SLAC-PUB-2845
November 1981
(T/E)

$SU_2 \times U_1$ Effects in e^+e^- Annihilation: Generalized Charge Asymmetry*

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ABSTRACT

The observability of the $SU_2 \times U_1$ electroweak charge asymmetry effects in e^+e^- annihilation at $\sqrt{s}$ = 29 GeV is addressed in the context of a Feynman-Field type fragmentation model. We assume three colors of five flavored quarks and one heavy lepton τ. We neglect, at this time, the hard gluon bremsstrahlung events. We take $b \to c + X$ as the b decay mode and we assume all τ's and heavy hadrons decay within the resolution of the detector so that only light hadrons and leptons are detected. Allowing all these decays to occur, we then compute the expected front-back asymmetry of negatively charged particles weighted with z^n for $z \geq 0.175$, where n = 0.5, 1, ..., 7, 10, and z is the light cone momentum fraction. We find, for example, that such an asymmetry is ~ 5% for n = 2 for $\sin^2\theta_W = 0.236$ and $\Lambda_{QCD} = 0.34$ GeV. In other words, due to the large number of charge particles produced per event, the $SU_2 \times U_1$ charge asymmetry may be accessible experimentally in $e^+e^- \to X$ already at PEP and PETRA energies.

Submitted to Nuclear Physics B

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

1. Introduction

The $SU_2 \times U_1$ model of Salam-Glashow and Weinberg [1] represents what appears to be a complete description of relatively low energy electroweak dynamics. By low energy we mean energies small compared to M_W and M_Z, the respective masses of the heavy vector gauge bosons W and Z in the model. What is of primary interest is to observe nontrivial manifestations of W and Z in higher energy electroweak interactions. A candidate for this type of observation is the process $e^+e^- \to X$ at center of momentum energies $\sqrt{s}$ not too far from M_Z. Such processes are currently under observation at PEP and PETRA, where we have $\sqrt{s} = 29$ GeV at PEP and $\sqrt{s} \cong 36$ GeV at PETRA. Recall that, for $\sin^2\theta_W = 0.236$, in the tree approximation, one most naively has

$$M_Z = \left[\left(\frac{4\pi\alpha}{8\sin^2\theta_W \cos^2\theta_W}\right)\frac{\sqrt{2}}{G_F}\right]^{\frac{1}{2}} \doteq 88 \text{ GeV}. \qquad (1)$$

Thus, it is not far fetched that effects due to Z boson exchange might be visible at PEP and PETRA [2].* Furthermore, such effects would surely be observable at SLC energies. Here, however, we wish to concentrate on PEP and PETRA energies. For definiteness, then, we will always fix $\sqrt{s}$ at $\sqrt{s_0} = 29$ GeV in what follows.

What we wish to do is to calculate what might be a reasonable physical quantity to use to observe the effects of Z boson exchange at PEP and

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* Indeed, recently, B. Adeva et al., Phys. Rev. Lett. 48 (1982) 1701, have reported the first "high" statistics front-back asymmetry for $e^+e^- \to \mu^+\mu^-$ at $\sqrt{s} \doteq 34.6$ GeV — their result agrees with the standard model in ref. 1.

PETRA. We want to emphasize that we are not the first to look for such a quantity [2]. We will conclude that such a quantity exists which may allow a "relatively" high statistics measurement with currently available data [3].

Our choice of physical parameter will be the generalized front-back charge asymmetry due to photon-Z interference in $e^+e^- \to f\bar{f}$, where $f = \mu, \tau$, u, d, s, c, b. Here, we have in mind three colors of each type of quark.

More specifically, we consider the following line of reasoning. The individual asymmetry for each "fundamental" $f\bar{f}$ pair has been given by several authors [2]. Indeed, the effects of QCD bremsstrahlung have also been calculated [2]. (We shall consistently neglect hard gluon bremsstrahlung effects here because they have been found to be small in the fundamental fermion asymmetries. We consider the soft glue effects to be contained in the quark fragmentation, our model for which we discuss presently.) Also, exclusive hadronic channels such as

$$e^+e^- \to h^+h^- \tag{2}$$

have been analyzed [2] with regard to their electroweak charge asymmetry. The natural next step is to sum over all possible observed charged particles (light leptons and light hadrons) in the final state and to consider the electroweak charge asymmetry of all charged particles produced in $e^+e^- \to X$. This will be the prototypical quantity computed in this paper. We will find that our prototypical quantity is not really optimal because slow hadrons† in the final state tend to wash-out the asymmetry of the underlying $f\bar{f}$ pair. Thus, we will find it convenient to weight

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† The author thanks R. Blankenbecler and S. Brodsky for useful discussions on this point.

each charged particle with a function

$$w(z)$$

such that

$$w(z) \to 0 \quad , \qquad z \to 0 \quad , \tag{3}$$

$$w(z) \to 1 \quad , \qquad z \to 1 \quad , \tag{4}$$

and

$$w(z) \geq 0 \quad , \tag{5}$$

where z is the light-cone momentum fraction of the respectively charged particle. Such weights have been considered elsewhere [2]. We shall refer to the front-back asymmetry of all light lepton and light hadron charged particles produced in $e^+e^- \to X$ and weighted with $w(z)$ as a generalized electroweak charge asymmetry. For definiteness, we will always take

$$w(z) = z^n \tag{6}$$

for some number $n \geq \frac{1}{2}$. The reader is invited to consider other choices for w.

Clearly, then, in general we will need a scheme for transforming $f\bar{f}$ into the observed light leptons and light hadrons. We choose to proceed as follows. For the three colors of u, d and s quarks we use a standard Feynman-Field fragmentation [4] in which, for simplicity, all mesons are pseudoscalar. Thus at $\sqrt{s} \sim 3$ GeV, the primordial fragmentation function of Field and Feynman is then

$$f(1-z) \;=\; .12 + 2.64(1-z)^2 \quad . \tag{7}$$

We shall subsequently consider the QCD evolution of this function $f(z)$. For the τ, c and b, we allow the heavy particles to decay, weakly, to light leptons and light hadrons.

More precisely, we follow the estimates in ref. 5 of charm production in deep inelastic neutrino-nucleon scattering. Thus, we compute the standard model [1] (here, X and X' denote "anything" as usual)

$$c \to s + X \tag{8}$$

$$b \to c + X, \quad c \to s + X' \tag{9}$$

$$\tau \to \nu_\tau + X \tag{10}$$

transitions and, then, use eq. (7) to develop the resulting light quarks into light hadrons. As one can see from eq. (9) we have used the results from ref. 6 to conclude that the dominant decay of the b quark involves the (b-c) transition as compared to the (b-u) transition. Let us remark that the use of eqs. (7), (8) and (9) to describe the fragmentation of the c and b quarks amounts to using a 'spectator' model for the decay of the heavy hadron which usually contains the primary heavy quark in $e^+e^- \to f\bar{f}$. While such spectator models were in serious doubt in the D^0, D^+ decay situation [7], recent experimental results [8] tend to indicate that the severity of this doubt was premature. Thus, we have reason to believe that eqs. (8), (9) and (10) are not unfounded. This completes the description of our basic strategy. To repeat, we will find that generalized electroweak asymmetries of the type considered here may very well be already observable at the PEP and PETRA energy $\sqrt{s_0}$ = 29 GeV.

Our work will be presented as follows. In the next section, we describe the precise physical quantities of interest to us. In sect. 3 we present our computation of these quantities. Two possible corrections to our work are discussed in sect. 4. Section 5 contains some concluding remarks.

2. Generalized Electroweak Asymmetry

We have interest in generalizations of the electroweak asymmetry with an eye toward observing such generalizations in the $SU_2 \times U_1$ model in e^+e^- annihilation at the PEP and PETRA energy $\sqrt{s_0} = 29$ GeV. In this section we wish to define these generalizations in detail.

More specifically, in the approximation of neglecting Higgs boson exchange, the tree level Feynman diagrams relevant to the electroweak charge asymmetry are the well-known diagrams in fig. 1 in the $SU_2 \times U_1$ model [2]. The corresponding asymmetries are by now well-known. For completeness, let us recall that, using the kinematics in fig. 1, the front-back charge asymmetry is defined as

$$a^f = \frac{\sigma_+^f - \sigma_-^f}{\sigma^f} \tag{11}$$

where we have defined σ^f as the total cross section for $e^+e^- \to f\bar{f}$ and $\sigma^f_{+(-)}$ as the cross section for $e^+e^- \to f\bar{f}$ with $\cos\theta_{c.m.} > (<)\ 0$ in fig. 1. Here, $\theta_{c.m.}$ is the center of momentum scattering angle between the e^- beam direction and the center of momentum produced fermion three momentum.

Continuing with this recapitulation, we may now note that, when the left-handed part of f, of charge Q (in units of the positron charge e), is the upper member of a weak $SU_2 \times U_1$ doublet, we have [2]

$$a^f \equiv a^Q = \frac{3s}{32Q \sin^2\theta_W \cos^2\theta_W \left(s - M_Z^2\right)} \tag{12}$$

and, when f is the lower member of such a weak isospin doublet and is of charge Q-1, we have

$$a^f \equiv a^{Q-1} = -\frac{3s}{32(Q-1) \sin^2\theta_W \cos^2\theta_W \left(s - M_Z^2\right)} \quad . \tag{13}$$

In eqs. (12) and (13), we are neglecting $(2m_f/\sqrt{s})^2$ compared to 1, and $s/(s - M_Z^2)$ compared to 1. We will continue to make these two approximations. For reference, let us note that the μ has Q = 0 in eq. (13) [it is the lower member of the respective $SU_2 \times U_1$ doublet] so that, at $\sqrt{s}$ = 29 GeV,

$$a^\mu = a^{0-1} = -.063 \quad . \tag{14}$$

Such an asymmetry is not inconsistent with the existing relatively low statistics data [3].[†] Here, we want to try to overcome this statistics limitation.

