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A MAGNIFYING MAGNETIC OPTICAL ACHROMAT*

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ABSTRACT

Scaling laws are developed to extend the unity magnification achromat principle [1] to magnetic optical systems possessing magnification. Examples are included to illustrate the methods developed.

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1. Introduction

Design criteria for a unity magnification second-order magnetic optical achromat have been described in a previous publication by K. L. Brown [1]. The achromat is a periodic array of n (n > 3) identical magnetic optical cells. The transformation matrix for the total system is the identity matrix and all second order transverse geometric and chromatic aberration terms are identically zero. Each cell of the achromat contains focusing and defocusing quadrupole components. One or more dipoles are added to each cell to provide momentum dispersion and two sextupole components are inserted into each cell, one for each transverse plane . One sextupole is positioned so as to couple more strongly to the x phase space and the other so as to couple more strongly to the y phase space. Corresponding x and y sextupoles in all cells are identical. Thus there are two families of sextupoles, one for the x plane and one for the y plane. The quadrupole components are adjusted to provide a unity first-order transformation matrix in each transverse plane. The two sextupole families are then adjusted to cancel one second order chromatic aberration term in each plane. When this is done all of the second order geometric and chromatic aberration terms vanish. The vanishing of the geometric terms is a direct consequence of the inherent symmetry in the cellular structure of the design. The chromatic terms vanish because the sextupoles are situated at the same position in each cell. A simple proof of this is given in the paper by Brown, and D. C. Carey has subsequently provided a more rigorous mathematical proof of the vanishing of the chromatic aberration terms [2].

The achromat described above is ideal for transporting a beam of charged particles where it is desired that the phase space configuration at the final point be a faithful reproduction of the beam at the entrance of the system. However there are many applications where it is desired to magnify or demagnify the beam size and at the same time have a system where the second order aberration terms either vanish or remain sufficiently small. It is the purpose of this paper to describe the design criteria for such a system which we shall call a "Magnifying Achromat". The notation used in this paper is that of TRANSPORT [3,4] and some traditional circular machine notation [5].

A typical cell structure for an achromat is the following:

$$Q_x-S_x---D---Q_y-S_y---D---$$

where Q_x is a quadrupole focusing in the x plane, Q_y is a quadrupole focusing in the y plane, S_x is a sextupole coupling predominantly to the x phase space, S_y is a sextupole coupling predominantly to the y phase space, and D is a dipole bending in the x plane. In this example each cell begins with Q_x and ends at the beginning of the next Q_x. Four or more such cells adjusted to a total phase shift of 2π (a unity transformation matrix) constitute a second-order achromat when S_x and S_y are adjusted so the second-order chromatic aberrations vanish. The transformation matrix for each cell (in each phase plane x and y) may be expressed as follows:

$$R_A = \begin{vmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{vmatrix} \qquad (1)$$

where β and α have the same values at the same relative position in each unit cell [5]. The transformation matrix for n such cells in sequence is

$$R_A^n = \begin{vmatrix} \cos n\mu + \alpha \sin n\mu & \beta \sin n\mu \\ -\gamma \sin n\mu & \cos n\mu - \alpha \sin n\mu \end{vmatrix} \qquad (2)$$

2. The magnifying achromat

The magnifying achromat is constructed in a similar manner to the unity magnification achromat described above but with some important differences. It is made of n cells $c_1 \ldots c_n$ which transport a beam so that the parameters $\beta_1\alpha_1, \beta_2\alpha_2 \ldots \beta_n\alpha_n$ describing the beam at the entrance of each cell satisfy the following relations:

$$\left(\frac{\beta_2}{\beta_1} \right)^{1/2} = \left(\frac{\beta_{i+1}}{\beta_i} \right)^{1/2} = \left(\frac{\beta_{n+1}}{\beta_n} \right)^{1/2} = r$$

and

$$\alpha_1 = \alpha_2 = \alpha_i = \alpha_n = \alpha \qquad (4)$$

These relations define a beam envelope magnification factor, r, from cell to cell. We now define a matrix M as follows

$$M = \begin{vmatrix} r & 0 \\ 0 & 1/r \end{vmatrix} \qquad (5)$$

The transformation matrices of the successive cells may now be expressed in the following way:

for the first cell

$$R_1 = MR_A \tag{6}$$

for the ith cell

$$R_i = M^{i-1} R_1 M^{-(i-1)} = M^i R_A M^{-(i-1)} \tag{7}$$

and the total transformation matrix for n cells is

$$R_T = R_n \cdots R_1 = M^n R_A^n \tag{8}$$

When the cells are adjusted so that $\mu = 2\pi/n$, the total phase shift is 2π and the transformation matrix R_T simplifies to

$$R_T = \begin{vmatrix} r^n & 0 \\ 0 & 1/r^n \end{vmatrix} \tag{9}$$

where r^n is the total optical magnification of the system. Appendix 2 contains a proof that such an array is always achromatic to first order.

