SLAC-PUB-2787 (Rev.)
August 1981
October 1981 (Rev.)
(T/E)

INITIAL STATE INTERACTIONS AND THE DRELL-YAN PROCESS*

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ABSTRACT

We show that initial state interactions violate the usual QCD factorization predictions for massive lepton-pair production in leading twist. The initial state collisions correct $d\sigma/dQ^2dx_F$ and also smear the lepton-pair transverse momentum distribution.

Submitted to Physical Review Letters

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* Work supported by the Department of Energy, contract DE-AC03-76SF00515.
** Work supported by the National Science Foundation.

A puzzling aspect of standard factorized QCD predictions[1] for massive lepton-pair production in hadronic collisions is the absence of any corrections to the cross section due to the effects of initial state interactions. For example, the elastic and inelastic collisions of a hadron propagating in a nuclear target might be expected to alter profoundly its constituents' transverse and longitudinal momentum distributions as well as their color quantum number correlations. Such initial-state interactions might destroy any simple connection between the Drell-Yan[2] cross sections and the projectile's structure functions as measured in deep inelastic scattering. It thus seems all the more remarkable that the standard QCD predictions for $d\sigma/dQ^2$ are quite consistent with experiment[3] (up to uncertain normalization factors) including the important feature that lepton-pair production in hadron-nucleus collisions is additive in the nucleon number A at high Q^2.

The standard derivations of QCD factorization for hard inclusive reactions are based on the organization of all collinear divergences into universal factors that can be incorporated into hadronic structure functions, and the demonstration that infrared divergencies cancel in the physical cross section. In this paper we are concerned with (s-independent) contributions from initial state interactions.[4] These come from the region of integration near the fermion poles and away from the collinear region. Such contributions correspond physically to the usual Glauber singularities, which occur, for example, when a fermion scatters in a target and then propagates nearly on-shell over a finite distance before annihilating. Because one is dealing with nearly on-shell scattering matrix-elements one cannot use Ward identities or a choice of gauge to eliminate these contributions.[5]

In order to illustrate the physics of the initial state interactions, we analyze the process $\pi N \to \mu^+ \mu^- X$ to all orders in perturbative QCD. We neglect only terms of higher order in $1/s$. The leading order gluon exchange contribution to initial state elastic scattering of the active $\bar{q}$ on a spectator quark in the nucleon is shown in Fig. 1(a). To leading order in s the energy denominator after the gluon exchange has the form $yr_\perp^2 - 2r_\perp \cdot \ell_\perp + i\varepsilon$, where $r_\perp^2 = s$, $\ell_\perp^2/1-y = -t$. The factor associated with the vector gluon exchange is $(r_\perp \cdot \ell_\perp / y\ell_\perp^2)dy$. The near on-shell Glauber region, which contributes to the leading twist cross section, corresponds to fixed momentum transfer $-t = \ell_\perp^2$ and $y = \ell^+/p^+ = \mathcal{O}\sqrt{-t/s}$.[6]

In this simple example one can already see a feature that has important consequences for phenomenology: the initial state interactions of the active q and $\bar{q}$ strongly modify the transverse momentum distribution $d\sigma/dQ^2 d^2Q_\perp$ of the lepton pair; i.e., the observed $Q_\perp$ distribution reflects not only the transverse momentum distribution intrinsic to the hadron wave functions, but also the transverse momentum exchange that occurs in the initial-state collisions. Accordingly, we predict that $\langle Q_\perp^2 \rangle$ should increase monotonically with the length L of a nuclear target, contrary to parton model predictions.[7]