More specifically, we consider the processes

$$e^+e^- \to \mu\bar{\mu},\ \tau\bar{\tau},\ u\bar{u},\ d\bar{d},\ s\bar{s},\ c\bar{c},\ b\bar{b} \quad , \tag{15}$$

where we allow all of $\tau\bar{\tau}$, $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$ to decay and/or fragment into light hadrons, which we will take to be π's and K's in the spirit of the simple version of Feynman-Field fragmentation as represented by eq. (7).

[†] See the footnote on page 2.

We then can compute the generalized electroweak charge asymmetry as

$$a(w) \equiv \frac{\sum\limits_{\substack{i \\ \cos\theta_{c.m.}>0}} ch_i^- w(z_i) - \sum\limits_{\substack{i \\ \cos\theta_{c.m.}<0}} ch_i^- w(z_i)}{\sum\limits_i ch_i^- w(z_i)} \tag{16}$$

where ch_i^- is 1 for the i-th negatively charged light leptonic or hadronic particle produced in $e^+e^- \to X$ and z_i is the light cone momentum fraction of this i-th lepton or hadron relative to the respective parent "fundamental" (anti-) fermion in eq. (15); ch_i^- is 0 otherwise. For definiteness, let us note that

$$z_i = \frac{p_i^0 + p_i^3}{\frac{\sqrt{s_0}}{2} + \left(\frac{s_0}{4} - m_f^2\right)^{\frac{1}{2}}} \tag{17}$$

if m_f is the mass of the parent (anti-) fermion in eq. (15), where the 3-axis is taken along the direction of $(\vec{q}_2)\vec{q}_1$ in fig. 1. Here, p_i^μ is the four momentum of hadron i in the e^+e^- center of momentum frame. We emphasize that the sums in eq. (16) are over almost all events produced at the PEP energy $\sqrt{s} = \sqrt{s_0} = 29$ GeV.

To make contact with the $SU_2 \times U_1$ model, we proceed as follows. We note that if we neglect transverse momentum effects the number of negatively charged light leptons and hadrons produced within solid angle $d\Omega$ about $\theta_{c.m.}$ and with z in the interval [z,z+dz] is simply proportional to

$$\frac{1}{\sigma_{tot}} \sum_f \left[\frac{d\sigma^f}{d\Omega} \sum_{ch^-} D_f^{ch^-}(z)dz + \frac{d\sigma^{\bar{f}}}{d\Omega} \sum_{ch^-} D_{\bar{f}}^{ch^-}(z)dz \right] \tag{18}$$

where σ_{tot} is the total cross section for eq. (15) and $D_f^{ch^-}$ and $D_{\bar{f}}^{ch^-}$ are well-known decay functions [4] for the quarks and antiquarks when ch^- is a light hadron and f = u, d, s, c or b. Here we generalize these decay functions to $ch^- \epsilon$ {all light leptons and light hadrons}. Thus, the numerator in eq. (16) is also the same as

$$\int_{\cos\theta_{c.m.}>0} d\Omega \int_0^1 \sum_f \left\{ \frac{1}{\sigma_{tot}} \frac{d\sigma^f}{d\Omega} \sum_{ch^-} D_f^{ch^-}(z)w(z) + \frac{1}{\sigma_{tot}} \frac{d\sigma^{\bar{f}}}{d\Omega} \sum_{ch^-} D_{\bar{f}}^{ch^-}(z)w(z) \right\} dz$$

$$- \int_{\cos\theta_{c.m.}<0} d\Omega \int_0^1 \sum_f \left\{ \frac{1}{\sigma_{tot}} \frac{d\sigma^f}{d\Omega} \sum_{ch^-} D_f^{ch^-}(z)w(z) + \frac{1}{\sigma_{tot}} \frac{d\sigma^{\bar{f}}}{d\Omega} \sum_{ch^-} D_{\bar{f}}^{ch^-}(z)w(z) \right\} dz$$

$$= \sum_f \sum_{ch^-} \int_0^1 dz \left(a^f D_f^{ch^-}(z)w(z) + a^{\bar{f}} D_{\bar{f}}^{ch^-}(z)w(z) \right) \frac{\sigma^f}{\sigma_{tot}}$$

$$= \sum_f \sum_{ch^-} \int_0^1 dz\, a^f \left(D_f^{ch^-}(z) - D_{\bar{f}}^{ch^-}(z) \right) w(z) \frac{\sigma^f}{\sigma_{tot}} \tag{19}$$

where we have used the identity

$$a^f = -a^{\bar{f}} \quad . \tag{20}$$

(Here the lower limit 0 in eq. (19) is only formal--we will ultimately impose an appropriate cut, z_{cut}, on all integrals over the various fragmentation functions which we consider. In the interim we will continue to use the formal lower limit 0 on these integrals--it is a mnemonic.) Evidently, the denominator of eq. (16) is the same as

$$\sum_f \sum_{ch^-} \int_0^1 dz \left(D_f^{ch^-}(z) + D_{\bar{f}}^{ch^-}(z) \right) w(z) \; \frac{\sigma^f}{\sigma_{tot}} \quad . \tag{21}$$

We have used the identity

$$\sigma^f = \sigma^{\bar{f}} \quad . \tag{22}$$

Hence, our generalized asymmetry is the same as

$$a(w) = \frac{\sum_f \sum_{ch^-} \int_0^1 dz \left(D_f^{ch^-}(z) - D_{\bar{f}}^{ch^-}(z) \right) w(z) \; a^f \frac{\sigma^f}{\sigma_{tot}}}{\sum_f \sum_{ch^-} \int_0^1 dz \left(D_f^{ch^-}(z) + D_{\bar{f}}^{ch^-}(z) \right) w(z) \; \frac{\sigma^f}{\sigma_{tot}}} \quad . \tag{23}$$

This expression (23) is the basic quantity which we wish to study.

We can simplify the result in eq. (23) somewhat further if we recall that, within our approximations,

$$\sigma^f = \frac{4\pi Q_f^2 \alpha^2}{3s} \tag{24}$$

where $Q_f e$ is the electric charge of f and $\alpha \equiv e^2/(4\pi)$ is the fine structure constant. Thus, we may also represent a(w) as

$$a(w) = \frac{\sum_f \sum_{ch^-} \int_0^1 dz \left(D_f^{ch^-}(z) - D_{\bar{f}}^{ch^-}(z) \right) w(z) \; a^f \, Q_f^2}{\sum_f \sum_{ch^-} \int_0^1 dz \left(D_f^{ch^-}(z) + D_{\bar{f}}^{ch^-}(z) \right) w(z) \; Q_f^2} \quad . \tag{25}$$

We will now analyze this expression (25) in some detail. This is done in the next section.

3. Computation of the Generalized $SU_2 \times U_1$ Charge Asymmetry

In this section we wish to compute the asymmetry functional a(w) defined in eq. (25). To compute eq. (25), we must then compute the various $D_f^{ch^-}$, $D_{\bar{f}}^{ch^-}$ in it in the simple view of the model of ref. 4 as typified by eq. (7). We consider the various choices for f in turn.

For the case $f = \mu$, there is no fragmentation. We write

$$D_\mu^\mu(z) = \delta(z-1) \quad , \qquad D_\mu^{\bar{\mu}}(z) \equiv 0 \quad . \tag{26}$$

Thus, we obtain the contribution to the numerator of eq. (25) of

$$\mathscr{N}^\mu \equiv a^\mu\, w(1)\, (-1)^2 = a^\mu\, w(1) = a^\mu \tag{27}$$

and the contribution

$$\mathscr{D}^\mu \equiv w(1)\, (-1)^2 = w(1) = 1 \tag{28}$$

to the denominator of (25). This completes the discussion of $f = \mu$.

For the case $f = \tau$, we assume the decays shown in fig. 2. From ref. 9, we have the following branching ratios B:

$$B(\tau \to \nu_\tau + e + \bar{\nu}_e) + B(\tau \to \nu_\tau + \mu + \bar{\nu}_\mu) \doteq .35 \tag{29}$$

$$B(\tau \to \nu_\tau + \pi^-) = .107 \tag{30}$$

$$B(\tau \to \nu_\tau + \rho^-) = .223 \tag{31}$$

$$B(\tau \to \nu_\tau + \text{"hadrons"}) = .320 \quad . \tag{32}$$

In eq. (32) by "hadrons" we intend all hadronic final states except π^- and ρ^-. What we need are the $D_\tau^{ch^-}(z)$ and $D_{\bar\tau}^{ch^-}(z)$ corresponding to eqs. (29)-(32). We consider each set of decays in turn.

