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SEMILEPTONIC DECAY RATES OF CHARMED PSEUDOSCALAR MESONS AND CHIRAL DYNAMICS*

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ABSTRACT

The hint of discrepancies between semileptonic and pure hadronic decay rates of the D leads us to consider the semileptonic decay modes of pseudoscalar mesons for producing one or two mesons in the final state in a chiral SU(4) × SU(4) model. The calculated leptonic "lifetimes" are about $\Gamma^{-1}(D^{\pm}, D \to \text{leptons} + X) \sim 4 \times 10^{-12}$s.

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The results of the charmed lifetime experiments,[1-5] especially the disagreement of $\tau(D^+)/\tau(D^o)$ with theoretical predictions,[6] has led to much theoretical interest in these decays. Most recent work has focused on the purely hadronic modes.[7] However, there are some indications that the hadronic rates differ from the semileptonic rates.[8] We calculate the semileptonic rates and spectra in this paper, applying the method of non-linear phenomenological lagrangians.[9] Our currents are derived in the massless limit and extrapolated to the observed masses to take the phase space restrictions into account. The application of this low energy theory to the phenomenology of τ-decay for $SU(2) \times SU(2)$ and $SU(3) \times SU(3)$ chiral groups led to a reasonable description of the multihadron decay process.[10] This success leads us to use the theory for the semileptonic decays of pseudoscalar mesons in chiral $SU(4) \times SU(4)$. These widths have previously been calculated in a different model by Ali and Yang.[11] We shall ignore the η' and η_c contributions and consider the η to be entirely octet in our calculations.

Our currents follow from our nonlinear realization of the chiral group, which determines the invariants and thus the lagrangian.[10] We explicitly allow for broken symmetry by allowing the decay constants to differ (we take $f_\eta = f_K$, $f_F = f_D$). Phenomenologically, the ratio $\lambda^{1/2} \equiv f_K/f \approx \lambda_D^{1/2} \equiv f_D/f_K \sim 1.1$.[12] The aim of this paper is to calculate the rates in the symmetry limit. Since our model contains only tree contributions to the currents, it must be considered in the context of the possibility of contributions of higher terms in the expansion (in terms of quark masses, for example). While this possibility is not considered in this paper, the calculation in the case of explicit symmetry breaking gives an indication of how sensitive the model is to gross changes in parameters (i.e., not very sensitive). The form of the currents in the symmetry limit is much simpler than in the broken symmetry case, so the currents are given in their more complicated form (to get the results in the symmetry limit the values $\lambda = \lambda_D = 1$ are taken).

$$J_\mu(\pi^+) = -i\sqrt{2}\, f_\pi\, p_{+\mu} \tag{1}$$

$$J_\mu(K^+) = -i\sqrt{2}\, f_K\, k_{+\mu} \tag{2}$$

$$J_\mu(D^+) = -i\sqrt{2}\, f_D\, d_{+\mu} \tag{3}$$

$$J_\mu(F^+) = -i\sqrt{2}\, f_D\, f_{+\mu} \tag{4}$$

$$J_\mu(\pi^+\pi) = \sqrt{2}\,(p_+ - p)_\mu \tag{5}$$

$$J_\mu(\bar{K}K^+) = (\bar{k} - k_+)_\mu \tag{6}$$

$$J_\mu(\bar{D}D^+) = (d_+ - \bar{d})_\mu \tag{7}$$

$$J_\mu(K\pi^+) = \left(\lambda^{1/2}k - \lambda^{-1/2}p_+\right)_\mu \tag{8}$$

$$J_\mu(K^+\pi) = 2^{-1/2}\left(\lambda^{1/2}k_+ - \lambda^{-1/2}p\right)_\mu \tag{9}$$

$$J_\mu(\eta K^+) = 3^{1/2}\, 2^{-1/2}\,(k_+ - \eta)_\mu \tag{10}$$

$$J_\mu(F^+\bar{D}) = (f_+ - \bar{d})_\mu \tag{11}$$

$$J_\mu(D\pi^+) = \left((\lambda\lambda_D)^{-1/2}p_+ - (\lambda\lambda_D)^{1/2}d\right)_\mu \tag{12}$$

$$J_\mu(D^+\pi) = 2^{-1/2}\left((\lambda\lambda_D)^{-1/2}d_+ - (\lambda\lambda_D)^{1/2}p\right)_\mu \tag{13}$$

