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Summing Soft Emission in QCD*

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ABSTRACT

We analyze multiple soft gluon emission in QCD. Leading and next-to-leading contributions to the quark form factor are resummed in impact parameter space at two-loop level. The cross section for $e^+e^- \to a + b + X$ is also given.

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Semi-inclusive processes [1] play an important role in perturbative QCD. Particularly interesting examples, both theoretically and experimentally, are lepton pair production in hadronic reactions (Drell-Yan) with the pair transverse momentum Q_T^2 large but much smaller than the pair mass Q^2 ($\Lambda^2 << Q_T^2 << Q^2$) and electron-positron annihilation into almost acollinear hadrons. The study of such processes is closely related to the understanding of the dynamics of soft gluon bremsstrahlung.

Common feature to those processes is the presence of large logarithmic corrections which are due to the incomplete compensation between real emission of soft gluons and virtual contributions. As a result an effective quark form factor appears. In evaluating physical quantities, such large corrections must be resumed to all orders to obtain meaningful answers [2].

After the pioneering work of ref. [1] and the important paper by Parisi and Petronzio [3] in which the leading contributions were correctly summed (see also refs. [4,5]), many authors [6-10] have proposed various resummation methods. Important developments have been given in refs. [6,8-10].

In spite of all these efforts an explicit derivation of the quark form factor including next-to-leading corrections has not been performed. Such an explicit derivation is the purpose of this note. We systematically analyze the leading and next-to-leading contributions. All the different sources of corrections at two-loop level are investigated and resummed. The resulting formula eq. (15) is given in the impact parameter space.

Bassetto, Ciafaloni and Marchesini [11], extending the jet-calculus approach [12] for inclusive many-parton distributions, have studied the impact of soft gluons by explicitly including the transverse momentum degrees of freedom into the Altarelli-Parisi [13] (AP) equation. The equation for the non-singlet part is in light-like gauge

$$Q^2 \frac{\partial}{\partial Q^2} D(Q^2,p_T,x) \tag{1}$$

$$= \int_x^1 \frac{dz}{z} \left[\frac{\alpha_s(Q^2)}{2\pi} P(z)\right]_+ \int \frac{d^2q_T}{\pi} \delta\left(z(1-z)\; Q^2 - q_T^2 - Q_o^2\right) D\left(Q^2, p_T - \frac{x}{z} q_T, \frac{x}{z}\right),$$

where the $[\;]_+$ notation indicates the regularization procedure and Q_o^2 is an infrared cut-off parameter [11]. In the limit $z \to 1$ the distribution is dominated by soft gluon emissions.* It has been shown [14] that in this limit of phase space it is important to use the correct scale in α_s. In fact the simple rescaling of the argument, $Q^2 \to Q^2(1-z)$ in α_s, resums [14, 18] the large kinematical corrections due to the soft gluon radiation. In the following we use the rescaled coupling constant in eq. (1).

In order to solve eq. (1) we make a Fourier transform into impact parameter (b_T) space [3,4,11]. Defining $D(Q^2,b,x) \equiv \int d^2p_T e^{-i\frac{1}{x}b_T \cdot p_T} \times D(Q^2,p_T,x)$ eq. (1) becomes

$$Q^2 \frac{\partial}{\partial Q^2} D(Q^2,b,x) \tag{2}$$

$$= \frac{C_F}{2\pi} \int_x^1 \frac{dz}{z} \int dq^2 \left[\alpha\left(Q^2(1-z)\right)\frac{1+z^2}{1-z}\right]_+ \delta\left(z(1-z)Q^2 - q^2\right) J_o\left(\frac{bq}{z}\right) D\left(Q^2, b, \frac{x}{z}\right)$$

with J_o the Bessel function of the first kind and $|b_T| = b$, $|q_T| = q$.** Using soft gluon limit approximation (i.e., $z \sim 1$ and $Q^2 \gg q^2$) eq. (2) can be solved to give

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*Here and in the following we only consider non-singlet channel which gives the largest contributions in the region we are interested in.

