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DISTINGUISHING BETWEEN DIRAC AND MAJORANA NEUTRINOS*
IN NEUTRAL CURRENT REACTIONS

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ABSTRACT

We calculate cross sections for neutral current reactions initiated by massive Dirac and Majorana neutrinos and propose such reactions as a new method of distinguishing between these types of neutrinos.

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The question of whether neutrinos are Majorana (M) fermions[1], identical to their antiparticles, or Dirac (D) fermions, distinct from their antiparticles, is a very fundamental one. For neutrinos which are massless and purely chirally coupled by V - A or V + A vertices (but not both), there is no difference between these two possibilities; that is, a massless, two-component chiral Weyl neutrino is equivalent to a massless M neutrino. Furthermore, as one would expect, the massless limit is a smooth one. Thus, chirally coupled D and M neutrinos with negligible masses behave in practically indistinguishable ways. It is true that if neutrinos interact by both V - A and V + A weak vertices, then the M and D cases can be distinguished, even when the neutrino mass is small or zero. However, there is at present no evidence for non V - A direct charged-current (CC) or neutral current (NC) couplings of neutrinos.

In this letter, we shall calculate the cross sections for NC reactions (including, in certain leptonic channels, CC contributions) initiated by D and M (anti)neutrinos and propose such reactions as a possible new method for distinguishing between these two types of particles. In particular, we shall show that even if they have only V - A couplings, D and M neutrinos behave very differently under the NC weak interactions if their masses are non-negligible. It should be stressed that the latter condition can be met even if the neutrinos are ultrarelativistic, as will be explained below.

1. Purely CC reactions can also be used to distinguish between D and M neutrinos (assuming that the respective reactions are allowed by charge conservation): an incident D "$\nu_\mu(\bar{\nu}_\mu)$"(see below) will produce only a $\mu^-(\mu^+)$, whereas an M $\nu_\mu = \bar{\nu}_\mu$ will produce a μ^- or μ^+, depending on its spin orientation and the Lorentz structure of the CC interaction. However, CC reactions have several shortcomings in this regard. The D-M differences are greatest

for relatively massive neutrinos. Thus, in particular, they would probably be clearest for $m_{\nu_i} > 0.52$ MeV, so that $i \neq$ i or 2 (i.e., ν_i cannot be the primary mass eigenstate in ν_e or ν_μ)[F1]. Hence, for these ν_i, which would be optimal for the test, the mixing-angle favored CC reaction could be kinematically suppressed or forbidden, and there would be very severe mixing-angle suppression of reactions such as $\overset{(-)}{\nu}_i d \to \ell^- pp$ or $\overset{(-)}{\nu}_i d \to \ell^+ nn$, where $\ell = e$ or μ, given the constraints that have been derived on the lepton mixing matrix for massive neutrinos.[2] In contrast, NC reactions do not suffer from either of these drawbacks.

2. We proceed to consider NC processes. We take the neutrinos to have only V - A direct weak couplings, in accord with the absence of any evidence to the contrary and with the successful standard $SU(2)_L \times U(1)$ theory. Lepton mixing has been discussed before[3]. The decays that yield the weak eigenstates ν_ℓ, $\ell = e, \mu, \tau\ldots$really consist of separate decays into the subset of all mass eigenstates $\{\nu_i\}$, $i = 1,..n$ allowed by phase space[2]. These $\overset{(-)}{\nu}_i$ propagate individually to the detector. Since we shall be studying reactions in which the neutrino mass is not negligible, and the subtleties of the approach to coherence[4] are not relevant, it is obvious that we must take the incident particle to be one of the mass eigenstates $\overset{(-)}{\nu}_i$, rather than $\overset{(-)}{\nu}_\ell$.

The difference between the NC D and M neutrino cross sections arises from the fact that if ψ_{ν_i} is an M field, the vector part of the neutrino current $\bar{\psi}_{\nu_i}\gamma_\lambda(1 - \gamma_5)\psi_{\nu_i}$ vanishes identically, while the axial vector part gives rise to a matrix element twice as large as in the case where ψ_{ν_i} is a D field. Empirically, neutrinos which are light (defined here to mean that $m_{\nu_i} \ll m_e$) and relativistic are almost completely chiral. That is, they contain a state preparation factor $(1 - \gamma_5)/2$, so that the matrix elements of $\bar{\psi}_{\nu_i}\gamma_\lambda(1-\gamma_5)\psi_{\nu_i}$ for the D and M cases become identical. This is the reason why it is necessary to use heavy (and possibly relativistic) or nonrelativistic neutrinos to see the difference.