$$J_\mu(D^+\eta) = 6^{-1/2}\left(\lambda_D^{-1/2}\eta - \lambda_D^{1/2}d_+\right)_\mu \tag{14}$$

$$J_\mu(F^+\bar{K}) = \left(\lambda_D^{-1/2}\bar{k} - \lambda_D^{1/2}f_+\right)_\mu \tag{15}$$

$$J_\mu(D^+K) = \left(\lambda_D^{-1/2}k - \lambda_D^{1/2}d_+\right)_\mu \tag{16}$$

$$J_\mu(DK^+) = \left(\lambda_D^{-1/2}k_+ - \lambda_D^{1/2}d\right)_\mu \tag{17}$$

$$J_\mu(F^+\eta) = 2^{1/2}\ 3^{-1/2}\ \left(\lambda_D^{1/2}f_+ - \lambda_D^{-1/2}\eta\right)_\mu \tag{18}$$

In the following

$$P^A_{\mu\nu}(\xi) = g_{\mu\nu}^{(\xi+3)}/4 - \frac{Q_\mu Q_\nu^{(\xi+1)/2}}{Q^2 - m_A^2} ,$$

where Q is the sum of the lepton momenta. Also, the form factor is given by

$$F_A(p) = \frac{m_A^2 - i\, m_A \Gamma_A}{m_A^2 - p^2 - i\, m_A \Gamma_A}$$

The form factor is inserted in such a way that the low energy limit is unaltered and the current (in the massless case) is conserved. (Recall in this context that $\delta f \propto \delta m$.[13])

$$J_\mu(K^+\pi^+\pi^-) = \left(2^{1/2}/3f_\pi\lambda^{1/2}\right)\Big\{F_\rho(p_+ + p_-)(p_+ - p_-)^\nu\ P^K_{\mu\nu}(\lambda) + \lambda k_+^\nu P^K_{\mu\nu}(1) - p_-^\nu P^K_{\mu\nu}(\lambda)\Big\} \tag{19}$$

$$J_\mu(K^+\pi_1\pi_2) = -\left(2^{1/2}/6f_\pi\lambda^{1/2}\right)\Big\{(p_1 + p_2)^\nu\ P^K_{\mu\nu}(\lambda) - 2\lambda k_+^\nu\ P^K_{\mu\nu}(1)\Big\} \tag{20}$$

$$J_\mu(K\pi^+\pi) = \left(1/f_\pi\lambda^{1/2}\right) F_\rho(p_+ + p)(p_+ - p)^\nu\ P^K_{\mu\nu}(\lambda) \tag{21}$$

$$J_\mu(D^+\pi^+\pi^-) = \left(2^{1/2}/3f_\pi(\lambda\lambda_D)^{1/2}\right)\Big\{F_\rho(p_+ + p_-)(p_+ - p_-)^\nu\ P^D_{\mu\nu}(\lambda\lambda_D) - p_-^\nu\ P^D_{\mu\nu}(\lambda\lambda_D) + \lambda\lambda_D d_+^\nu\ P^D_{\mu\nu}(1)\Big\} \tag{22}$$

$$J_\mu(D^+\pi_1\pi_2) = -\left(2^{1/2}/6f_\pi(\lambda\lambda_D)^{1/2}\right)\left\{(p_1 + p_2)^\nu\, P^D_{\mu\nu}(\lambda\lambda_D) - 2\lambda\lambda_D d^\nu_+\, P^D_{\mu\nu}(1)\right\} \tag{23}$$

$$J_\mu(D^+K\bar{K}) = \left(2^{1/2}/6f_\pi(\lambda\lambda_D)^{1/2}\right)\left\{F_\phi(k + \bar{k})(\bar{k} - k)^\nu\, P^D_{\mu\nu}(\lambda_D) + (\bar{k} - 3k)^\nu\, P^D_{\mu\nu}(\lambda_D) + 2\lambda_D d_+{}^\nu\, P^D_{\mu\nu}(1)\right\} \tag{24}$$

$$J_\mu(D^+\eta\pi) = \left(6^{1/2}/18f_\pi\lambda\lambda_D^{1/2}\right)\left\{p^\nu\, P^D_{\mu\nu}(\lambda\lambda_D) + \lambda\eta^\nu\, P^D_{\mu\nu}(\lambda_D) - 2\lambda\lambda_D d^\nu_+\, P^D_{\mu\nu}(1)\right\} \tag{25}$$