Let us now consider the integrated cross section $d\sigma/dQ^2$. The interference diagram of Fig. 1(b) contributes in the same order of perturbation theory as Fig. 1(a) and acts to renormalize the zeroth-order distribution. In fact, explicit computation of the leading twist contributions to $d\sigma/dQ^2$ shows that in an Abelian theory, for Q^2

sufficiently large, the contributions of Figs. 1(a) and 1(b) completely cancel. This cancellation of elastic scattering contributions persists in all orders of perturbation theory, and is a consequence of the exponentiation of the (imaginary) Glauber elastic amplitude. It reflects the fact that the spectrum of incident of antiquarks, although modified in transverse momentum, is unchanged in flux normalization and only negligibly changed in longitudinal momentum. Thus, $d\sigma/dQ^2$ is unchanged by elastic initial state interactions -- provided that $x_{\bar{q}} s \gg |t|$, and the antiquark beam remains coherent over the region in the target within which the annihilating quark is confined; i.e., if $(\Delta p_z^{\bar{q}})_{lab} \cdot L_N \ll 1$ where L_N is the nucleon size and $\Delta p_z^{\bar{q}} \cong \sqrt{-t/s}\, M_N$ is the average change of the antiquark's longitudinal momentum upon passage through the target. Therefore the cancellation occurs in the Abelian theory for $s \gg |\bar{t}|(M_N L_N)^2$.

Although the total flux of $\bar{q}$ is unchanged by initial state interactions in the non-Abelian theory, the color correlations <u>are</u> changed by gluon exchange. Consequently, the elastic initial state corrections do not cancel. For example, the ratio of color factors for hadron-hadron scattering for Figs. 1(a) and 1(b) is $-1/(n_c^2 - 1) = -1/8$. Thus, these contributions do not cancel, but add. (Cross diagrams and vertex diagrams with trigluon couplings have the wrong phase to contribute to the Glauber singularities in this order.) Their net effect in the non-Abelian theory is to change the overall normalization of $d\sigma/dQ^2 dx_F$ by a new initial state factor $I_{e\ell} \neq 1$, without changing the Q^2 or x_F dependence of the cross section. The factor $I_{e\ell}$ tends to cancel the $1/n_c$ factor in the Drell-Yan cross section because the color

exchange introduces a color correlation between the quark and antiquark before they annihilate. Thus we expect $1 \leq I_{e\ell} \leq 3$. Despite this change in the πN muon pair cross section, the initial state interactions do not affect nucleon number additivity in hadron nucleus lepton pair production, i.e., $(d\sigma/dQ^2)_A \cong A(d\sigma/dQ^2)_N$. This is because the interactions of the active antiquark with spectators in a (color singlet) nucleon that does not contain the active quark cancel as in the Abelian case.

Although the elastic initial state interactions have only a minor effect on the x-distributions of the annihilating constituents, one might expect inelastic reactions to alter the x-distributions significantly. For example, the Feynman scaling bremsstrahlung associated with initial state collisions can remove an arbitrarily large fraction of the momentum of the incident particles. We shall show, however, that such bremsstrahlung is suppressed at large Q^2.

Let us consider the initial-state bremsstrahlung graphs shown in Fig. 2. The amplitude of Fig. 2(a), in which the gluon is emitted before any initial state interactions, has a numerator coupling $\varepsilon_\perp \cdot j_\perp$, whereas all the other diagrams lead to numerators of the form $\varepsilon_\perp \cdot (j_\perp + \ell_\perp)$. Let us define the j-part of each amplitude to be the piece obtained by keeping only the $\varepsilon_\perp \cdot j_\perp$ term in the numerator and dropping all cross terms of the form $\ell_\perp \cdot j_\perp$ in the energy denominators. The remainder of the leading twist contribution is denoted by the ℓ-part. The ℓ-part is proportional to the momentum transfer $\ell_\perp$, and so contains the part of the bremsstrahlung that is induced by the active-spectator interactions.