A comparative study of D and M cross sections is worthwhile because of the theoretical insights that it yields. However, the results of the study go farther; they may provide a new and hitherto unexploited method to determine whether a neutrino is of D or M type, using experiments which, although quite demanding, do not require very high energies or new accelerators. We suggest several specific ways of obtaining fluxes of massive $\overset{(-)}{\nu}_i$'s which are tagged; i.e., have known momenta and polarizations. First, assume that in the peak search test proposed in Ref. 2 for $M_{\ell 2}$ decays (where $M = \pi$ or K and $\ell = e$ or μ), one discovers a peak in $|\vec{p}_\ell|$ corresponding to the emission of a heavy but rarely produced ν_i. From the measurement of $|\vec{p}_\ell|$, one then determines m_{ν_i} and, using the testable assumption of V − A couplings, the polarization of the $\overset{(-)}{\nu}_i$. In general, the polarization $P(\overset{(-)}{\nu}_i)$ will be significantly different from the $m_{\nu_i} = 0$ value of -1 $(+1)$; in particular, if $m_{\nu_i} > m_\ell$ (which is possible in $K_{\mu 2}$ and M_{e2} decays) $\mathrm{sgn}(P(\overset{(-)}{\nu}_i)) = -\mathrm{sgn}(P(\overset{(-)}{\nu}_i; m_{\nu_i} = 0))$. This fact is quite important for distinguishing between D and M neutrinos. Secondly, having determined m_{ν_i}, one can set up an experiment involving $M_{\ell 2}$ decay in flight, with $\vec{p}_{\nu_i}$ tagged by detection of the accompanying $\ell^\pm$. For a heavy $\overset{(-)}{\nu}_i$, one may wish to boost its laboratory energy by selecting events in which it was emitted roughly forward parallel to $(\hat{p}_M)_{lab}$. For light $\overset{(-)}{\nu}_i$, one would set $|\vec{p}_M|_{lab}$ and the direction of $\ell^\pm$ so as to select decays in which the $\overset{(-)}{\nu}_i$ is emitted backward and thus has very low $(\beta_{\nu_i})_{lab}$.

We consider first the leptonic reactions $\overset{(-)}{\nu}_i e \to \overset{(-)}{\nu}_i e$, since these illustrate our ideas in a simple context and provide a basis for our later analysis of more practical (semileptonic) reactions. Neglecting small mixing effects and assuming single Z (and, for i = 1, also W) exchange, we can write the effective Lagrangian for these processes as

$$-\mathcal{L}(\overset{(-)}{\nu}_i e \to \overset{(-)}{\nu}_i e) = \frac{G_0}{\sqrt{2}} \Big\{ [\bar{\psi}_{\nu_i} \gamma_\lambda (1-\gamma_5) \psi_{\nu_i}][\bar{\psi}_e \gamma^\lambda (g_V^Z - g_A^Z \gamma_5) \psi_e]$$

$$+ \delta_{i1} [\bar{\psi}_e \gamma_\lambda (1-\gamma_5) \psi_{\nu_i}][\bar{\psi}_{\nu_i} \gamma^\lambda (1-\gamma_5) \psi_e] \Big\}$$

$$= \frac{G_0}{\sqrt{2}} [\bar{\psi}_{\nu_i} \gamma_\lambda (1-\gamma_5) \psi_{\nu_i}][\bar{\psi}_e \gamma^\lambda (g_V^{(i)} - g_A^{(i)} \gamma_5) \psi_e] \tag{2.1}$$

where $g_X^{(i)} = g_X^Z + \delta_{i1}$, $X = V,A$. Existing data are consistent with the assignments of the standard $SU(2)_L xU(1)$ electroweak theory: $g_A^Z = -1/2$ and $g_V^Z = -1/2 + 2\sin^2\theta_W \simeq -0.04$ for $\sin^2\theta_W \simeq 0.23$. Define momenta and spins according to