$$J_\mu(D^+\eta_1\eta_2) = -\left(2^{1/2}/18f_\pi(\lambda\lambda_D)^{1/2}\right)\left\{(\eta_1 + \eta_2)^\nu\, P^D_{\mu\nu}(\lambda\lambda_D) - 2\lambda_D d^\nu_+\, P^D_{\mu\nu}(1)\right\} \tag{26}$$

$$J_\mu(D^+K\pi) = \left(1/3f_\pi\lambda\lambda_D^{1/2}\right)\left\{F_{K*}(p + k)(p - k)^\nu\, P^F_{\mu\nu}(\lambda\lambda_D) + p^\nu P^F_{\mu\nu}(\lambda\lambda_D) - \lambda\lambda_D d^\nu_+\, P^F_{\mu\nu}(1)\right\} \tag{27}$$

$$J_\mu(D^+K^+\pi^-) = \left(2^{1/2}/3f_\pi\lambda\lambda_D^{1/2}\right)\left\{F_{K*}(k_+ + p)(k_+ - p_-)^\nu\, P^F_{\mu\nu}(\lambda\lambda_D) + k^\nu_+\left(\lambda\, P^F_{\mu\nu}(\lambda_D) - P^F_{\mu\nu}(\lambda\lambda_D)\right) - p^\nu_-\, P^F_{\mu\nu}(\lambda\lambda_D) + \lambda\lambda_D d^\nu_+\, P^F_{\mu\nu}(1)\right\} \tag{28}$$

$$J_\mu(D^+K\eta) = \left(3^{1/2}/9f_\pi(\lambda\lambda_D)^{1/2}\right)\left\{\lambda\, P^F_{\mu\nu}\left(\lambda_D\lambda^{-1}\right)(5k - 4\eta)^\nu - \lambda d^\nu_+\, P^F_{\mu\nu}(1)\right\} \tag{29}$$

$$J_\mu(D\pi^+\pi) = -\left(1/4f_\pi(\lambda\lambda_D)^{1/2}\right)F_\rho(p + p_+)(p - p_+)^\nu\, P^D_{\mu\nu}(\lambda\lambda_D) \tag{30}$$

$$J_\mu(DK^+\bar{K}) = \left(2^{1/2}/3f_\pi(\lambda\lambda_D)^{1/2}\right)\left\{(\bar{k} - 2k_+)^\nu\, P^D_{\mu\nu}(\lambda_D) + \lambda_D d^\nu\, P^D_{\mu\nu}(1)\right\} \tag{31}$$

$$J_\mu(D\pi^+\eta) = -\left(3^{1/2}/9f_\pi\lambda\lambda_D^{1/2}\right)\left\{p^\nu_+\, P^D_{\mu\nu}(\lambda\lambda_D) + \lambda\eta^\nu\, P^D_{\mu\nu}(\lambda_D) - 2\lambda\lambda_D d^\nu\, P^D_{\mu\nu}(1)\right\} \tag{32}$$

$$J_\mu(DK\pi^+) = -\left(2^{1/2}/3f_\pi\lambda\lambda_D^{1/2}\right)\Big\{F_{K*}(p_+ + k)(p_+ - k)^\nu P^F_{\mu\nu}(\lambda\lambda_D)$$

$$+ p_+^\nu P^F_{\mu\nu}(\lambda\lambda_D) + k^\nu\left[P^F_{\mu\nu}(\lambda\lambda_D) - \lambda P^F_{\mu\nu}(\lambda_D)\right]$$

$$- \lambda\lambda_D d^\nu P^F_{\mu\nu}(1)\Big\} \tag{33}$$

$$J_\mu(DK^+\pi) = -\left(1/3f_\pi\lambda\lambda_D^{1/2}\right)\Big\{F_{K*}(p + k_+)(p - k_+)^\nu P^F_{\mu\nu}(\lambda\lambda_D)$$

$$+ p^\nu P^F_{\mu\nu}(\lambda\lambda_D) + k_+^\nu\left[P^F_{\mu\nu}(\lambda\lambda_D) - \lambda P^F_{\mu\nu}(\lambda_D)\right]$$