We now must examine how such a system can be built. A system of n identical cells whose transformation matrices R_A are adjusted so that $R_A^n = I$ is called a unity magnification achromat. Let us denote it by acr1. The system made of the n cells whose transformation matrices are $R_1, R_2, \ldots R_n$ (as defined in [6] and [7]) will be referred to as the corresponding magnifying achromat associated with acr1.

3. First order properties of the magnifying achromat.

Consider the transformation matrix of the ith cell, including the dispersive terms d and d'

$$R_i = \begin{vmatrix} c & s & d \\ c' & s' & d' \\ 0 & 0 & 1 \end{vmatrix} \qquad (10)$$

and extend the magnification matrix M to a 3×3 matrix as follows

$$M = \begin{vmatrix} r & 0 & 0 \\ 0 & 1/r & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Let us now design successive cells so that their matrices scale as shown in eq. (7). Then

$$R_{i+1} = M R_i M^{-1} = \begin{vmatrix} c & sr^2 & dr \\ c'/r^2 & s' & d'/r \\ 0 & 0 & 1 \end{vmatrix} \qquad (11)$$

Formulae (10) and (11) determine the first-order scaling laws from cell to cell. These scaling laws also apply to the corresponding subarray of the successive optical cells and in particular to each element of the cells. Now let us develop the scaling laws for typical building blocks.

3.1 Scaling law for a drift distance.

The matrix for a drift of length ℓ is

$$R_{drift} = \begin{vmatrix} 1 & \ell \\ 0 & 1 \end{vmatrix} \qquad (12)$$

Comparing eq. (12) with eq. (11), we conclude that the length ℓ scales with the matrix element R_{12}. So the length of a drift in cell 2 must be r^2 times its length in cell 1, and so forth for subsequent cells.

3.2 Scaling law for a quadrupole.

The matrix for a quadrupole is

$$R_{quad} = \begin{vmatrix} \cos k\ell & \sin k\ell/k \\ -k \sin k\ell & \cos k\ell \end{vmatrix} \tag{13}$$

where sin and cos become sinh and cosh for the defocusing plane. R_{11} is constant from cell to cell, therefore $k\ell$ must be constant. Since R_{12} scales as r^2, k and $k^2\ell$ must scale as r^{-2}.

3.3 Scaling for a uniform field wedge bending magnet.

The matrix for a uniform field wedge bending magnet referred to the center of the magnet is

$$R_{bend} = \begin{vmatrix} 1 & 0 & 0 \\ -\sin\alpha/\rho & 1 & \sin\alpha \\ 0 & 0 & 1 \end{vmatrix} \tag{14}$$

The above matrix must be premultiplied and postmultiplied by the matrix of a drift of length $\rho\tan\alpha/2$ to obtain the first order transformation matrix of a wedge magnet. The physical length of the dipole is equal to $\ell = \rho\alpha$. Correct first order scaling of the focusing and dispersive properties of the dipole is achieved when $\sin\alpha$ scales as r^{-1} and ρ scales as r. This implies that ℓ scales according to a more complex law for large α. However, for small angles $\sin\alpha \approx \alpha$ and ℓ is approximately constant. Since distances between centers of successive elements must scale as r^2, additional drift spaces have to be added on each side of the dipoles in successive cells. In the vertical plane the wedge dipole acts like a drift space of length ℓ. This length should scale as r^2 which conflicts with the exact scaling needed for the horizontal plane. Therefore for large bending angles the first order fitting will only be approximate. This can be observed in example 2, given below. However for most high energy applications $\tan\alpha = \sin\alpha = \alpha$ is a valid approximation. In this limit, the dipole scaling laws are quite satisfactory as is illustrated in example 1.

