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NATURALNESS AND SUPERSYMMETRY*

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ABSTRACT

Possibilities to solve the hierarchy problem in a natural way are discussed. We point out that supersymmetry has several interesting properties that could lead to a solution of this notorious problem. Models of Supercolor and supersymmetric grand unified models are reviewed. A particular solution to the strong CP-problem is presented.

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1. Mass Scales

The highest scale in physics we know of is the Planck scale $M_p \sim 10^{19}$ GeV. It is obtained from the gravitational interactions via Newton's constant. One ultimate goal would be an understanding of all physical scales from M_p. This would require a unified theory of the gravitational with strong and electroweak interactions. We are still far away from this goal. Supergravity[1] might be a candidate to marry gravitation and quantum field theory but there is nothing (apart from speculations) that relates these theories to the world of the strong, weak and electromagnetic interactions.

With the hope that we can possibly describe physics far below the Planck scale without an explicit understanding of gravity, we proceed to discuss other mass scales in physics.

The Fermi constant G_F provides us with a scale $M_W \sim 250$ GeV for the weak interactions. Hadron masses are of the order of $M_C \sim 1$ GeV related to Λ_{QCD}. The mass of the electron is ~1 MeV and neutrino masses perhaps of the order of ~eV. There might be other scales related to CP-violation, flavor changing neutral currents, etc.

If the proton decays as predicted by grand unified theories this gives us a scale $M_X \sim 10^{15}$ associated with the extremely weak baryon number violating interactions.

Grand unified models (take SU(5)) for example) serve as an example of how one can and cannot understand a small scale from a large scale. SU(5) is characterized by the scale M_X (the breakdown of SU(5)) and a coupling constant α_X. With the given fermion representation and an asymptotically free color SU(3), we can qualitatively understand the scale of the hadron masses $M_C \sim 1$ GeV in this model. The huge hierarchy $M_X/M_C \sim 10^{15}$ is understood from the logarithmic growth of $\alpha_{SU(3)}$ approaching the infrared region. This is a remarkable result.

The scale $M_W \sim 250$ GeV is not understood in grand unified theories. It is put in by hand. The fact that $M_X/M_W \sim 10^{13}$ is called the hierarchy problem. Technically the $SU(2)_2 \times U(1)$ breakdown at M_X achieved via vacuum expectation values of scalar (Higgs) fields. Parameters in the Higgs potential have to be chosen with extreme care to arrive at $M_X/M_W \sim 10^{13}$. Moreover, the ratio is not stable in perturbation theory. As a result, we do not understand M_W in terms of M_X.

Let us now discuss the masses of quarks and leptons. In the standard $SU(2) \times U(1)$ model these masses are protected by the $SU(2) \times U(1)$ symmetry, i.e., they can only receive masses after $SU(2) \times U(1)$ is broken. This is the scale M_W. Thus we could understand the mass of the hypothetical top quark (M_t). The mass of the electron is small, $M_e/M_W \sim 10^{-6}$, but does not constitute a hierarchy as huge as discussed before. The region between M_t and M_e is populated by various fermions and fermion masses might be understood via low order perturbation theory or by new scales related to extended Technicolor.

The neutrino masses are zero in the standard SU(5) model (due to B-L conservation). We know that $M\nu_e < 40$ eV. Small masses for ν_e of order eV could be related to a huge mass scale where lepton number is violated.

There is nothing more that can be said about the fermion masses without discussing specific models. The situation, however, that we do not understand M_W and the fermion mass generation via Yukawa couplings is unsatisfactory.

2. Naturalness

It was observed by K. Wilson and L. Susskind[2] that scalar particles pose problems when inserted in models. Mass terms of scalars are quadratically divergent in perturbation theory. Thus the only "natural" mass scale for fundamental scalars would be the cutoff scale: M_X in grand unified theories (i.e., the hierarchy problem). In the standard electroweak model the Higgs mass should not be larger than 1 TeV in order to keep the model perturbatively well defined. This and related subjects are discussed in great detail by 't Hooft.[3] We define naturalness in the sense of 't Hooft. A scale M_1 is naturally much smaller than a scale M_2 if there is a reason for it. The reason can be a symmetry such that M_1 tends to zero in the symmetry limit. Fermion masses could be protected by a chiral symmetry (e.g., M_t by $SU(2)_L \times U(1)$) spin-1 bosons by a gauge symmetry (e.g., the photon by $U(1)_{em}$). Scalars can be protected by supersymmetry.[4]

In view of the discussed problems with scalar particles there are two solutions: (i) no scalars (i.e., more explicitly, no fundamental scalars) and (ii) supersymmetry. We will first discuss (i) in the next section and then (ii) in the remainder of the paper. It turns out that the discussion of naturalness in supersymmetric theories is very delicate and interesting.

