = 5$ Gev^2, $\Lambda = 50$ Mev
b) 5 , 200
c) 2.5 , 50
d) 2.5 , 200
$\frac{s}{\beta} \frac{d\sigma}{dx}$ (μb Gev^2)
d
b
c
a
.2
.4
.6
.8
1.0
x

Figure 11

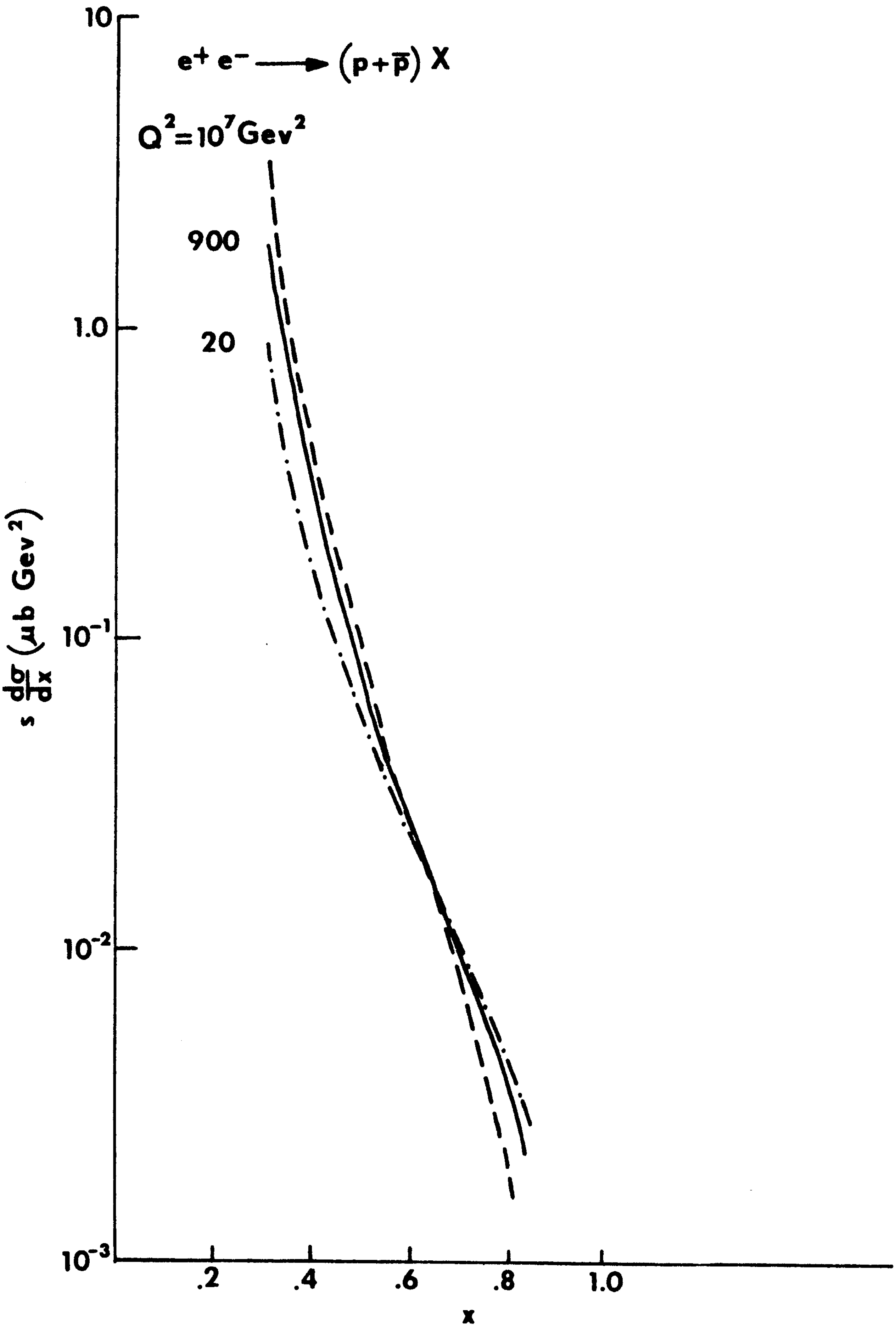

10
1.0
10^{-1}
10^{-2}
10^{-3}
$e^+ e^- \longrightarrow (p+\bar{p})\,X$
$Q^2=10^7 \mathrm{Gev}^2$
900
20
$s\,\frac{d\sigma}{dx}$ (μb Gev^2)
.2
.4
.6
.8
1.0
x