Consider first the j-parts. The j-parts of the amplitudes simply combine to give the factorized structure of a Drell-Yan amplitude with gluon emission (including wave function evolution) times the same elastic initial state scattering amplitude as we considered above. For example, for the j-parts of the Abelian graphs of Figs. 2(a) and 2(b) the denominators combine as follows:

$$B^{-1}\left(A^{-1} + C^{-1}\right) = A^{-1}C^{-1} , \qquad (1)$$

where A is the denominator associated with the emission of a gluon and C is the denominator associated with the elastic Glauber scattering. In the case of the non-Abelian theory we must also take into account the triple-gluon coupling graph, Fig. 2(c). Aside from the color factor, its j-part is identical to that of Fig. 2(a). The color factor is such that, when added to the color factor of Fig. 2(a), it yields the color factor of Fig. 2(b). Thus, the factorization of the j-part into a bremsstrahlung amplitude times an elastic scattering amplitude goes through in the non-Abelian case as well. This result is easily generalized to include multiple bremsstrahlung gluons and multiple exchange gluons to arbitrary order in perturbation theory. Because of the factorization property, the color factor is always computed with the bremsstrahlung (real or virtual) after the Glauber scattering [as in Fig. 2(b)]. Thus, the color traces are different in the real and virtual bremsstrahlung cases. As pointed out by Mueller,[8] this implies that, in the non-Abelian theory, the Sudakov suppression due to the virtual corrections to the $q\bar{q} \to \gamma$ vertex is not cancelled by the real

bremsstrahlung corrections. Physically the Sudakov suppression is uncancelled because the color correlation enhancement, $I_{e\ell} > 1$, is reduced by subsequent bremsstrahlung of color. The net result is that the usual QCD prediction for $d\sigma/dQ^2dx_F$ is multiplied by a factor $I(Q^2,x_1,x_2) = (I_{e\ell} - 1)|S_A(Q^2,x_1,x_2)|^2 + 1$. A leading log analysis suggests that $S_A(Q^2,x_1,x_2)$ decreases as a fractional power of Q^2 as $Q^2 \to \infty$. Consequently, we expect that $I(Q^2,x_1,x_2)$ ultimately goes to unity as $Q^2 \to \infty$.

Now let us examine the ℓ-parts, which arise from gluon emission internal to the active-spectator exchanges. We note that bremsstrahlung internal to the elastic exchanges tends to be suppressed because of cancelling contributions from the Glauber singularities in the energy denominators on either side of the quark-real gluon vertex. For example, the energy denominators A and B in Fig. 2(b) are of the form $A \cong (y - y_A + i\varepsilon)r_\perp^2$, $B \cong (y - y_B + i\varepsilon)r_\perp^2$, with $y_A - y_B = \mathcal{M}^2/s$, where $\mathcal{M}$ is the invariant mass of the antiquark-gluon system. The leading twist contribution to the lepton-pair cross section due to the ℓ-part amplitudes comes from the region $\mathcal{M}^2 \lesssim \ell_\perp^2 \ll Q^2$. If the wave function $\psi(x-y, k_\perp-\ell_\perp)$ is a slowly varying function of y, then the leading twist contributions from $y \sim y_A$ and $y \sim y_B$ cancel in the integral over y. The dependence of $\psi(x)$ on the longitudinal momentum fraction of the constituent $x = (k^0 + k^3)/(p^0 + p^3)$ is controlled by the longitudinal size of the target: $\psi \tilde{=} \psi(xML)$, where L is the length of the target. For example, in a nonrelativistic bound state $x = (m + k^3)/M$ and $\psi(k_3) \sim e^{ik_3L}$ for constituents fixed at separation L. The cancellation of the two Glauber contributions to the bremsstrahlung thus holds only if $\mathcal{M}^2/x_{\bar{q}}s \leq \langle\ell_\perp^2\rangle/x_{\bar{q}}s \ll (M_NL)^{-1}$, i.e.,

$$Q^2 \tilde{=} x_q x_{\bar{q}} s \geq x_q \langle \ell_\perp^2 \rangle M_N L \ . \qquad (2)$$

This is a new condition for the validity of the QCD prediction for the $x_{\bar{q}}$ dependence of the cross section.

The suppression of radiation over a finite length can be understood in terms of the uncertainty principle. The induced bremsstrahlung changes the spectator laboratory momentum by an amount $\Delta p_z^{spec} \cong \mathcal{M}^2 M_N / x_{\bar{q}} s$. In order to detect the radiation specifically induced by the active-spectator interactions, one must have $\Delta p_z^{spec} L > 1$. This leads immediately to Eq. (2) as the condition for no induced radiation in the target.