3. Technicolor[2,5]

These are theories without fundamental scalars. They are an attempt to understand the weak scale M_W via asymptotic freedom (in the same way we understand M_C).

Technicolor is QCD scaled to $\Lambda_{TC} \sim 1$ TeV and the breakdown of the weak interactions $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$ is given by fermion-antifermion condensates and the related F_{TC} (the analog of $f\pi$ in chiral symmetry breakdown). Technicolor is viewed as an asymptotically free gauge theory that becomes strong at a TeV. The Higgs mass is of order of 1 TeV (a $Q\overline{Q}$ boundstate of techniquarks) and there are several low mass pseudogoldstone bosons. Attempts to unify TC and $SU(3) \times SU(2) \times U(1)$ (excluding gravity) have been unsuccessful.

Technicolor is not the whole story. One has to explain the fermion masses and there are no parameters available (like Yukawa couplings) to adjust the masses. A new scale and model, Extended Technicolor,[6,7] could solve this problem. The concept of "tumbling gauge theories"[8] might even explain the so-called fermion mass hierarchies but there are phenomenological problems: most notably, the smallness of the $K_L - K_S$ mass difference. Models[9,10,11] with extended color or extended electroweak (instead of extended Technicolor) interactions seem to suffer from the same problems.

The main problem of these models is a lack of understanding of the condensation process in strongly interacting gauge theories, which does not allow to make quantitative predictions. It might even be that the above-mentioned problems are not real but just a byproduct of our ignorance. There is no model that unifies TC and $SU(3) \times SU(2) \times U(1)$ at

$M_X \sim 10^{15}$. Maybe gravity is needed. Another way would lead to composite fermions. But there we are limited even more severely by our ignorance about strongly interacting gauge theories. The only island of security might be 't Hooft's anomaly conditions.[3]

4. Supersymmetry and Its Stability in Perturbation Theory

Models of local supersymmetry (supergravity) are the only known candidates for quantum field theories of gravity.[1] They might even provide us with a natural explanation of the absence of a cosmological constant. We know, however, that supersymmetry has to be broken since the known low energy particle spectrum is not supersymmetric. The work of Fayet[12] and others implies that the scale of supersymmetry breakdown is not smaller than the breakdown of the weak interactions. In the following we will not discuss supergravity but assume that a global $N = 1$ supersymmetry could be present as low as $M_S \sim 1$ TeV and we assume that the theory can be essentially treated without the inclusion of gravity. This is a strong assumption. (The goldstino of broken supersymmetry is most likely to give mass to the gravitino in a super-Higgs effect.) We will be especially interested if there is a relation between M_S and M_W and the speculation that supersymmetry might be able to solve the hierarchy problem.

We first discuss supersymmetry in perturbation theory. Witten[13] has given a beautiful discussion of this subject and we will partially repeat it here.

The Hamiltonian of a supersymmetric model can be written as

$$H = \sum_\alpha Q_\alpha^2 \qquad (1)$$

where Q_α' are the supersymmetric charges

$$Q_\alpha = \int J_\alpha^0 \, d^3x \tag{2}$$

α is a spiner index. It follows that the spectrum of H is positive. The groundstate is annihilated by Q_α

$$Q_\alpha \, |0\rangle = 0 \tag{3}$$

implying $\langle 0|H|0\rangle = 0$ as the groundstate energy. If the energy of the groundstate is different from zero, e.g.,

$$\langle 0|H|0\rangle = f^2 \tag{4}$$

supersymmetry is broken and the supersymmetry current can create a Goldstone fermion (goldstino) out of the vacuum.

$$\left\langle 0 \middle| J_\alpha^\mu \middle| \psi_\beta \right\rangle = f(\gamma^\mu)\alpha\beta \tag{5}$$

Supersymmetry gives a meaning to the vacuum energy without the inclusion of gravity.