Figure 12

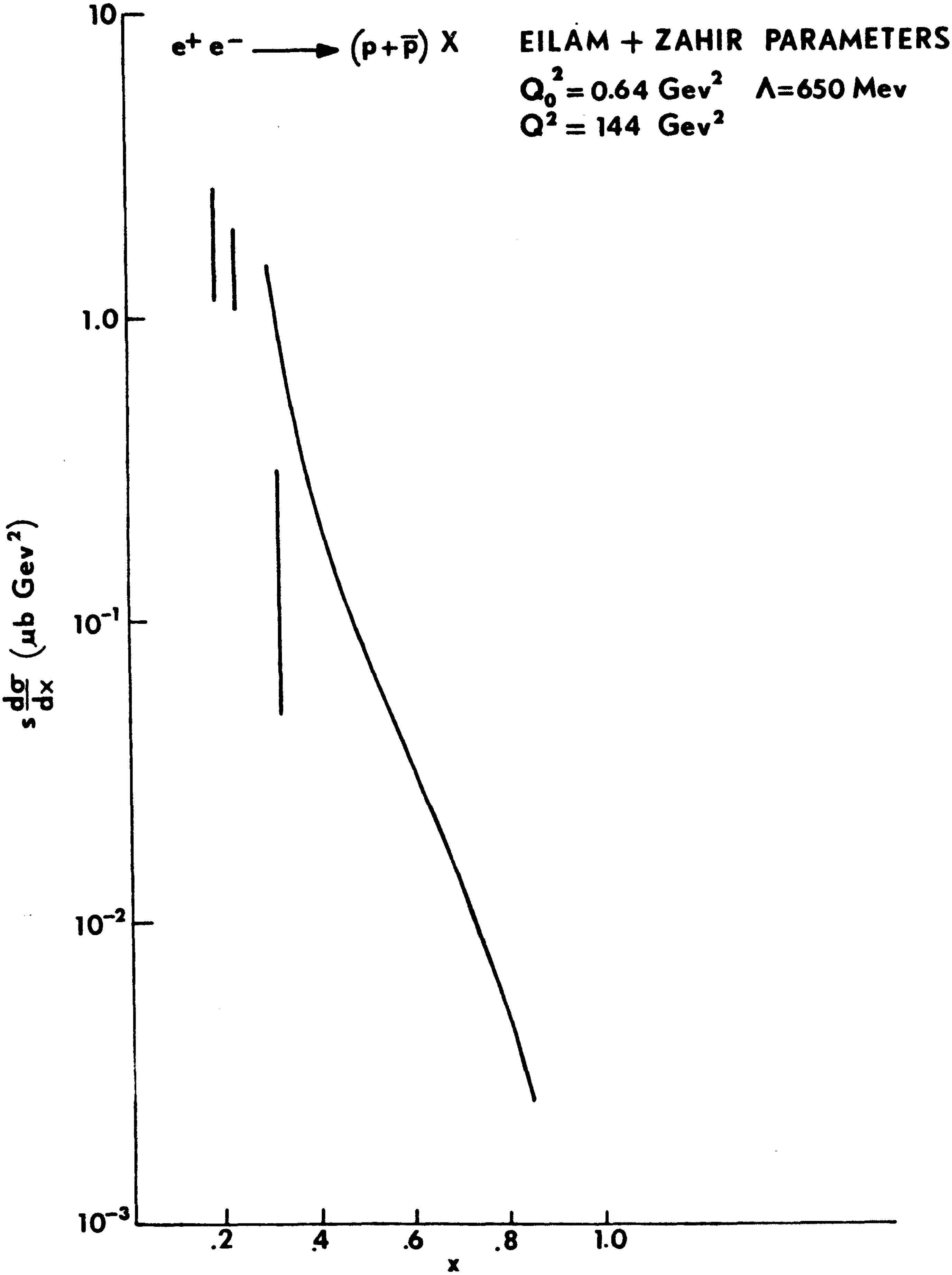
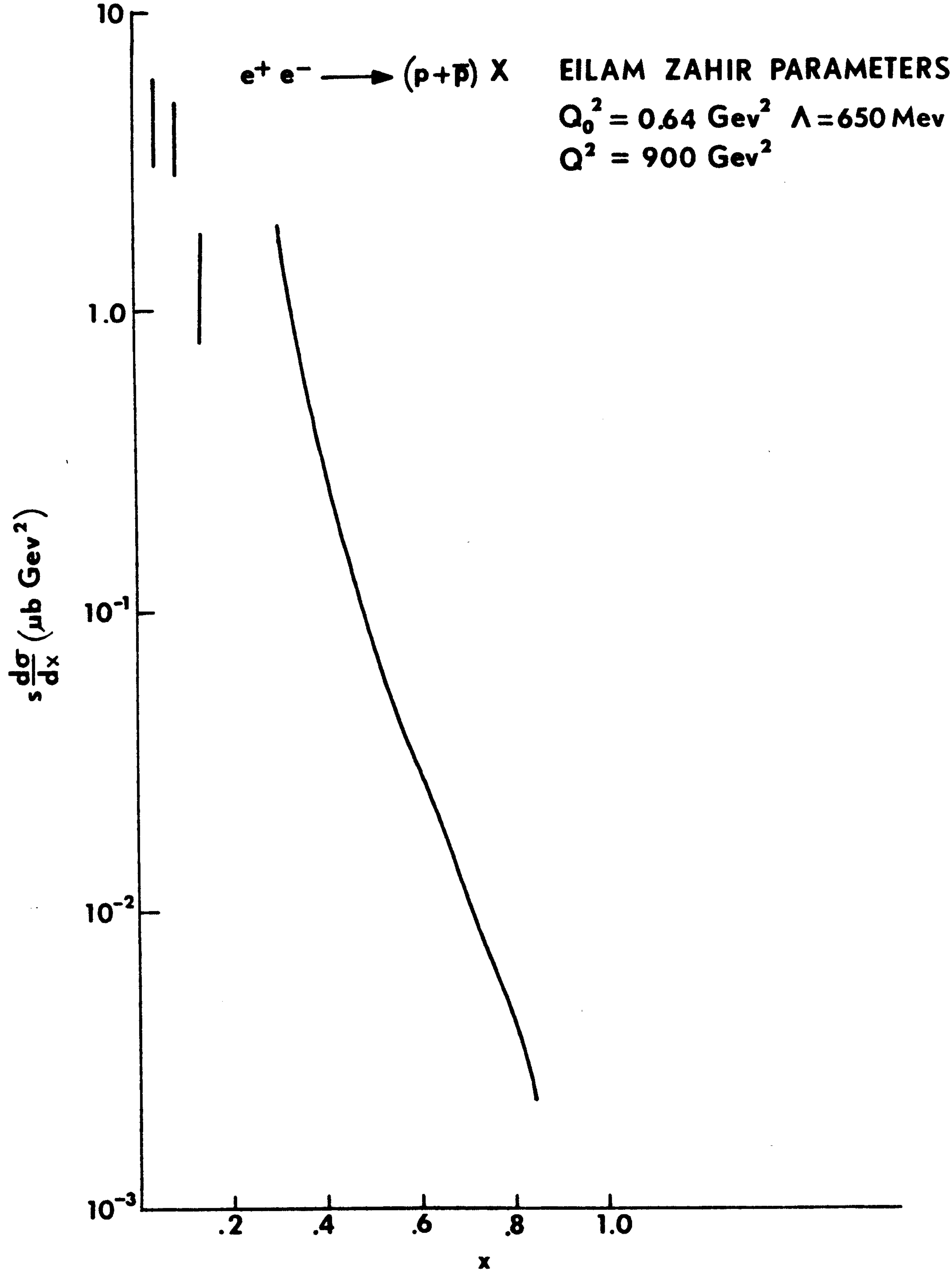
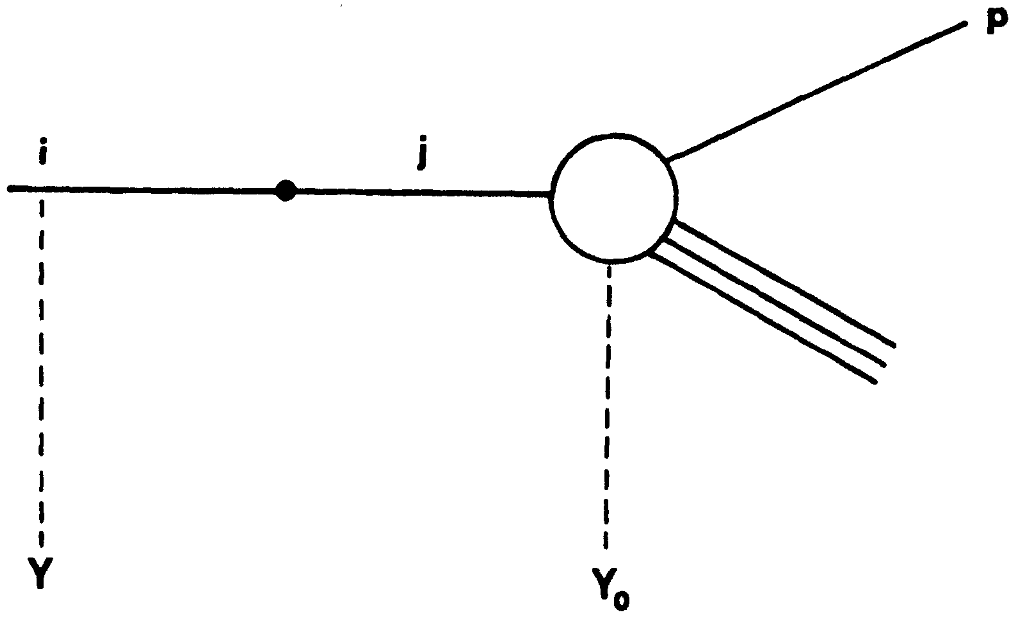

