SLAC-PUB-2800
September 1981
(N)

FERMION MASSES AND STRONG $SU(2)_L$*

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ABSTRACT

A class of semirealistic models with fermion mass generation via nonstrong interaction is shown to lead to strong "weak" interactions. Problems of strong $SU(2)_L$ are discussed.

Submitted for Publication

*Work supported by the Department of Energy, contract DE-AC03-76SF00515.
†By Fellowship from Deutsche Forschungsgeminschaft.

1. Fermion Mass Generation and Strong $SU(2)_L$

There has recently been some discussion of fermion mass generation via nonstrong interactions[1,2,3] in the context of models of dynamical symmetry breakdown. In this mechanism the extended Technicolor[4] interactions are replaced by extended color and/or electroweak interactions. Light fermions obtain masses through a feeddown from condensates via these interactions. The masses are typically $\sim\alpha\Lambda$, $\alpha^2\Lambda$ where Λ is the scale of condensation. The mass mechanism may involve instantons of the strong gauge group responsible for the condensation process.[2,3,5] Using complementarity[6] it has been shown that certain aspects of the mass mechanism can be most easily understood in the so-called confining (or symmetric) picture,[2,3] which leads us naturally to the discussion of light composite fermions. No realistic models have been found so far in this context.

In the following we discuss a class of models which incorporate the mass mechansim discussed and try to make contact with reality. We will show that we can at least partially succeed. One property of these models is the identification of a Technicolor group as the extension of the color or electroweak group, i.e., at a scale Λ_1. A group G (with weak coupling) at this scale is broken dynamically to, for example, $G'(\text{Technicolor}) \times G''(\text{color})$. Technicolor becomes strong at a scale $\Lambda_2 << \Lambda_1$ and the coset interaction $G/G' \times G''$ are responsible for the light fermion masses. The models will allow an interpretation in the symmetric picture with composite fermions. The investigation of these models led us to the observation that a strong weak interaction might appear naturally. This strong weak interaction is identical to the one recently proposed by Abott and Farhi.[7] $SU(2)_L$ is supposed to become strong at ~100 GeV and the light left-handed fermions are boundstates of a fundamental fermion and a scalar. The scalar in our case is a Technicolor fermion-antifermion boundstate. The concept of a strong $SU(2)_L$ was obtained[7] by using complementarity[6] in its most extreme form. With certain assumptions about strong interaction dynamics it has been shown that the phenomenology of the strong weak interactions may be compatible with all low energy data. There is, however, one aspect we might not feel very comfortable about. This is the large value of the weak mixing angle

$$\sin^2\theta_W = \frac{1}{4} \cdots \frac{1}{5} \tag{1}$$

It is the angle that mixes the "weak" and electromagnetic interactions. If now the weak interactions are strong, one would expect $\sin^2\theta_W$ to be of order $\alpha = 1/137$. We do not know if the dynamics of the strong interactions might solve this problem. We know about one strong interaction: QCD. Let us see what we can learn. From Ref. 7 we know that

$$\sin^2\theta_W = \frac{2e^2k}{g^2} \tag{2}$$

in QCD g would correspond to $f\rho NN$ and $k = f\rho NN/2f\rho$ (compare the book of Sakurai[8]) and $\alpha = e^2/4\pi$. These values are measured in QCD.[8] $k \approx 1/2$ and $f\rho N^2N = 4\pi(3 \pm 1)$. This leads to

$$\sin^2\theta_W = \frac{\alpha}{3} \tag{3}$$

as expected. Thus the dynamics of $SU(2)_L$ should differ from $SU(3)_C$ in order to solve this problem. The concept of a strong weak interaction appears to us nontheless attractive.

2. The Models and the Evolution of the Coupling Constants

The models we investigate are of the type SU(N) × SU(N+4) × U(1) with the fermion representations.

$$\left(\frac{N(N+1)}{2}, \quad 1, \quad -(N+4)\right)$$
$$\left(\bar{N}, \quad \overline{(N+4)}, \quad N+2\right)$$
$$\left(1, \quad \frac{(N+4)(N+3)}{2}, \quad -N\right) \tag{4}$$

where the last entry denotes the U(1) charge that is conserved by SU(6) as well as SU(10) instantons. For definiteness we will discuss the case N = 6 and will mention the properties for models N ≠ 6 at the end. Notice that the representation (4) is completely anomaly free. There are two additional U(1) charges which are conserved up to instanton processes. We will come back to these charges when we discuss the generation of fermion masses.