4. Second order properties of the magnifying achromat.

For this analysis we refer the reader to the theory and notation developed in reference [4] and in particular to tables 1 and 8 which define the notation and the formalism which we shall now use.

4.1 Scaling of the sine-like, cosine-like and the dispersion functions.

In reference [4], the R_{12} matrix element of eq. (10) above is called the sine-like function s, the R_{11} matrix element is the cosine-like function c, and R_{13} is the dispersion function d. There is no scaling law from cell to cell for the s, c and d functions in a given achromat. However, if we compare the functions s_o, c_o and d_o as defined for the nonmagnifying achromat, acrl, with the functions s, c and d of the corresponding magnifying achromat as we move from cell to cell, the ratios s/s_o, c/c_o, and d/d_o scale as r and the ratios s'/s'_o, c'/c'_o and d'/d'_o scale as r^{-1}. This permits us to determine the second order scaling laws.

4.2 Scaling law for the sextupole strengths.

The integrated strength of a sextupole is $S_j = B_o L/a^2 B\rho$. Let us look at the second order geometric aberration terms in table VII of ref [4] and consider only the contribution coming from the sextupoles. What must be the scaling applied to their strengths so that the second order geometric aberration terms are zero at the end of a 2π phase shift? These geometric terms are all of the form:

$$\sum S_j \, s^n \, c^m \tag{15}$$

where

$$n + m = 3$$

and where the summation extends over the entire achromat. Comparing this sum to the one that would be obtained for the nonmagnifying achromat we observe that the function ratios c/c_o and s/s_o scale as r from cell to cell. In order to obtain the same contribution as for the non magnifying achromat, S_j, the integrated sextupole strength, must scale as r^{-3}. It is easily verified that this scaling law will guarantee an exact cancellation of all geometric terms arising from the presence of the sextupoles. The sextupole strengths may now be adjusted to cancel the chromatic terms arising from the quadrupoles and the dipoles as in the nonmagnifying achromat. The length of the sextupoles should scale as r^2 although this is not rigorously necessary in most applications since the effective position of the sextupole is at its center.

4.3 Second order aberrations caused by the dipoles.

The second order geometric and chromatic terms produced by the dipole magnets in a magnifying achromat will not vanish at the 2π phase shift point in contrast to the unity magnifying achromat. This is verified by using ref. [4] and is observed in the computer simulations of magnifying achromat designs. The importance of these residual terms increases with the dipole strengths. Example 1 and example 2 given in the following paragraph illustrate this fact clearly.

5. Examples of uses of the magnifying achromat.

5.1 We now present here an example of a magnifying achromat to illustrate the scaling laws and to demonstrate its aberration properties with a TRANSPORT run output.

The achromat example shown is made of five cells each characterized by a magnification factor of $r = 1.2$. The total achromat has a magnification factor of $r^5 = 2.48832$ equal to its optical magnification. In order to observe the scaling laws most clearly, the output should be scanned from the last element upwards. For the benefit of the reader, we have added to the standard TRANSPORT output the values of $\sin\alpha$ for the dipoles and the integrated strengths of the quadrupoles and sextupoles.

The reader should observe that the following scaling laws are used in the examples shown:

All lengths, except those of the dipoles and their immediately adjoining drifts, scale as $r^2 = (1.2)^2 = 1.44$.

The radius of curvature in the dipoles scale as $r = 1.2$.

The field strength in the dipoles scales as $r^{-1} = 1/1.2$.

The value of $\sin\alpha$ in the dipoles scales as $r^{-1} = 1/1.2$.

The quadrupole strengths $k^2\ell = B_o\ell/aB\rho$ scale as $r^{-2} = (1.2)^{-2}$.

The sextupole strengths $S = B_oL/a^2B\rho$ scale as $r^{-3} = (1.2)^{-3}$.

The sum $D1+L_B+D2$ of the equivalent dipole length, L_B, and its two adjoining drift lengths scale as $r^2 = 1.44$. Note that for scaling purposes $L_B = 2\rho\tan\alpha/2$, where ρ is the bending radius of the dipole.

Note that in the TRANSPORT runs the scaling of the quadrupole strengths was achieved by scaling the aperture, a, according to r^4 and keeping the field, B, constant. Similarly the scaling of the sextupole strengths was achieved by scaling the aperture, a, according to $r^{5/2}$ and keeping the field B constant. This set up allows TRANSPORT to link the elements of one family from cell to cell and thereby find a solution to the fitting problem.