$e^+e^- \longrightarrow (p+\bar{p})\,X$
EILAM + ZAHIR PARAMETERS
$Q_0^2 = 0.64$ Gev^2 $\Lambda = 650$ Mev
$Q^2 = 144$ Gev^2
$s\frac{d\sigma}{dx}$ (μb Gev^2)
10
1.0
10^{-1}
10^{-2}
10^{-3}
.2
.4
.6
.8
1.0
x

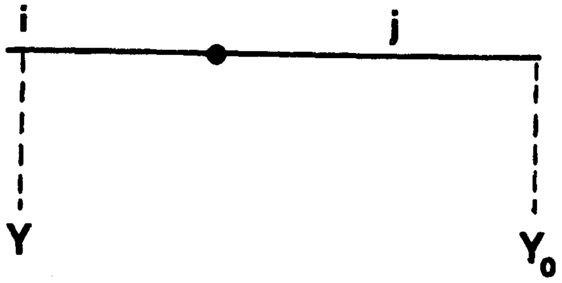
Figure 13a



$e^+ e^- \longrightarrow (p+\bar{p})\ X$
EILAM ZAHIR PARAMETERS
$Q_0^2 = 0.64$ Gev^2 $\Lambda = 650$ Mev
$Q^2 = 900$ Gev^2
10
1.0
10^{-1}
10^{-2}
10^{-3}
$s\frac{d\sigma}{dx}$ (μb Gev^2)
.2
.4
.6
.8
1.0
x