$$- \lambda\lambda_D d^\nu P^F_{\mu\nu}(1)\Big\} \tag{34}$$

$$J_\mu(DK^+\eta) = \left(3^{1/2}/9f_\pi(\lambda\lambda_D)^{1/2}\right)\Big\{\lambda(5k_+ - 4\eta)^\nu P^F_{\mu\nu}\left(\lambda_D\lambda^{-1}\right) - \lambda_D d^\nu P^F_{\mu\nu}\Big\} \tag{35}$$

$$J_\mu(F^+K^-\pi^+) = \left(2^{1/2}/3f_\pi\lambda\lambda_D\right)\Big\{F_{K*}(p_+ + k_-)(p_+ - k_-)^\nu P^D_{\mu\nu}(\lambda)$$

$$+ k_-^\nu\left[P^D_{\mu\nu}(\lambda) - 2\lambda\lambda_D P^D_{\mu\nu}\left(\lambda_D^{-1}\right)\right] + \lambda\lambda_D f_+^\nu P^D_{\mu\nu}(1)\Big\} \tag{36}$$

$$J_\mu(F^+\bar{K}\pi) = \left(1/3f_\pi\lambda\lambda_D^{1/2}\right)\Big\{F_{K*}(p + \bar{k})(\bar{k} - p)^\nu P^D_{\mu\nu}(\lambda\lambda_D)$$

$$+ \bar{k}^\nu P^D_{\mu\nu}(\lambda\lambda_D) - \lambda\lambda_D f_+^\nu P^D_{\mu\nu}(1)\Big\} \tag{37}$$

$$J_\mu(F^+\bar{K}\eta) = \left(3^{1/2}/9f_\pi(\lambda\lambda_D)^{1/2}\right)\Big\{(5\eta - 4\bar{k})^\nu P^D_{\mu\nu}(\lambda_D) - \lambda_D f_+^\nu P^D_{\mu\nu}(1)\Big\} \tag{38}$$

$$J_\mu(F^+K^+K^-) = \left(2^{1/2}/3f_\pi(\lambda\lambda_D)^{1/2}\right)\Big\{\lambda(k_+ - 2k_-)^\nu P^F_{\mu\nu}\left(\lambda_D\lambda^{-1}\right)$$

$$+ \lambda_D f_+^\nu P^F_{\mu\nu}(1)\Big\} \tag{39}$$

$$J_\mu(F^+K\bar{K}) = \left(2^{1/2}/3f_\pi(\lambda\lambda_D)^{1/2}\right)\Big\{\lambda(k - 2\bar{k})^\nu P^F_{\mu\nu}\left(\lambda_D\lambda^{-1}\right)$$

$$+ \lambda_D f_+^\nu P^F_{\mu\nu}(1)\Big\} \tag{40}$$

$$J_\mu\left(F^+\eta_1\eta_2\right) = -\left(2^{3/2}/9f_\pi(\lambda\lambda_D)^{1/2}\right)\left\{\lambda(\eta_1+\eta_2)^\nu\, P^F_{\mu\nu}\left(\lambda_D\lambda^{-1}\right) - 2\lambda_D f^\nu_+\; P^F_{\mu\nu}(1)\right\} \tag{41}$$

$$J_\mu(F^+D^-\pi^+) = \left(2^{1/2}/3f_\pi\lambda\lambda_D\right)\left\{F_{D*}(p_+ + d_-)(p_+ - d_-)^\nu\, P^K_{\mu\nu}(\lambda) + d^\nu_-\left[P^K_{\mu\nu}(\lambda) - 2\lambda\lambda_D\, P^K_{\mu\nu}\left(\lambda_D^{-1}\right)\right] + \lambda\lambda_D f^\nu_+\, P^K_{\mu\nu}\left(\lambda_D^{-1}\right)\right\} \tag{42}$$

$$J_\mu(F^+\bar{D}\pi) = \left(1/3f_\pi\lambda\lambda_D\right)\left\{F_{D*}(p+\bar{d})(p-\bar{d})^\nu\, P^K_{\mu\nu}(\lambda) + \bar{d}^\nu\left[P^K_{\mu\nu}(\lambda) - 2\lambda\lambda_D\, P^K_{\mu\nu}\left(\lambda_D^{-1}\right)\right] + \lambda\lambda_D f^\nu_+\, P^K_{\mu\nu}\left(\lambda_D^{-1}\right)\right\} \tag{43}$$

