SLAC-PUB-2802
September 1981
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EXCEPTIONAL GROUPS AND ELEMENTARY PARTICLE STRUCTURES*

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ABSTRACT

We construct a new finite-dimensional quantum mechanical space over the complex octonionic plane using the recently developed algebraic techniques of Jordan pairs and inner ideals. The automorphism group of this structure is $E_6 \times U(1)$, realized on precisely two E_6 irreps (27,27*), which we abstract as a (topless) model for grand unification.

I. Introduction

There are two very different ways in which the exceptional groups are currently applied in particle physics, which may be called the "algebraic approach" and the "gauge symmetry approach" to elementary particle structure.

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* Supported in part by the National Science Foundation and the U. S. Department of Energy under contract number DE-AC03-76SF00515.

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(Invited paper presented at the Tenth International Colloquium on Group Theoretical Methods in Physics, University of Kent, Canterbury, England, 31/VIII - 4/IX 1981.)

The first approach is possibly the more fundamental, and is based on the hope that the unusual nature of quark degrees of freedom can be found in algebraic models - in particular in non-associative algebras - where these strange properties are to appear naturally and not as artifacts. The algebraic approach traces its origins to the early researches of Jordan which led to the first exceptional quantum mechanical structure[1], the 26-dimensional Jordan algebra (M_3^8) of Hermitian 3×3 matrices over octonions, which has as its automorphism group the exceptional Lie group F_4. Current interest in the use of octonions, and the exceptional groups, stems from Gürsey[2], who noted that specializing one of the seven non-scalar units (to play the role of i) automatically achieves a rationale for $SU(3)^{color}$. In particular, the five exceptional Lie groups exhibit color-flavor structure:

G_2: $SU(3)^c$ F_4: $SU(3)\times SU(3)^c$ E_7: $SU(6)\times SU(3)^c$

E_6: $SU(3)\times SU(3)\times SU(3)^c$ E_8: $E_6\times SU(3)^c$

Gürsey[3] emphasized that the non-associativity of octonions may be connected with the problem of confinement.

The second approach is more heuristic and based on quantum field theory. One constructs, in the standard way, a quantum field theory of massless fermions interacting through a gauge field of massless bosons, with the Lagrangian having the gauge group symmetry G. When implemented with the techniques of spontaneous symmetry breaking, this approach has been remarkably successful in explaining and correlating an enormous amount of experimental data. The role of the exceptional groups in this program is limited, but basic, namely to suggest that the proper symmetry group is one of the five expectional groups.

In principle, these two structures based on the exceptional groups are not necessarily distinct, for one might hope that they fit together with the fundamental algebraic structure supplying a finite dimensional charge space "lying over" every space-time point, in the manner, say, of a fibered manifold. To date, no one has been able to implement this idea, and the two structures remain distinct.

The present talk will discuss both structures:

(a) For the algebraic structure we will present a new model for charge space, a complex octonionic plane having the automorphism group $E_{6,0}\times U(1)$, and realized by the reducible representation $(27)\dot{+}(27^*)$. This structure is not a projective geometry and the associated quantum mechanics has new and unusual features; for example, the validity of the superposition principle is restricted, but without the existence of superselection rules[4].

(b) For application to the gauge symmetry approach we will abstract from this algebraic model two features: (1) the automorphism group of the structure will be taken as the gauge symmetry group, and (2) the dimension of the charge space will be taken to imply that fundamental fermions fill out two irreps of $E_{6,0}$ (the (27) and (27*)). In this way, we obtain one of the current topless models for grand unification, which we shall discuss in some detail.

II. The Complex Octonionic Plane

A. Algebraic Preliminaries and New Algebraic Concepts

The Jordan algebraic approach attempted to capture the essence of the Hermitian matrix algebra of quantum mechanics by eliminating all reference to the underlying wave function concept, by focussing attention only on the algebraic properties of observables, and by eliminating the explicit use of the imaginary unit i. (This latter via the "formally real axiom": $a^2 + b^2 = 0 \Rightarrow a = b = 0$.)