Consider thus N = 6. We gauge a subgroup SU(5) ⊂ SU(6) and will assume this interaction to become strong. The fermions are in $SU(5) \times SU(10) \times U(1)_{E_6} \times U(1)_A \times U(1)_B \times U(1)_C$ notation

$$\chi_{AB}\ (15,\ 1,\ 2,\ 10,\ 5,\ 0)$$
$$\chi_{6A}(5,\ 1,\ -4,\ 10,\ 5,\ 0)$$
$$\chi_{66}(1,\ 1,\ -10,\ 10,\ 5,\ 0)$$
$$\chi^{A\alpha}(\bar{5},\ \overline{10},\ -1,\ -8,\ -4,\ -4)$$
$$\chi^{6\alpha}(1,\ \overline{10},\ 5,\ -8,\ -4,\ -4)$$
$$\omega_{\alpha\beta}(1,\ 45,\ 0,\ 6,\ 0,\ 3) \tag{5}$$

where

$$A,B = 1, \ldots 5, \quad \alpha,\beta = 1, \ldots 10,$$

$$E_6 = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1_{-5} \end{pmatrix}_{SU(6)} \tag{6}$$

$U(1)_{E_6}$ and $U(1)_A$ are conserved whereas $U(1)_B$, $U(1)_C$ are broken by SU(5), SU(10) instantons. One might omit at this stage the field χ_{66} (it will correspond to a right-handed neutrino).

Part of the SU(10) will be gauged weakly. We cannot gauge SU(10) completely. This would lead to proton decay as we will see later on. We have to separate weak and color interactions at this stage. We thus assume as gauge group

$$SU(8)_{EC} \times SU(2)_L \times U(1)_Y \subset SU(10)$$

SU(8) contains $SU(3)_{color}$. (An alternative way would be $SU(3)_C \times SU(7)_{EW} \times U(1)_Y$, corresponding to an extended weak interaction.) Y is the usual hypercharge. The coupling constant will be denoted by g_8, g_2, g_Y, respectively.

We now assume the SU(5) interactions to become strong at a scale Λ and the formation of condensates in the most attractive channel (MAC).[9] In this case this corresponds to a condensate

$$\phi_B^\alpha = \langle \chi_{AB}\, \psi^{A\alpha} \rangle \tag{7}$$

There is still some ambiguity and we will discuss a particular pattern

$$\phi_B^\alpha = \Lambda^3\, \delta_B^\alpha \qquad \alpha = 1, \ldots, 5 \tag{8}$$

which breaks SU(5) × SU(10) to $SU(5)_D$ × SU(5) where the D indicates a diagonal subgroup. In the following we will denote indices in $SU(5)_D$ by α and indices in SU(5) by x, thus replacing our old $\alpha = 1, \ldots, 10$ by (α, x). Part of the SU(10) has been gauged. The discussion of the subgroup alignment problem[10] tells us that the unbroken gauge group is

$SU(5)_D \times SU(3)_C \times SU(2)_L \times U(1)_Y$. Since we had assumed that $g_S >> g_8$ we obtain for the gauge coupling at Λ: $g_8 = g_3 \approx g_{5D}$. Using this equality and the assumption that $SU(3)_C$ becomes strong at Λ_C = 500 MeV, we can deduce Λ and Λ_{5D} from the given fermion content and the one loop β-function. We obtain

$$\Lambda \approx 500 \text{ TeV}$$

$$\Lambda_{5D} \approx 500 \text{ GeV} \tag{9}$$

Thus $SU(5)_D$ becomes strong at 500 GeV; we call it the Technicolor group. The coupling g_2 is in principle a free parameter in the model. Assuming that g_2 is not too different from g_8 at Λ allows g_2 to become strong at $\Lambda_L \approx 100$ GeV, which leads us to the strong weak interactions.

3. Light Fermions and Mass Generation

Technicolor $SU(5)_D$ becomes strong at 500 GeV. It leads to the following condensates

$$\chi_{6A}\, \psi^{6\alpha} \; ; \quad \psi^{[A\alpha]}\, \omega_{\alpha\beta} \; ; \quad \psi^{AX}\, \omega_{\alpha X} \tag{10}$$

These condensates do not break $SU(5)_D$ but several approximate global symmetries, leading to Pseudogoldstone bosons. The situation here is similar to the one in the usual Technicolor models. We have checked that there are no low-lying states that are in conflict with present low energy phenomenology.

We observe that one would have to make weird assumptions in order that the condensates in (10) break $SU(2)_L$. In the context of this model we therefore consider the appearance of a strong confining $SU(2)_L$ to appear "naturally." The Higgs bosons in the usual model are here TC boundstates

$$H^X = \psi^{AX} \chi_{6A}$$

$$H_X^1 = \omega_{\alpha X} \psi^{6\alpha} \tag{11}$$

We now list the massless fermions:

	Y	Z	X
$\bar{\nu} = \chi_{66}$	0	10	25
$\bar{d} = \psi^{6a}$	1/3	-6	9
$\bar{u} = \omega_{ab}$	-2/3	2	-43
$\bar{e} = \omega_{rs}$	1	2	-43
$e = \psi^{6r} H^s \varepsilon_{rs}$	-1	-2	43
$\nu = \psi^{6r} H'_r$	0	-10	-25
$d = \omega_{ar} H^r$	-1/3	6	-9
$u = \omega_{ar} H'_s \varepsilon^{rs}$	2/3	-2	-7

(12)

where we have split x = 1, ...,5 = (r,a) r = 1,2 and a = 1,2,3 in an obvious way.