For the purely leptonic decays involved in eq. (29), one has, from fig. 2, the amplitude, neglecting lepton masses compared to M_W,

$$\mathcal{M} = \frac{-i}{M_W^2}\left(\frac{g}{2\sqrt{2}}\right)^2 \bar{u}_{\nu_\tau}(p_1)\, \gamma^\mu(1-\gamma_5)\, u_\tau(k)\, \bar{u}_\ell(p_2)\, \gamma_\mu(1-\gamma_5)\, v_{\nu_\ell}(p_3)$$
$$\times \sqrt{\frac{m_\tau}{k^0}}\sqrt{\frac{m_{\nu_\tau}}{p_1^0}}\sqrt{\frac{m_\ell}{p_2^0}}\sqrt{\frac{m_{\nu_\ell}}{p_3^0}} \tag{33}$$

where g is the weak SU_2 coupling constant and $\ell = e, \mu$. The kinematics is summarized in fig. 2 and we use the notation of ref. 10 for the Dirac matrices and spinors. As usual, m_A is the rest mass of A, $A = W, \tau, \nu_\tau, \ell, \nu_\ell$. The function $D_\tau^\ell(z)$ is then readily determined as follows. The differential distribution corresponding to eq. (33) is, in the τ rest frame,

$$d^9\Gamma = (2\pi)^4\, \delta^4(k-p_1-p_2-p_3)\, |\mathcal{M}|^2\, \frac{d^3p_1 d^3p_2 d^3p_3}{(2\pi)^9} \quad . \tag{34}$$

On using the standard methods[10] for evaluating $|\mathcal{M}|^2$ and integrating over d^3p_1 we have, neglecting m_ℓ/m_τ compared to 1,

$$d^6\Gamma \propto \left(\frac{1}{2}\, m_\tau^2 - m_\tau p_3^0\right) \frac{d^3p_2 d^3p_3}{p_1^0\, p_2^0}\, \delta\left(m_\tau - p_3^0 - p_2^0 - p_1^0\right) \quad . \tag{35}$$

The result (35) may now be used to determine the distribution in dz for the lepton ℓ in fig. 2, which has four momentum p_2, $\ell = e, \mu$.

More precisely, the integrations over d^3p_3 and $d^2p_2^\perp$ (here $\vec{p}^\perp$ is the component of momentum transverse to $\vec{q}_1$ in fig. 1) allow us to write

$$d\Gamma \propto \left[\frac{m_\tau^2 p_2^+ - p_2^{+3}}{4} - \frac{p_2^+}{9m_\tau} \left(m_\tau^3 - p_2^{+3} \right) \right] \frac{dp_2^+}{p_2^+}$$

$$= m_\tau^3 \left[\frac{z - z^3}{4} - \frac{z}{9} (1 - z^3) \right] \frac{dz}{z} \quad , \tag{36}$$

where p_2^+ is the light cone momentum of the lepton in fig. 2 .

Thus, we conclude

$$D_\tau^\mu(z) + D_\tau^e(z) = c \left(\frac{1 - z^2}{4} - \frac{1 - z^3}{9} \right) \tag{37}$$

for some constant c.

On requiring

$$\int_0^1 dz \left[D_\tau^\mu(z) + D_\tau^e(z) \right] = .35 \quad , \tag{38}$$

we obtain

$$c = .35\,(12) = 4.2 \quad . \tag{39}$$

This allows us to write, finally,

$$D_\tau^{\mu+e}(z) \equiv D_\tau^\mu(z) + D_\tau^e(z) = 4.2\left(\frac{1-z^2}{4} - \frac{1-z^3}{9}\right) \equiv .35\ D_2(z) \quad , \tag{40}$$

where we have introduced the normalized form of $D_\tau^{\mu+e}(z)$, $D_2(z)$, for future reference.

For the $\bar{\tau}$, we have from fig. 2 immediately

$$D_{\bar{\tau}}^\mu(z) = D_{\bar{\tau}}^e(z) = 0 \quad . \tag{41}$$

Evidently, then, the decays in eq. (29) make the contribution

$$\begin{aligned} \mathcal{N}_{e+\mu}^\tau &= \int_0^1 dz \left[D_\tau^e(z) + D_\tau^\mu(z) - D_{\bar{\tau}}^e(z) - D_{\bar{\tau}}^\mu(z) \right] w(z)\ (-1)^2\ a^\tau \\ &= 4.2\ a^\tau \left[\frac{5}{36(n+1)} - \frac{1}{4(n+3)} + \frac{1}{9(n+4)} \right] \quad , \end{aligned} \tag{42}$$

for $w(z) = z^n$, to the numerator of $a(w)$ and the contribution

$$\begin{aligned} \mathcal{D}_{e+\mu}^\tau &= \int_0^1 dz \left[D_\tau^e(z) + D_\tau^\mu(z) + D_{\bar{\tau}}^e(z) + D_{\bar{\tau}}^\mu(z) \right] w(z)\ (-1)^2 \\ &= 4.2 \left[\frac{5}{36(n+1)} - \frac{1}{4(n+3)} + \frac{1}{9(n+4)} \right] \end{aligned} \tag{43}$$

to the denominator of $a(w)$. This completes the discussion of the decays in eq. (29).

Turning next to the decay $\tau \to \nu_\tau + \pi^-$, one evidently has the diagram illustrated in fig. 3a. The corresponding distribution is well known:

$$dz\, D^{\pi^-}_{\tau,\mathrm{direct}}(z) \propto \frac{dz}{z} \quad , \qquad \frac{m_\pi^2}{m_\tau^2} \leq z \lesssim 1 \quad , \tag{44}$$

where we distinguish this $\nu_\tau-\pi$ contribution $D^{\pi^-}_{\tau}(z)$ from that arising from the fragmentation of $\bar{u}$ and d in fig. 2 with the subscript direct. We normalize eq. (44) so that

$$\int_{m_\pi^2/m_\tau^2}^{1} dz\, D^{\pi^-}_{\tau,\mathrm{direct}}(z) = .107 \tag{45}$$

in accordance with eq. (30). Then,

$$D^{\pi^-}_{\tau,\mathrm{direct}}(z) = \frac{.107}{2\,\ell n\,(m_\tau/m_\pi)}\,\frac{1}{z} \quad . \tag{46}$$

Clearly, from fig. 3b

$$D^{\pi^-}_{\bar{\tau},\mathrm{direct}}(z) = 0 \quad . \tag{47}$$

We therefore have from eq. (30) the contribution

$$\begin{aligned} \mathcal{N}^{\tau}_{\pi^-,\mathrm{direct}} &= \frac{a^\tau(.107)}{2\,\ell n\,(m_\tau/m_\pi)} \int_{m_\pi^2/m_\tau^2}^{1} \frac{dz}{z}\, w(z) \\ &= \frac{a^\tau(.107)}{2\,\ell n\,(m_\tau/m_\pi)} \left(\frac{1}{n} - \frac{m_\pi^{2n}}{n m_\tau^{2n}} \right) \\ &\doteq \frac{a^\tau(.107)}{2n\,\ell n\,(m_\tau/m_\pi)} \end{aligned} \tag{48}$$

to the numerator of a(w) for $w = z^n$. The corresponding contribution to the denominator of a(w) is

$$\mathscr{D}^{\tau}_{\pi^-,\mathrm{direct}} \doteq \frac{.107}{2\,\ln\,(m_\tau/m_\pi)}\;\frac{1}{n} \quad . \tag{49}$$

This completes the discussion of the decay in eq. (30).

Turning next to the decay $\tau \to \nu_\tau + \rho^-$, we allow the ρ^- to decay to $\pi^-\pi^o$. Thus, the situation is illustrated in fig. 4. For the step $\tau \to \nu_\tau + \rho^-$, we have

$$D^{\rho^-}_{\tau,\mathrm{direct}}(z) = \frac{.223}{2\,\ln\,(m_\tau/m_\rho)}\;\frac{1}{z} \tag{50}$$

in complete analogy with eq. (46). For the step $\rho^- \to \pi^-\pi^o$ we have

$$D^{\pi^-}_{\rho^-}(z) = \frac{1}{2\,\ln\,(m_\rho/m_\pi)}\;\frac{1}{z} \quad . \tag{51}$$

Following refs. 4 and 11, we have finally, for the distribution $D^{\pi^-}_{\tau;\rho^-}$ of π^- due to eq. (31),

$$\begin{aligned} D^{\pi^-}_{\tau;\rho^-}(z) &= \int_z^1 \frac{d\eta}{\eta}\; D^{\pi^-}_{\rho^-}\left(\frac{z}{\eta}\right)\; D^{\rho^-}_{\tau,\mathrm{direct}}(\eta) \\ &= \frac{.223}{4\,\ln\,(m_\tau/m_\rho)\,\ln\,(m_\rho/m_\pi)} \int_z^1 \frac{d\eta}{\eta}\;\frac{1}{\eta}\;\frac{\eta}{z} \\ &= \frac{.223}{4\,\ln\,(m_\tau/m_\rho)\,\ln\,(m_\rho/m_\pi)} \begin{cases} -\dfrac{\ln z}{z} \;, & z > \dfrac{m_\rho^2}{m_\tau^2} \\[2ex] -\dfrac{\ln\,(m_\rho^2/m_\tau^2)}{z} \;, & \dfrac{m_\rho^2}{m_\tau^2} \geq z \geq \dfrac{m_\pi^2}{m_\tau^2} \end{cases} \quad . \end{aligned} \tag{52}$$

Clearly,

$$D^{\pi^-}_{\bar{\tau};\rho^+}(z) = 0 \quad , \tag{53}$$

where $D^{\pi^-}_{\bar{\tau};\rho^+}$ is the distribution of π^- due to the decay sequence $\bar{\tau} \to \bar{\nu}_\tau + \rho^+, \rho^+ \to \pi^+ + \pi^o$; that $D^{\pi^-}_{\bar{\tau};\rho^+}$ is trivial is a tautology.

