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QCD AND CHIRAL $SU(N) \otimes SU(N)$*

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ABSTRACT

We report on the observation that the restriction of the indices $|\ell(R)| \leq 1$ (solitariness) for massless composite fermions in an $SU(3)_C \times SU(N)_L \times SU(N)_R \times U(1)$ model does not allow any solution to 't Hooft's anomaly conditions. Arguments for solitariness are discussed and possible consequences for models of composite quarks and leptons are presented.

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Recently 't Hooft[1] has proposed a set of consistency conditions which must be satisfied by the massless composite fermions in a strongly interacting gauge theory. The conditions are a consequence of an unbroken chiral symmetry. They are of two types. The first is a set of anomaly conditions, which require that the massless composite fermions give the same contribution to the triangle anomalies of the chiral currents as the fundamental fermions in the theory. These conditions have recently been proven to be a direct consequence of analyticity and unitarity[2]. The second set is suggested by the decoupling theorem of Appelquist and Carrazone[3] but is not a direct consequence of this theorem. They instead follow from the "persistent mass hypothesis" suggested by Preskill and Weinberg.[4]

The persistent mass hypothesis (PMH) states that the massless bound state solutions of the anomaly conditions obtain a small mass of order m when any of the constituents is given a small bare mass of order m. PMH can then be used to derive a new set of conditions for massless composite fermions by a simple continuity argument (most clearly presented by I. Bars[5]): Given a solution to the anomaly conditions for the full chiral symmetry group G, one then introduces a bare mass parmeter $M \ll \Lambda$, for some subset of fundamental fermions, which explicitly breaks G to a subgroup G'. One then demands that the remaining massless bound states (consistent with PMH) are a solution of the anomaly conditions for the unbroken chiral group G'. 't Hooft has shown that as a consequence of these two sets of conditions the chiral $SU(N) \otimes SU(N)$ flavor symmetry of QCD must be spontaneously broken. This follows directly from the fact that there are no simultaneous solutions to these conditions.[1,4,5] This is such an extraordinary result that every part of the argument should be well understood. However, the second set of conditions following directly from PMH is crucial for the conclusion and is unfortunately not on as firm a footing as the anomaly conditions. Let us elaborate: (i) There are known solutions to the anomaly conditions alone for QCD and for example chiral SU(2) $\otimes$ (SU(2) or SU(4) $\otimes$ SU(4); (ii) It is not unreasonable to believe that composite fermions may remain massless even though their constitutents have small bare masses (i.e., $m_{bare} << \Lambda_{QCD}$). Moreover, when $m_{bare} \simeq \Lambda_{QCD}$ the theory undergoes a phase transition such that for $m_{bare} >> \Lambda_{QCD}$ the composite fermions containing these massive constitutents become massive as required by the decoupling theorem.[3] There are known examples which exhibit this behavior, one being a Jona Lasinio type model of Ref. 4. A second one is due to Savas Dimopolous which shows that the behavior of (ii) appears naturally in a renormalizable model. We will reproduce this model in Appendix B.

This criticism led Preskill and Weinberg to the conclusion that it is still an open possibility that the chiral symmetries of QCD are unbroken.

On this note we would like to suggest the following "hypothesis of solitariness" as an alternative. We propose that the index $\ell(R)$ of the representation R of composite fermions should only take on the value ±1 or zero. This index can in principle take on any integer value. We recall that it corresponds to the multiplicity of massless states with identical Lorentz and SU(N) × SU(N) × U(1) quantum numbers. Thus, if, for a particular representation R, $|\ell(R)| = n$ with $n > 1$, we have the situatión that at least n - 1 (orbitally or radially) excited states are required to be massless. This, however, contradicts the common belief that the excited states should rather have a mass of the order of the binding scale.

In nonrelativistic boundstate systems where one can define the concept of a boundstate wave function it can usually be shown that groundstate and excited states are nondegenerate. The systems considered in this paper are, however, extremely relativistic and these arguments do not apply.

The only relativistic model we know of, where these questions could be studied is two-dimensional quantum-chromodynamics in the 1/N-expansion. It turns out that in this model the concept of a wave function can be defined in the infinite momentum frame and it turns out that excited states are massive, even if the groundstate is massless. Although we cannot prove this in general for relativistic systems we nonetheless consider it as a reasonable assumption. This then leads us to the condition of solitariness.

We now apply this condition QCD: $SU(3)_C \times SU(N)_L \times SU(N)_R \times U(1)$ where $N \geq 3$. There are five indices possible for the nonexotic states discussed and the anomaly conditions can be written as (see Appendix A for details)

$$XN^2 + YN + 3Z = 0 \qquad (1)$$

$$XN^2 + \tilde{Y}N + Z = 0 \qquad (2)$$

where X, Y, C are functions of the indices. Instead of inspecting these two equations it is convenient to consider first the difference of the two equations

$$(Y - \tilde{Y})N + 2Z = 0 \qquad (3)$$

It can then be shown that there are no simultaneous solutions to the anomaly and solitariness conditions. The proof is given in Appendix A.