The axioms for a Jordan algebra were taken to be: (1) $x \cdot y = y \cdot x$ (commutativity), and (2) $(x^2 \cdot y) \cdot x = x^2 \cdot (y \cdot x)$ (non-associativity).[1]

It is remarkable that this technique - which, by contrast to the Dirac approach, exchanges commutativity for non-commutativity and non-associativity for associativity - is essentially identical to standard quantum mechanics. The one exception, M_3^8, is the first known example of a quantum mechanics for which there is no Hilbert space, and no wave function.

Although the Jordan program began in physics, most of the interest, and developments, in Jordan algebras have been in mathematics[5]; progress here has led to fundamental changes in the basic viewpoints, changes which we will show are the key to developing the complex octonionic plane. Let us summarize now the two developments we will utilize:

(a) the concept of a quadratic Jordan algebra[6], and the related concept of inner ideals[7]; and

(b) the concepts of structural group[8] and of Jordan pairs[9].

Consider first a quadratic Jordan algebra. The idea here is to model everything on the product: $U_x(y) = xyx$, taking (for example) an associative algebra. This new product is quadratic in x and linear in y - rather than bi-linear as in the Jordan product. The axioms for quadratic Jordan algebras were given by McCrimmon:

(Q1) $U_I = Id$, (Q2) $U_x V_{y,x} = V_{x,y} U_x$, (Q3) $U_{U_x(y)} = U_x U_y U_x$,

where $V_{x,y}(z) = (U_{x+z} - U_x - U_z)(y)$ and I is the unit in J.

These axioms no doubt appear very complicated, and it is not clear that they really constitute a step forward! This is indeed the case, however, since:

(1) Nothing is lost - quadratic Jordan algebras are categorically equivalent to linear Jordan algebras whenever the latter are defined.

(2) There is a structure theory for the quadratic algebras which is closely analogous to that for associative algebras.

Let us explain this last point. For a physicist the Jordan approach is unhandy largely because it banishes the concept of wave function (more precisely, bra and ket vectors). In mathematical terms the concept of a ket vector is the concept of a (left) ideal, a subset N of the associative algebra A such that: $A \cdot n \subset N$ if $n \in N$. In a non-associative algebra there is no such concept at all. What replaces it is a concept that exists only in a quadratic algebra: the concept of an inner ideal, a subset M of a quadratic algebra J such that: $U_x(J) \subset M$ if $x \in M$.

The projective geometry of the space of n-tuples over a field F (in physical terms an n-component wave function) is isomorphic to the geometry of left ideals in the (associative) algebra of n×n matrices over F. For quadratic (non-associative) algebras, inner ideals play an equivalent role in the construction of geometries: the geometrical objects are identified with inner ideals, and the incidence relation is automatically given by set containment[7]. Actually the geometrical objects are better identified with the principal inner ideals, that is, the inner ideals B generated by a single element b in J: $U_b(J) = B$. The principal inner ideal plays the same role, in quadratic Jordan algebra, as that of the (one-sided) ideal (bra or ket vector) in the associative case.

Consider the second conceptual development: the concepts of a structural group and Jordan pairs. The automorphisms of a given physical structure are a well-known approach to the intrinsic properties of the structure. For an algebra, one studies the automorphisms which preserve the algebraic laws; accordingly, such transformations always map the unit element into itself.

How could one change the unit element? If u has an inverse, let us replace the product xy in an associative algebra A by: $xy \to xu^{-1}y$. The new unit element and its inverse are easily computed: $1^{(u)} = u, x^{-1(u)} = ux^{-1}u$.

For associative algebras this new algebra $A^{(u)}$ is, in fact, isomorphic to A but, remarkably, for non-associative algebras this shift of the unit can produce a different algebra. Such a new algebra is called an isotope $J^{(u)}$ of the original algebra J.

The desire to study not only the Jordan algebra J but all its isotopes as a single entity leads to the concepts of structural group and of Jordan pair. The structural group, Str(J), is the group of isomorphic mappings of a Jordan algebra J and its isotopes onto itself.

The automorphism group Aut(J) is the subset of such mappings fixing the unit element.