Thus, we have that eq. (31) contributes, for $w(z) = z^n$,

$$\mathcal{N}^\tau_{\rho^-;\pi^-} = \frac{.223 \quad a^\tau}{4 \,\ell n\,(m_\tau/m_\rho)\,\ell n\,(m_\rho/m_\pi)} \left[\frac{1}{n^2} \left(1 - \frac{m_\rho^{2n}}{m_\tau^{2n}}\right) + \frac{2\,\ell n\,(m_\rho/m_\tau)}{n} \left(\frac{m_\pi^2}{m_\tau^2}\right)^n \right] \tag{54}$$

to the numerator of $a(w)$ and

$$\mathcal{D}^\tau_{\rho^-;\pi^-} = \frac{\mathcal{N}^\tau_{\rho^-;\pi^-}}{a^\tau} \tag{55}$$

to the denominator of $a(w)$. This completes the discussion of the decay in eq. (31). We turn next to $\tau \to \nu_\tau$ + "hadrons".

For $\tau \to \nu_\tau$ + "hadrons", we compute the diagram shown in fig. 2 for $\tau \to \nu_\tau + \bar{u} + d$ (we neglect the Cabibbo angle) and use the Field-Feynman model for subsequent hadronization of $\bar{u}$ and d. Now, from fig. 2, we can say immediately that, neglecting m_u/m_τ ,m_d/m_τ and m_ℓ/m_τ compared to 1, the z-distribution of d in the decay $\tau \to \nu_\tau + \bar{u} + d$ is the same as the z-distribution of ℓ in the decay $\tau \to \nu_\tau + \bar{\nu}_\ell + \ell$, $\ell = e,\mu$. Thus, we conclude that

$$\begin{aligned} D^d_\tau(z) &= \frac{.320}{.35} \; D^{e+\mu}_\tau(z) \\ &= 3.84 \left(\frac{1-z^2}{4} - \frac{1-z^3}{9} \right) \quad . \end{aligned} \tag{56}$$

Indeed, from fig. 2, it is evident that the matrix element for $\tau \to \nu_\tau + \bar{u} + d$ is obtained from eq. (33) by the replacements $(\bar{\nu}_\ell,\ell) \to (\bar{u},d)$ respectively everywhere in eq. (33). On making these replacements and using the well-known methods [10], we find, by reasoning entirely analogous to that underlying eqs. (34)-(40),

$$D_{\tau}^{\bar{u}}(z) = 1.92 \left[1 - z^2 - \frac{2}{3}\left(1 - z^3\right)\right]$$
$$\equiv .320 \quad D_3(z) \qquad , \tag{57}$$

where we have introduced the normalized form, $D_3(z)$, of $D_{\tau}^{\bar{u}}(z)$ for future reference.

To relate eq. (56) and eq. (57) to a(w), observe that the mean number of negatively charged particles, that is to say the mean number of π^- and K^- particles, arising from eqs. (56) and (57) is

$$D_{\tau;d+\bar{u}}^{\pi^-+K^-}(z) = \int_z^1 \frac{dy}{y}\left[D_d^{\pi^-+K^-}\left(\frac{z}{y}\right)D_{\tau}^{d}(y) + D_{\bar{u}}^{\pi^-+K^-}\left(\frac{z}{y}\right)D_{\tau}^{\bar{u}}(y)\right] \tag{58}$$

where $D_q^{\pi^-+K^-}(z)$ is the sum of Field-Feynman fragmentation functions for q into π^- and K^- respectively. Note also that the distribution of u in $\bar{\tau}$ can be inferred from eq. (33) by observing that the matrix element for $\bar{\tau} \to \bar{\nu}_{\tau} + u + \bar{d}$ may be obtained from eq. (33) by the replacements $(u_{\tau}, u_{\nu_{\tau}}) \to (v_{\bar{\nu}_{\tau}}, v_{\bar{\tau}})$, $(\bar{v}_{\ell}, \ell) \to (\bar{d}, u)$ in eq. (33). On doing this, we find

$$D_{\bar{\tau}}^{\bar{d}}(z) = D_{\tau}^{\bar{u}}(z) \tag{59}$$

$$D_{\bar{\tau}}^{u}(z) = D_{\tau}^{d}(z) \qquad . \tag{60}$$

Thus,

$$D_{\bar{\tau};u+\bar{d}}^{\pi^-+K^-}(z) = \int_z^1 \frac{dy}{y}\left[D_u^{\pi^-+K^-}\left(\frac{z}{y}\right)D_{\bar{\tau}}^{u}(y) + D_{\bar{d}}^{\pi^-+K^-}\left(\frac{z}{y}\right)D_{\bar{\tau}}^{\bar{d}}(y)\right]$$
$$= \int_z^1 \frac{dy}{y}\left[D_u^{\pi^-+K^-}\left(\frac{z}{y}\right)D_{\tau}^{d}(y) + D_{\bar{d}}^{\pi^-+K^-}\left(\frac{z}{y}\right)D_{\tau}^{\bar{u}}(y)\right] \qquad . \tag{61}$$

In this way, we obtain the result that the decays in eq. (32) give the contribution

$$\mathcal{N}^{\tau}_{\text{"hadrons"}} = a^{\tau} \int_0^1 dz \int_z^1 \frac{dy}{y} \left[\left(D_d^{\pi^-+K^-}\left(\frac{z}{y}\right) - D_u^{\pi^-+K^-}\left(\frac{z}{y}\right)\right) D_{\tau}^{d}(y) + \left(D_{\bar{u}}^{\pi^-+K^-}\left(\frac{z}{y}\right) - D_{\bar{d}}^{\pi^-+K^-}\left(\frac{z}{y}\right)\right) D_{\tau}^{\bar{u}}(y)\right] w(z) \tag{62}$$

to the numerator of a(w) and the contribution

$$\mathcal{D}^{\tau}_{\text{"hadrons"}} = \int_0^1 dz \int_z^1 \frac{dy}{y} \left[\left(D_d^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_u^{\pi^-+K^-}\left(\frac{z}{y}\right)\right) D_{\tau}^{d}(y) + \left(D_{\bar{u}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_{\bar{d}}^{\pi^-+K^-}\left(\frac{z}{y}\right)\right) D_{\tau}^{\bar{u}}(y)\right] w(z) \tag{63}$$

to the denominator of a(w).

To complete the discussion of $\tau \to \nu_{\tau}$ + "hadrons", we must specify the Field-Feynman functions $D_q^{\pi^-+K^-}$. We have,* from ref. 4,

$$D_d^{\pi^-+K^-}(z) - D_u^{\pi^-+K^-}(z) = .4\ f(1-z) \tag{64}$$

$$D_{\bar{u}}^{\pi^-+K^-}(z) - D_{\bar{d}}^{\pi^-+K^-}(z) = .6\ f(1-z) \tag{65}$$

$$D_d^{\pi^-+K^-}(z) + D_u^{\pi^-+K^-}(z) = .48\,\bar{F}(z) + .4\ f(1-z) = .48\,F(z) - .08\ f(1-z) \tag{66}$$

$$D_{\bar{u}}^{\pi^-+K^-}(z) + D_{\bar{d}}^{\pi^-+K^-}(z) = .48\,\bar{F}(z) + .6\ f(1-z) = .48\,F(z) + .12\ f(1-z)\ , \tag{67}$$

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*Strictly speaking, the function D_q defined in ref. 4 is already the sum of D_q and $D_{\bar{q}}$ as defined here. Since we are only making an estimate of a(w) to ± 25 %, in general, we will suppress this in what follows.

where, hère [4],

$$\bar{F}(z) \equiv F(z) - f(1 - z) \quad , \tag{68}$$

with f(1 - z) given by eq. (7) and with F(z) given by

$$\begin{aligned} F(z) &= \frac{3}{[3-2(.88)]z} + \frac{3(.88)\,z}{2(.88)-1} + \frac{2(.88)[2(.88)^2-3(.88)-2]z^{2-2(.88)}}{[3-2(.88)][2(.88)-1]} \\ &\cong \frac{2.42}{z} + 3.47\,z - 5.77\,z^{.24} \quad . \end{aligned} \tag{69}$$

Thus,

$$\mathcal{N}^{\tau}_{\text{"hadrons"}} = a^{\tau} \int_0^1 dz \int_z^1 \frac{dy}{y} \left[.4\, D^{d}_{\tau}(y) + .6\, D^{\bar{u}}_{\tau}(y) \right] f\left(1 - \frac{z}{y}\right) \; w(z) \tag{70}$$

and

$$\begin{aligned} \mathcal{D}^{\tau}_{\text{"hadrons"}} = \int_0^1 dz \int_z^1 \frac{dy}{y} \Bigg\{ & .48 \left[D^{d}_{\tau}(y) + D^{\bar{u}}_{\tau}(y) \right] \; F\left(\frac{z}{y}\right) \\ & + \left[-\,.08\, D^{d}_{\tau}(y) + .12\, D^{\bar{u}}_{\tau}(y) \right] \; f\left(1 - \frac{z}{y}\right) \Bigg\} \; w(z) \quad . \end{aligned} \tag{71}$$

These expressions [eqs. (70) and (71)] will be evaluated presently. This completes the discussion of the contributions of $\tau\bar{\tau}$ production to a(w).