$$\overset{(-)}{\nu}_i (p_\nu, s_\nu) + e(p_e) \to \overset{(-)}{\nu}_i (p_\nu') + e(p_e') \tag{2.2}$$

and a coordinate system according to Fig. 1. Thus, the polarization vector of the incident neutrino in its rest frame is

$$\vec{s}_\nu = |\langle \vec{\sigma}_\nu \rangle| (\sin\theta_s, 0, \cos\theta_s)$$

$$\equiv (|\vec{s}_\perp|, 0, s_\parallel) \tag{2.3}$$

Further, define $s \simeq (p_\nu + p_e)^2$. In the case of D neutrinos, we shall write the cross sections for incident ν_i; in both the D and M cases, the corresponding cross sections for $\bar{\nu}_i$ are given by

$$d\sigma(\bar{\nu}_i e \to \bar{\nu}_i e) = d\sigma(\nu_i e \to \nu_i e\ ;\ s_\nu \to -s_\nu,\ g_V^{(i)} g_A^{(i)} \to -g_V^{(i)} g_A^{(i)}) \tag{2.4}$$

From the definition $\psi_{\nu_i^M} = (\psi_{\nu_i^M})_c$, it follows that

$$d\sigma(\nu_i^M e \to \nu_i^M e) = d\sigma(\nu_i^M e \to \nu_i^M e \; ; \; s_\nu \to -s_\nu, \; g_V^{(i)} g_A^{(i)} \to -g_V^{(i)} g_A^{(i)}) \tag{2.5}$$

The differential cross sections are most simply expressed in the center-of-mass frame:

$$\begin{aligned}
\frac{d\sigma}{d\Omega}(\nu_i^D e \to \nu_i^D e) = \left(\frac{G_0^2}{8\pi^2 s}\right)\Big[& \Big\{ (E_{\nu_i} E_e + |\vec{p}|^2)^2 (g_V^{(i)} + g_A^{(i)})^2 \\
& + (E_{\nu_i} E_e + |\vec{p}|^2 \cos\theta)^2 (g_V^{(i)} - g_A^{(i)})^2 + m_e^2 (E_{\nu_i}^2 - |\vec{p}|^2 \cos\theta)(g_A^{(i)2} - g_V^{(i)2}) \Big\} \\
- |\vec{p}| \Big\{ & s^{1/2} (E_{\nu_i} E_e + |\vec{p}|^2) s_\parallel (g_V^{(i)} + g_A^{(i)})^2 \\
& + (E_{\nu_i} E_e + |\vec{p}|^2 \cos\theta)\left[(E_e + E_{\nu_i}\cos\theta) s_\parallel + m_{\nu_i} |\vec{s}_\perp| \sin\theta\cos\varphi\right](g_V^{(i)} - g_A^{(i)})^2 \\
& + m_e^2 \left[E_{\nu_i}(1-\cos\theta) s_\parallel - m_{\nu_i} |\vec{s}_\perp| \sin\theta\cos\varphi \right](g_A^{(i)2} - g_V^{(i)2}) \Big\} \Big]
\end{aligned} \tag{2.6}$$

and

$$\begin{aligned}
\frac{d\sigma}{d\Omega}(\nu_i^M e \to \nu_i^M e) = \left(\frac{G_0^2}{4\pi^2 s}\right)\Big[& \Big\{ (E_{\nu_i} E_e + |\vec{p}|^2)^2 + (E_{\nu_i} E_e + |\vec{p}|^2 \cos\theta)^2 \\
& + m_{\nu_i}^2 (E_e^2 - |\vec{p}|^2 \cos\theta) \Big\} (g_V^{(i)2} + g_A^{(i)2}) \\
& + m_e^2 (E_{\nu_i}^2 - |\vec{p}|^2 \cos\theta + 2 m_{\nu_i}^2)(g_A^{(i)2} - g_V^{(i)2}) \\
& - 2 g_V^{(i)} g_A^{(i)} |\vec{p}| \left(2 E_{\nu_i} E_e + |\vec{p}|^2 (1+\cos\theta)\right) \Big\{ E_{\nu_i} s_\parallel (1-\cos\theta) \\
& \qquad - m_{\nu_i} |\vec{s}_\perp| \sin\theta\cos\varphi \Big\} \Big]
\end{aligned} \tag{2.7}$$