To within the accuracy allowed by the single precision IBM run of TRANSPORT , the first order achromaticity and the cancellation of all second order chromatic aberrations are verified. This is illustrated in example 1.

5.2 Example 2.

Example 2 is similar to example 1, except that the field strength of the first dipole has been increased by a factor of ten. The four other dipoles are scaled according to the scaling laws given above. The purpose of this example is to illustrate that the residual second-order geometric aberration terms increase with the strength of the dipoles, and that the first-order fitting is not perfect in the y plane because $\sin \alpha \neq \alpha$.

5.3 Use of the magnifying achromat in a circular machine.

The authors have used the principle of the magnifying achromat as a basis for studying the design of low beta insertions in storage rings [6,7].

References

[1] K.L. Brown, "A second-order magnetic optical achromat," SLAC-PUB-2257 (February 1979).

[2] D.C. Carey, "Why a second-order magnetic optical achromat works," FERMILAB-PUB-79/61-EXP, 2042, Nucl. Instrum. Methods 189 (1981) 365-367.

[3] K.L. Brown, D.C. Carey, Ch. Iselin and F. Rothacker, "TRANSPORT a computer program for designing charged particle beam transport systems," SLAC 91 (1973 rev.), NAL 91, and CERN 80-04.

[4] K.L. Brown, "A first- and second-order matrix theory for the design of beam transport systems and charged particle spectrometers," SLAC Report No. 75, or Advances in Particle Physics. 1,(1967) 71-134.

[5] K.L. Brown, "Beam envelope matching for beam guidance systems," SLAC-PUB-2370 (August 1980); also Nucl. Instrum. and Methods 187 (1981) 51-65; Presented at the Conference on charged particle optics, Giessen, West Germany, Sept. 8-12, 1980.

[6] K.L. Brown and R.V. Servranckx, "Chromatic corrections for large storage rings, the 11th Int. Conf. on High Energy Accelerators," CERN, Geneva, Switzerland, 1980, p. 656-660.

[7] R.V.Servranckx, "Chromatic correction schemes for large circular machines"; Nucl. Instrum. and Methods 187 (1981) 67-73; Presented at the Conference on charged particle optics, Giessen, West Germany Sept. 8-12, 1980.

Appendix 1.

Let T_A be the 3×3 transformation matrix representing each cell in a unity magnification achromat.

$$T_A = \begin{vmatrix} c & s & d \\ c' & s' & d' \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} R_A & v \\ 0 & 1 \end{vmatrix} \qquad (16)$$

where v is the dispersion vector $\begin{pmatrix} d \\ d' \end{pmatrix}$, then

$$T_A^n = \begin{vmatrix} R^n & \left(R_A^{n-1} + R_A^{n-2} + \ldots + I\right)v \\ 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} R^n & \left[\left(R_A^n - I\right)\left(R_A - I\right)^{-1}\right]v \\ 0 & 1 \end{vmatrix} \qquad (17)$$

We conclude that T_A^n is achromatic to first order if and only if

$$\left[\left(R_A^n - I\right)\left(R_A - I\right)^{-1}\right]v = 0 , \qquad (18)$$

so, if and only if, either

(a) $v = 0$:(every cell is achromatic),

or

(b) $R_A^n = I$:(the phase space advance is a multiple of 2π).

Appendix 2

Denote by T_A the 3×3 transformation matrix of each cell of a nonmagnifying achromat. Appendix 1 showed that T_A^n is achromatic when $T_A^n = I$.

The transformation matrices of the corresponding magnifying achromat are

$$\begin{aligned} T_1 &= M\,T_A \\ T_2 &= M^2\,T_A\,M^{-1} \\ &\;\;\vdots \\ T_n &= M^n\,T_A\,M^{-(n-1)} \end{aligned} \tag{19}$$

Then the total transformation matrix is

$$T_t = T_n \ldots T_1 = M^n\,T_A^n \tag{20}$$

Since M^n is a diagonal matrix, we can state:

T_t is achromatic if and only if T_A^n is achromatic

and using appendix 1, we have:

T_t is achromatic if and only if

(a) each cell is achromatic,

or

(b) the total phase advance is a multiple of 2π.