We consider next $u\bar{u}$ production. To the numerator of a(w) we receive the contribution

$$\mathcal{N}^u = \int_0^1 dz \left(D_u^{\pi^-+K^-}(z) - D_{\bar{u}}^{\pi^-+K^-}(z) \right) w(z)\; 3\left(\frac{2}{3}\right)^2 a^u$$

$$= \frac{4}{3}\; a^u \int_0^1 dz \left(D_u^{\pi^-+K^-}(z) - D_{\bar{u}}^{\pi^-+K^-}(z) \right) w(z) \quad . \tag{72}$$

From ref. 4 we have

$$D_u^{\pi^-+K^-}(z) - D_{\bar{u}}^{\pi^-+K^-}(z) = -.6\; f(1-z) \quad . \tag{73}$$

Thus,

$$\mathcal{N}^u = \frac{4}{3}\; a^u \int_0^1 dz\; (-.6)\; f(1-z)\; w(z)$$

$$= -.8\; a^u \left[\frac{2.76}{n+1} - \frac{5.28}{n+2} + \frac{2.64}{n+3} \right] \tag{74}$$

for $w(z) = z^n$.

Similarly, $u\bar{u}$ production makes the contribution

$$\mathcal{D}^u = \int_0^1 dz \left(D_u^{\pi^-+K^-}(z) + D_{\bar{u}}^{\pi^-+K^-}(z) \right) w(z)\; 3\left(\frac{2}{3}\right)^2$$

$$= \frac{4}{3} \int_0^1 dz \left(.48\; \bar{F}(z) + .6\; f(1-z) \right) w(z)$$

$$= \frac{4}{3} \int_0^1 dz \left(.48\; F(z) + .12\; f(1-z) \right) w(z) \tag{75}$$

$$= \frac{4}{3} \left[.48 \left(\frac{2.42}{n} + \frac{3.47}{n+2} - \frac{5.77}{n+1.24} \right) + .12 \left(\frac{2.76}{n+1} - \frac{5.28}{n+2} + \frac{2.64}{n+3} \right) \right]$$

$$= \frac{1.55}{n} + \frac{.442}{n+1} + \frac{1.38}{n+2} + \frac{.422}{n+3} - \frac{3.69}{n+1.24}$$

to the denominator of a(w). We should remark that in eqs. (74) and (75) we have included the factor of 3 for the sum over color. This completes the discussion of $u\bar{u}$ production.

Turning next to the $d\bar{d}$ and $s\bar{s}$ production, we proceed in complete analogy with our discussion of $u\bar{u}$ production. In this way, we find, using the results of ref. 4, the contributions

$$\mathcal{N}^d = \frac{1}{3}(.4)\, a^d \int_0^1 dz\ f(1-z)\ w(z) = .133\ a^d \left[\frac{2.76}{n+1} - \frac{5.28}{n+2} + \frac{2.64}{n+3}\right] \quad (76)$$

and

$$\mathcal{N}^s = \frac{1}{3}(.4)\, a^s \int_0^1 dz\ f(1-z)\ w(z) = .133\ a^s \left[\frac{2.76}{n+1} - \frac{5.28}{n+2} + \frac{2.64}{n+3}\right] \quad (77)$$

to the numerator of a(w) and the contributions

$$\mathcal{D}^d = \frac{1}{3} \int_0^1 dz \left[.48\ \bar{F}(z) + .4\ f(1-z)\right] w(z)$$

$$= \frac{1}{3}\left[.48\left(\frac{2.42}{n} + \frac{3.47}{n+2} - \frac{5.77}{n+1.24}\right) - .08\left(\frac{2.76}{n+1} - \frac{5.28}{n+2} + \frac{2.64}{n+3}\right)\right]$$

$$\cong \frac{.387}{n} - \frac{.0736}{n+1} + \frac{.697}{n+2} - \frac{.0704}{n+3} - \frac{.924}{n+1.24} \quad (78)$$

and

$$\mathcal{D}^s = \frac{1}{3}\left[.48 \int_0^1 dz\ w(z)\ F(z) - .08 \int_0^1 dz\ w(z)\ f(1-z)\right] = \mathcal{D}^d \quad (79)$$

to the denominator of a(w). This completes the discussion of $d\bar{d}$ and $s\bar{s}$ production.

We consider next $c\bar{c}$ production. At this point, we have a decision to make: namely, "How will we allow c and $\bar{c}$ to become hadrons?" Our view is that of ref. 5. In particular, rather than to attempt to model the change we need to make to f in eq. (7) for heavy hadron fragmentation [12], what we will do is to take a spectator-view of the decay of heavy hadrons so that (we continue to neglect the Cabibbo angle — we will always do this)

$$\begin{aligned} c &\to s + u + \bar{d} \\ &\to s + \bar{e} + \nu_e \\ &\to s + \bar{\mu} + \nu_\mu \end{aligned} \tag{80}$$

as is illustrated in fig. 5. We then use eqs. (7), (68),(69) and the results in ref. 4 to determine the final light hadron distributions. Hence, from our discussion of the $\tau\bar{\tau}$ case, we can write

$$\begin{aligned} \mathscr{N}^c = 3\left(\frac{2}{3}\right)^2 a^c \int_0^1 dz\, w(z) \Bigg\{ \int_z^1 \Bigg[& D_c^s(y)\, D_s^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_c^u(y)\, D_u^{\pi^-+K^-}\left(\frac{z}{y}\right) \\ & + D_c^{\bar{d}}(y)\, D_{\bar{d}}^{\pi^-+K^-}\left(\frac{z}{y}\right) - D_{\bar{c}}^{\bar{s}}(y)\, D_{\bar{s}}^{\pi^-+K^-}\left(\frac{z}{y}\right) - D_{\bar{c}}^{\bar{u}}(y)\, D_{\bar{u}}^{\pi^-+K^-}\left(\frac{z}{y}\right) \\ & - D_{\bar{c}}^{d}(y)\, D_d^{\pi^-+K^-}\left(\frac{z}{y}\right) \Bigg] \frac{dy}{y} - D_{\bar{c}}^{e}(z) - D_{\bar{c}}^{\mu}(z) \Bigg\} \end{aligned} \tag{81}$$

and

$$\begin{aligned} \mathscr{D}^c = 3\left(\frac{2}{3}\right)^2 \int_0^1 dz\, w(z) \Bigg\{ \int_z^1 \frac{dy}{y} \Bigg[& D_c^s(y)\, D_s^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_c^u(y) D_u^{\pi^-+K^-}\left(\frac{z}{y}\right) \\ & + D_c^{\bar{d}}(y)\, D_{\bar{d}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_{\bar{c}}^{\bar{s}}(y)\, D_{\bar{s}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_{\bar{c}}^{\bar{u}}(y)\, D_{\bar{u}}^{\pi^-+K^-}\left(\frac{z}{y}\right) \\ & + D_{\bar{c}}^{d}(y)\, D_d^{\pi^-+K^-}\left(\frac{z}{y}\right) \Bigg] + D_{\bar{c}}^{e}(z) + D_{\bar{c}}^{\mu}(z) \Bigg\} \end{aligned} \tag{82}$$

where $\mathcal{N}'^{c}$ is the contribution of $c\bar{c}$ production to the numerator of a(w) and $\mathcal{D}^{c}$ is the contribution of $c\bar{c}$ production to the denominator of a(w). Thus, we need to determine $D_q^f(z)$ for $q = c$ and $f = s, u, \bar{d}$ and for $q = \bar{c}$ and $f = \bar{s}, \bar{u}, d, e$ and μ. These z-distributions can be obtained from analyses completely analogous to that associated with eq. (33) and our results for the $\tau\bar{\tau}$ case.

More specifically, effecting the analoga of our $\tau\bar{\tau}$ analysis for eq. (33) for $c \to s + u + \bar{d}$, $\bar{c} \to \bar{s} + \bar{u} + d$ and $\bar{c} \to \bar{s} + \ell + \bar{\nu}_\ell$, we find that the normalized distributions of u in c, d in $\bar{c}$ and ℓ in $\bar{c}$ are equal to $D_2(z)$, $\ell = e,\mu$, and the normalized distributions of $\bar{d}$ in c and $\bar{u}$ in $\bar{c}$ are equal to $D_3(z)$. Since the branching fractions for c-decay are [13], in the spectator model,

$$B\left(\bar{c} \to \bar{s} + e + \bar{\nu}_e\right) + B\left(\bar{c} \to \bar{s} + \mu + \bar{\nu}_\mu\right) \doteq .164 \qquad (83)$$

and

$$B\left(c \to s + \text{"hadrons"}\right) \doteq .836 \quad , \qquad (84)$$

we have

$$D_{\bar{c}}^{e+\mu}(z) = .164\, D_2(z) = 1.97 \left(\frac{1 - z^2}{4} - \frac{1 - z^3}{9}\right) \qquad (85)$$

$$D_c^u(z) = D_{\bar{c}}^d(z) = .836\, D_2(z) = 10.0 \left(\frac{1 - z^2}{4} - \frac{1 - z^3}{9}\right) , \qquad (86)$$

and

$$D_c^{\bar{d}}(z) = D_{\bar{c}}^{\bar{u}}(z) = .836\, D_3(z) = 5.02 \left[1 - z^2 - \frac{2}{3}\left(1 - z^3\right)\right]. \qquad (87)$$

Thus, for the evaluation of eqs. (81) and (82) there only remain the computations of $D_c^s(z)$ and $D_{\bar{c}}^{\bar{s}}(z)$. We now turn to these computations.