Figure 13b





denotes $D_{ji}(Y-Y_0)$

Figure 14

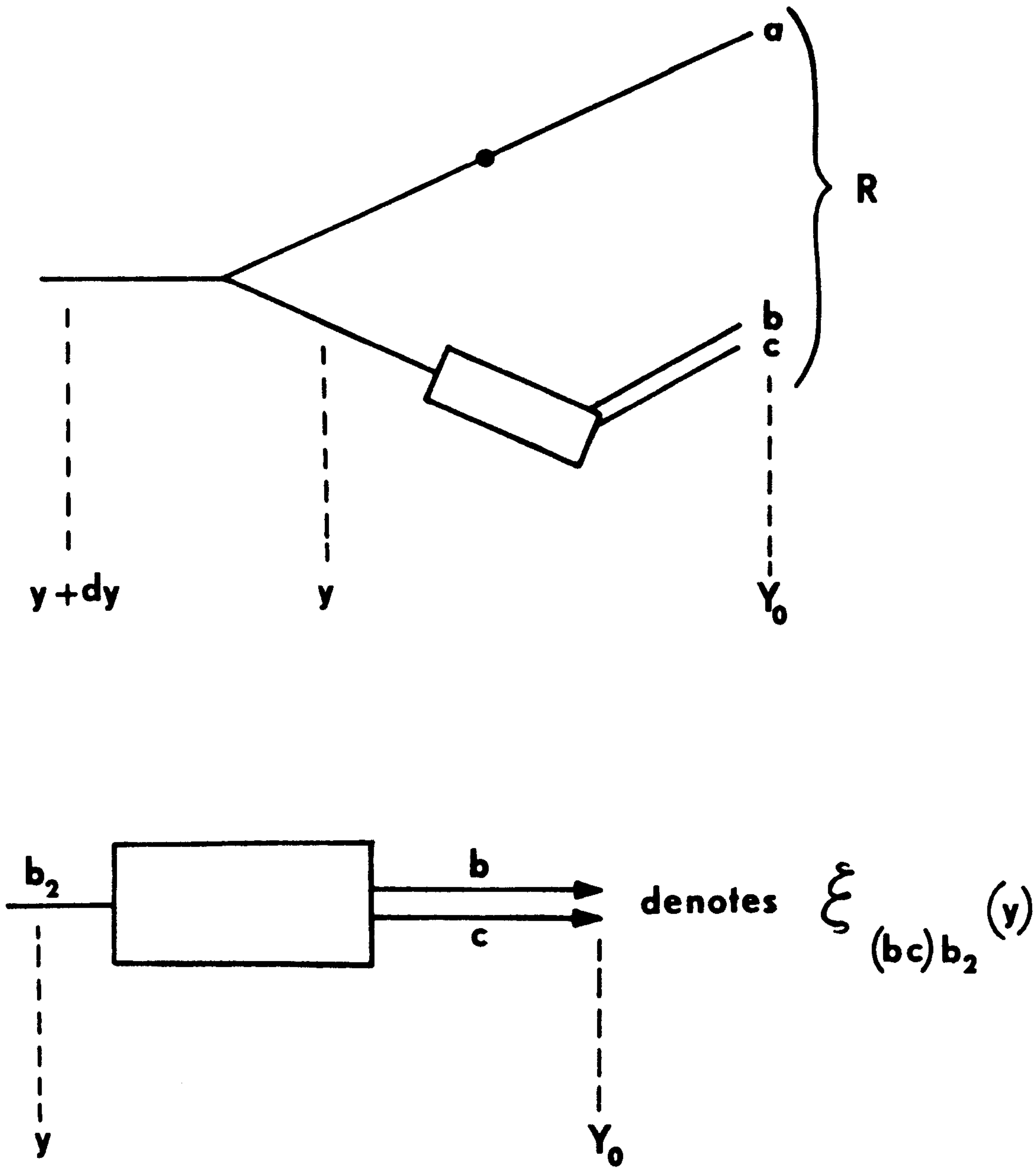


Figure 15

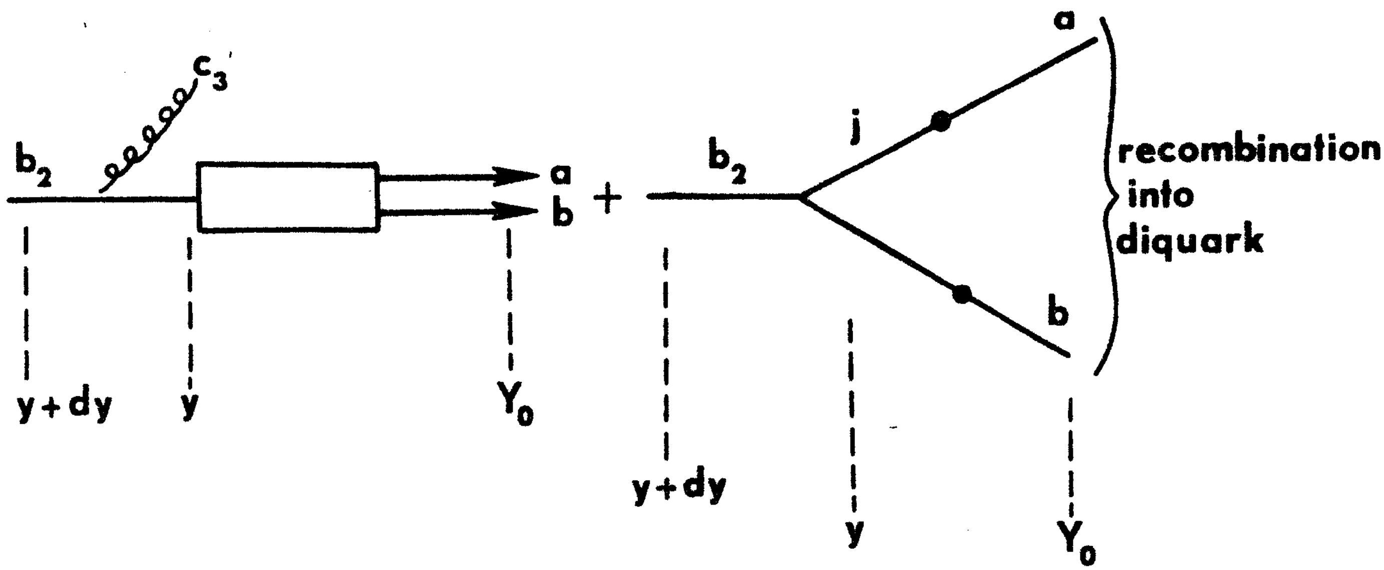


Figure 16

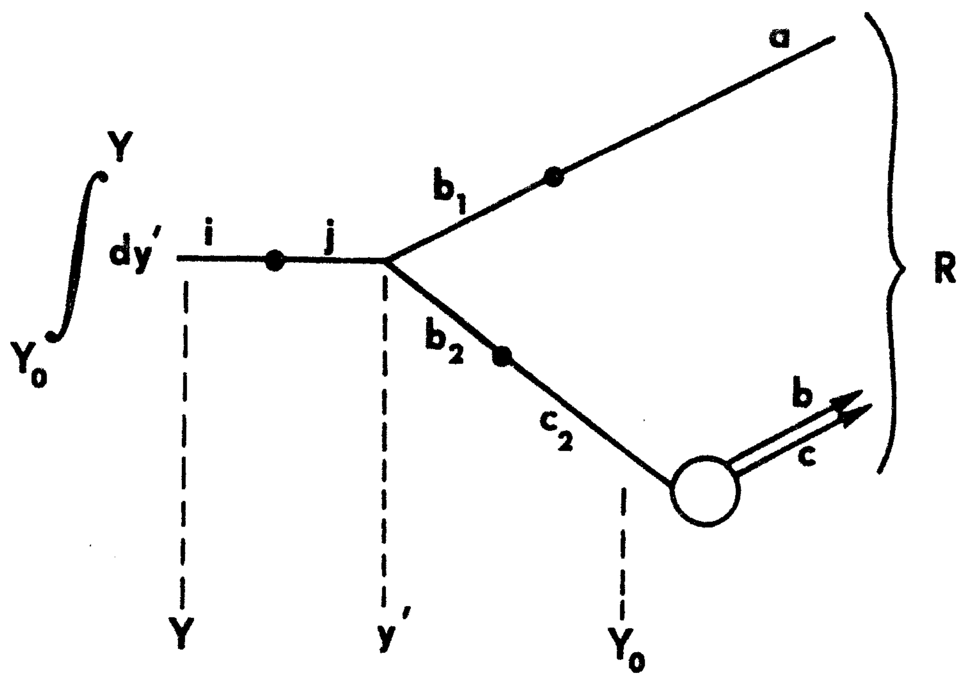