The form factors have been added in the minimal way, e.g., (22) was given by

$$J_\mu(D^+\pi^+\pi^-)_{\text{low energy}} = \left(2^{1/2}/3f_\pi(\lambda\lambda_D)^{1/2}\right)\left\{(p_+ - 2p_-)^\nu\, P^D_{\mu\nu}(\lambda\lambda_D) + \lambda\lambda_D d^\nu_+\, P^D_{\mu\nu}(1)\right\} .$$

The widths for the decay $P_1 \to \ell\nu$, $P_2\ell\nu$, $P_2P_3\ell\nu$ are given trivally in terms of the currents Eq. (1)-(43) by

$$\Gamma = \frac{Q^2 f(\theta_c)}{128\,\pi^8\, m_1}\int \frac{d^3k_\ell}{2k^0_\ell}\,\frac{d^3k_\nu}{2k^0_\nu}\left(k^\sigma_\ell k^{\sigma'}_\nu + k^{\sigma'}_\ell k^\sigma_\nu - k_\ell\cdot k_\nu\, g^{\sigma\sigma'}\right) \times\int \prod_i \frac{d^3p_i}{2p^0_i}\,\delta^4\!\left(p_1 - Q - \sum_j p_j\right) J^+_\sigma\left(\left\{p_i\right\}\right) J_{\sigma'}\left(\left\{p_i\right\}\right) \tag{44}$$

In Eq. (44), $f(\theta_c)$ is $\sin^2\theta_c$ ($\cos^2\theta_c$) for the 4 ± i5, 11 ± i12 (13 ∓ i14) current(s). The spectra, $d\Gamma/dz$, are given in terms of $z \equiv 1-2k^\sigma_\nu/m_1$, which is a measure of the observed energy. Several representative spectra are presented in Figs. 1 and 2. The widths

$$\Gamma = \int_{\left(\frac{\sum_i m_i + m_\ell}{m_1}\right)^2}^{1} dz \, \frac{d\Gamma}{dz}$$

and corresponding rates τ^{-1} are given in Table I. Naturally, the Cabibbo-favored decays dominate the rates.

The rates for D^+, D, and F, to the extent calculated, namely for processes containing up to two final-state pseudoscalars, are $(\lambda = 1)\tau^{-1}_{D^+\ \text{leptonic}} = 2.46 \times 10^{11} s^{-1}$, $\tau^{-1}_{D\ \text{leptonic}} = 2.49 \times 10^{11} s^{-1}$, and $\tau^{-1}_F = 2.39 \times 10^{11} s^{-1}$. Hence as we expect all the decay "lifetimes" to leptons are about the same $\tau \approx 4 \times 10^{-12}$s. The neglect of higher numbers of pseudoscalar particles seems well-justified, as the widths fall by about an order of magnitude each time an extra particle is added.

The calculated inverse rate is to be compared to measured lifetimes of ~4 or ~10 × 10^{-13}s.[1-5] Using a branching ratio of 20% for leptonic D decays, we obtain from our above result ~4 × 10^{-13}s for the D lifetime, in good agreement with observation.

It should be noted that the model seems to work well despite the limitations on the applicability of the theory to this problem. It would be of interest to study why the results seem so good. It could also be of interest to extend this technique to B meson semileptonic decays.

As a final remark, the rate for the process $F \to \tau\nu_\tau$ is not given in the table, but is given by

$$\frac{\Gamma(F \to \nu_\tau \tau)}{\Gamma(F \to \nu_\mu \mu)} = 64.7$$

so that $\tau^{-1}(F \to \nu_\tau \tau) = 1.8 \times 10^{11}$s.

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TABLE I

Decay rate τ^{-1} in $\sec^{-1}$ compared to experimental values for the processes considered, compared to experimental values taken from the Review of Particle Properties, Rev. Mod. Phys. 52, S1 (1980). The muon values are in parentheses. The table values for K_L have been used to compare the $\bar{K}$ decay rates to experiment.