To get an intuitive understanding of the Jordan pair structure, note first that a Jordan algebra may be considered as a way to multiply symmetric matrices. Similarly, the Jordan pair is a way to multiply rectangular matrices: let V^+ be the set of m×n matrices and V^- the set of n×m matrices. Then the quadratic product: $U_x(y) = xyx$, $x \in V^{\pm}$, $y \in V^{\mp}$ is the desired multiplication.

The axioms for the Jordan pair $V = (V^+, V^-)$ have been given by Loos[9]:

JP1) $$V_{x^+,y^-}U_{x^+} = U_{x^+}V_{y^-,x^+}$$

JP2) $$V_{U_{x^+}y^-,y^-} = V_{x^+,U_{y^-}x^+}$$

JP3) $$U_{U_{x^+}y^-} = U_{x^+}U_{y^-}U_{x^+}$$

where

$$V_{x^+,y^-}(z^+) = (U_{x^+ + z^+} - U_{x^+} - U_{z^+})(y^-)$$

the same holding with the signs interchanged.

We can also get a Jordan pair by doubling a Jordan algebra, that is, we take $V^+ = V^- = J$ and the same quadratic map U defined on J. This is the construction used for the complex octonionic plane, below.

If J is a Jordan algebra and V = (J,J) is the Jordan pair obtained by doubling J, then there exists a one-to-one correspondence between the structural group of J, Str(J), and the automorphism group of V, Aut(V).

Jordan pairs are strictly related to three-graded Lie algebras[10]. Any three-graded Lie algebra $L = L_1 + L_0 + L_{-1}$ $([L_i, L_j] \subset L_{i+j})$ can be obtained from a Jordan pair and, conversely a Jordan pair can be obtained from L by setting:

$$L_1 = V^+, L_{-1} = V^-, V_{x^+,y^-}(z^+) = [[x^+,y^-],z^+] .$$

The map U is then obtained by:

$$U_{x^+}(y^-) = \tfrac{1}{2} V_{x^+,y^-}(x^+) .$$

We will obtain the complex-octonionic plane by a three-grading of the (complex) Lie algebra E_7; this yields the pairing of a complex M_3^8.

The construction of a quantum mechanics over a complex octonionic plane was begun by Gürsey[3], but without using the concepts of inner ideals or Jordan pairs. Let us indicate how these new concepts afford a more natural approach.

First recall that the work of Jordan, von Neumann and Wigner, was categoric; within their axioms M_3^8 is the <u>only</u> possible new quantum mechanics. Thus to go further one must drop one (or more) of their axioms: in the present case we drop the axiom of formal reality. The price one pays for this (in a direct approach, such as in Gürsey) is that the elements of the algebra become complex octonionic 3×3 matrices, which are Hermitian only under octonionic conjugation, but <u>not</u> under complex conjugation. This destroys at once the <u>raison d'etre</u> for the Jordan algebraic approach, that is, the study of <u>algebras of observables</u>!

The use of Jordan pairs nicely remedies this difficulty: the pair consists of <u>two</u> complex M_3^8 structures, and the concept of observable becomes the concept of <u>Hermitian pairs</u>[4].

Similarly the use of Jordan pairs allows one to take over the language of inner ideals and, equally importantly, the concept of a Peirce decomposition. It is through this latter concept that we are able to achieve, in a natural way, an orthocomplementation for the complex octonionic plane[4].

B. <u>The Explicit Construction</u> (cf. Ref. 4)

(1) We consider the Jordan pair obtained by doubling the Jordan algebra J of 3×3 Hermitian matrices over the complex octonions. (Hermiticity is considered with respect to the octonions only.)

$$x = \begin{pmatrix} \alpha_1 & a & \bar{b} \\ \bar{a} & \alpha_2 & c \\ b & \bar{c} & \alpha_3 \end{pmatrix}$$ where the bar is the octonionic conjugation

with $\alpha_i \in ¢$; $a,b,c \in \mathscr{C}$ (Cayley) over complex scalars.

The Jordan product is the symmetrized product: $x\cdot y = \frac{1}{2}(xy + yx)$ for $x,y \in J$, with xy the ordinary matrix product of x and y.