where $E_{\nu_i} = p^o_{\nu_i} = p'^o_{\nu_i}$, $E_e = p_e{}^o = p_e'{}^o$, and $|\vec{p}| = |\vec{p}_{\nu_i}| = |\vec{p}_e| = |\vec{p}'_{\nu_i}| = |\vec{p}'_e|$ refer to center-of-mass quantities. Observe that in the D cross section s_ν terms can be either of the form $(V_\nu A_\nu)(V_e^2$ or $A_e^2)$ or of the form $(V_\nu^2$ or $A_\nu^2)$ $(V_e A_e)$, in an obvious shorthand. However, since $V_\nu = 0$ in the M case, the s_ν terms in the M cross section must be, and are, of the form $A_\nu^2(V_e A_e)$ only. Analysis of Eqs. (2.6) and (2.7) shows that in general they are significantly different.

For $m_{\nu_i} = 0$,

$$\frac{d\sigma}{d\Omega}\left(\nu_i^D e \to \nu_i^D e \; ; \; m_{\nu_i} = 0\right) = \frac{G_0^2 s}{16\pi^2}\left(\frac{1-s_{\parallel}}{2}\right)(1-\delta_e)^2\Big[\left(g_V^{(i)} + g_A^{(i)}\right)^2$$
$$+ \left(\cos^2\frac{\theta}{2} + \delta_e \sin^2\frac{\theta}{2}\right)^2\left(g_V^{(i)} - g_A^{(i)}\right)^2 + 2\delta_e \sin^2\frac{\theta}{2}\left(g_A^{(i)2} - g_V^{(i)2}\right)\Big]$$
$$\equiv \left(\frac{1-s_{\parallel}}{2}\right)\frac{d\sigma}{d\Omega}\left((\nu_i^D)_L e \to (\nu_i^D)_L e \; ; \; m_{\nu_i} = 0\right) \tag{2.8}$$

where $\delta_a \equiv m_a^2/s$. Further,

$$\frac{d\sigma}{d\Omega}\left(\nu_i^M e \to \nu_i^M e \; ; \; m_{\nu_i} = 0\right) = \left(\frac{1-s_{\parallel}}{2}\right)\frac{d\sigma}{d\Omega}\left((\nu_i^D)_L e \to (\nu_i^D)_L e \; ; \; m_{\nu_i} = 0\right)$$
$$+ \left(\frac{1+s_{\parallel}}{2}\right)\frac{d\sigma}{d\Omega}\left((\bar{\nu}_i^D)_R e \to (\bar{\nu}_i^D)_R e \; ; \; m_{\nu_i} = 0\right) \tag{2.9}$$

With the assumption that the neutrinos have only V - A weak couplings, $(s_{\parallel})_\nu = -(s_{\parallel})_{\bar{\nu}} = -1$ in the Dirac case. From (2.8) and (2.9), one sees that the cross sections for ν_i^D and $(\nu_i^M)_L$ are then identical, as are those for $\bar{\nu}_i^D$ and $(\nu_i^M)_R$. This is in accord with the general theorem on the equivalence of massless, chiral D and M neutrinos noted earlier. Furthermore, it is obvious that these cross section identities also apply approximately to the case of non-zero neutrino mass if $m_{\nu_i}^2/s \ll 1$ and $m_{\nu_i}^2/m_e^2 \ll 1$.

Now consider a $\overset{(-)}{\nu}_i$ with $m^2_{\nu_i} \gg m^2_e$ emitted in M_{e2} decay at or nearly at rest[F2]. Because $(s_\parallel)_{\nu_i} = -(s_\parallel)_{\bar{\nu}_i}$ is very close to 1 rather than the $m_{\nu_i} = 0$ value of -1, the D NC (and CC) cross section is severely suppressed, and consequently, there is a drastic difference between it and the corresponding M cross section. It is this type of situation where m_{ν_i} is not negligible, even though the neutrino is ultrarelativistic. In the limit $|\vec{p}| \gg m_{\nu_i}, m_e$,

$$\frac{d\sigma}{d\Omega}(\nu_i^D e \to \nu_i^D e) = \frac{G_0^2 |\vec{p}|^2}{8\pi^2}\Big[(1-s_\parallel)(g_V^{(i)} + g_A^{(i)})^2 + (g_V^{(i)} - g_A^{(i)})^2 \times$$