Process	Decay Rate ($\sec^{-1}$)		Experiment
	$\lambda = \lambda_D = 1$	$\lambda = \lambda_D = 1.2$	
$\pi^- \to \ell\bar{\nu}$	4.69×10^3 (3.69×10^7)	4.69×10^3 (3.69×10^7)	4.9×10^3 (3.8×10^7)
$K^- \to \ell\bar{\nu}$	3.57×10^3 (3.33×10^7)	4.28×10^3 (4.00×10^7)	(5.1×10^7)
$D^- \to \ell\bar{\nu}$	9.47×10^2 (1.37×10^8)	1.36×10^3 $(1.97 \times 10^8$	
$F^- \to \ell\bar{\nu}$	1.66×10^3 (2.83×10^9)	2.39×10^2 (4.08×10^9)	
$\pi^- \to \pi\ell\bar{\nu}$	0.370	0.370	0.392
$K^- \to \pi\ell\bar{\nu}$	3.54×10^6 (2.29×10^6)	3.58×10^6 (2.35×10^6)	3.9×10^6 (2.6×10^6)
$\bar{K} \to \pi^+\ell\bar{\nu}$	7.17×10^6 $(4.63 \times 10^6$	7.24×10^6 (4.76×10^6)	7.5×10^6 (5.2×10^6)
$K \to K^+\ell\bar{\nu}$	9.38×10^{-2}	9.38×10^{-2}	
$\eta \to K^+\ell\bar{\nu}$	3.48×10^3	3.48×10^3	
$D^- \to \pi\ell\bar{\nu}$	4.56×10^9 (4.45×10^9)	4.74×10^9 (4.65×10^9)	
$D^- \to K\ell\bar{\nu}$	1.06×10^{11} (1.03×10^{11})	1.07×10^{11} (1.04×10^{11})	
$D^- \to \eta\ell\bar{\nu}$	8.44×10^8 (8.16×10^8)	8.53×10^8 (8.26×10^8)	
$D^- \to \bar{D}\ell\bar{\nu}$	0.297	0.297	
$\bar{D} \to \pi^+\ell\bar{\nu}$	8.96×10^9 (8.76×10^9)	9.33×10^9 (9.15×10^9)	
$\bar{D} \to K^+\ell\bar{\nu}$	1.05×10^{11} (1.02×10^{11})	1.06×10^{11} (1.03×10^{11})	
$F^- \to \bar{K}\ell\bar{\nu}$	9.49×10^9 (9.25×10^9)	9.59×10^9 (9.36×10^9)	
$F^- \to \eta\ell\bar{\nu}$	1.08×10^{11} (1.05×10^{11})	1.09×10^{11} (1.07×10^{11})	
$F^- \to \bar{D}\ell\nu$	7.93×10^5 (1.25×10^5)	7.93×10^5 (1.25×10^5)	

TABLE I (Continued)

Process	Decay Rate (sec^{-1})		Experiment
	$\lambda = \lambda_D = 1$	$\lambda = \lambda_D = 1.2$	
$K^- \to \pi^+\pi^-\ell\bar{\nu}$	1.07×10^3 (1.56×10^2)	1.09×10^3 (1.72×10^2)	
$K^- \to \pi\pi\ell\bar{\nu}$	1.08×10^3 (2.09×10^2)	1.19×10^3 (2.33×10^2)	1.5×10^3
$\bar{K} \to \pi^+\pi\ell\bar{\nu}$	7.37×10^2 (6.91×10^1)	6.78×10^2 (6.36×10^1)	3.2×10^3 (7.3×10^2)
$D^- \to \pi^+\pi^-\ell\bar{\nu}$	4.05×10^8 (3.87×10^8)	4.06×10^8 (3.89×10^8)	
$D^- \to \pi\pi\ell\bar{\nu}$	1.63×10^8 (1.55×10^8)	2.00×10^8 (1.91×10^8)	
$D^- \to K\bar{K}\ell\bar{\nu}$	2.19×10^7 (2.00×10^7)	1.89×10^7 (1.72×10^7)	
$D^- \to \eta\pi\ell\bar{\nu}$	1.77×10^7 (1.65×10^7)	1.91×10^7 (1.78×10^7)	
$D^- \to \eta\eta\ell\bar{\nu}$	8.15×10^5 (6.98×10^5)	7.49×10^5 (6.42×10^5)	
$D^- \to K\pi\ell\bar{\nu}$	4.44×10^9 (4.17×10^9)	3.37×10^9 (3.17×10^9)	
$D^- \to K^+\pi^-\ell\bar{\nu}$	8.44×10^9 (7.74×10^9)	6.27×10^9 (5.90×10^9)	
$D^- \to K\eta\ell\bar{\nu}$	8.90×10^7 (7.41×10^7)	8.90×10^7 (7.41×10^7)	
$\bar{D} \to \eta^+\pi\ell\bar{\nu}$	6.63×10^7 (6.34×10^7)	5.67×10^7 (5.43×10^7)	
$\bar{D} \to K^+\bar{K}\ell\bar{\nu}$	1.70×10^7 (1.48×10^7)	1.51×10^7 (1.32×10^7)	
$\bar{D} \to \eta\pi^+\ell\bar{\nu}$	3.41×10^7 (3.17×10^7)	3.66×10^7 (3.41×10^7)	
$\bar{D} \to K\pi^+\ell\bar{\nu}$	8.14×10^9 (7.65×10^9)	6.05×10^9 (5.69×10^9)	
$\bar{D} \to K^+\pi\ell\bar{\nu}$	4.25×10^9 (4.00×10^9)	3.15×10^9 (2.97×10^9)	
$\bar{D} \to \eta K^+\ell\bar{\nu}$	5.24×10^8 (4.37×10^8)	5.73×10^8 (4.39×10^8)	