The quadratic operator defining the pair structure is

$$U_{x^\sigma} y^{-\sigma} = tr(x^\sigma, y^{-\sigma})x^\sigma - x^{\sigma\#} \times y^{-\sigma}$$

where $\sigma = \pm$ distinguishes the two copies of J. Here we have used the definitions:

$$tr(x,y) = tr(x\cdot y)$$

$$x^\# = x^2 - x\,tr(x) - \tfrac{1}{2}I(tr(x^2) - tr(x)^2)$$

$$x\times y = (x+y)^\# - x^\# - y^\# = 2x\cdot y - x\,tr(y) - y\,tr(x) - I(tr(x\cdot y) - tr(x)tr(y))$$

where I is the identity in J.

(2) An element (x,y) of the Jordan pair is an idempotent if $U_x(y) = x$ and $U_y(x) = y$.

Corresponding to an idempotent (x,y) one can define the Peirce decomposition of the algebra with respect to the pair (x,y). The space V = (J,J) is split into three subspaces:

$$V = V_2 + V_1 + V_0, \qquad V_i = (V_i^+, V_i^-) \ .$$

Without entering into the details, we give, as example, the Peirce decomposition with respect to the idempotent (E_1, E_1), where:

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

we get

$$V_2^+ = V_2^- = \begin{pmatrix} \mathbb{C} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V_1^+ = V_1^- = \begin{pmatrix} 0 & \mathscr{C} & \bar{\mathscr{C}} \\ \bar{\mathscr{C}} & 0 & 0 \\ \mathscr{C} & 0 & 0 \end{pmatrix}$$

$$V_0^+ = V_0^- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbb{C} & \mathscr{C} \\ 0 & \bar{\mathscr{C}} & \mathbb{C} \end{pmatrix}$$

It is possible to prove that V_2^σ and V_0^σ are principal inner ideals. We say that two non-zero idempotents are orthogonal if one is in the V_0-space of the other. An idempotent is primitive if it cannot be written as the sum of two orthogonal idempotents. It is possible to prove that a primitive idempotent (x,y) satisfying the normalization condition

$tr(x,y) = 1$ can only have the form:

$$y = x^* \quad \text{with} \quad x^\# = 0$$

where * denotes the complex conjugation.

(3) The geometry. Given a primitive normalized idempotent $x = (x^+, x^-)$, we associate to it:

i) a point $x_\square = V_2(x)$;

ii) a line $x^\square = V_0(x)$.

The incidence relation in this geometry takes the form:

$$x_\square \text{ is incident to } y^\square \text{ when } V_2(x) \subset V_0(y) \ .$$

This plane has been investigated in [4], where a quantum mechanical interpretation has been given. It belongs to the class of the so-called Hjelmslev-Moufang planes[11], and is not a projective geometry. There are lines in it, in fact, which do not intersect in a point but in a 5-dimensional manifold of points, which obey a relation called connectedness. The existence of connected points is due to the fact that the underlying algebra of the complex octonions is not a division algebra.

(4) The fact that the primitive normalized idempotents are the proper elementary objects (pure states) implies an important restriction on the structural group of J (the automorphism group of the Jordan pair V). We must require for the physically interesting transformations to map the set of all primitive normalized idempotents onto themselves. The structural group of J has 79 generators, coming from the three grading of the complex Lie algebra of E_7. Of these 79 generators, 78 form the complex Lie algebra of E_6 and 1 is the generator of complex

scale changes.

The requirement that the primitive normalized idempotents map onto themselves implies that we get the compact real form: $E_{6,0}\times U(1)$, as the physically interesting automorphism group. We can prove the following important results[4]:

a) The group $E_{6,0}$ acts transitively on points and on triples of mutually orthogonal points;

b) The maximal subgroup of $E_{6,0}$ leaving a point invariant is $SO(10)\times U(1)$;

c) The complex octonionic plane is the homogeneous space $E_{6,0}/SO(10)\times U(1)$.