The statement that $\langle 0|H|0\rangle = 0$ in the case of supersymmetry is potentially unstable in perturbation theory. One could imagine that with an arbitrarily small perturbation one could introduce terms in the effective potential that left the vacuum energy (as a solution of the potential) by a tiny amount, enough to break supersymmetry spontaneously. This augmentation is actually only true with a qualification: one needs a massless fermion that plays the role of the Goldstone fermion after supersymmetry breakdown, since an arbitrarily small perturbation cannot change a massive to a massless fermion. In realistic models there is, however, always at least one such fermion, the fermionic partner of the photon. Thus realistic supersymmetric models are potentially unstable in perturbation theory.

To proceed the discussion of the potential we have to be more explicit in our notation. We will consider theories with chiral fields

$$\phi(x,\theta) = \frac{\varphi(x)}{\sqrt{2}} + \theta^{\alpha}\psi_{\alpha}(x) + (\theta^{\alpha}\theta_{\alpha})\,\frac{F(x)}{\sqrt{2}} \qquad (6)$$

and vector superfields (in the Wess-Zurnino gauge)

$$V_A(x,\theta,\bar\theta) = -\theta\sigma_{\mu}\,\bar\theta\,V^{\mu}_A + i(\theta\theta)\bar\theta\,\bar\lambda_A - i(\bar\theta\bar\theta)\,\theta\lambda_A + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D_A \qquad (7)$$

where A is a group index of the adjoint representation. F and D are auxiliary fields that can be eliminated by their equations of motion.

In supersymmetric actions there are two different types of terms to distinguish:

(i) terms $$\int d^4x \int d^4\theta\; K(x,\theta,\bar\theta) \qquad (8)$$

where K might be a superfield or a derivative of a superfield. An example is the kinetic energy of a chiral superfield

$$\int d^4x \int d^4\theta\; \phi^+(x,\bar\theta)\,\phi\,(x,\theta) \qquad (9)$$

(ii) terms $$\int d^4x \int d^2\theta\; K(x,\theta) \qquad (10)$$

An example is a mass term for a chiral superfield.

$$m\int d^4x \int d^2\theta\; \phi^2(x,\theta) + \text{h.c.} \qquad (11)$$

The potential in supersymmetric models of chiral and vector superfields can in general be written as (D is real)

$$V = \frac{1}{2}\,\text{Tr}\,D^2 + \sum_i F_i^* F_i \qquad (12)$$

It is obvious from this expression that supersymmetry is spontaneously broken if, and only if, auxiliary fields acquire a nonvanishing vacuum expectation value. The supersymmetry transformations

$$\langle 0|\{Q_\alpha,\bar{\lambda}_{A\beta}\}|0\rangle \sim \gamma_{5\alpha\beta} \langle 0|D_A|0\rangle$$

$$\langle 0|\{Q_\alpha,\psi_\beta\}|0\rangle \sim \gamma_{5\alpha\beta} \langle 0|F|0\rangle \tag{13}$$

indicate that λ or ψ is the corresponding goldstino. Thus the vacuum expectation values of the two terms in (12) must separately vanish for supersymmetry to be unbroken.

After eliminating the auxiliary fields in (12) via their equation of motion, the first term becomes

$$\frac{1}{2} \mathrm{Tr}\ D^2 = \sum_A g_A \varphi_i^* T^A_{ij} \varphi_j \tag{14}$$

where the T^A are group generators and g_A is the gauge coupling. We have here tacitly assumed that the gauge group contains no U(1) factor. We will discuss the U(1) case later.

The second term is not related to the gauge couplings, but only to terms which are of type (10)

$$F^*F = \sum_i \left|\frac{\partial k}{\partial \phi_i}\right|^2 \tag{15}$$

where

$$k(\phi) = \alpha_i \phi^i + a_{ij} \phi^i \phi^i + a_{ijk} \phi^i \phi^j \phi^k \tag{16}$$

There is now a theorem that in any order of perturbation theory only terms of the form (8) and not terms of form (10) are generated. This implies immediately that if $\langle 0|F|0\rangle = 0$ at the tree level, it will vanish in all order of perturbation theory.