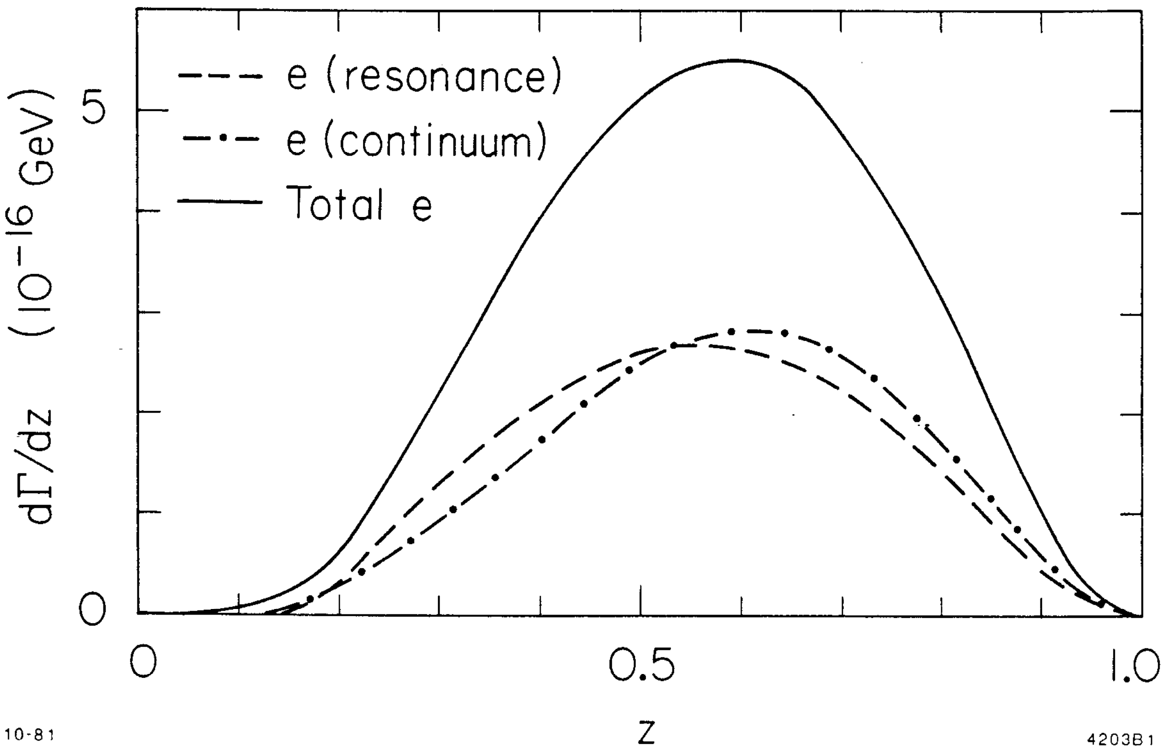
TABLE I (Concluded)