III. A Model for Grand Unification Abstracted from the Complex Octonionic Plane

The use of exceptional groups as gauge groups in Grand Unified Theories (inelegantly called GUTs) is very actively pursued at the present time, and we can do little more than give a suggestion of this large field[2], calling attention to the papers on GUTs by Serdaroğlu, by King and by Sorba at this conference. We have seen that the automorphism group of the complex octonionic plane is $E_{6,0}\times U(1)$, realized on two irreps (27) and (27*) of $E_{6,0}$. Abstracting this structure as a model for gauge group symmetry, we obtain precisely the topless E_6 model for GUTs, currently studied in the literature[15]-[19]. We will discuss briefly how such a model exemplifies grand unification, and how this model in particular fits current data.

The basic idea of grand unification, as is well known, is to embed the standard model for the strong and electro-weak interactions: $SU(3)^{color} \times SU(2)^{weak} \times U(1)^{EM}$ in a simple[3] Lie group G, using the Higgs-Kibble mechanism to spontaneously break the symmetry, first to hadro-electro-weak, and then to the $SU(3)^{c} \times U(1)^{EM}$, the absolutely valid gauge symmetries.

The first model[20] for a GUT used the group SU(5) and had some immediate successes: predicted the Weinberg angle ($\sin^2\theta_W$) and the b-quark to τ-lepton mass ratio[21], as well as implying a finite lifetime for the proton[22].

More interestingly this model led to the concept of fermion families (of left-handed two-component fermions) using SU(5) irreps:

$$\text{family} = (5^*) + (10) \qquad [SU(5) \text{ irreps}]$$

$$\text{electron family} = \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \hat{d}^i_R \right\} + \left\{ \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L , \hat{u}^i_R , \hat{e}_R \right\}$$

$$\text{muon family} = \left\{ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \hat{s}^i_R \right\} + \left\{ \begin{pmatrix} c^i \\ s^i \end{pmatrix}_L , \hat{c}^i_R , \hat{\mu}_R \right\}$$

$$\text{tau family} = \left\{ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L , \hat{b}^i_R \right\} + \left\{ \begin{pmatrix} t^i \\ b^i \end{pmatrix}_L , \hat{t}^i_R , \hat{\tau}_R \right\}$$

where: i stands for the 3 colors, $\hat{\psi}_R = i\sigma_2\psi^*_R$ (transforms as left-handed) and ( ) denotes a weak $SU(2)^W$ doublet, the rest being singlets.

These three families comprise 45 left-handed fermions, for which there exists good experimental evidence for all but the t-quark (6 left-handed states).

The "standard model" consists of these three ("superfluously replicated") families, unified (at least partially) by SU(5). (The use of a reducible representation is required to eliminate triangle anomalies).

The grand unification scheme leaves the choice of the gauge group as an arbitrary element. The exceptional groups have several features which make them most attractive as gauge groups for this type of model building[4]:

(1) We have already noted the unique algebraic and geometric structures with which the exceptional groups are associated; this could conceivably be fundamental.

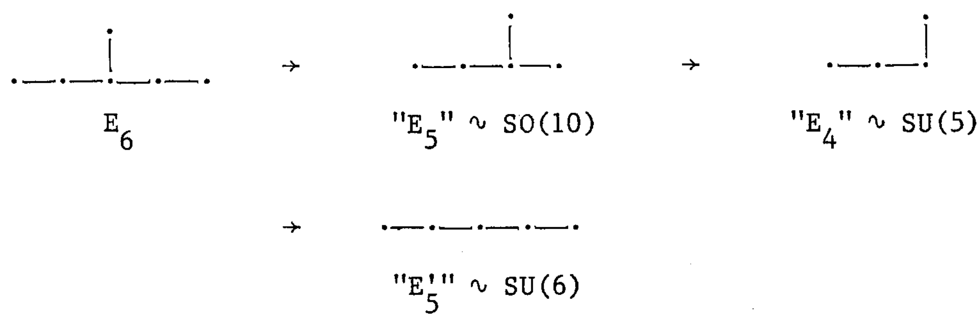
(2) The exceptional groups, as also noted, provide (through the octonions) a natural explanation of the otherwise mysterious origin of $SU(3)^{color}$. Of the five exceptional groups, the two smallest, G_2 and F_4, are eliminated as candidates for grand unification. [G_2 because it is too small for a flavor structure; F_4 because it leads to an unacceptable Weinberg angle, as well as too small a flavor group $SU(3)^{fl}$ only).]
<u>This leaves only the E-series as possibilities</u>.