Figure 17

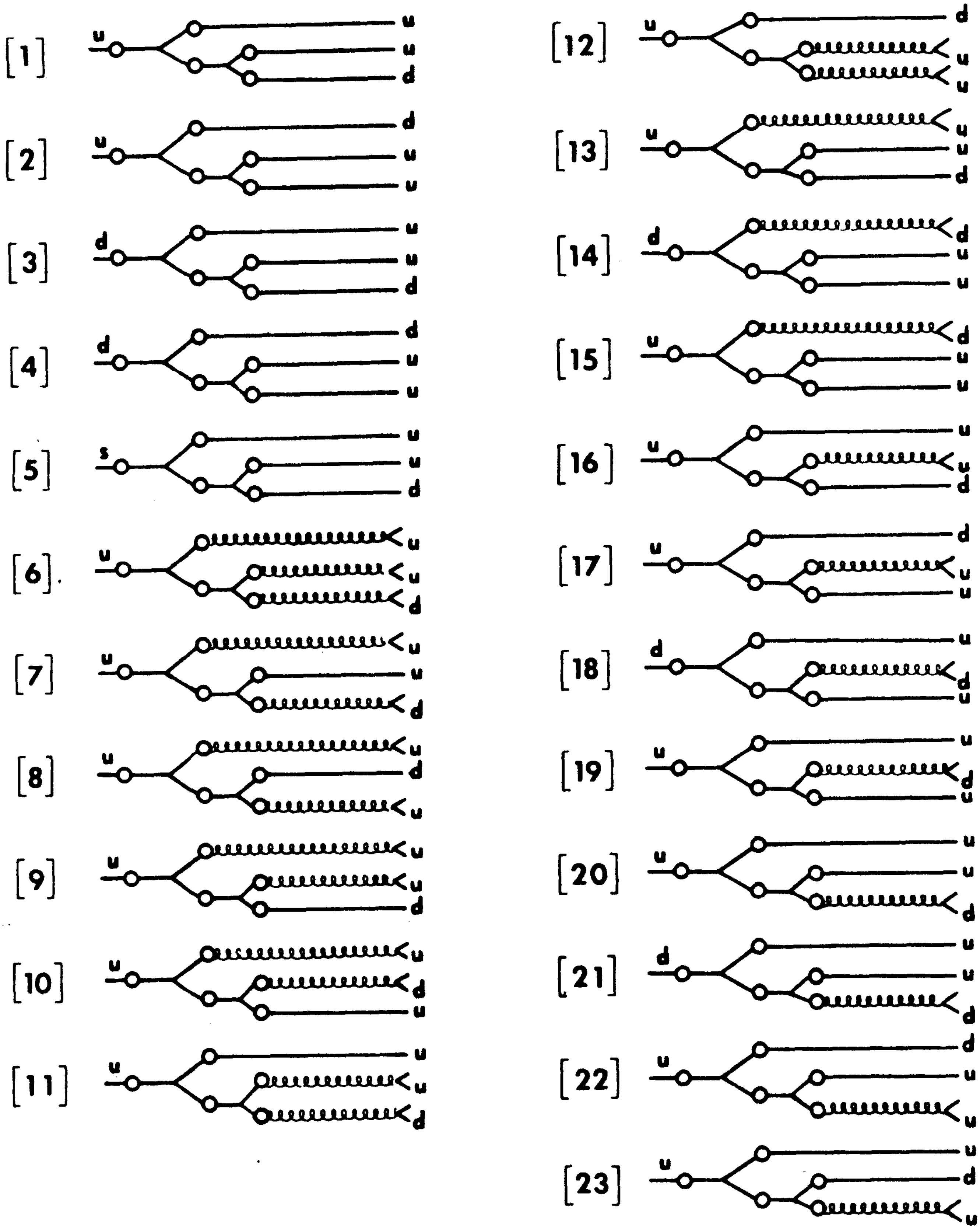
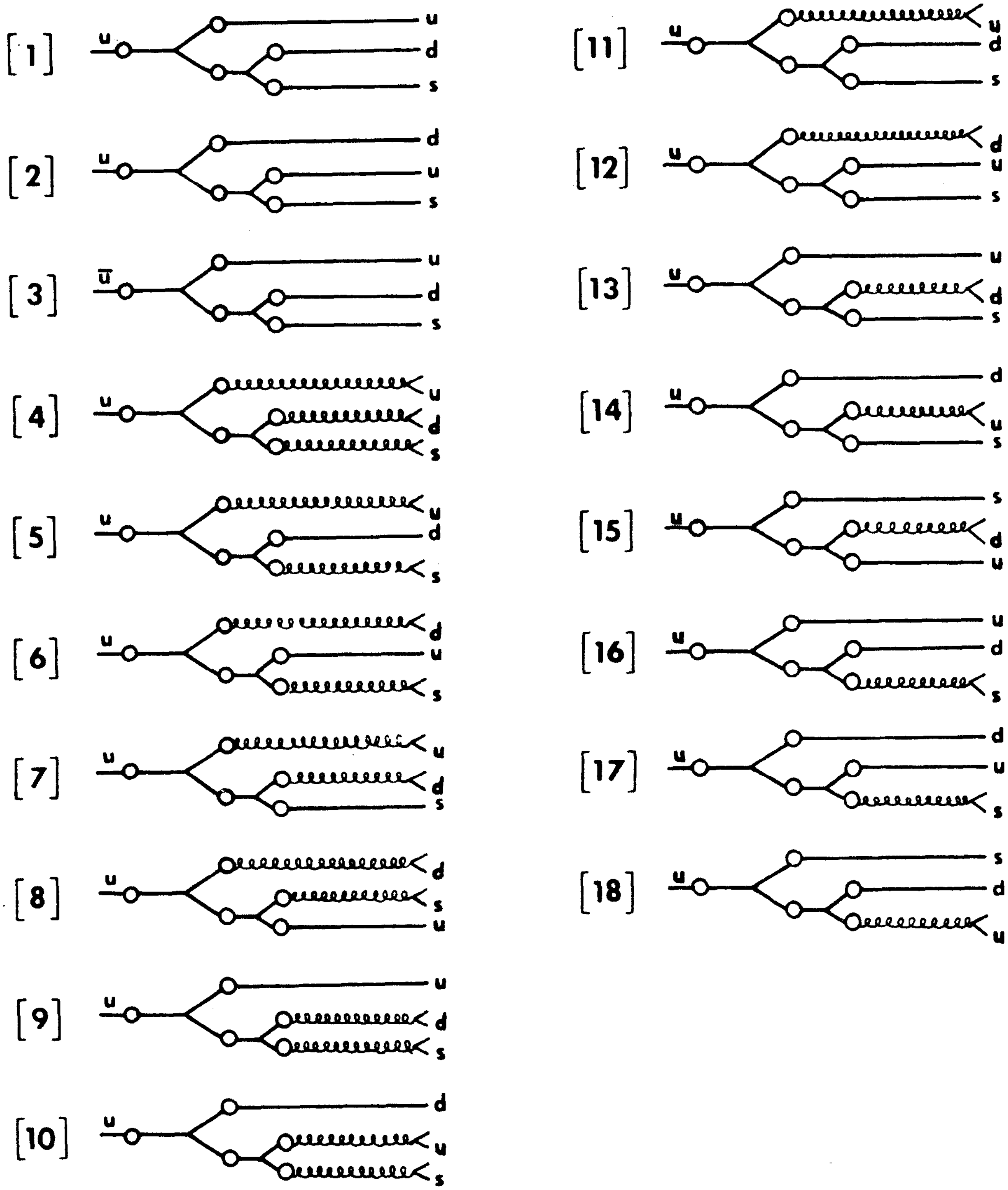


Figure A1


[1]
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d
s
[2]
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[3]
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[4]
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[5]
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Figure B1