1#EXAMPLE1// MAGNIFYING 2ND-ORDER ACHROMAT// R V S 5/4/81

	Length	B_O	Aperture	Strength	Rho	Alpha	sin(alpha)
SEXT	1.28994 M	6.45725 KG	309.58618 MM	2.605E-01 M-2			
QUAD	4.29981 M	7.25172 KG	924.42065 MM	1.011E-01 M-1			
DRIFT	8.59963 M						
SEXT	1.28994 M	-14.39964 KG	309.58618 MM	-5.810E-01 M-2			
QUAD	4.29981 M	-8.31280 KG	924.42065 MM	-1.159E-01 M-1			
DRIFT	6.85453 M						
BEND	3.49011 M	1.60862 KG			207.360 M	0.964 DEG	1.682E-02
DRIFT	6.85453 M						
SEXT	0.89579 M	6.45725 KG	196.25885 MM	4.502E-01 M-2			
QUAD	2.98598 M	7.25172 KG	445.80469 MM	1.456E-01 M-1			
DRIFT	5.97196 M						
SEXT	0.89579 M	-14.39964 KG	196.25885 MM	-1.004 M-2			
QUAD	2.98598 M	-8.31280 KG	445.80469 MM	-1.669E-01 M-1			
DRIFT	4.22684 M						
BEND	3.49018 M	1.93034 KG			172.800 M	1.157 DEG	2.019E-02
DRIFT	4.22684 M						
SEXT	0.62208 M	6.45725 KG	124.41599 MM	7.780E-01 M-2			
QUAD	2.07360 M	7.25172 KG	214.99081 MM	2.097E-01 M-1			
DRIFT	4.14720 M						
SEXT	0.62208 M	-14.39964 KG	124.41599 MM	-1.735 M-2			
QUAD	2.07360 M	-8.31280 KG	214.99081 MM	-2.404E-01 M-1			
DRIFT	2.40197 M						
BEND	3.49029 M	2.31641 KG			144.000 M	1.389 DEG	2.424E-02
DRIFT	2.40197 M						
SEXT	0.43200 M	6.45725 KG	78.87201 MM	1.344 M-2			
QUAD	1.44000 M	7.25172 KG	103.67998 MM	3.019E-01 M-1			
DRIFT	2.88000 M						
SEXT	0.43200 M	-14.39964 KG	78.87201 MM	-2.998 M-2			
QUAD	1.44000 M	-8.31280 KG	103.67998 MM	-3.461E-01 M-1			
DRIFT	1.13465 M						
BEND	3.49044 M	2.77970 KG			120.000 M	1.667 DEG	2.909E-02
DRIFT	1.13465 M						
SEXT	0.30000 M	6.45725 KG	50.00000 MM	2.323 M-2			
QUAD	1.00000 M	7.25172 KG	50.00000 MM	4.348E-01 M-1			
DRIFT	2.00000 M						
SEXT	0.30000 M	-14.39964 KG	50.00000 MM	-5.180 M-2			
QUAD	1.00000 M	-8.31280 KG	50.00000 MM	-4.984E-01 M-1			
DRIFT	0.25449 M						
BEND	3.49066 M	3.33564 KG			100.000 M	2.000 DEG	3.490E-02
DRIFT	0.25449 M						

0.009	13.021 MM					
0.002	0.622 MR	0.000				
0.0	0.832 MM	0.0	0.0			
0.0	0.636 MR	0.0	0.0	-0.001		
-0.000	0.011 CM	-0.000	0.000	0.0	0.0	
0.0	1.600 PC	-0.000	-0.000	0.0	0.0	-0.998

TRANSFORM 1

0.40188	0.00004	0.0	0.0	0.0	-0.00010
0.00000	2.48833	0.0	0.0	0.0	-0.00003
0.0	0.0	0.40194	-0.00013	0.0	0.0
0.0	0.0	-0.00020	2.48789	0.0	0.0
-0.00000	0.00000	0.0	0.0	1.00000	-0.00703
0.0	0.0	0.0	0.0	0.0	1.00000