(3) The universality of the quark and leptonic weak and electromagnetic charges, as well as the 1/3 integral charges for the quarks, are consequences of the group structure[24] (assuming leptons to have charge 0, $\pm$ 1 only).

(4) For E_6, the flavor group is $SU(3)_L \times SU(3)_R \times SU(3)^c$ which provides a natural explanation for chiral symmetry and an intrinsic lepton-quark-antiquark symmetry[25].

(5) The E-series of exceptional groups consists of E_6, E_7 and E_8. By removing nodes from the Dynkin diagram of E_6, one may define[15]

"continuations of the E-series" which are isomorphic to classical groups. Thus one finds:



It is interesting that all of these groups have been proposed for GUTs: SO(10) by Fritzsch and Minkowski[26]; SU(6) by Segre and Weldon[27]; and SU(5) by Georgi and Glashow[20].

(6) The exceptional groups are all triangle anomaly-free (since they all have no third rank Casimir invariant).

(7) As many have emphasized, if the t-quark is <u>not</u> discovered, the exceptional group E_6 affords the most natural explanation, with the fundamental fermions belonging to a reducible representation of <u>two</u> 27's. <u>The complex octonionic plane</u>, as we have noted, <u>leads to an explanation of the need for precisely two 27-plets</u>.

(8) If, on the other hand, the t-quark <u>is</u> discovered the exceptional group E_6 can still be used, by taking the fermions to lie in a reducible representation of <u>three</u> 27's (one for each family). This will work, but there is no explanation of why precisely three generations enter.

Let us indicate in more detail how the fundamental fermions fit into the scheme of two 27-plets as suggested if the internal space is modelled on a complex octonionic plane. The flavor-color sub-group reduction is given by:

$$E_6 \supset SU(3)_L \times SU(3)_R \times SU(3)^{color}$$
$$\rightarrow SU(2)_L^{weak} \times SU(2)_R^{neutral} \times U(1)^{EM} \times SU(3)^{color} \ .$$

The fundamental irrep (27) splits:

$$(27) = \underset{\text{leptons}}{(3^*,3,1^c)} + \underset{\text{quarks}}{(3,1,3^c)} + \underset{\text{antiquarks}}{(1,3^*,3^{*c})} \ .$$

One can easily comprehend the lepton assignments from the "nonet" diagram, Fig. 1. In the diagram a solid line indicates a weak SU(2) doublet and a dotted line indicates a "neutral SU(2)" doublet.

The lepton symbols in this diagram have their usual meaning, with $\bar{N}$ being a new $SU(2)^W$ singlet neutral lepton.

The quark assignments $(3,1,3^c)$ are given by a $(SU(2)^W)$ doublet and a singlet of left-handed colored (i) quarks:

$$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L , \ b_L^i$$

and the anti-quarks $(1,3^*,3^{*c})$ are given by three left-handed $SU(2)^W$ singlets:

$$\hat{u}_R^i \ , \ \hat{d}_R^i \ , \ \hat{b}_R^i \ .$$

(Under the neutral SU(2) subgroup $\hat{d}_R^i$ and $\hat{b}_R^i$ form a doublet.)

There is a similar assignment for the second irrep[5] with the substitutions: $e \rightarrow \mu$, $\tau \rightarrow \tau'$, $N \rightarrow N'$, $u \rightarrow c$, $d \rightarrow s$ and $b \rightarrow h$ (where h is a (heavy) charge -1/3 quark).

<u>How well does this model fit the presently known data?</u>

(1) All currently known leptons are accommodated. The model

predicts a sixth quark (h) with charge -1/3, a fourth charged lepton (τ), and several neutral leptons (some massive).

(2) The (unrenormalized) Weinberg angle is: ($\sin^2\theta_w$) = 3/8, which is satisfactory.[6]

(3) Proton decay can be accommodated either way. (Since 27 and 27^* enter, a mechanism discussed by Gell-Mann et al.[24] can be used to stabilize the proton if desired.)