Terms contributing to (14) are of type (8) but they cannot lead to a supersymmetry breakdown with an unbroken gauge group in the nonabelian case. Thus we have the remarkable situation that terms are not generated in any order of perturbation theory although they are not forbidden by any symmetry. It is these "miraculous" cancellations that give us the stability of supersymmetry in perturbation theory.

The case of an abelian U(1) gauge-group is somewhat anomalous. This is because of the Fayet-Iliopoulos D-term

$$\xi \int d^4\theta\, V = \xi\, D(x) \tag{17}$$

where ξ has dimension of $(\mathrm{mass})^2$. This term is gauge invariant and supersymmetric. In absence of parity conservation (D is pseudoscalar) it can be generated in perturbation theory with a quadratically divergent coefficient, e.g., at the one loop level

$$\sum_i g_i \int \frac{d^4k}{k^2} \tag{18}$$

The generation of such a term can break supersymmetry in perturbation theory. Dimopolous and Raby[14] and Witten[13] independently realized the potential danger of such a term.[15] Witten immediately solved the problem (at least partially), stating the following theorem[13]: If the U(1)-group is embedded at an arbitrary scale in a nonabelian group, the D-term cannot be generated in perturbation theory. This could be used as an argument for a grand unification of hypercharge in supersymmetric models. It was speculated that high mass, low mass cancellations appear in a very peculiar way to prevent the D-term from appearing in perturbation theory.

Facing these interesting possibilities, we investigated the appearance of the D-term more closely.[16] The result is less exciting than anticipated. The D-term can only be generated at the one loop level. All contributions in higher order perturbation theory cancel order by order, without involving high mass-low mass cancellations. The one loop term is proportional to TrQ. TrQ is obviously zero in grand unified models, but does not imply grand unification. It just implies that the "D-anomaly" TrQ vanishes.

We have seen that (except for this anomaly) supersymmetric theories are stable in perturbation theory due to remarkable cancellations of certain terms in the effective potential. We emphasize that these results are only proven in perturbation theory.

5. Nonperturbative Effects

The speculation that the mentioned results do not hold beyond perturbation theory is extremely exciting. Nonperturbative effects could give coefficients in the effective potential that are smaller than any power of the coupling constant and thus allow for huge hierarchies.

It has indeed been shown that in a supersymmetrical quantum mechanical model supersymmetry breakdown occurs due to tunneling effects, supporting the speculation that "miraculous" cancellation only appears in perturbation theory.[13] The nonperturbative breaking of supersymmetry could lead to huge mass hierarchies if there are terms induced in the effective potential that are allowed by the symmetries but are nontheless not generated in perturbation theory. This is exactly what happens in the case of the D-term and terms of the form $\int d^2\theta k(x,\theta)$. Witten observed that in a supersymmetric SU(5) model SU(5)/SU(3) × SU(2) coset instantons could give us a mass scale

$$m^2 \sim M_X^2 \exp\left(-\frac{1}{\alpha_x}\right) \tag{19}$$

with $m^2/M_X \sim 10^{-13}$. However it seems unlikely that instantons can break supersymmetry in four dimensions. Nonetheless, supersymmetry breaking by arbitrarily small effects remains an interesting possibility, and we will discuss supersymmetric grand unified models in the next section.

A way to dynamically break supersymmetry is Supercolor.[14,17] It is a gauge interaction (like Technicolor) that becomes strong in the 1-10 TeV range and is assumed to lead to fermion-antifermion condensates that break supersymmetry.

Suppose there is a Supercolor group SU(N) and chiral left-handed superfields $S = (\varphi,\psi,F)$, $\bar{S} = (\bar{\varphi},\bar{\psi},\bar{F})$ that transform as N, $\bar{N}$ representations of the group. The condensation of

$$\langle \psi\bar{\psi} \rangle = \Lambda_S^3 \tag{20}$$

leads in general to a breakdown of supersymmetry since

$$\{Q,\ \varphi\bar{\psi} + \bar{\varphi}\psi\} = \psi\bar{\psi} + \ldots \tag{21}$$

implying that $\varphi\bar{\psi} + \bar{\varphi}\psi$ is the goldstino. One should, however, keep in mind that this condensation process is an assumption. The situation here is even more ambiguous than in Technicolor models. It could be that only $\varphi\bar{\varphi}$ condense, which in general does not lead to supersymmetry breakdown. Moreover the scalars could screen the fermions and lead to a low energy spectrum of composite particles that respect supersymmetry. These problems are under investigation.[18]

