SLAC-PUB-2842
October 1981
(T/E)

Threshold Factors and the Top Quark Mass*

Giora J. Tarnopolsky

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

The ratio of the number of hadronic events to that of muon pairs in electron - positron annihilation at the mass of the weak neutral boson permits a determination of the top quark mass.

(Submitted to Physical Review Letters)

---

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

It is now known that the postulated sixth, or top (t), quark has a mass above m=18.5 GeV[1] ,beyond the reach of the operational electron - positron storage rings. With an new generation of accelerators being planned specifically to produce the neutral weak boson Z^0 at resonance, there is a distinct possibility that evidence for the t quark be first obtained in the analysis of decay products of the Z^0. This possibility becomes probable by the realization that the production, by hadron beams, of systems containing the t quark proceeds at such low rates[2] as to be lost in the background, and by considering the rate of production of a resonant ($t\bar{t}$) system in e^+e^- annihilation. The expected electronic width[3] of such resonances,the inverse square dependance of the cross section on center-of-mass energy (E_{cm}), the energy resolution and luminosity of the accelerator[4] ,all presage an arduous search. It is thus important to restrict such a search to an energy interval as limited as possible.

Consider now the production of $t\bar{t}$ pairs at E_{cm}=M, the Z^0 mass. In all that follows we adopt the standard model[5] of weak and electromagnetic interactions, in which the mass and partial widths of the Z^0 all are functions of a single parameter, the sine of Weinberg's angle $\sin\theta_w$. In this model three generations of colored quarks (q), forming weak isospin doublets, have both vector and axial-vector couplings[6] to the Z^0. The partial width for $Z^0 \to t\bar{t}$ is calculated to be

$$\Gamma_t(\beta)=3\cdot(M/24\pi)\cdot[g_v{}^2\beta(3-\beta^2)+2g_a{}^2\beta^3] \qquad (1)$$

where $\beta=\sqrt{1-(2m/M)^2}$ is the t velocity in units of the speed of light and the couplings[7] are $g_v=(e/2\sin\theta_w\cos\theta_w)(\frac{1}{2} - (4/3)\sin^2\theta_w)$ and

$g_a=e/(4\sin\theta_W\cos\theta_W)$ with e the unit charge. The measured value[8] $\sin^2\theta_W=0.22\pm0.01$ results in $(g_a/g_v)^2=5.8\pm0.8$, with a threshold factor dependance predominantly cubic for velocities $\beta \gtrsim \sqrt{3g_v^2/(2g_a^2+g_v^2)}=0.49$. Thus the rate of $t\bar{t}$ production at the Z^0 shall be noticeably modulated by the threshold factor as indicated in Fig.1 where the ratio $\rho=\Gamma_t(\beta)/\Gamma_u$ of the top quark partial width to that of a massless up quark is shown as a function of m. With the present lower bound on m, the threshold suppression of $t\bar{t}$ production is about 22%.

Within the standard model, ρ can be expressed in terms of the experimental ratio r of the number N_h of hadronic events to that of muon pair events $N_{\mu\mu}$ produced at $E_{cm}=M$,

$$\rho = 1 + r\,\frac{\Gamma_{\mu\mu}}{\Gamma_u} - \frac{\Gamma_h}{\Gamma_u} \qquad (2)$$

where Γ_h is the hadronic width calculated for three quark generations of massless, colored, weak isospin doublets, $\Gamma_{\mu\mu}$ is the $\mu^+\mu^-$ partial width and Γ_u is the partial width into u quarks . The ratios of widths in Eq. (2) are smooth functions[9] of $\sin^2\theta_W$, and the main contribution to the error in ρ arises from the error in the experimental value $r=N_h/N_{\mu\mu}$ rather than from the uncertainty in $\sin^2\theta_W$ measured at the Z^0. Considering a sample of 10^6 Z^0's, $N_{\mu\mu}\simeq 31{,}000$, $N_h\simeq 730{,}000$ and $\sin^2\theta_W=0.22\pm0.001$ results in the t quark mass error determination shown in Fig. 2.