(4) The scalar (Higgs) representations to be used in breaking the symmetry to effect the complicated mass structure of the fermions and gauge bosons is a very difficult problem and the subject of much current research [15],[16],[18],[19],[25],[30]. The current consensus[15] is that effective Higgs fields in the 27, 78,(induced)351' irreps can simultaneously give small left-handed neutrino masses, very heavy right-handed neutrinos, charged leptons and quarks with masses $\gtrsim$ GeV, suitable weak bosons ($W^{\pm}, Z_0$) and superheavy leptoquarks - i.e., the desired scenario.

(5) Weak decays of the b-quark is problematic and will be discussed in the next section.

The Neutral Weak Current Model of Georgi and Glashow

The absence of the t-quark has led to a most interesting, but radical, speculation by Georgi and Glashow[31],[32] which is of special interest in that it fits naturally with the model we have abstracted from the complex-octonionic plane. The problem is that the b and h quarks, which, as shown above, are $SU(2)^W$ singlets, have no weak interactions whatsoever except via mixing, (with the s,d quarks) and this leads necessarily to unwanted flavor-changing neutral currents.

To correct this, Georgi and Glashow suggested the reduction of the flavor group: $SU(3)_L \times SU(3)_R \rightarrow SU(2)^{weak} \times SU(2)^{neutral} \times U(1)^{EM}$, thereby introducing, besides the standard electro-weak group, a new neutral interaction, $SU(2)^{neutral}$, which is taken to be an appreciably weaker ($G_N \sim \frac{1}{10} G_F$) type of weak interaction. Any mixing of the b-quark in this model is strictly forbidden. (Weak, right-handed, decays of the b-quark then occur through the $\begin{pmatrix} d_R \\ b_R \end{pmatrix} SU(2)^{neutral}$ doublet.) One prediction of this model is striking: the weak decays of the b-quark are semi-leptonic.

Experimental data on the weak decays of the b-quark have now been reported[33],[34] based on the weak decays of the B mesons produced at the 4s upsilon resonance. The results are quite clear: b decays primarily to c, as in the standard model, and the $SU(2)^{neutral}$ model of Georgi and Glashow is ruled out.

Although all models without a t-quark - such as the complex octonionic plane discussed here - are in trouble, the data cannot yet rule them out. Achiman[17], in a preliminary analysis of the data, shows that the E_6 model based on two 27's can still be made to fit, using a carefully controlled mixing of the b,s,d,h quarks.[7]

Whether or not this particular E_6 model is satisfactory for a grand unified theory should be definitely settled in the near future. If, as appears likely, this model is indeed ruled out, we still would claim that the complex octonionic plane itself is of interest. The unusual features of this plane, achieved within the framework of a quantum mechanical interpretation, are intriguing enough to merit further investigation.

Footnotes

1. The rôle of this second axiom is exactly the same as the Jacobi axiom in Lie algebras; it ensures that one has an integration process (the Jordan analog to the Baker-Campbell-Hausdorff identity).
2. A computer search (SPIRES), noted in a recent GUTs review by Goldman[12], yielded over a thousand papers. Other recent reviews are by Slansky[13] and by Ramond[14].
3. Semi-simple G×G with a discrete symmetry is also used.
4. Okubo[23] has recently proven the uniqueness of SU(5) and SO(10) for GUTs. His explicit assumption that all Q = +1 leptons are weak isospin singlets results in eliminating the E-series of exceptional groups from the start.
5. The second irrep is a 27^* which we interpret physically as a right-handed fermion family. More precisely, the 27^* irrep is interpreted as the CP conjugate of a left-handed family (so that we effectively have two 27 irreps). The $E_{6,0}$ structure permits C to be defined[28] by the subgroup having Cartan index = 2,[29]. The CP operation corresponds to the (unique) "unfolded" subgroup of Cartan index = 6, for $E_{6,0}$ (in general, CP corresponds to the subgroup of Cartan index = rank of simple group).
6. $\sin^2\theta_W$ is a group invariant, since it is defined by the relative normalization of the weak isospin generator (I_3^W) to the EM charge generator (Q) in the unifying group algebra. It does not follow that $\sin^2\theta_W$ is a property of the group, since it depends on how the $SU(2)^W$ and Q are embedded.