As a consequence we might conclude that in QCD, the $SU(N)_L \times SU(N)_R$ chiral symmetry must be spontaneously broken. We have to mention here that the proof in Appendix A only considers nonexotic boundstates. The inclusion of exotics (e.g., $qqq\bar{q}\bar{q}$) as candidates for massless fermions would require an extension of this proof. We do not however feel strongly about this possibility.

A proof of a similar result for models of the type $SU(n) \times SU(N)_L$ $SU(N)_R \times U(1)$ for $n = 2\ell + 1$, $\ell \geq 2$ is exceedingly difficult to obtain. Studies of $n = 5$, however, seem to indicate the result to be true for this case as well. Should the result (that all solutions to the anomaly conditions violate solitariness) be true for general n, this could have consequences for models of composite quarks and leptons based on the $SU(N) \times SU(N)$ chiral structure. The only way in which these models could make sense is the identification of $\ell(R) = k > 1$ with the number of generations. One would then have to interpret for example the μ and the τ as excited states of the electron, and these particles would be degenerate in the chiral limit.

In order to arrive at a realistic model one would then first encounter the problem of lifting this mass degeneracy. This is difficult since there are no quantum lnumbers that distinguish between these three states. But even if it would be possible to invent such a mechanism one would then have to face the situation that in general $\mu \to e\gamma$ transitions would be allowed at too large a rate.

We conclude with the remarks that our result has only been proven for $n = 3$, and it is still a question if it holds for $n > 3$. In view of the speculation[6] that generation number comes from a discrete subgroup of an anomalous U(1) it would perhaps be worthwhile to try to extend our proof including exotics.

Our main result is, however, the observation that the condition of solitariness (in connection with the anomaly condition) is sufficient to prove spontaneous chiral symmetry breakdown in QCD.

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APPENDIX A

Consider $SU(3)_C \times SU(N)_L \times SU(N)_R \times U(1)$ with the fundamental fermions in the representations $(N > 3)$

$$q_L = \left(3,\ N,\ 1,\ \frac{1}{3}\right)$$
$$\bar{q}_R = \left(\bar{3},\ 1,\ \bar{N},\ -\frac{1}{3}\right)$$

The following list of ten representations are the possible spin 1/2 nonexotic $SU(3)_L$ singlet boundstates (compare Ref. 1)

$r_1,\ \ell_1\ \ \boxed{L}\boxed{L}\boxed{L}$	$r_6,\ -\ell_1\ \ \boxed{R}\boxed{R}\boxed{R}$
$r_2,\ \ell_2\ \ \begin{array}{c}\boxed{L}\\\boxed{L}\\\boxed{L}\end{array}$	$r_7,\ -\ell_2\ \ \begin{array}{c}\boxed{R}\\\boxed{R}\\\boxed{R}\end{array}$
$r_3,\ \ell_3\ \ \boxed{L} \times \boxed{R}\boxed{R}$	$r_8,\ -\ell_3\ \ \boxed{R} \times \boxed{L}\boxed{L}$
$r_4,\ \ell_4\ \ \boxed{L} \times \begin{array}{c}\boxed{R}\\\boxed{R}\end{array}$	$r_9,\ -\ell_4\ \ \boxed{R} \times \begin{array}{c}\boxed{L}\\\boxed{L}\end{array}$
$r_5,\ \ell_5\ \ \begin{array}{ll}\boxed{L}&\boxed{L}\\\boxed{L}&\end{array}$	$r_{10},\ -\ell_5\ \ \begin{array}{ll}\boxed{R}&\boxed{R}\\\boxed{R}&\end{array}$

where $r_1 \ldots r_5$ are left-handed and $r_6 \ldots r_{10}$ are right-handed. $\ell_1 \ \ldots\ \ell_5$ are the indices of the representations r_1 to r_5. Note that the indices of $r_6 \ldots r_{10}$ are $-\ell_1, \ldots, -\ell_5$. We now compute the chiral anomalies. It is sufficient to consider the anomalies A corresponding to triangle graphs of three $SU(N)_L$ currents, and A_1 corresponding to the triangle graphs of two $SU(N)_L$ and one U(1) current.