In summary, we have shown that the relative magnitudes of the vector and axial vector couplings of the weak neutral boson to weak isospin $+\frac{1}{2}$, charge +2/3 quarks leads to a supression of $t\bar{t}$ Z^0 decays that follows the third power of the quark velocity. The threshold suppression reduces appreciably the total Z^0 width, and its magnitude can be used, within the standard model, to estimate the t quark mass. Furthermore, the error in the mass determination, while considerable, is of the same order as the energy level separation[3] of the excited vector meson resonances. It thus circumscribes the resonance search to the relevant energy interval and may greatly facilitate the study of $t\bar{t}$ spectroscopy. Finally, the magnitude of the threshold effect will have to be experimentally established before attempting to infer the number of neutrinos from the total Z^0 width.

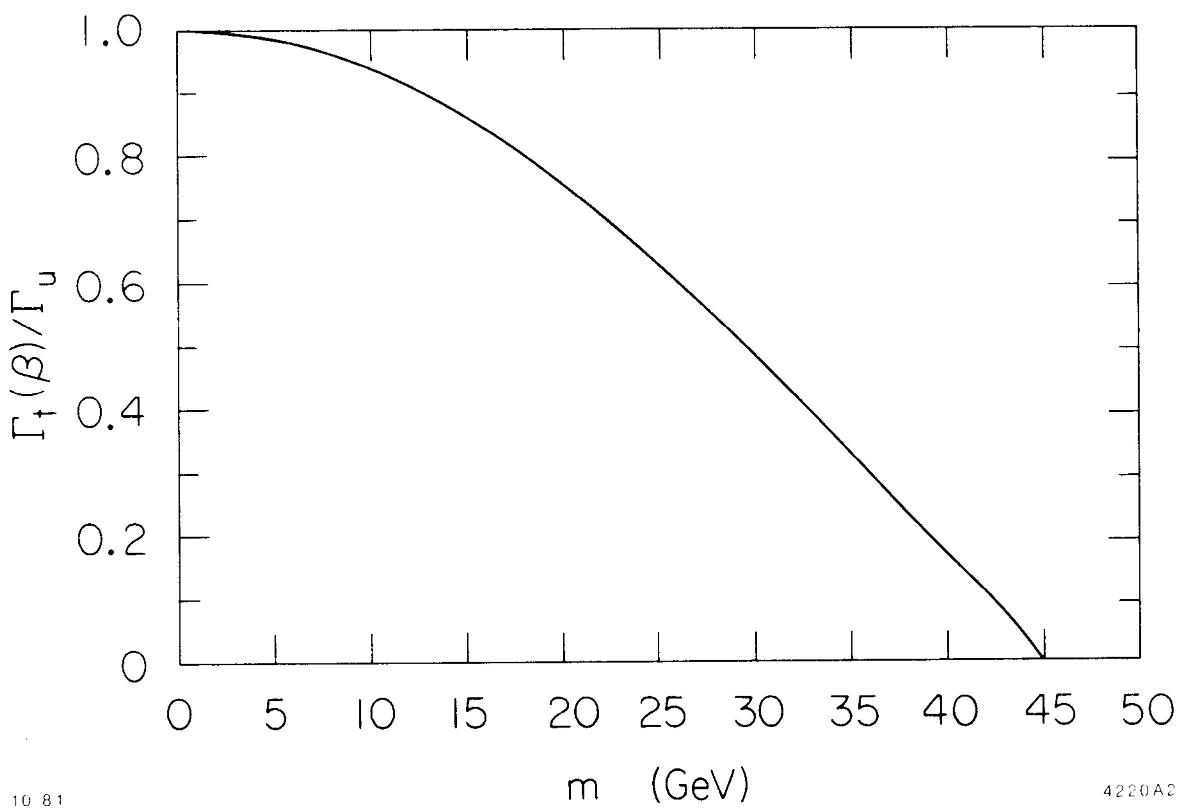
Discussions with Prof. S. Brodsky and encouragement from Profs. D.W.G.S. Leith and G.J. Feldman are gratefully acknowledged. This work was supported by the Department of Energy, contract DE-AC03-76SF00515.