$$\Big\{ (1-s_\parallel)\cos^4\frac{\theta}{2} - \frac{m_{\nu_i}}{2|\vec{p}|}|\vec{s}_\perp| \cos^2\frac{\theta}{2}\sin\theta\cos\varphi + \frac{1}{8}\sin^2\theta\Big[(1-s_\parallel)\frac{m_e^2}{|\vec{p}|^2} + \frac{m_{\nu_i}^2}{|\vec{p}|^2}\Big]\Big\}$$

$$+ \frac{1}{2}(1-s_\parallel)\frac{m_e^2}{|\vec{p}|^2}\sin^2\frac{\theta}{2}\,(g_A^{(i)2} - g_V^{(i)2})\Big] \qquad (2.10)$$

For a wide range of m_{ν_i}, $(1 - s_\parallel)|\vec{p}|^2 \ll m^2_{\nu_i}$, so that

$$\frac{d\sigma}{d\Omega}(\nu_i^D e \to \nu_i^D e) \simeq \frac{G_0^2 m_{\nu_i}^2}{64\pi^2}(g_V^{(i)} - g_A^{(i)})^2 \sin^2\theta \qquad (2.11)$$

Then

$$\frac{\sigma(\nu_i^M e \to \nu_i^M e)}{\sigma(\overset{(-)}{\nu}{}_i^D e \to \overset{(-)}{\nu}{}_i^D e)} \simeq \frac{6S}{m_{\nu_i}^2}\left[1 + \frac{1}{3}\left(\frac{g_V^{(i)} \pm g_A^{(i)}}{g_V^{(i)} \mp g_A^{(i)}}\right)^2\right]$$

$$\simeq \frac{8S}{m_{\nu_i}^2} \gg 1 \qquad (2.12)$$

which is a huge ratio. However, $\sigma(\nu_i^M e \to \nu_i^M e)$ is rather small, rendering the test difficult.

Accordingly, we next proceed to semileptonic reactions and consider first $\overset{(-)}{\nu}_i + N \to \overset{(-)}{\nu}_i + N$, where N = p or n (in practical experiments, N = p).

Now for neutrinos from $M_{\ell 2}$ decays at rest, the maximum momentum transfer, $(-q^2)_{max}$, is sufficiently small, so that for illustrative purposes one can neglect the induced structure terms $i\sigma_{\lambda\rho}q^{\rho}/(2m_N)$ and $q_\lambda \gamma_5/m_N$ in the hadronic matrix element and take

$$\langle N'; p_N' | J_\lambda^Z | N; p_N \rangle \simeq \bar{u}(p_N') \gamma_\lambda \left(g_{V,N}^Z(0) - g_{A,N}^Z(0)\gamma_5 \right) u(p_N) \tag{2.13}$$

where

$$g_{V,N}^Z(0) = \left(\tfrac{1}{2} - 2\sin^2\theta_W\right) N_p - \tfrac{1}{2} N_n \tag{2.14}$$

and

$$g_{A,N}^Z(0) = \tfrac{1}{2}\left(N_p - N_n\right) g_A(0)$$

with $g_A(0) \simeq 1.25$. (2.15)

One can then take over Eqs. (2.6) and (2.7) with the replacements $m_e \to m_N$ and $g_K^{(i)} \equiv g_{K,e}^{(i)} \to g_{K,N}^Z(0)$, $K = V,A$ to obtain the approximate cross sections for the reactions $\overset{(-)}{\nu}_i N \to \overset{(-)}{\nu}_i N$.

As in the $\overset{(-)}{\nu}_i e$ case, the D and M cross sections are, in general, significantly different and, in particular, this difference is very great if the $\nu_i(\bar{\nu}_i)$ has a polarization near to 1 (-1) when emitted in M_{e2} decay. Furthermore, $\sigma(\nu_i^M N \to \nu_i^M N) \sim 10^{-38}$ cm^2 for $E_\nu \sim 0.3$ GeV, so that if a heavy ν_i exists, the measurement of the $\nu_i N$ NC cross section should provide a method for determining if it is of D or M type. It is straightforward to calculate the full cross sections with structure terms included; the conclusions remain qualitatively the same and we omit the details. If m_{ν_i} is such that a ν_i can be emitted in $M_{\mu 2}$ decays, these may constitute a more copious source than M_{e2} decays. Although the s_ν effects are not so drastic for $M_{\mu 2}$ decays (compare Figs. 6 and 8 with 7 and 9 in Ref. 2), they still cause substantial differences between the D and M cross sections.