Given these assumptions of $\psi\bar{\psi}$ condensation and supersymmetry breakdown, Supercolor models are in good shape. Realistic models have been constructed in models with Supercolor and Technicolor,[14,17] and in models with Supercolor alone.[19] Especially the latter types are attractive since they relate the breakdown of the weak interactions to the supersymmetry breakdown. If one does not try to solve the strong CP-problem at these energies, there are no problems with light axioms. One should point out that it is easier to obtain "realistic" models with Supercolor than with Technicolor. In order to give masses to fermions in TC-models one is forced to introduce new interactions. In Supercolor models one has scalars and Yukawa couplings that can transmit the masses of the condensates to the light fermions.

Supercolor is certainly a possibility that one has to take seriously into account.

6. Supersymmetric Grand Unified Models

We discuss for simplicity a supersymmetric version of the standard SU(5) GUT. We introduce a real supergauge field V_G and the following complex left-handed superfields.[20]

$$
\begin{array}{ll}
\phi_{24}(x,\theta) & (24) \\
H(x,\theta) & (5) \\
\bar{H}(x,\theta) & (\bar{5}) \\
\bar{X}_i(x,\theta) & (\bar{5}) \\
Y_i(x,\theta) & (10)
\end{array}
\tag{22}
$$

where $i = 1,2,3$ labels the three generations of quarks and leptons. ϕ_{24} vacuum expectation values are assumed to break $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$.

It would be nice if the scalar partners of the fermions in the $\bar{5}$ could serve as Higgs particles. This is not possible since this would imply that in order to avoid proton decay the scalar partners of, let us say, the d-quarks have to be of the order of 10^{15} GeV. Thus supersymmetry would be broken at that scale. We have therefore to introduce $H(x,\theta)$ (and in addition $\bar{H}(x,\theta)$ to cancel anomalies).

It was first pointed out by DRW[21] that in this model the unification scale is not 10^{15} GeV but rather 10^{17} GeV as a result of the gauge fermions in V_G. The weak mixing angle is essentially unchanged (only slightly affected by $\bar{H}$). 10^{17} GeV is already not that far from the Planck mass, and one has to be aware of the fact that gravity might enter the game at that stage.

The Lagrangian for the model can be written as

$$\mathscr{L} = \mathscr{L}_{\text{gauge}} - V \tag{23}$$

where $\mathscr{L}_{\text{gauge}}$ represents the pure gauge Lagrangian plus all the kinetic terms for matter fields and also matter gauge couplings. The term V contains all the necessary Yukawa couplings and scalar interactions. We introduce a singlet chiral superfield ϕ_1 and write V in a scale invariant form.[22]

$$V = \int d^2\theta \Big\{ \lambda_1 \phi_1^3 + \lambda_2 \phi_1 \phi_{24}^2 + \lambda_3 \phi_{24}^3 + h_1 \bar{H}\, \phi_1\, H + h_2 \bar{H}\, \phi_{24}\, H + + g_{ij}\, Y_i\, Y_j\, H + g'_{ij}\, Y_i\, \bar{X}_j\, \bar{H} \Big\} + \text{Hermitean conj.} \tag{24}$$

This is the most general potential which is compatible with classical scale invariance, gauge symmetry, supersymmetry and renormalizability.

$\mathscr{L}$ is invariant under two global abelian symmetries, $U(1)_X \otimes U(1)_Z$ (see Table I). $U(1)_Z$ is anomaly free and connected to Baryon-Lepton number conservation in the usual way

$$B\text{-}L = \frac{1}{5}\,(2Q_Y + Q_Z) \tag{25}$$

where Q_Y is hypercharge. Q_X is, however, only conserved up to an SU(5) anomaly. This symmetry can be used to rotate away the QCD angle θ associated with $F\tilde{F}$; i.e., $U(1)_X$ is a Peccei-Quinn symmetry.[23]

Notice that the occurrence of this symmetry is forced upon us by the assumption of scale invariance. Explicit mass terms $m \int d^2\theta \phi_{24}^2$ would break this symmetry. The singlet ϕ_1 was introduced to allow for the SU(5) breakdown without the inclusion of mass terms in the Lagrangian. A solution of the CP-problem has also been found in nonsupersymmetric models.[24,20] There it was essential to introduce a complex ϕ_{24}. In a supersymmetric model this is automatic.