REFERENCES

1. JADE collaboration, W. Barlet *et al*., Phys. Lett. B 100, 364(1981). Other recent PETRA results appear in R. Felst, presented at the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn(1981).
2. Sandip Pakvasa, Proceedings of Neutrino '79,Bergen,Sweden,1979, ed. by A. Haatuft and C. Jarlskog, v. 2,p.605.
3. D. P. Stanley and D. Robson,Phys.Rev.D 22, 165(1980)
4. SLAC-Report-229, June 1980, p.17 and p.69;CERN/ISR-LEP/79-33, August 1979,p. 11 and p.39.
5. S. L. Glashow, Nucl. Phys. 22, 579(1961); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam,Proc. Eight Nobel Symposium on Elementary Particle Physics, Stockholm, Sweden,1968, ed. by N. Svartholm (Almqvist and Wiksell),Stockholm, 1968),p.367.
6. J. D. Bjorken, Proceedings of the Summer Institute on Particle Physics, SLAC Report 198,November 1976.
7. The effective couplings of quarks connected to a weak current are not significantly affected by strong interaction vertex corrections at the high t mass. See M.B. Gavela *et al*., Physical Review D 22, 2906 (1980)
8. J. E. Kim *et al*., Rev. Mod. Phys. 53, 211(1981).
9. These functions are $\Gamma_{\mu\mu}/\Gamma_u=3(1-4x+8x^2)/(9-24x+32x^2)$ and $\Gamma_h/\Gamma_u=6(9-18x+20x^2)/(9-24x+32x^2)$, where $x=\sin^2\theta_W$.

FIGURE CAPTIONS

1. The suppression factor of $t\bar{t}$ decays of the Z^0 as a function of the top quark mass for $\sin^2\theta_W=0.22$, M=90 GeV.
2. The error in the determination of the top quark mass as a function of the top quark mass for sample of 10^6 Z^0's.



1.0
0.8
0.6
0.4
0.2
0
$\Gamma_t(\beta)/\Gamma_u$
0
5
10
15
20
25
30
35
40
45
50
m (GeV)
10 81
4220A2

Fig. 1

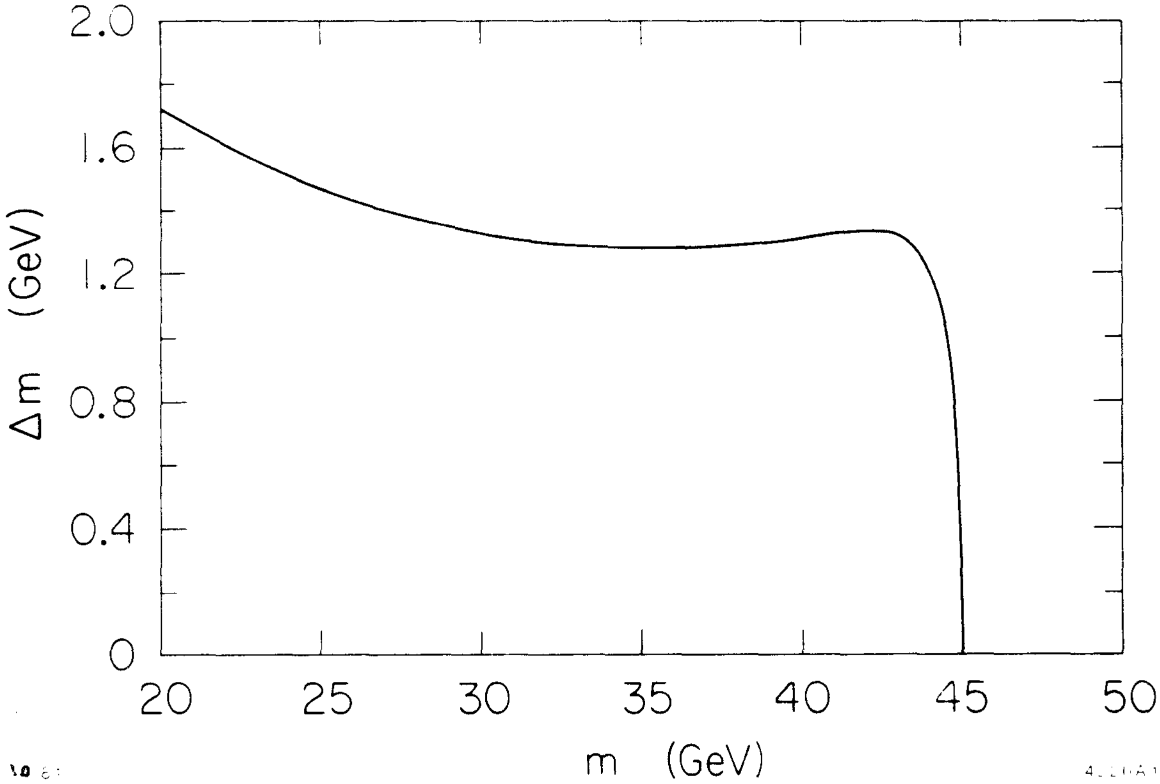


Fig. 2