**In eq. (2) and in the following we put $Q_o^2 = 0$. Infrared singularities are regularized dimensionally.

$$D(Q^2,b,x) = D\left(Q_1^2,b,x\right)e^{T_1\left(Q^2,Q_1^2,b\right)} . \tag{3}$$

T_1 is*

$$T_1\left(Q^2,Q_1^2,b\right) = -\frac{C_F}{\pi}\int_{Q_1^2}^{Q^2}\frac{dk^2}{k^2}\int_0^{k^2}\frac{dq^2}{q^2}\,\alpha_s(q^2)\left(1-\frac{q^2}{k^2}+\frac{q^4}{2k^4}\right)\left[1-J_o(bq)\right]. \tag{4}$$

The integral of the first term in the parentheses in eq.(4) corresponds to the result obtained in ref. [3].

The scale Q_1^2 which must be larger than the lower limit of the perturbative analysis $\left(Q_1^2 > \Lambda^2\right)$, can be chosen arbitrarily and b is free to vary from 0 to ∞. To absorb the possible large corrections of the type $\ln Q_1^2 b^2$ we chose $Q_1^2 = \frac{1}{b^2}$. This choice with the condition $Q_1^2 > \Lambda^2$ fixes the upper limit on b^2 to be $b^2 < \frac{1}{\Lambda^2}$. Equation (3) then becomes

$$D(Q^2,b,x) = D\left(\frac{1}{b^2},b,x\right)e^{T_1(Q^2,b)}$$

and

$$T_1(Q^2,b) = -\frac{C_F}{\pi}\int_{\frac{1}{b^2}}^{Q^2}\frac{dq^2}{q^2}\left[\alpha_s(q^2)\left(\ln\frac{Q^2}{q^2}-\frac{3}{4}\right)+2\ln\frac{e^{\gamma_E}}{2}\,\alpha_s\!\left(\frac{1}{b^2}\right)\right] . \tag{5}$$

with γ_E the Euler constant. The condition $Q_T^2 << Q^2$ in semi-inclusive processes corresponds to $Q^2b^2 >> 1$ in b space [3, 8, 10]. The leading contribution [3] in eq. (5) is given by the sum of terms of the type $B(B/L)^n (n \geq 1)$ where $B \equiv \ln Q^2b^2$ and $L \equiv \ln Q^2/\Lambda^2$. Next-to-leading logarithmic corrections are of the type $(B/L)^n$ and the next are proportional to $1/L(B/L)^n$ etc.** In order to include the next-to-leading terms $(B/L)^n$

*The use of the exact kinematics in the integration of q^2 does not change the final result within the approximation we are considering (see next footnote).

**To this classification we will add intermediate terms of the type $\ln L(B/L)^n$ at two-loop level. See eq. (15). In eq. (5) and in the following we neglect terms of the type $(1/L)\ (B/L)^n(\gamma \geq 1,\ n \geq 0)$ and inverse powers of Q^2.

let us first examine eq. (5). By a straightforward integration it appears that some terms of the type $(B/L)^n$ are already present. However this is not the only source of such corrections. In fact, as we shall see, the inclusion of two-loop AP probability into eqs. (1) and (2) produce terms of the same order.

By using the two-loop AP probability calculated in ref. [15] (see also ref. [17]) in light-like gauge ($\overline{MS}$ scheme), eq. (1) becomes

$$Q^2 \frac{\partial}{\partial Q^2} D(Q^2,b,x) \tag{6}$$

$$= \int_x^1 \frac{dz}{z} \int dq^2 \left[\frac{C_F}{2\pi} \alpha_s \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_2(z)\right]_+ \delta\left(z(1-z)Q^2 - q^2\right) J_o\left(\frac{bq}{z}\right) D\left(Q^2,b,\frac{x}{z}\right) ,$$

where $P_2 \equiv C_F \cdot K \frac{1+z^2}{1-z}$ and $K = C_G\left(\frac{67}{18} - \frac{\pi^2}{6}\right) + N_F T_F \left(-\frac{10}{9}\right)$.

To include the corrections of the form $(B/L)^n$ it is sufficient to consider only the terms proportional to $1/(1-z)$ in two-loop AP probability. In eq. (6) the argument of α_s^2 is the rescaled one. It has been proven in ref. [18] that the rescaled coupling constant resums all the large corrections in the $z \sim 1$ region at the order $\alpha_s(Q^2(1-z))$. Similar proof has not yet been given for the higher orders in $\alpha_s(Q^2(1-z))$ which give non-dominant corrections in the $z \sim 1$ region. Here we make the assumption that the rescaled coupling is the correct choice also at order $\left[\alpha_s(Q^2(1-z))\right]^2$. Explicit calculation of order $\alpha_s^3(Q^2)$ can give a non-trivial check of the above assumption. Nevertheless the observation [1] that it is the transverse virtualness which controls semi-inclusive processes supports this choice. Solving eq. (6) we have