It is interesting to record the nonrelativistic limits of the cross sections for $\overset{(-)}{\nu}_i N \to \overset{(-)}{\nu}_i N$ reactions:

$$\lim_{|\vec{p}|\to 0} \frac{d\sigma}{d\Omega}\left(\overset{(-)}{\nu}{}_i^D N \to \overset{(-)}{\nu}{}_i^D N\right) = \frac{G_0^2\, m_{red}^{(\nu N)\,2}}{8\pi^2}\left(g^Z_{V,N}(0)^2 + 3\, g^Z_{A,N}(0)^2\right) \tag{2.16}$$

and

$$\lim_{|\vec{p}|\to 0} \frac{d\sigma}{d\Omega}\left(\nu_i^M N \to \nu_i^M N\right) = \frac{3 G_0^2\, m_{red}^{(\nu N)\,2}}{2\pi^2}\, g^Z_{A,N}(0)^2 \tag{2.17}$$

where $m_{red}^{(\nu a)} \equiv m_{\nu_i} m_a/(m_{\nu_i} + m_a)$. For the practical case $N = p$, $g_V^Z(0)^2 << g_A^Z(0)^2$ so that the M cross section is a factor of 4 larger than the D one. Unfortunately, nonrelativistic NC reactions are extremely hard to detect experimentally.

Other exclusive NC reactions such as $\overset{(-)}{\nu}_i N \to \overset{(-)}{\nu}_i N'\pi$ would also be sensitive to D-M differences.

A particularly simple semileptonic reaction is $\overset{(-)}{\nu}_i + (N_p,N_n)_{J=0} \to \overset{(-)}{\nu}_i + (N_p,N_n)_{J=0}$, where $(N_p,N_n)_{J=0}$ denotes a spin zero nucleus with N_p protons and N_n neutrons. For such a nucleus, $\langle N_p,N_n;p_N'|J_\lambda^Z|N_p,N_n;p_N\rangle \simeq g^Z_{V,N}(0) f_+(q^2) \times (p_N + p_N')_\lambda$, where $f_+(q^2)$ denotes the nuclear form factor, with $f_+(0) = 1$, and the $f_-(q^2)(p_N - p_N')_\lambda$ term is negligible by CVC. We calculate (again in the center-of-mass frame)

$$\frac{d\sigma}{d\Omega}\left(\nu_i^D + (N_p,N_n)_{J=0} \to \text{''}\right) = \frac{G_0^2\, g^Z_{V,N}(0)^2 f_+(q^2)}{8\pi^2 s} \times$$

$$\Big[\left\{ m_{\nu_i}^2 E_N^2 + (2s - m_{\nu_i}^2)|\vec{p}|^2 \cos^2\tfrac{\theta}{2} \right\}$$

$$- |\vec{p}|\, s^{1/2} \left\{ 2 s_\parallel \left(E_{\nu_i} E_N + |\vec{p}|^2\right)\cos^2\tfrac{\theta}{2} + m_{\nu_i} E_N |\vec{s}_\perp| \sin\theta\cos\varphi \right\} \Big] \tag{2.18}$$

and

$$\frac{d\sigma}{d\Omega}\left(\nu_i^n + (N_p, N_n)_{J=0} \rightarrow \text{''} \right) = \frac{G_o^2 \, g_{V,N}^Z(0)^2 \, f_+(q^2)^2}{2\pi^2} |\vec{p}|^2 \cos^2\frac{\theta}{2}$$

(2.19)

Since only the vector part of the nuclear neutral current contributes, the D cross section is of the form $(V_\nu^2 + A_\nu^2 + V_\nu A_\nu s_\nu) V_N^2$ in an obvious shorthand, while the M cross section is simply of the form $A_\nu^2 V_N^2$, since in the M case, $V_\nu = 0$. It follows that $d\sigma_M/d\Omega$ has no dependence on s_ν.