More precisely, on repeating the analoga of the steps from eq. (35) to eq. (38) for the four-momentum p_1 in the matrix elements for $c \to s + \bar{f}' + f$ and $\bar{c} \to \bar{s} + \bar{f} + f'$, we find

$$D_{\bar{c}}^{\bar{s}}(z) = D_c^s(z) = D_2(z) = 12\left(\frac{1 - z^2}{4} - \frac{1 - z^3}{9}\right) = D_1(z) \quad . \tag{88}$$

The results in eqs. (85)-(88) taken together with the results in ref. 4 for $D_q^{\pi^- + K^-}(z)$, $q = u, \bar{u}, d, \bar{d}, s, \bar{s}$, are sufficient to evaluate eqs. (81) and (82). We shall do this presently. For the moment, this completes the discussion of $c\bar{c}$ production.

We turn finally to $b\bar{b}$ production. We follow refs. 6, 12 and 14 and consider the b to decay as shown in fig. 6. We note that the τ, $\bar{\tau}$, c and $\bar{c}$ in fig. 6 must then be allowed to decay according to the diagrams in figs. 2 - 5. If one allows for corrections due to masses, one has [6, 14], from fig. 6, the approximations

$$B\left(b \to c + e + \bar{\nu}_e\right) + B\left(b \to c + \mu + \bar{\nu}_\mu\right) = \frac{2}{5.8} \doteq .345 \tag{89}$$

$$B\left(b \to c + \tau + \bar{\nu}_\tau\right) = \frac{.2}{5.8} \doteq .0345 \tag{90}$$

$$B(b \to c + d + \bar{u}) \doteq .517 \tag{91}$$

and

$$B(b \to c + s + \bar{c}) \doteq .103 \quad . \tag{92}$$

The masses which correspond to eqs. (89)-(92) are approximately

$$m_b \doteq 5.1 \text{ GeV} \tag{93}$$

$$m_c \doteq 1.8 \text{ GeV} \tag{94}$$

$$m_d = m_u \doteq .33 \text{ GeV} \tag{95}$$

$$m_s \doteq .50 \text{ GeV} \tag{96}$$

and

$$m_\tau \doteq 1.78 \text{ GeV} \quad . \tag{97}$$

These are not unreasonable mass parameters. What we see is that ~ 85 % of the b decay is to the channels where the mass corrections are small. Thus, we may use the distributions D_1, D_2 and D_3 for the final fermions and antifermions in eqs. (89)-(92) without making a large error. We will do this.

Turning now to the relationship between eqs. (89)-(92) and a(w) in eq. (25), we note that the numerator in a(w) receives the contribution, from the b-decay,

$$\mathcal{N}^b = a^b\, 3\left(-\frac{1}{3}\right)^2 \int_0^1 dz \left\{ \int_z^1 \frac{dy}{y} \left[D_b^c(y)\, D_c^{ch^-}\left(\frac{z}{y}\right) + D_b^\tau(y)\, D_\tau^{ch^-}\left(\frac{z}{y}\right) \right.\right.$$

$$+ D_b^d(y)\, D_d^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_b^{\bar{u}}(y)\, D_{\bar{u}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_b^s(y)\, D_s^{\pi^-+K^-}\left(\frac{z}{y}\right)$$

$$\left. + D_b^{\bar{c}}(y)\, D_{\bar{c}}^{ch^-}\left(\frac{z}{y}\right)\right] + D_b^{e+\mu}(z) - \int_z^1 \frac{dy}{y} \left[D_{\bar{b}}^{\bar{c}}(y)\, D_{\bar{c}}^{ch^-}\left(\frac{z}{y}\right) \right.$$

$$+ D_{\bar{b}}^{\bar{\tau}}(y)\, D_{\bar{\tau}}^{ch^-}\left(\frac{z}{y}\right) + D_{\bar{b}}^{\bar{d}}(y)\, D_{\bar{d}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_{\bar{b}}^{u}(y)\, D_u^{\pi^-+K^-}\left(\frac{z}{y}\right)$$

$$\left.\left. + D_{\bar{b}}^{\bar{s}}(y)\, D_{\bar{s}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_{\bar{b}}^{c}(y)\, D_c^{ch^-}\left(\frac{z}{y}\right)\right] - D_{\bar{b}}^{e+\mu}(z) \right\} w(z) \qquad (98)$$

and that the denominator of a(w) in eq. (25) receives the corresponding contribution

$$\mathcal{D}^b = 3\left(-\frac{1}{3}\right)^2 \int_0^1 dz \left\{ \int_z^1 \frac{dy}{y} \left[D_b^c(y)\, D_c^{ch^-}\left(\frac{z}{y}\right) + D_b^\tau(y)\, D_\tau^{ch^-}\left(\frac{z}{y}\right) \right.\right.$$

$$+ D_b^d(y)\, D_d^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_b^{\bar{u}}(y)\, D_{\bar{u}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_b^s(y)\, D_s^{\pi^-+K^-}\left(\frac{z}{y}\right)$$

$$\left. + D_b^{\bar{c}}(y)\, D_{\bar{c}}^{ch^-}\left(\frac{z}{y}\right)\right] + D_b^{e+\mu}(z) + \int_z^1 \frac{dy}{y} \left[D_{\bar{b}}^{\bar{c}}(y)\, D_{\bar{c}}^{ch^-}\left(\frac{z}{y}\right) \right.$$

$$+ D_{\bar{b}}^{\bar{\tau}}(y)\, D_{\bar{\tau}}^{ch^-}\left(\frac{z}{y}\right) + D_{\bar{b}}^{\bar{d}}(y)\, D_{\bar{d}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_{\bar{b}}^{u}(y)\, D_u^{\pi^-+K^-}\left(\frac{z}{y}\right)$$

$$\left.\left. + D_{\bar{b}}^{\bar{s}}(y)\, D_{\bar{s}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_{\bar{b}}^{c}(y)\, D_c^{ch^-}\left(\frac{z}{y}\right)\right] + D_{\bar{b}}^{e+\mu}(z) \right\} w(z) \qquad (99)$$

where $D_c^{ch^-}(z)$, $D_{\bar{c}}^{ch^-}(z)$, $D_\tau^{ch^-}(z)$ and $D_{\bar{\tau}}^{ch^-}(z)$ may be inferred from eqs. (36), (46), (52), (58), (61), (81), (82) and (85)-(88):

$$D_{\tau}^{ch^-}(z) = D_{\tau}^{\mu+e}(z) + D_{\tau,direct}^{\pi^-}(z) + D_{\tau;\rho}^{\pi^-}(z) + D_{\tau;d+\bar{u}}^{\pi^-+K^-}(z) \tag{100}$$

$$D_{\bar{\tau}}^{ch^-}(z) = D_{\bar{\tau};u+\bar{d}}^{\pi^-+K^-}(z) \tag{101}$$

$$D_{c}^{ch^-}(z) = \int_z^1 \frac{dy}{y}\left[D_c^s(y)\, D_s^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_c^u(y)\, D_u^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_c^{\bar{d}}(y)\, D_{\bar{d}}^{\pi^-+K^-}\left(\frac{z}{y}\right)\right] \tag{102}$$

$$D_{\bar{c}}^{ch^-}(z) = D_{\bar{c}}^{e+\mu}(z) + \int_z^1 \frac{dy}{y}\left[D_{\bar{c}}^{\bar{s}}(y)\, D_{\bar{s}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_{\bar{c}}^{\bar{u}}(y)\, D_{\bar{u}}^{\pi^-+K^-}\left(\frac{z}{y}\right) + D_{\bar{c}}^{d}(y)\, D_{d}^{\pi^-+K^-}\left(\frac{z}{y}\right)\right] \quad . \tag{103}$$