Note that for very long targets induced radiation does occur. Indeed, for macroscopic size targets one must allow for the depletion of the incident beam and the production of secondary hadrons. In the case of a nucleus, an estimate of the condition for no induced radiation is

$$Q^2 \geq x_q M_N L_A \langle \ell_\perp^2 \rangle_A \tilde{=} x_q M_N (1.2 \text{ fm}) \ A^{1/2} \langle \ell_\perp^2 \rangle_N \ A^{1/3} \tilde{=} .25 \text{ GeV}^2 \ A^{2/3} \qquad (3)$$

where we have used the following estimate for the average momentum exchange in a quark-nucleon collisions: $\langle \ell_\perp^2 \rangle_N^{1/2} \sim 200$ MeV. This value for $\langle \ell_\perp^2 \rangle_N$ is consistent with the CIP data[9] for $\pi A \to \mu^+ \mu^- X$, which shows some increase of $\langle Q_\perp^2 \rangle$ with A. Note that for a uranium target one requires $Q^2 \gtrsim 10$ GeV^2 before radiation losses can be neglected.

In summary, we predict two important effects arising from initial state interactions at large Q^2: (1) a new contribution to $d\sigma/dQ^2dX_F$ for hadron-hadron collisions as compared to standard factorization predictions, and (2) a smearing of the transverse momentum distribution $d\sigma/dQ^2dQ_\perp^2$. Similarly, in the case of direct photon production at large p_T, we predict an enhancement in the cross section, due to initial state collisions, which grows with nuclear size. For very large p_T, the A-dependent relative correction is $\langle \ell_\perp^2 \rangle_N A^{1/3}/p_T^2$. More generally, all large p_T hadron and jet production QCD predictions are modified by initial and final state collisions. For example, in the case of jet fragmentation processes in deep-inelastic scattering $\ell A \to \ell' HX$, the final state collisions modify the transverse momentum distributions $D(z,p_T)$ of the produced hadrons. In addition, we expect the inelastic final state collisions of soft particles to increase the hadron multiplicity. Finally, we note that these predictions do not depend critically upon the detailed nature of the color-changing Pomeron-like interaction[10] between active and spectator quarks, and consequently these effects are expected to occur quite generally.

ACKNOWLEDGEMENTS

We wish to thank G. Bertsch, J. Collins, J. Gunion, S. Gupta, A. Mueller, G. Sterman and D. R. Yennie for helpful conversations.

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6. The singularity at $\ell_\perp = 0$ cancels when one sums over the amplitudes for the interaction of a gluon with all constituents of a neutral system.

7 See, for example, R. P. Feynman and R. D. Field, Phys. Rev. D15, 2590 (1977).

8. A. H. Mueller, private communication.

9. G. E. Hogan, Ph.D. Dissertation, Princeton University (1979).

10. For example, any elastic or inelastic interaction with an amplitude $T \sim sf(t)$, where $tf(t) \to 0$ for t large, leads to our results.

FIGURE CAPTIONS

Fig. 1. Lowest order QCD contributions to elastic scattering initial state collisions of the active and spectator quarks in $\pi N \to \mu^+\mu^- X$.

Fig. 2. Lowest order amplitudes for initial state bremsstrahlung in $\pi N \to \mu^+\mu^- X$.