In (12) we have displayed three U(1) charges: Y, Z, and X. Y is usual hypercharge and coincides with electric charge in the strong $SU(2)_L$ model. Z is a conserved global charge. (Z + 8Y)/10 corresponds to B-L conservation. The global charge X is conserved by all condensates, but broken by instantons. We observe already at this stage that a mass term for the u-quark has to include an instanton interaction. Masses for d and e, however, do conserve the X quantum number.

We will first discuss the mass of the d-quark. We have $d = \omega_{ar} H^r = \omega_{ar} \psi^{Ar} \chi_{6A}$ and $\bar{d} = \psi^{6a}$. A mass term is given in Fig. 1. It involves the TC-condensates $\omega_{ir} \psi^{ir}$ and $\chi_{6i} \psi^{6i}$ as well as the extended color interactions of $SU(8)/SU(5) \times SU(3)$. There is no instanton present.

The right-handed electron is $\bar{e} = \omega_{rs}$. Although a mass term for the electron is allowed by the U(1) quantum number in (12), the electron can only receive a mass if there are extended electroweak interactions that can transform r or s to $i \subset SU(5)_D$, for example. This would be possible if we would have gauged the whole SU(10). Because of proton decay we know, however, that $SU(2)_L$ and $SU(3)_L$ can only be unified at a high energy $\sim 10^{15}$ GeV in the usual way. An extension of our model (to allow for an electron mass) would thus require first the introduction of extended electroweak interactions that commute with the extended color interactions. A unification of both interactions should then occur at a higher energy scale. We have not yet attempted to construct such an extended model.

A mass term for the u-quark requires the breakdown of $U(1)_X$ since $\Delta X = -43 - 77 = -120$. $SU(5)_S$ instantons break $U(1)_X$ by $\Delta X = 120$. We thus need an $SU(5)_S$ antiinstanton. The mass mechanism is bizarre, as can be seen in Fig. 2. The $SU(5)_D$ instanton and the TC-condensates do not break $U(1)_X$.

4. Not Enough Symmetry?

We now want to mention a problem of our N = 6 model. It is this absence of a custodial[11] SU(2) that in the standard model would ensure the relation $M_W = M_Z \cos\theta_W$ naturally. In the model of strong $SU(2)_L$ the "W-bosons" are a triplet of this symmetry and are therefore equal in mass.[7] In the N = 6 there is no SU(2) at the TC-level that would ensure this degeneracy in a natural way. This is the most serious problem of our model. It could be that it is solved in an extended version of our model that includes extended electroweak interactions.

The model with N = 4 does not suffer from this problem. Here we have as gauge symmetries $SU(3)_S \times SU(6)_{EC} \times SU(2)_L \times U(1)$ where SU(6) is extended color. We assume the breakdown to occur to $SU(3)_D \times SU(3)_C \times SU(2)_L \times U(1)$. Notice that this assumption is in contradiction to what we generally assume to be a solution of the subgroup alignment problem.[10] $SU(3)_D$ plays the role of Technicolor. The TC-condensates are (compare (10)):

$$\chi_{4A}\ \psi^{4A}\ ;\quad \psi^{[A\alpha]}\ \omega_{\alpha\beta}\ ;\quad \psi^{Ax}\ \omega_{\alpha X} \tag{13}$$

where $A,\alpha,\beta = 1,2,3$. There is an SU(2) global symmetry such that χ_{4A} and $\bar{\psi}_{[A\alpha]}$ transforms as a doublet under this symmetry. Observe that this is not the case in the N = 6 model since χ_{6A} and $\bar{\psi}_{[A\alpha]}$ are in different $SU(5)_D$ representations. This symmetry in the N = 4 model would ensure the degeneracy of the "W-bosons" naturally. The N = 4 problem (as mentioned) suffers, however, from the subgroup alignment problem.