The distributions $D_b^{\mu+e}(y)$, $D_b^{\tau}(y)$, $D_b^{d}(y)$, $D_b^{\bar{u}}(y)$, $D_b^{s}(y)$, $D_b^{\bar{c}}(y)$ and $D_b^{c}(y)$ are easily seen from the analogy between figs. 2, 5 and 6 and from the results in eqs. (40), (57) and (88) and the branching ratios in eqs. (89)-(92) to be

$$\begin{aligned}
D_b^{\mu+e}(y) &= .345\, D_2(y) \quad , & D_b^{\bar{u}}(y) &= .517\, D_3(y) \quad , \\
D_b^{c}(y) &= D_1(y) = D_2(y) \quad , & D_b^{s}(y) &= .103\, D_2(y) \quad , \\
D_b^{\tau}(y) &= .0345\, D_2(y) \quad , & D_b^{\bar{c}}(y) &= .103\, D_3(y) \quad ,
\end{aligned} \tag{104}$$

and

$$D_b^{d}(y) = .517\, D_2(y) \quad .$$

The corresponding results for $D_{\bar{b}}^{\mu+e}(y)$, $D_{\bar{b}}^{\bar{c}}(y)$, $D_{\bar{b}}^{\bar{\tau}}(y)$, $D_{\bar{b}}^{\bar{d}}(y)$, $D_{\bar{b}}^{u}(y)$, $D_{\bar{b}}^{\bar{s}}(y)$ and $D_{\bar{b}}^{c}(y)$ are easily seen to be

$$D_{\bar{b}}^{\mu+e}(y) = 0 \quad , \qquad D_{\bar{b}}^{u}(y) = .517\ D_2(y) \quad ,$$

$$D_{\bar{b}}^{\bar{c}}(y) = D_b^{c}(y) = D_2(y) \quad , \qquad D_{\bar{b}}^{\bar{s}}(y) = .103\ D_3(y) \quad ,$$

$$D_{\bar{b}}^{\bar{\tau}}(y) = .0345\ D_3(y) \quad , \qquad D_{\bar{b}}^{c}(y) = .103\ D_2(y) \quad , \tag{105}$$

and

$$D_{\bar{b}}^{\bar{d}}(y) = .517\ D_3(y) \quad .$$

Thus, the computations of eqs. (98) and (99) are now completely specified in our simple fragmentation model. We now turn to this calculation and the calculations of $\mathscr{N}^{\tau}_{\text{"hadrons"}}$, $\mathscr{D}^{\tau}_{\text{"hadrons"}}$, $\mathscr{N}^{c}$ and $\mathscr{D}^{c}$ in order that we may compute the quantity of interest*

$$a(w) = \frac{\sum\limits_{f=\mu,\tau,u,d,s,c,b} \mathscr{N}^{f}}{\sum\limits_{f=\mu,\tau,u,d,s,c,b} \mathscr{D}^{f}} \quad ; \tag{106}$$

here, we use the symbolic notation $\mathscr{N}^{f}$ ($\mathscr{D}^{f}$) to denote all of the respective contributions to the numerator (denominator) in eq. (25) which are generated by the production of $f\bar{f}$ in $e^+e^- \to f\bar{f}$.

In computing the expression for a(w) in eq. (106), we have used the SLAC computer. All integrals which were not given explicitly in our discussion above were evaluated using standard Monte Carlo techniques; specifically, using the Neyman construction [15] with 500 points per multidimensional integral. In these integrals, as we promised in sect. 2, a cut was placed on the lowest value of z, due to the limit

$$z_{cut} = \frac{m_K}{\sqrt{s_0/4} + \sqrt{s_0/4 - m_b^2}} \doteq .0175 \tag{107}$$

---

* Note that, according to the footnote on page 19, all $\mathscr{N}^{f}$ and $\mathscr{D}^{f}$ contributions in (106) which involve f(1-z) must be reduced by a factor of 2 relative to the respective quantities $\mathscr{N}^{\mu}$ and $\mathscr{D}^{\mu}$.

for the decay $b \to K + \ldots$, where z_{cut} is the lowest value of z allowed for K in these decays if we require $p^3(K) > 0$; here, $p^3(K)$ is the component of the K 3-momentum along the direction of the 3-momentum of the parent b. In this way, we find the results in table 1. These are the basic results of this communication.

What we can see very clearly is that, in our model calculation, this quantity a(w) is measurable for an appropriate choice of w. Thus, it is desirable to discuss the level of completeness of our calculation. We turn to this in the next section.

4. Possible Corrections to a(w)

The most obvious physical effect which has been neglected in our calculation of a(w) is the effect of QCD corrections. As we stated in the introduction, it has been shown by Zerwas et al. in ref. 2 that the QCD corrections to the individual asymmetries a^f are in fact small, $\lesssim 10\%$. However, one can ask about the QCD corrections to the Field-Feynman fragmentation model as used here. We wish to address this issue in the present section. In addition there are the recent observations of a large number of baryons and antibaryons in the final states at PEP and PETRA [16]. We will also discuss the ramifications of these latter observations for our work in the present section. We consider, first, QCD effects.

More precisely, we follow refs. 17 and implement QCD evolution by evolving the function f(1-z) as follows:

$$f(1-z) \to \frac{e^{.69\delta/4}}{\Gamma(\delta+1)}(-\ln z)^{\delta}\left[.12 + \frac{6(.88)(1-z)^2}{(1+\delta)(\delta+2)} + c_0(1-z)^3\right] \equiv f'(z,s) \quad , \tag{108}$$

where, for example, for five flavors [17] and a QCD scale* $\Lambda = 0.34$ GeV,

$$\delta = \frac{16}{23} \ln \left(\frac{\ln\left(\frac{\sqrt{s}}{.34}\right)}{\ln\left(\frac{3}{.34}\right)} \right) , \tag{109}$$

if we assume eq. (7) is valid for $\sqrt{s} = 3$ GeV. The parameter c_0 is such that

$$\int_0^1 dz\, f'(z,s) = 1 \quad . \tag{110}$$

There would then be a corresponding change in $\bar{F}$ and F in eqs. (68) and (69). We will not need these changes here. For definiteness, we note that, for the case of interest, $\sqrt{s} \doteq 29$ GeV, we have

$$\delta \doteq .502 \tag{111}$$

$$c_0 \doteq .292 \tag{112}$$

so that

$$f'(z,841) \doteq 1.23(.12 + 1.41(1-z)^2 + .292(1-z)^3)(-\ln z)^{.502} . \tag{113}$$

To proceed, we observe that what we are interested in is the number of particles with $z > z_{cut}$, in general weighted with z^n, $n > 0$. For simplicity we consider $n = 10$ first. Since the main effect of QCD

* Here we simply use an average value of several experimental measurements for the one-loop (5-quark) QCD scale, as discussed for example in B. F. L. Ward, Phys. Rev. D25 (1982) 1330. We are aware that this parameter continues to change with time.

is to move particles from higher to lower z values in the f(1-z) distribution, it is clear that $n = 10$ will not underestimate the effect of QCD for $n = \frac{1}{2}, 1, \ldots, 7$, which are the remaining values of n in table 1. For $z_{cut} = .0175$, we find

$$\int_{.0175}^{1} dz\ f(z)\ z^{10} = .0140 \tag{114}$$

$$\int_{.0175}^{1} dz\ f'(z,841)\ z^{10} = .00458 \quad . \tag{115}$$

Thus the effect of QCD is to cause our $n = 10$ moment of f to decrease from its value for eq. (7) by a factor of ~ 3 at $\sqrt{s}$ = 29 GeV. For $n = \frac{1}{2}$, the corresponding change in the respective integral over f is $\sim 16\%$. Note also that it is expected that $a(w) \to a^{\mu}$ for $n \to \infty$. Hence, we can argue that QCD only affects our moments in the regime in moment space where the heavy particle and hadronic contributions to a(w) are minimal. [Our result for a(w) at n = 10 is already close to a^{μ}.] Thus, we do not expect QCD to change our results substantially — but this should be investigated in more detail. We will not pursue this further here. We turn now to baryon-antibaryon production in $e^+e^- \to X$ at $\sqrt{s}$ = 29 GeV.

Namely, the recent [16] high energy $e^+e^- \to X$ data appear to involve a large number of baryons and antibaryons in the final state. This effect was not anticipated explicitly by the Field-Feynman model and is not explicitly taken into account in our calculation. However, the baryons tend to have small z and do not make the dominant contribution to the number of charged particles. Thus, we do not expect them to have a significant effect on our results.

In summary, our position is that the uncertainty in f(1-z) due to the data from which it is abstracted does not warrant the consideration of QCD effects in our calculation. In addition, while there is anomalous production of baryons and antibaryons in $e^+e^- \to X$ at $\sqrt{s} = 29$ GeV, mesons still are the dominant charged particles produced so that our results should not be significantly affected by this phenomenon.

5. Discussion

We have found that the generalized $SU_2 \times U_1$ charge asymmetry may indeed be tractable from the point of view of observation. The key point is to sum over all charged particles of a given sign in an event independent of whether they be leptons or light hadrons. In this way, one can hope to use all events and all charged particles in this event so as to enhance greatly, perhaps, the data sample at the PEP energy $\sqrt{s} = 29$ GeV compared to the currently available $\mu\bar{\mu}$ sample.

Indeed, in our discussion, we discuss the case of negative particles explicitly. Evidently an entirely analogous discussion applies for positively charged particles. Thus the two different experimental quantities $a(w)^-$ and $a(w)^+$ may be combined with an appropriate sign to enhance even further the statistics, where $a(w)^{\binom{+}{-}}$ denotes the respective asymmetry for particles with positive (negative) charge. For, although the $\mu\bar{\mu}$ contributions to $a(w)^-$ and $a(w)^+$ correspond to the same measurement, whenever there is fragmentation, the remaining contributions to $a(w)^-$ and $a(w)^+$ are actually independent physical events— events which can be used to verify the $SU_2 \times U_1$ model.

We conclude by emphasizing that we do feel then that generalized asymmetries of the type discussed in this paper may very well be observable already with the current statistics. We await the experimental results.