The fundamental fermions have $A = 3$ and $A_1 = 1$. The composites have:

$$A(r_1) = \frac{(N+3)(N+6)}{2} \qquad A_1(r_1) = \frac{(N+2)(N+3)}{2}$$

$$A(r_2) = \frac{(N-3)(N-6)}{2} \qquad A_1(r_2) = \frac{(N-2)(N-3)}{2}$$

$$A(r_3) = \frac{N(N+1)}{2} \qquad A_1(r_3) = \frac{N(N+1)}{2}$$

$$A(r_4) = \frac{N(N-1)}{2} \qquad A_1(r_4) = \frac{N(N-1)}{2}$$

$$A(r_5) = N^2 - 9 \qquad A_1(r_5) = N^2 - 3$$

$$A(r_8) = N(N+4) \qquad A_1(r_8) = N(N+2)$$

$$A(r_9) = N(N-4) \qquad A_1(r_9) = N(N-2)$$

Thus we have the following anomaly conditions:

$$A = 3 = \frac{\ell_1}{2}(N+3)(N+6) + \frac{\ell_2}{2}(N-3)(N-6) + \ell_5(N^2-9) + \ell_3\left(\frac{N(N+1)}{2} - N(N+4)\right) + \ell_4\left(\frac{N(N-1)}{2} - N(N-4)\right)$$

$$A_1 = 1 = \frac{\ell_1}{2}(N+2)(N+3) + \frac{\ell_2}{2}(N-2)(N-3) + \ell_5(N^2-3) + \ell_3\left(\frac{N(N+1)}{2} - N(N+2)\right) + \ell_4\left(\frac{N(N-1)}{2} - N(N-2)\right)$$

We rewrite these equations in the form

$$XN^2 + YN + 3Z = 0 \tag{A.1}$$

and

$$XN^2 + \tilde{Y}N + Z = 0 \tag{A.2}$$

where

$$X = \ell_1 + \ell_2 - \ell_3 - \ell_4 + 2\ell_5$$

$$Y = 9\ell_1 - 9\ell_2 - 7\ell_3 + 7\ell_4$$

$$\tilde{Y} = 5\ell_1 - 5\ell_2 - 3\ell_3 + 3\ell_4$$

and

$$Z = 6\ell_1 + 6\ell_2 - 6\ell_5 - 2$$

We first consider the difference of the two anomaly conditions

$$(Y - \tilde{Y})N + 2Z = 0 \tag{A.3}$$

which leads to

$$N(\ell_1 - \ell_2 - \ell_3 + \ell_4) = 1 + 3\ell_5 - 3\ell_1 - 3\ell_2$$

We now demand the condition of solitariness: $\ell_i = \pm 1,\ 0$ and define

$$A = \ell_1 - \ell_2 - \ell_3 + \ell_4$$

$$B = 1 + 3\ell_5 - 3\ell_1 - 3\ell_2$$

Inspection of (A.3) and some algebra tells us that there are only possible solutions of (A.3) for $|A| = 1, 2$ and $N = 4, 5, 7, 8, 10$ that fulfill solitariness. It can however be shown that all these solutions do not solve (A.1) and (A.2) separately. We have thus shown that there are no solutions that simultaneously fulfill the anomaly constraints and the condition of solitariness.

APPENDIX B

We here give details of the model of S. Dimopoulos. It is based on an SU(5) × U(1) gauge theory where SU(5) is strong and U(1) weak. Fermions are chosen to be in the following representations: $(\bar{5}, 0)$; (10, 1); (10, 0); $(\overline{10}, 0)$; (24, -3/10) and (1, Q) where Q should be chosen to cancel the $U(1)^3$ anomalies.

We first discuss the broken picture[7,8] and assume condensation in the maximally attractive channel. As a consequence, 24 will self-condense and become heavy of order Λ_5. At the next step we have two possibilities of condensation

a) $(\overline{10}, 0) \times (\overline{10}, 0)$ and $(10, 1) \times (10, 1)$

b) $(\overline{10}, 0) \times (\overline{10}, 1)$ and $(10, 0) \times (10, 0)$

Subgroup alignment calculations[9] favor solution b) since there the U(1) gauge group is broken less strongly. We now turn to the symmetric picture[7,8]. From the analysis in Ref. 7 we know that there is one massless composite fermion in the game. In case a) this would be $(10, 1) \times (10, 1) \times [(\bar{5}, 0)]^*$ and in case b) $(10, 0) \times (10, 0) \times [(\bar{5}, 0)]^*$. We assume solution b) to be realized because of the mentioned subgroup alignment considerations.

Now suppose we give a small explicit mass $m[(\overline{10}, 0) \times (10, 0)]$ to the fundamental fermions. This mass will compete with the U(1) coupling α in the subgroup alignment problem but we choose m small enough, such that the solution b) is still favored. The boundstate $(10, 0) \times (10, 0) \times (\bar{5}^*, 0)$ now contains massive constituents, but is still massless. Thus this example violates the persistent mass hypothesis.

Suppose that we now increase m. We do not know what happens in an intermediate region, but we know what will happen for $m >> \Lambda_5$. If m is that large the alignment will switch to solution a) since (10, 1) and $(\overline{5}, 0)$ are the only low energy states. Thus $(10, 1) \times (10, 1)$ condensation will occur. This could lead to a massless composite fermion of the type $(10, 1) \times (10, 1) \times (\overline{5}^*, 0)$. We thus have the very interesting possibility of a phase transition at a certain value of m, which doesn't change the low energy spectrum. In both phases there will be a massless composite fermion. In one of the phases this fermion will consist of massive constitutents.