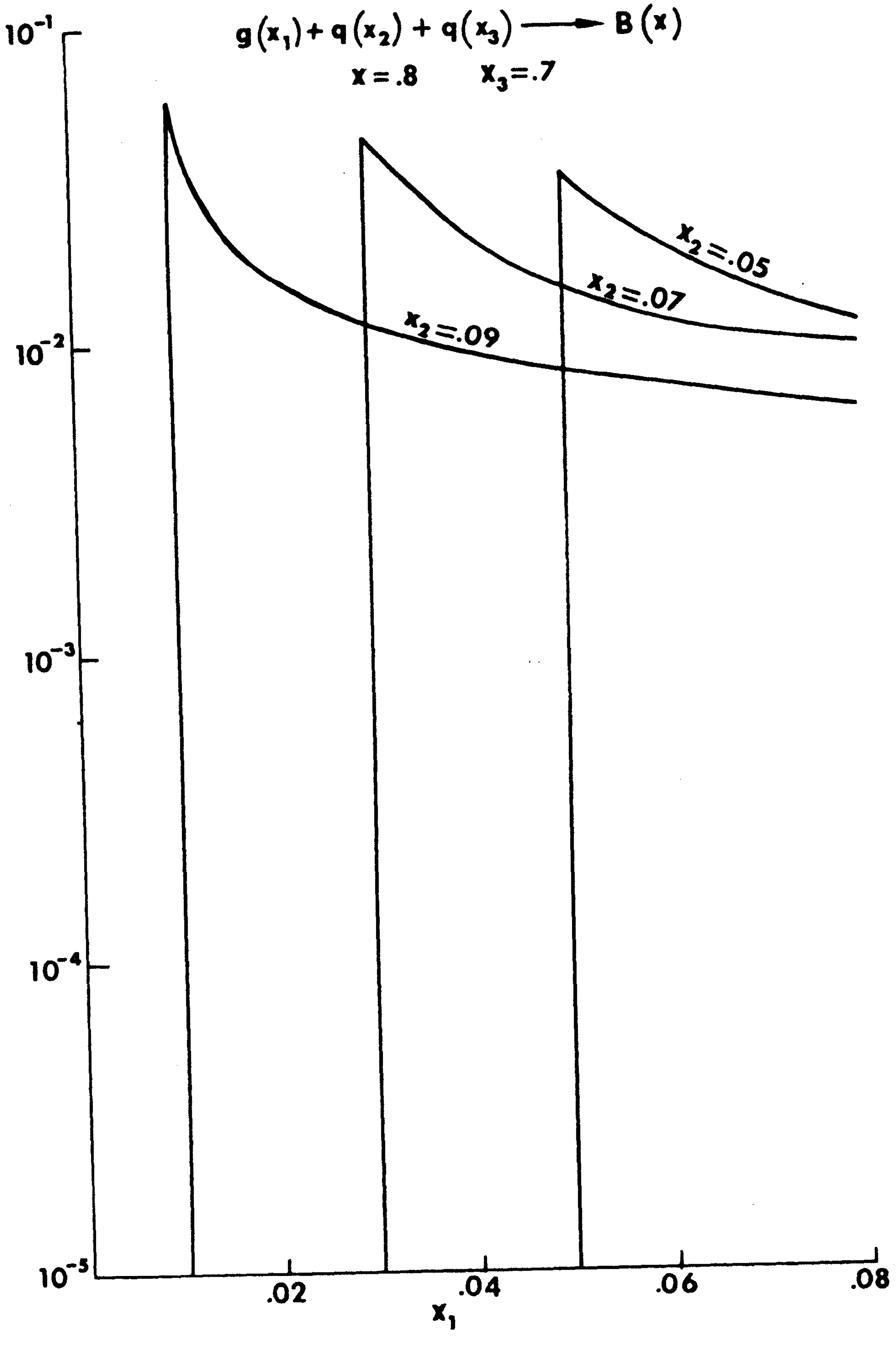
g(x1) + q(x2) + q(x3) → B(x)
x = .8 X3 = .7
10^-1
10^-2
10^-3
10^-4
10^-5
x2 = .05
x2 = .07
x2 = .09
.02
.04
.06
.08
x1

Figure C1

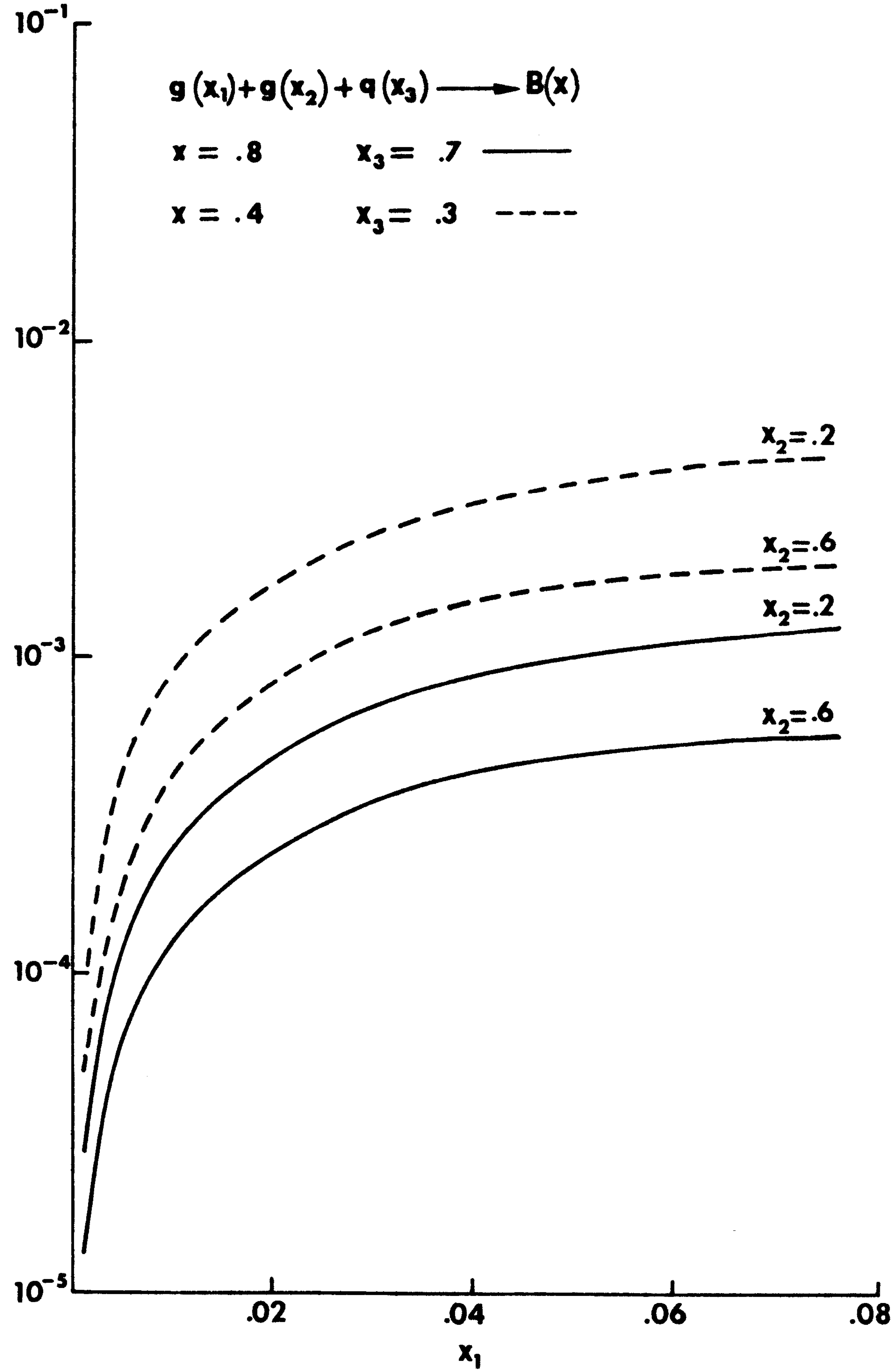
$g(x_1)+g(x_2)+q(x_3) \longrightarrow B(x)$
$x = .8$ $x_3 = .7$
$x = .4$ $x_3 = .3$
$x_2=.2$
$x_2=.6$
$x_2=.2$
$x_2=.6$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
.02
.04
.06
.08
x_1