7. Professor R. J. Oakes (private communication) points out that the more recent data tend to disagree with Achiman's fit. See also the recently published paper[35].

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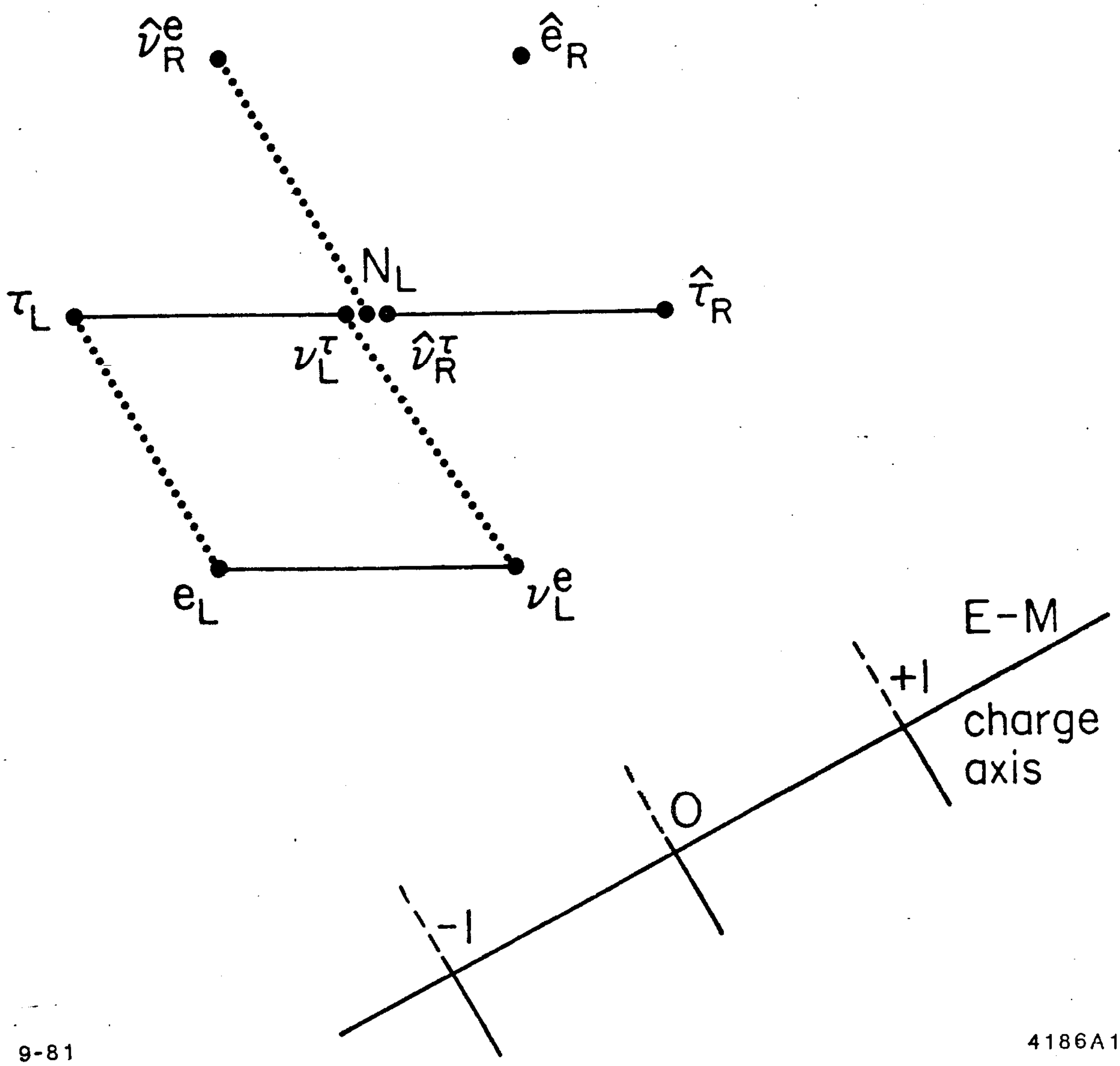
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Fig. 1