Note added:

The question naturally arises as to whether the charged particle distributions in our model agree with the available data. Using data provided by the Mark II Group at PEP (D. Schlatter, private communication, 6/25/82), we have computed

$$r_n \equiv \frac{\int_{z_{cut}}^{1} dz\, z^n\, D_{Exp}^{ch^-}(z)}{\int_{z_{cut}}^{1} dz\, D_{Theory}^{ch^-}(z)\, z^n}$$

where $D_{Theory}^{ch^-}$ is the z-distribution for negatively charged particles in our model, $D_{Exp}^{ch^-}$ is the z-distribution for negatively charged particles as determined from this Mark II PEP data and $n = \frac{1}{2}, 1, \ldots, 7$ and 10. We find r_n = .98, .93, .86, .9, .92, .94, .95, .96 and .98 for $n = \frac{1}{2}, 1, \ldots, 7$ and 10, respectively. Thus we do feel that our simple model has an adequate representation of the charged particle distribution in z at PEP.

Acknowledgements

The author is gratefully indebted to Prof. S. D. Drell for the hospitality of the SLAC Theory Group in general and for the use of the SLAC computer in particular. The author also gratefully acknowledges useful conversations with Prof. M. Perl.

The author thanks Dr. D. Schlatter for communicating to him the most recent PEP results on $sd\sigma/dx$ from the Mark II Group.

This work was supported by the Department of Energy, under contract DE-AC03-76SF00515.

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Table 1.

Generalized $SU_2 \times U_1$ asymmetry results. The value of n is the value of the power of z in $w(z) = z^n$; a(w) is the corresponding value of the asymmetry in eq. (25).

n	a (w)
.5	- .0176
1	- .0319
2	- .0488
3	- .0552
4	- .0576
5	- .0591
6	- .0601
7	- .0607
10	- .0618

FIGURE CAPTIONS

Fig. 1. Feynman diagram and kinematics for $e^+e^- \to f\bar{f}$ in the $SU_2 \times U_1$ model. Here, f may be μ, τ, u, d, s, c or b. As usual $p^- \cdot q_1 = s/4 - (s/4 - m_e^2)^{1/2}\,(s/4 - m_f^2)^{1/2}\cos\theta_{c.m.} \to (s/4)(1 - \cos\theta_{c.m.})$ where $s \equiv (p^- + p^+)^2$, for $4m_f^2/s \to 0$.

Fig. 2. τ and $\bar{\tau}$ decay, in the $SU_2 \times U_1$ model, into leptons and light quarks. We neglect the Cabibbo angle.

Fig. 3. τ and $\bar{\tau}$ decay into the $\overset{(-)}{\nu}_\tau \pi$ final states.

Fig. 4. The decay $\tau \to \nu_\tau + \rho^-$
$$\hookrightarrow \pi^-\pi^0 \quad .$$
The cut on the ρ-propagator indicates that it is on-shell.

Fig. 5. The c and $\bar{c}$ decays in the spectator view of the decay of heavy hadrons. We neglect the Cabibbo angle.

Fig. 6. The b and $\bar{b}$ decays in the spectator $SU_2 \times U_1$ model, assuming only b–c transitions (as indicated by ref. 6) and taking the Cabibbo angle to be trivial.

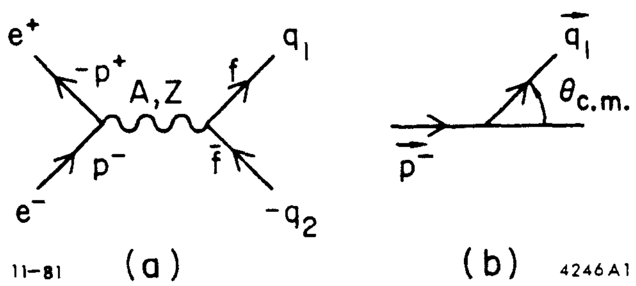

Fig. 1

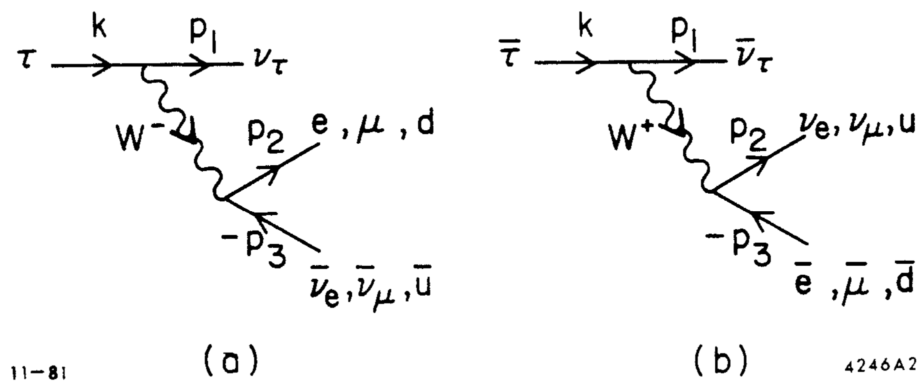
τ
k
p_1
ν_τ
W^-
p_2
e, μ, d
$-p_3$
$\bar{\nu}_e, \bar{\nu}_\mu, \bar{u}$
$\bar{\tau}$
k
p_1
$\bar{\nu}_\tau$
W^+
p_2
ν_e, ν_μ, u
$-p_3$
$\bar{e}, \bar{\mu}, \bar{d}$
11-81
(a)
(b)
4246A2

Fig. 2

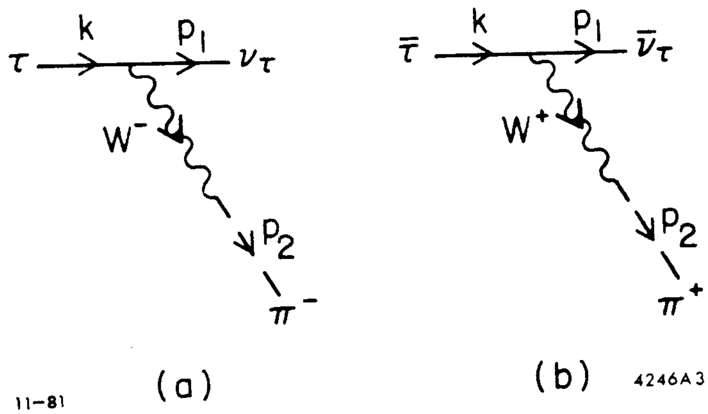

Fig. 3

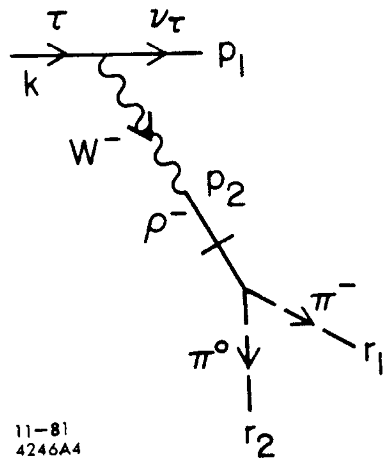

Fig. 4

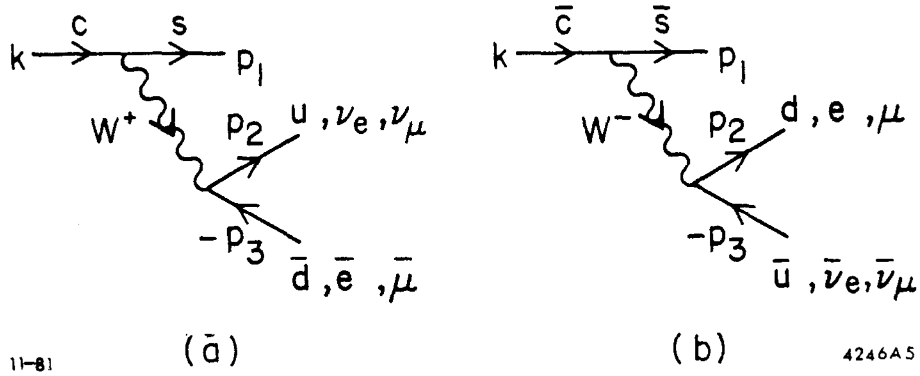

Fig. 5

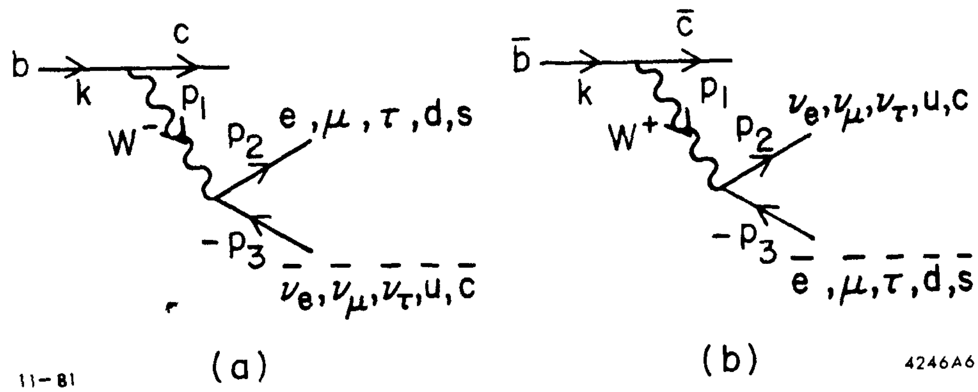
b
c
k
p1
W⁻
p2
e, μ, τ, d, s
−p3
ν̄e, ν̄μ, ν̄τ, ū, c̄
(a)
11-81
b̄
c̄
k
p1
W+
p2
νe, νμ, ντ, u, c
−p3
ē, μ̄, τ̄, d̄, s̄
(b)
4246A6

Fig. 6