A solution to the potential that preserves supersymmetry and breaks SU(5) to SU(3) × SU(2) × U(1) is subject to the following constraints

$$\frac{3\lambda_3}{15} V_2 + 2\lambda_2 V_1 = 0$$

$$3\lambda_1 V_1^2 + \lambda_2 V_2^2 = 0 \qquad (26)$$

where $V_1 = \langle 0|\varphi_1|0\rangle$ and $V_2 = \langle 0|\varphi_{24}^Y|0\rangle$ and

$$Y = \frac{1}{\sqrt{60}} \left(-2_{-2_{-2}}\ _3\ _3\right) \qquad (27)$$

Since ϕ_{24} transforms nontrivial under $U(1)_X$ this symmetry is broken at M_X spontaneously and gives rise to a Weinberg-Wilzcek[25] axion of mass.

$$m_a \sim \frac{\Lambda_{QCD}^2}{M_X} \qquad (28)$$

which couples to ordinary matter proportional to m/Fa and therefore effectively decouples.[20]

The superfields H, $\bar{H}$ obtain masses through the vacuum expectation values via h_1 and h_2.

The doublet

$$m_2 = h_1V_1 + \frac{3}{\sqrt{60}} h_2V_2 \tag{29}$$

The triplet

$$m_3 = h_1V_1 - \frac{2}{\sqrt{60}} h_2V_2$$

We have to fine tune at this point. We choose

$$m_2 = 0 \tag{30}$$

Observe, however, two differences of this fine tuning to the one in usual GUT's

(i) $m_2 = 0$, not 10^{-13} M_X, and this result could come from a unified Clebsch-Gordan coefficient of a unified Higgs potential[26]

(ii) $m_2 = 0$ is stable in supersymmetric perturbation theory and can only receive contribution from nonperturbative effects.

How can we break supersymmetry at the TeV scale? One way is an explicit breakdown. Dimopoulos and Georgi[27] undertook this adventure of soft explicit breakdown. They introduce the following parameters:

(i) masses for scalar partners of quarks and leptons

(ii) masses for Higgs fermions

(iii) masses for gauge fermions

(iv) mass matrix for Higgs bosons

They arrive at a realistic model. With respect to the gauge hierarchy problem one has, however, gained nothing. One does not understand the M_S/M_X scale ratio.

Another way in the nonperturbative breakdown à la Witten.[13] It is a spontaneous breakdown since the effective potential respects supersymmetry. The ground state might be asymmetric: a combination of D and F terms might acquire vacuum expectation values. Spontaneous breakdown causes problems with the low energy spectrum in the conventional models. This happens because there are mass relations[28,12]

$$\sum_J (-1)^J \, m_J^2 (2J + 1) = 0 \tag{31}$$

in spontaneously broken supersymmetry at the tree graph level. At this level vacuum expectation values of D and F do not give masses to the fermions and split the masses of the complex scalars in opposite directions. These mass relations do not hold in higher order of perturbation theory.[29]

The argument that with a small coupling constant, higher order corrections are small is however not true.[30] Take for example the model of Ref. 29. There the perturbation parameter is not g but $g\xi/m^2$, which need not be small. Not much progress has been made in using this fact for realistic models. It has been stated that in the SU(3) × SU(2) × U(1) model it is not possible to obtain a realistic model of spontaneous supersymmetry breakdown at the TeV level.[12]

This led Fayet to introduce a new low energy U'(1) gauge theory where he assigns positive charges to all quarks and leptons. This allows him to give large positive $(\text{mass})^2$ to all scalar partners of quarks and leptons. The model has one drawback. The U'(1) has anomalies in triangular graphs of three U'(1) currents as well as in graphs with two SU(3) and a U'(1) current.[31] To arrive at an anomaly free theory one would have to introduce at least one new chiral superfield with non-trivial U'(1) quantum numbers. The fermions in these new superfields ought to get a large mass to remove them from the low energy spectrum. Since these fermions are in nonreal representations of U'(1), mass terms for these fermions would break U'(1) explicitly.