$$D(Q^2,b,x) = D\left(\frac{1}{b^2},b,x\right) e^{T_1(Q^2,b) + T_2(Q^2,b)} \tag{7}$$

with

$$T_2(Q^2,b) = -\frac{C_F}{\pi}\frac{K}{2\pi}\int_{\frac{1}{b^2}}^{Q^2} \frac{dq^2}{q^2}\,\ell n\,\frac{Q^2}{q^2}\,\alpha_s^2(q^2) \quad . \tag{8}$$

Let us consider now the process $e^+e^- \to a+b+X$. Due to the factorization of soft bremsstrahlung [9,19] the cross section is written in the following form (fig. (1))

$$\frac{1}{\sigma}\frac{d\sigma}{dx_1\,dx_2\,dQ_T^2} \tag{9}$$

$$= \frac{1}{2}\int db\, b\, J_o(bQ_T)\left[D_q^a\,(Q^2,b,x_1)\,D_{\bar{q}}^b\,(Q^2,b,x_2) + (q \leftrightarrow \bar{q})\right] S_D(Q^2,b)$$

where Q_T^2 ($<<Q^2$) is the relative transverse momentum of hadrons a and b. S_D (Q^2,b) represents the set of 2PI diagrams including external hard vertices. Using the light-like gauge with the gauge vector being parallel to the $\bar{q}$ momentum, the quark distribution D_q is given by eq. (7). Due to this choice of the gauge vector the expression for $D_{\bar{q}}$ is different and the two evolutions of q and $\bar{q}$ become asymmetric. In fact the soft gluons attached to $\bar{q}$ are suppressed [20] and only the collinear singularities remain. The evolution for the antiquark can be computed in an analogous way to the quark

$$D_{\bar{q}}\,(Q^2,b,x) = D_{\bar{q}}\left(\frac{1}{b^2},b,x\right) e^{\bar{T}(Q^2,b)}$$

with

$$\bar{T}(Q^2,b) = -\frac{C_F}{4\pi}\int_{\frac{1}{b^2}}^{Q^2} \frac{dq^2}{q^2}\,\alpha_s(q^2) \tag{10}$$

$S_D(Q^2,b)$ can be calculated in the same way as for the coefficient function in deep inelastic scattering [15,16]. One has

$$S_D(Q^2,b) = 1 + C_F\,\frac{\alpha_s(Q^2)}{\pi}\,\ell n\,Q^2 b^2 + (\text{const. term}) + \mathcal{O}(\alpha_s^2) \tag{11}$$

The large logarithm $\ell n\,Q^2 b^2$ is due to the choice of Q^2 as the renormalization point. To get rid of such term, which can spoil the perturbative expansion, eq. (11) must be resummed to all orders. This may be performed by changing the renormalization point from Q^2 to $\frac{1}{b^2}$. Such procedure suggests*

$$S_D\,(Q^2,b) = \left(1 + \mathcal{O}\left(\alpha_s(\tfrac{1}{b^2})\right)\right) e^{\frac{C_F}{\pi}\int_{\frac{1}{b^2}}^{Q^2} \frac{dq^2}{q^2}\,\alpha_s\,(q^2)} \tag{12}$$

Now we combine eqs. (7), (10) and (12) into eq. (9). The result is

$$\frac{1}{\sigma}\,\frac{d\sigma}{dx_1\,dx_2\,dQ_T^2} \tag{13}$$

$$= \frac{1}{2}\int db\,b\,J_o(bQ_T)\left[D_q^a\left(\frac{1}{b^2},b,x_1\right) D_{\bar{q}}^b\left(\frac{1}{b^2},b,x_2\right) + (q \leftrightarrow \bar{q})\right] e^T$$

where exp [T] represents the effective quark form factor with

$$T \equiv T_1 + T_2 + \bar{T} + \frac{C_F}{\pi}\cdot\int_{\frac{1}{b^2}}^{Q^2} \frac{dq^2}{q^2}\,\alpha_s\,(q^2),\ \text{i.e.},$$

*We presume that eq. (12) has to be preferred to, e.g., the naive exponentiation of eq. (11). A rigorous derivation of eq. (12) would need more detailed analysis.