5. The Symmetric Picture

We will now discuss briefly the complementary picture.[6,3] I thas the property that $SU(5)_S$ is unbroken and that the low energy states are $SU(5)_S$ singlets. We first observe that χ_{66} and $\psi^{6\alpha}$ are already $SU(5)_S$ singlets and survive at low energies in the symmetric as well as in the broken picture. The remaining low energy states will be composite $SU(5)_S$ singlets. According to the results of Ref. 3 we can read of these states by inspection of the condensates. We define

$$\xi^{\alpha\beta} = \psi^{i\alpha} \chi_{ij} \psi^{j\beta} - \psi^{i\beta} \chi_{ij} \psi^{i\alpha} \tag{14}$$

and

$$\xi_{6\alpha} = \chi_{6i} \chi^{*ij} \psi^{*}_{j\alpha} \tag{15}$$

We use indices m for SU(5), a for SU(3) and r for SU(2). The condensate that breaks SU(8) to SU(5) × SU(3) but leaves $SU(5)_S$ unbroken is

$$\theta^{12345} = \left\langle \chi_{i_1 j_1} \chi_{i_2 j_2} \chi_{i_3 j_3} \chi_{i_4 j_4} \chi_{i_5 j_5} \varepsilon^{j_1 \cdots j_5} \psi i_1 1 \, \psi i_2 2 \, \psi i_3 3 \, \psi i_4 4 \, \psi i_5 5 \right\rangle \tag{16}$$

We first observe that ξ^{ab}, ξ^{rs}, ξ^{ar} do not contain fermion combinations that are condensates in the broken picture. Thus they will be massive in the symmetric picture. ξ^{mn}, ξ^{ma}, ξ^{mr} however do. Thus we can identify

$$\begin{aligned} \xi^{mn} &\mathrel{\hat{=}} \psi^{[ij]} = (\overline{10}, 1, 1) \\ \xi^{ma} &\mathrel{\hat{=}} \psi^{ia} = (\overline{5}, \overline{3}, 1) \\ \xi^{mr} &\mathrel{\hat{=}} \psi^{ir} = (\overline{5}, 1, \overline{2}) \end{aligned} \tag{17}$$

The same discussion applies to $\xi_{6\alpha}$. ξ_{6m} contains a condensate and can be identified

$$\xi_{6m} \mathrel{\hat{=}} \chi_{6i} \tag{18}$$

with χ_{6i} in the broken picture. The composite fermions fulfill 't Hooft's anomaly conditions.[12]

6. Conclusion

We have attempted to construct a semirealistic model that allows the generation of light fermion masses through nonstrong interactions. It turned out that these models lead naturally to a strong $SU(2)_L$ interaction. The main problem of these models is the absence of a custodial SU(2) symmetry, which in the case of weak $SU(2)_L$ would ensure the relation $M_W = M_Z \cos\theta_W$. This problem occurs in addition to the usual problems which exist in models of strong weak interactions.

The models indicate that one needs extended color as well as extended electroweak interactions to give masses to all light fermions. The problem of the custodial SU(2) could possibly be solved in the framework of these extended models. This would be the case if the extended electroweak sectors behave as the $\bar{N} = 4$ model.

ACKNOWLEDGEMENTS

This work has been prepared for the Deutsche Forschungsgemeinschaft. The results have been obtained in collaboration with Stuart Raby. I would like to thank L. Susskind for interesting discussions and suggestions. The hospitality of the Aspen Center for Physics is gratefully acknowledged.

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FIGURE CAPTIONS

Fig. 1: The d-quark receives a mass through TC-condensates and extended color interactions.

Fig. 2: The u-quark mass through a combination of instantons, TC-condensates and extended color interactions.

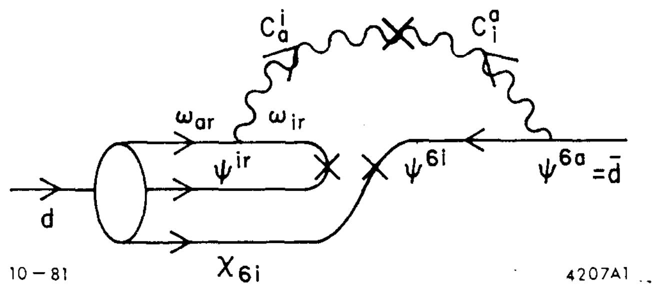
C_a^i
C_i^a
ω_{ar}
ω_{ir}
ψ^{ir}
ψ^{6i}
$\psi^{6a} = \bar{d}$
d
χ_{6i}
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Fig 1

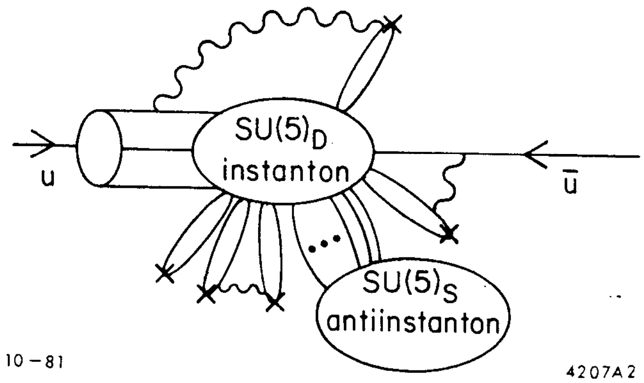

Fig. 2