For $|\vec{p}| = 0$, there is a striking difference between the D and M cross sections. While the former approaches the constant $G_o^2 m_{red}^{(\nu N)2} g_{V,N}^Z(0)^2/(8\pi^2)$, the latter vanishes. This can easily be understood; in the limit $|\vec{p}| \rightarrow 0$, the M neutrino current $\bar{\psi}_{\nu_i}\gamma_\lambda\gamma_5\psi_{\nu_i} \propto \delta_{\lambda j}\chi_{\nu_i}^\dagger \sigma_j \chi_{\nu_i}$. But the matrix element of the nuclear neutral current is $\propto (p_N + p_N')^0 = 2m_N$, so that there is no remaining three-vector to contract with the σ_j, and hence the amplitude must vanish.

If $m_{\nu_i} = 0$ or is negligibly small, there is again an equivalence between the D and M cross sections:

$$\frac{d\sigma}{d\Omega}\left(\nu_i^n + (N_p, N_n)_{J=0} \rightarrow \text{''} ; m_{\nu_i} = 0 \right) = \frac{G_o^2 \, s \, g_{V,N}^Z(0)^2 \, f_+(q^2)^2}{8\pi^2} (1 - s_N)^2 \cos^2\frac{\theta}{2}$$

$$= \left(\frac{1-s_{11}}{2}\right) \frac{d\sigma}{d\Omega}\left((\nu_i^D)_L + (N_p, N_n)_{J=0} \rightarrow \text{''} ; m_{\nu_i} = 0 \right) + \left(\frac{1+s_{11}}{2}\right) \frac{d\sigma}{d\Omega}\left((\bar{\nu}_i^D)_R + (N_p, N_n)_{J=0} \rightarrow \text{''} ; m_{\nu_i} = 0 \right)$$

(2.20)

Our results on $\overset{(-)}{\nu}_i e \rightarrow \overset{(-)}{\nu}_i e$ can easily be transcribed to $\overset{(-)}{\nu}_i q \rightarrow \overset{(-)}{\nu}_i q$ to treat the case of deep inelastic scattering. To leading order, in valence quark approximation

$$\frac{d\sigma}{dx\,dy}\left(\nu_i^D(\bar\nu_i^D)N \to \nu_i^D(\bar\nu_i^D)X\right) = \frac{G_0^2 s}{\pi}\left(\frac{1 \mp s_{\parallel}}{2}\right)\sum_j x q_j(x)\left[\left(\frac{g^Z_{V,j} \pm g^Z_{A,j}}{2}\right)^2 + (1-y)^2\left(\frac{g^Z_{V,j} \mp g^Z_{A,j}}{2}\right)^2\right] \tag{2.21}$$

while

$$\frac{d\sigma}{dx\,dy}\left(\nu_i^M N \to \nu_i^M X\right) = \frac{G_0^2 s}{\pi}\sum_j x q_j(x)\left[\left(\frac{1-s_{\parallel}}{2}\right)* \left\{\left(\frac{g^Z_{V,j}+g^Z_{A,j}}{2}\right)^2 + (1-y)^2\left(\frac{g^Z_{V,j}-g^Z_{A,j}}{2}\right)^2\right\} + \left(\frac{1+s_{\parallel}}{2}\right)\left\{\left(\frac{g^Z_{V,j}-g^Z_{A,j}}{2}\right)^2 + (1-y)^2\left(\frac{g^Z_{V,j}+g^Z_{A,j}}{2}\right)^2\right\}\right] \tag{2.22}$$

where x and y are the standard Bjorken scaling variables, and the sum over the j'th flavor quark distributions depends on N. Since $(s_{\parallel})_\nu = -(s_{\parallel})_{\bar\nu}$ could be substantially different from -1, there would in general be a significant difference between the D and M cross sections at arbitrarily high energies. If $s \gg m^2_{\nu_i}, m_N^2$, but m_{ν_i} is sufficiently large that $(1 - s_{\parallel})s \ll m^2_{\nu_i}$ (and $|\vec{s}_\perp| = 0$), then the analogue of Eq. (2.11) would read

$$\frac{d\sigma}{dx\,dy}\left(\nu_i^D(\bar\nu_i^D)N \to \nu_i^D(\bar\nu_i^D)X\right) \simeq \frac{G_0^2 m^2_{\nu_i}}{\pi}\, y(1-y)\sum_j q_j(x)\left(\frac{g^Z_{V,j} \mp g^Z_{A,j}}{2}\right)^2 \tag{2.23}$$

Of course, for $(\beta_{\nu_i})_{lab} \to 1$, it would become increasingly difficult to separate the heavy $\overset{(-)}{\nu}_i$ from the much larger flux of light (anti)neutrinos by the tagging (and implied precise timing) method presented above, especially in view of the nonzero momentum spread of the initial M beam.