$$T = -\frac{C_F}{\pi}\int_{\frac{1}{b^2}}^{Q^2} \frac{dq^2}{q^2}\left[\ell n\, \frac{Q^2}{q^2}\left[\alpha_s(q^2) + \frac{K}{2\pi}\,\alpha_s^2(q^2)\right] + 2\ell n\, \frac{e^{\gamma_E}}{2}\,\alpha_s\left(\frac{1}{b^2}\right) - \frac{3}{2}\,\alpha_s(Q^2)\right] \tag{14}$$

Also the running coupling constant must be considered at two-loop level,

$$\alpha_s(q^2) = \frac{1}{\beta_o \ell n\, \frac{q^2}{\Lambda^2}} - \frac{\beta_1 \ell n(\ell n \frac{q^2}{\Lambda^2})}{\beta_o^3\, \ell n^2 \frac{q^2}{\Lambda^2}}\,, \text{ with } \beta_o = \frac{(33 - 2N_F)}{12\pi} \text{ and } \beta_1 = \frac{(153 - 19N_F)}{24\pi^2} \quad .$$

Using the above α_s, the explicit form of T is

$$\begin{aligned} T = {} & \frac{C_F}{\pi\beta_o}\left[L\,\ell n\left(1 - \frac{B}{L}\right) + B\right] + \frac{2C_F}{\pi\beta_o}\,K(1)\,\frac{B}{L - B} \\ & - \frac{C_F K}{2\pi^2\beta_o^2}\left[\ell n\left(1 - \frac{B}{L}\right) + \frac{B}{L - B}\right] - \frac{3C_F}{2\pi\beta_o}\,\ell n\left(1 - \frac{B}{L}\right) \\ & + \frac{C_F\beta_1}{\pi\beta_o^3}\left[\frac{B}{L - B} + \frac{L}{L - B}\,\ell n\left(1 - \frac{B}{L}\right) + \frac{B}{L - B}\,\ell n\, L \right. \\ & \left. + \frac{1}{2}\,\ell n^2\left(1 - \frac{B}{L}\right) + \ell n\left(1 - \frac{B}{L}\right)\ell n\, L\right] \end{aligned} \tag{15}$$

with $K(1) = \ell n 2 - \gamma_E$.

To summarize let us examine eq. (15). It contains all the contributions of the type $B(B/L)^n$ (leading) and $(B/L)^n$ (next-to-leading). The intermediate $\ell nL \cdot (B/L)^n$ due to the two loop form of α_s are also included. In particular the two-loop AP probability P_2 in eq. (6) is important for the correct evaluation of the $(B/L)^n$ type contributions. Equations (13) and (15) are free from large perturbative corrections as long as b is kept between $1/Q$ and $1/\Lambda$. The neglected corrections are (i) terms of order $\mathcal{O}\left[(1/L)^\gamma (B/L)^n\right]$ ($\gamma \geq 1$, $n \geq 0$), (ii) inverse powers of Q^2.

The b space formulation has the following advantages: the evolution equation (eq. (1)) can be solved analytically. Transverse momentum conservation is satisfied.* Due to this fact, small total transverse momentum configurations (including $Q_T = 0$) generated by multiple gluon emission [8] are naturally included in our result.

We will consider elsewhere the problem of taking the Fourier transform (eq. (13)) and the comparison with the PEP and PETRA experimental data.

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Figure Caption

Fig. (1) The process $e^+e^- \rightarrow a + b + X$; single (double) line represents quark or antiquark (hadron).

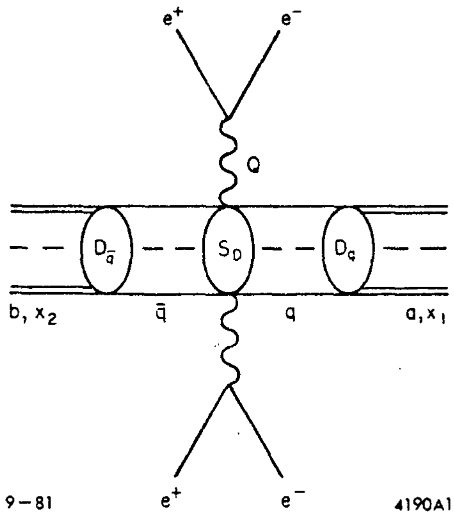

Fig. 1