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NEUTRINO MASS AND NEUTRINO COSMOLOGY*

Chao-Hsi Chang**
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305
and
Institute of Theoretical Physics
Beijing, China

ABSTRACT

The theoretical progress on neutrino mass and neutrino cosmology made in china during the period 1980-81 is reviewed.

Wide interest in neutrino mass problem has arisen in China recently and many research works have been done. I address myself to talk about them here.

1. On the Experiments of Measuring the Electron-Neutrino Mass and of Observing the Neutrino Oscillation

If the neutrinos are massive, then how much is their mass, how to measure neutrino mass with an adequate accuracy and looking for the best way to observe the neutrino oscillation are further interesting problems.

(a) In China, Ching and Ho have considered the atomic effects in the decay of $H^3 \to He^{3+} + e^- + \tilde{\nu}$, which has been used to measured the neutrino mass (m_{ν_e}). They took into account the contribution of the three lowest states of He^{3+} exactly and for the rest they used the Closure Approximation. Therefore a more accurate formula of spectrum including the atomic effect is obtained:[1]

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$$
\begin{aligned}
dN(E_\beta) = {} & AF\left(Z, E_\beta - \bar{V}_n\right) \frac{E_n - \bar{V}_n}{E_\beta} \left[\frac{\left(E_\beta - \bar{V}_n\right)^2 + \bar{V}_n^2 - m_e^2}{E_\beta^2 - m_e^2} \right] \\
& \times \left\{ \sum_{n=1}^{3} |\langle f_n | i \rangle|^2 \left(W_n - E_\beta\right)\left[\left(W_n - E_\beta\right)^2 - m_\nu^2\right]^{1/2} \right. \\
& + \left(1 - \sum_{n=1}^{3} |\langle f_n | i \rangle|^2\right)\left[\left(W_0 + \bar{V}_4 - E_\beta\right)^2 - \bar{V}_4^2 - \bar{V}_4^2 - \frac{1}{2} m_\nu^2\right] \\
& \left. \times\ \theta\left(W_0 + \bar{V}_4 - E_\beta - m_\nu\right) \right\} p_\beta\, E_\beta\, dE_\beta\ , \qquad (1)
\end{aligned}
$$

where

$$V_n = \langle f_n | H_i - H_f | f_n \rangle\ ,$$

$$W_n = M_i - M_{f_n} + E_i - E_{f_n} - E_R\ ,$$

M_i and M_f are the masses of the initial and final nuclei respectively; E_i and E_{f_n} the binding-energies, H_i and H_f the Hamiltonians of the initial and final atomic systems H^3 and He^{3+} respectively, E_R is the recoil energy, $F(Z, E_\beta - V_n)$ is the Fermi function, A is a factor including the nuclear transition matrix element and the related constants, and the bars mean in average. Comparing (1) with the theoretical formulae used by Lyubimov et al.[2] it is certain that (1) is more accurate.
In addition, Ching and Ho have also pointed out that in determining the end point of the β spectrum, a systematic error of about 13 eV had been brought in the analysis of Lyubimov. Therefore the uncertainty of the same order would also be involved in their final result.

(b) Ching, Ho and Chang have analyzed what is the "best" experiment for detecting neutrino oscillations.[3] They thought that the "best" way to observe the neutrino oscillation is to use the monochromatical ν_μ beam obtained from the stopped K^+-mesons, or, more realistically, to use the ν_μ, $\tilde{\nu}_\mu$ and ν_e beam obtained from the stopped

$$\begin{array}{l} \pi^+ \to \mu^+ + \nu_\mu \\ \qquad\quad \hookrightarrow e^+ + \nu_e + \tilde{\nu}_\mu \end{array} \qquad (2)$$

cascade decay in a beam dump facility of π-meson factory, because both of the spectrum E and the distance L which the neutrinos have travelled are more definite, compared with the others. Since there is no $\tilde{\nu}_e$ in the cascade decay process (2), either in the beam unless there is

$$\nu_\mu \to \tilde{\nu}_e \qquad \text{and/or} \qquad \nu_e \to \tilde{\nu}_e \qquad (3)$$

oscillations, therefore one can detect the neutrino oscillation (3) and determine the oscillating angle ϕ_{ij} by means of measuring e^+ produced by the neutrino beam colliding with a hydrogen or other nuclear target. As the cross section of neutrino increases with E^2,[4] one can expect to have the same rate for collecting the events as that of a high power reaction experiment (e.g., F. Reines et al.[25]), since the high energy component in such a neutrino beam is richer than in a reactor neutrino beam although its intensity is lower. In this way a preliminary bound of the oscillation (3) has been obtained by P. Nemethy et al.[26]

2. On Some Problems of Massive Neutrinos

If the neutrinos are massive ($m_\nu \simeq 30$ eV) some special modifications should be made for some of the simplest models (e.g., the $SU_L(2) \otimes U(1)$ GWS model,[5] the SU(5) GUT model,[6] etc.). On the other hand, the physical neutrinos may be different from the weak-current neutrinos, due to nonzero neutrino mass, namely, there may exist a KM matrix between the physical and weak-current neutrinos. Therefore the study of the generation problem can be grounded not only on the level of grand unification but also on the level of electroweak unification, e.g., one can try to work out the generation problem by introducing a horizontal symmetry in the quark section and lepton section separately.