Process	Decay Rate (sec^{-1})		Experiment
	$\lambda = \lambda_D = 1$	$\lambda = \lambda_D = 1.2$	
$F^- \rightarrow K^-\pi^+\ell\bar{\nu}$	7.62×10^8 (7.30×10^8)	5.06×10^8 (4.84×10^8)	
$F^- \rightarrow \bar{K}\pi\ell\bar{\nu}$	3.83×10^8 (3.67×10^8)	2.96×10^8 (2.84×10^8)	
$F^- \rightarrow \eta\bar{K}\ell\bar{\nu}$	1.69×10^7 (1.48×10^7)	1.37×10^7 (1.21×10^7)	
$F^- \rightarrow K^+K^-\ell\bar{\nu}$	9.58×10^8 (8.70×10^8)	9.58×10^8 (8.70×10^8)	
$F^- \rightarrow K\bar{K}\ell\bar{\nu}$	9.21×10^8 (8.35×10^8)	9.21×10^8 (8.35×10^8)	
$F^- \rightarrow \eta\eta\ell\bar{\nu}$	7.76×10^8 (6.96×10^8)	7.76×10^8 (6.96×10^8)	
$F^- \rightarrow D\pi^+\ell\bar{\nu}$	5.66×10^1	3.01×10^1	
$F^- \rightarrow \bar{D}\pi\ell\bar{\nu}$	2.46×10^2	1.31×10^2	

Figure Captions

Fig. 1 Spectrum $d\Gamma/dx$ in terms of the variable $z = 1-2k_\nu^o/m_{D^-}$ for $D^- \to \pi^+\pi^- e\bar{\nu}$ showing resonant part (dashed line) and continuum part (dotted line). λ is taken to be 1.2.

Fig. 2 Typical spectrum $d\Gamma/dx$ (for $\bar{D} \to K^+\pi\ell\bar{\nu}$) showing small lepton mass effect (μ curve is dashed line). λ is taken to be 1.2.



e (resonance)
e (continuum)
Total e
5
0
dΓ/dz (10^-16 GeV)
0
0.5
1.0
Z
10-81
4203B1

Fig. 1

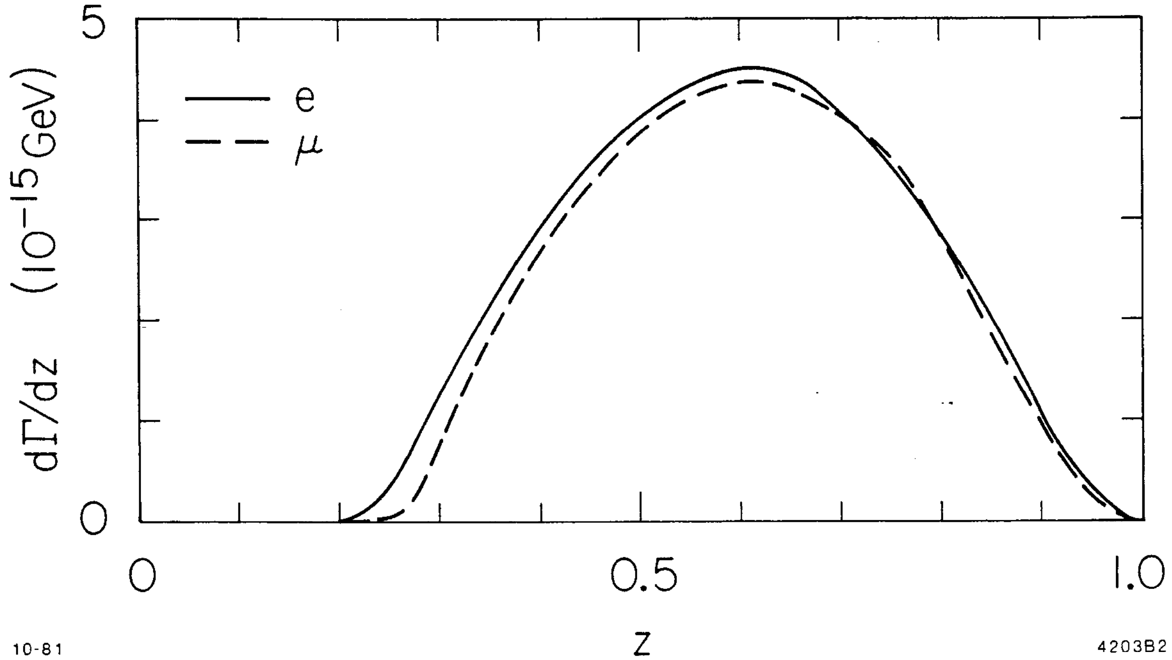


Fig. 2