0*2ND ORDER TRANSFORM*

1 11	8.277E-06								
1 12	-4.032E-04	1 22	4.196E-03						
1 13	0.0	1 23	0.0	1 33	3.046E-05				
1 14	0.0	1 24	0.0	1 34	1.748E-04	1 44	-2.047E-04		
1 15	0.0	1 25	0.0	1 35	0.0	1 45	0.0	1 55	0.0
1 16	1.470E-05	1 26	-4.531E-07	1 36	0.0	1 46	0.0	1 56	0.0
								1 66	-2.682E-05
2 11	1.986E-06								
2 12	-1.129E-04	2 22	1.314E-03						
2 13	0.0	2 23	0.0	2 33	6.717E-06				
2 14	0.0	2 24	0.0	2 34	1.719E-05	2 44	-1.076E-04		
2 15	0.0	2 25	0.0	2 35	0.0	2 45	0.0	2 55	0.0
2 16	6.651E-06	2 26	-9.350E-05	2 36	0.0	2 46	0.0	2 56	0.0
								2 66	-2.806E-06
3 11	0.0								
3 12	0.0	3 22	0.01						
3 13	-2.657E-06	3 23	1.713E-04	3 33	0.0				
3 14	2.932E-05	3 24	-3.161E-04	3 34	0.0	3 44	0.0		
3 15	0.0	3 25	0.0	3 35	0.0	3 45	0.0	3 55	0.0
3 16	0.0	3 26	0.0	3 36	-2.596E-05	3 46	-2.078E-04		
4 11	0.0								
4 12	0.0	4 22	0.0						
4 13	1.297E-05	4 23	-3.769E-04	4 33	0.0				
4 14	1.623E-05	4 24	-1.048E-03	4 34	0.0	4 44	0.0		
4 15	0.0	4 25	0.0	4 35	0.0	4 45	0.0	4 55	0.0
4 16	0.0	4 26	0.0	4 36	8.897E-05	4 46	1.631E-04		
5 11	-7.724E-09								
5 12	2.032E-07	5 22	2.260E-07						
5 13	0.0	5 23	0.0	5 33	-1.762E-07				
5 14	0.0	5 24	0.0	5 34	-5.879E-07	5 44	-3.138E-06		
5 15	0.0	5 25	0.0	5 35	0.0	5 45	0.0	5 55	0.0
5 16	2.140E-08	5 26	-1.024E-06	5 36	0.0	5 46	0.0	5 56	0.0
								5 66	-1.728E-04

1#EXAMPLE2// MAGNIFYING 2ND-ORDER ACHROMAT// R V S 5/4/81

	Length	B_o	Aperture	Strength	Rho	Alpha	sin(alpha)
SEXT	1.28994 M	0.63766 KG	309.58618 MM	2.572E-02 M-2			
QUAD	4.29981 M	7.12808 KG	924.42065 MM	9.941E-02 M-1			
DRIFT	8.59963 M						
SEXT	1.28994 M	-1.44317 KG	309.58618 MM	-5.823E-02 M-2			
QUAD	4.29981 M	-8.28693 KG	924.42065 MM	-1.155E-01 M-1			
DRIFT	6.95412 M						
BEND	3.43590 M	16.08620 KG			20.736 M	9.494 DEG	1.649E-01
DRIFT	6.95412 M						
SEXT	0.89579 M	0.63766 KG	196.25885 MM	4.446E-02 M-2			
QUAD	2.98598 M	7.12808 KG	445.80469 MM	1.431E-01 M-1			
DRIFT	5.97196 M						
SEXT	0.89579 M	-1.44317 KG	196.25885 MM	-1.006E-01 M-2			
QUAD	2.98598 M	-8.28693 KG	445.80469 MM	-1.664E-01 M-1			
DRIFT	4.29782 M						
BEND	3.44293 M	19.30344 KG			17.280 M	11.416 DEG	1.979E-01
DRIFT	4.29782 M						
SEXT	0.62208 M	0.63766 KG	124.41599 MM	7.683E-02 M-2			
QUAD	2.07360 M	7.12808 KG	214.99081 MM	2.061E-01 M-1			
DRIFT	4.14720 M						
SEXT	0.62208 M	-1.44317 KG	124.41599 MM	-1.739E-01 M-2			
QUAD	2.07360 M	-8.28693 KG	214.99081 MM	-2.397E-01 M-1			
DRIFT	2.44911 M						
BEND	3.45320 M	23.16412 KG			14.400 M	13.740 DEG	2.375E-01
DRIFT	2.44911 M						
SEXT	0.43200 M	0.63766 KG	78.87201 MM	1.328E-01 M-2			
QUAD	1.44000 M	7.12808 KG	103.67998 MM	2.968E-01 M-1			
DRIFT	2.88000 M						
SEXT	0.43200 M	-1.44317 KG	78.87201 MM	-3.005E-01 M-2			
QUAD	1.44000 M	-8.28693 KG	103.67998 MM	-3.450E-01 M-1			
DRIFT	1.15926 M						
BEND	3.46828 M	27.79698 KG			12.000 M	16.560 DEG	2.850E-01
DRIFT	1.15926 M						
SEXT	0.30000 M	0.63766 KG	50.00000 MM	2.294E-01 M-2			
QUAD	1.00000 M	7.12808 KG	50.00000 MM	4.274E-01 M-1			
DRIFT	2.00000 M						
SEXT	0.30000 M	-1.44317 KG	50.00000 MM	-5.193E-01 M-2			
QUAD	1.00000 M	-8.28693 KG	50.00000 MM	-4.968E-01 M-1			
DRIFT	0.25449 M						
BEND	3.49066 M	33.35637 KG			10.000 M	20.000 DEG	3.420E-01
DRIFT	0.25449 M						