Figure C2

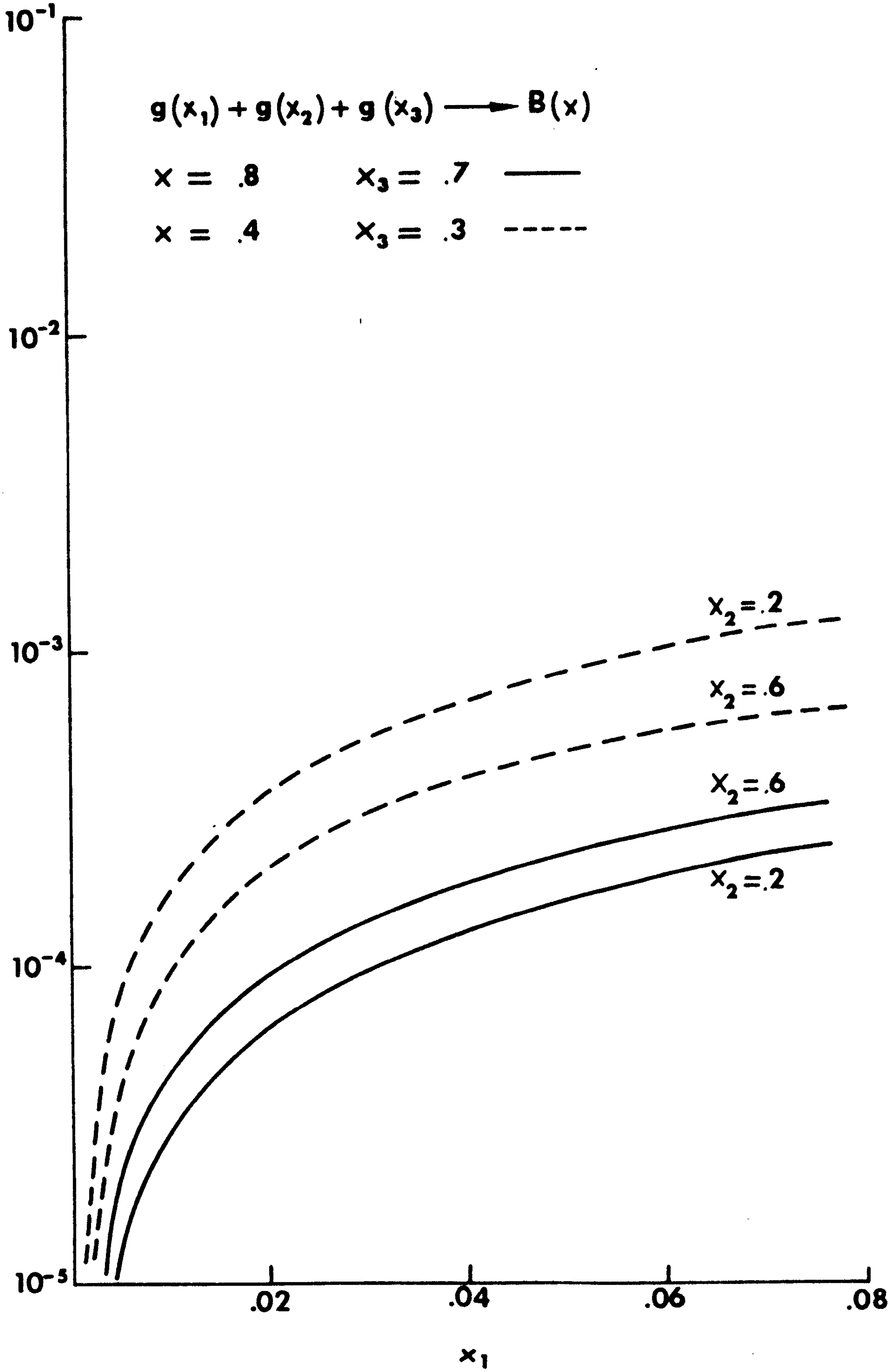
$g(x_1) + g(x_2) + g(x_3) \longrightarrow B(x)$
$x = .8$ $x_3 = .7$
$x = .4$ $x_3 = .3$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$x_2 = .2$
$x_2 = .6$
$x_2 = .6$
$x_2 = .2$
.02
.04
.06
.08
x_1

Figure C3