An interesting way[30] out of this problem would be the introduction of an anomaly free U'(1) where all scalar partners of quarks and leptons have positive charges and the Higgses have negative charge. If supersymmetry is broken by a vacuum expectation value of $D' = \xi$ (of U'(1)) all scalar partners of quarks and leptons would obtain large positive $(\text{mass})^2$ $(+g'\xi)$. whereas the Higgses get $m^2 = -g'\xi$ $(g'\xi > 0)$. Thus through radiative correction we would obtain a weak breakdown as a result of the spontaneous supersymmetry breakdown. Details of this scenario have to be worked out.

A grand unified version of this type of model is not yet available. One candidate for a U'(1) of this type would be the U(1) of $E_6/SO(10)$.[20,30]

$$E_6 \to SO(10) \quad U(1)$$

$$27 \to (16,1) + (10,-2) + (1,4)$$

the (16,1) are the usual fermions, and in an SU(5) notation the 10 would correspond to our $5 + \bar{5}$ of Higgses. The color triplets (in the $5 + \bar{5}$) have however to receive a high mass in order to avoid proton decay. Thus the U(1) has to be broken at M_X and cannot survive down to low energies.

This problem seems to occur whenever SU(5) is present in a SSGUT. We do not yet know how to solve this problem. It might be that the relation (30) is connected to this problem.

7. Conclusion

The stability of supersymmetry in perturbation theory for no apparent reasons suggests a unique possibility to understand the hierarchy problem. We do, however, not know how SS is broken by nonperturbative effects. Grand unified supersymmetric models nearly automatically solve the strong CP-problem. There are two problems one hopes to solve in the near future:

(i) A higher unification that gives the relation (30) as a result of Clebsch-Gordan coefficients.

(ii) A U'(1) that allows a realistic low energy spectrum and induces the $SU(2) \times U(1) \times U'(1)$ breakdown through radiative correction of the supersymmetry breakdown.

As indicated, a solution to (i) and (ii) might be related.

The attempt of Supercolor models leads to realistic models.

As a result there are hopes, but one has to face the potential danger that one cannot proceed without the inclusion of gravity.

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26. By this we mean that in a larger group ϕ_1, ϕ_{24}, H and $\bar{H}$ could belong to a single representation. Relations like

$$\frac{V_1}{V_2} = - \frac{3h_2}{\sqrt{60}\, h_1} = - \frac{3\lambda_3}{\sqrt{60}\, \lambda_2}$$

could then possibly be obtained as Clebsch-Gordan coefficients.

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31. In order to make the U'(1) anomaly free, one is forced to introduce new superfields that transform nontrivially under $SU(3)_C$ in addition to the fields introduced by Fayet.

TABLE I

$\underline{U(1)_X}$

$$\Phi_1(x,\theta) \rightarrow e^{i\alpha}\, \Phi_1(x, e^{\frac{3}{2}i\alpha}\theta)$$

$$\Phi_{24}(x,\theta) \rightarrow e^{i\alpha}\, \Phi_{24}(x, e^{\frac{3}{2}i\alpha}\theta)$$

$$H(x,\theta) \rightarrow e^{i\alpha}\, H(x, e^{\frac{3}{2}i\alpha}\theta)$$

$$\bar{H}(x,\theta) \rightarrow e^{i\alpha}\, \bar{H}(x, e^{\frac{3}{2}i\alpha}\theta)$$

$$Y_i(x,\theta) \rightarrow e^{i\alpha}\, Y_i(x, e^{\frac{3}{2}i\alpha}\theta)$$

$$\bar{X}_i(x,\theta) \rightarrow e^{i\alpha}\, \bar{X}_i(x, e^{\frac{3}{2}i\alpha}\theta)$$

$$V_G(x,\theta,\theta^*) \rightarrow V_G(x, e^{\frac{3}{2}i\alpha}\theta, e^{-\frac{3}{2}i\alpha}\theta^*)$$

$\underline{U(1)_Z}$

$$H(x,\theta) \rightarrow e^{-2i\alpha}\, H(x,\theta)$$

$$\bar{H}(x,\theta) \rightarrow e^{2i\alpha}\, \bar{H}(x,\theta)$$

$$Y_i(x,\theta) \rightarrow e^{i\alpha}\, Y_i(x,\theta)$$

$$\bar{X}_i(x,\theta) \rightarrow e^{-3i\alpha}\, \bar{X}_i(x,\theta)$$

All other fields remaining unchanged.