0.004	13.021 MM						
0.005	0.623 MR	0.000					
0.0	0.846 MM	0.0	0.0				
0.0	0.627 MR	0.0	0.0	-0.068			
-0.041	1.128 CM	0.000	0.001	0.0	0.0		
0.0	1.600 PC	-0.000	-0.000	0.0	0.0	-0.999	

```
*TRANSFORM 1*
              0.40187    0.00020    0.0        0.0        0.0        -0.00010
              0.00000    2.48835    0.0        0.0        0.0        -0.00003
              0.0        0.0        0.40852   -0.01438    0.0         0.0
              0.0        0.0       -0.01918    2.44849    0.0         0.0
              0.00000   -0.00000    0.0        0.0        1.00000    -0.70422
              0.0        0.0        0.0        0.0        0.0         1.00000
0*2ND ORDER TRANSFORM*
   1 11  4.463E-05
   1 12 -1.969E-03   1 22  1.658E-02
   1 13  0.0         1 23  0.0         1 33  2.436E-04
   1 14  0.0         1 24  0.0         1 34  1.691E-03   1 44 -1.749E-03
   1 15  0.0         1 25  0.0         1 35  0.0         1 45  0.0         1 55  0.0
   1 16  1.037E-03   1 26  1.314E-03   1 36  0.0         1 46  0.0         1 56  0.0
                                                                  1 66 -1.745E-02

   2 11  9.348E-06
   2 12 -5.530E-04   2 22  6.093E-03
   2 13  0.0         2 23  0.0         2 33  4.984E-05
   2 14  0.0         2 24  0.0         2 34  1.510E-04   2 44 -9.404E-04
   2 15  0.0         2 25  0.0         2 35  0.0         2 45  0.0         2 55  0.0
   2 16  4.802E-04   2 26 -6.418E-03   2 36  0.0         2 46  0.0         2 56  0.0
                                                                  2 66 -1.904E-03

   3 11  0.0
   3 12  0.0         3 22  0.0
   3 13 -2.534E-05   3 23  1.736E-03   3 33  0.0
   3 14  3.077E-04   3 24 -3.491E-03   3 34  0.0         3 44  0.0
   3 15  0.0         3 25  0.0         3 35  0.0         3 45  0.0         3 55  0.0
   3 16  0.0         3 26  0.0         3 36 -1.773E-03   3 46  2.127E-03

   4 11  0.0
   4 12  0.0         4 22  0.0
   4 13  9.928E-05   4 23 -3.050E-03   4 33  0.0
   4 14  1.341E-04   4 24 -1.013E-02   4 34  0.0         4 44  0.0
   4 15  0.0         4 25  0.0         4 35  0.0         4 45  0.0         4 55  0.0
   4 16  0.0         4 26  0.0         4 36  5.154E-04   4 46  1.051E-02

   5 11 -9.648E-07
   5 12  2.581E-05   5 22  1.645E-05
   5 13  0.0         5 23  0.0         5 33 -4.911E-07
   5 14  0.0         5 24  0.0         5 34 -4.337E-05   5 44  2.855E-05
   5 15  0.0         5 25  0.0         5 35  0.0         5 45  0.0         5 55  0.0
   5 16  1.529E-05   5 26 -8.686E-04   5 36  0.0         5 46  0.0         5 56  0.0
                                                                  5 66 -1.569E-02
```