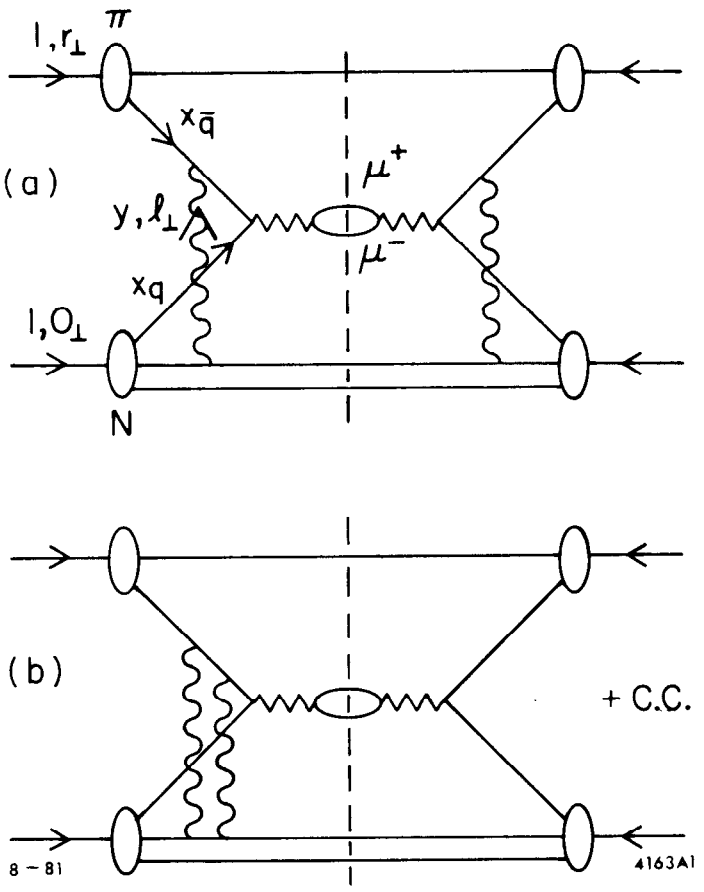

Fig. 1

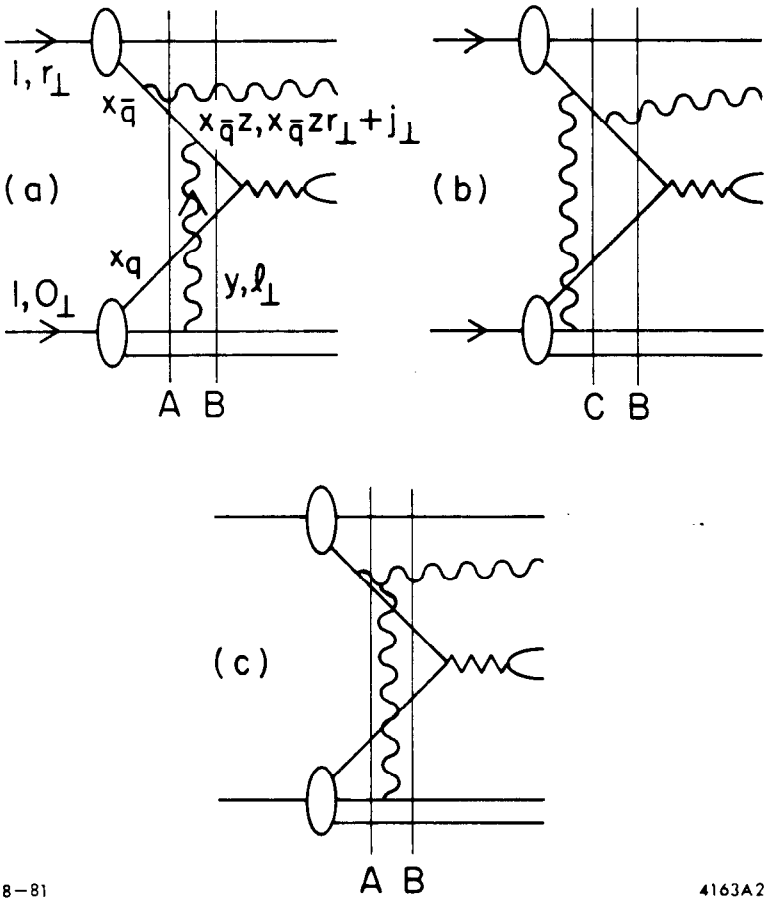
1, $r_\perp$
$x_{\bar{q}}$
$x_{\bar{q}}z, x_{\bar{q}}zr_\perp + j_\perp$
(a)
x_q
1, $0_\perp$
$y, \ell_\perp$
A B
(b)
C B
(c)
A B
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Fig. 2