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FOOTNOTES

F1. The transformation of the n mass eigenstates ν_i to the corresponding weak eigenstates is given by $\nu_\ell = \sum_{i=1}^{n} U_{\ell i}\nu_i$.

F2. For typical models, there is a stringent upper limit on $|U_{1i}|^2$ from bounds on neutrinoless double beta decay; see W. Haxton et al., Phys. Rev. Lett. 47 (1981) 153; M. Doi et al., Osaka preprints OS-GE-80-27, 81-29; and S. P. Rosen, in Proceedings of the Neutrino '81 Conference. In some models, however, this bound does not apply; see L. Wolfenstein, Carnegie-Mellon preprint COO-3066-180.

REFERENCES

[1] For an early discussion and references, see K. M. Case, Phys. Rev. 107 (1957) 307.

[2] R. E. Shrock, Phys. Lett. 96B (1980) 159; Phys. Rev. D24 (1981) 1232, 1275.

[3] Z. Maki, N. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28 (1962) 870; B. Pontecorvo, Zh. Eks. Teor. Fiz. 53 (1967) 1717 [J.E.T.P. 26 (1968) 984]. For the current experimental status, see C. Baltay, in Proceedings of the Neutrino '81 Conference.

[4] B. Kayser, Phys. Rev. D24 (1981) 110.

FIGURE CAPTION

1. Coordinate system for neutrino reactions

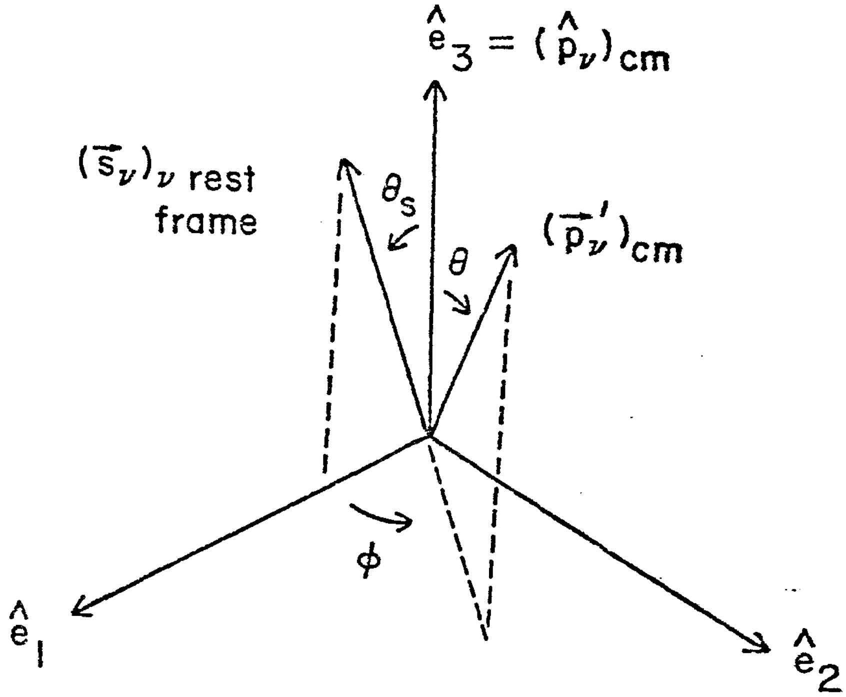
$\hat{e}_3 = (\hat{p}_\nu)_{cm}$
$(\vec{s}_\nu)_{\nu\ \text{rest frame}}$
θ_s
θ
$(\vec{p}_\nu{}')_{cm}$
ϕ
$\hat{e}_1$
$\hat{e}_2$

Fig. 1