Based on the $SU_L(2) \otimes SU_R(2) \otimes U(1)$, Wu and Cao have investigated a kind of horizontal discrete symmetry.[7] Several mass formulas of leptons were obtained. One of the interesting results is that a kind of neutrino (ν_τ?)

with mass ≈250 MeV and lifetime $\approx 10^{-6}$ sec, is allowed in their theory. The effects on all related processes and the method of detecting it were also discussed, they indicated that in order to find it one should look for the decay $\nu_\tau(?) \to e^\pm + \pi^\mp$, which should be seen in the beam dump experiments.

However if one believes $SU_L(2) \otimes SU_R(2) \otimes U_{B-L}(1)$, etc. models in which there are not only left-handed neutrinos (ν) but also right-handed neutrinos (N) in doublets and the left-handed neutrino obtains mass of the order $1\text{-}10^2$ eV owing to the right-handed neutrino has extremely heavy mass ($m_N \simeq m_{W_R}$),[8-10] then it is very possible that a big splitting may occur when considering the mixing among the various generations. Thanks to the big splitting, one of the extremely heavy "right-handed neutrinos" may have a mass of order of a few GeV or even lower so that it may have remarkable mixture with the "left-handed" neutrinos (later on we call this as a two-step mixing) as here all neutrinos have Majorana masses and the mass difference becomes smaller by the big splitting. This kind possibility has been pointed out and investigated.[11] As an example, a horizontal discrete symmetry

$$\begin{array}{llll} R: & \psi_{\mu L} \to -\psi_{\mu L}, & \psi_{\mu R} \to -\psi_{\mu R} & \\ & \psi_{eL} \to \psi_{eL}, & \psi_{\mu R} \to \psi_{eR} & \\ & \psi_{\tau L} \to \psi_{\tau L}, & \psi_{\tau R} \to \psi_{\tau R} & (4) \end{array}$$

is employed,* then it is easy to demonstrate the possibility and the so-called two-step mixing in $SU_L(2) \otimes SU_R(2) \otimes U_{B-L}(1)$ model.

After the two-step mixing the neutrino doublets have the following formulation generally:

$$\begin{pmatrix} c_1\nu_1 + c_2 s_1 \nu_2 + s_1 s_2 N \\ e \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \quad \begin{pmatrix} s_1\nu_1 + c_1 c_2 \nu_2 + c_1 s_2 N \\ \tau \end{pmatrix}_R$$

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* We employ R, that means only ν_e-ν_τ can mix, for the neutrino oscillation and $\mu \to e+\gamma$ experiments show ν_μ-ν_e mixes much smaller than ν_e-ν_τ does, either for simplicity.

and

$$\begin{pmatrix} \frac{1}{\sqrt{2}}\left(-s_2\nu_2 + c_2N + D\right) \\ e \end{pmatrix}_R \ , \quad \begin{pmatrix} N_\mu \\ \mu \end{pmatrix}_R \ , \quad \begin{pmatrix} \frac{1}{\sqrt{2}}\left(s_2\nu_2 - c_2N + D\right) \\ \tau \end{pmatrix}_R$$

where

$$c_{1,2} = \cos\theta_{1,2} \quad , \quad s_{1,2} = \sin\theta_{1,2} \quad , \quad D = \frac{1}{\sqrt{2}}\left(N_e + N_\tau\right) \ .$$

Now none of the neutrino oscillation experiments could decide the mixing angles $\theta_{1,2}$, because of difficulties. If the above mechanism works, to discover the N and measure its productions and decays so that to measure out $\theta_{1,2}$ may be easier than the neutrino oscillation experiments. As a matter of fact if we identify the N with M^o(?), a possible neutral lepton (mass $\simeq$ 1.6-2.1 GeV, lifetime $\simeq 6 \times 10^{-12}$ sec) discovered in 1977 in a bubble chamber experiment,[12]

$$\nu + \text{nucl.} \to M^o + P + 2\pi^o + \ldots \quad ,$$
$$\hookrightarrow e^- + \mu^+ + \nu$$

namely, we can use $m_N \simeq 2.0$ GeV, $\tau_N \simeq 10^{-11}$-10^{-12} sec as input then the properties of N^o, in particular, the fact, why it is so difficult to be discovered and why it may be easier observed in a bubble chamber filled with freon, can be explained, and $s_1^2 s_2^2 \simeq 10^{-2}$ is obtained. The biggest cross section for producing N is of e-production but the best way to detect N is using a high energy beam dump facility to produce a high intensity neutrino beam with more ν_e and ν_τ components to collide with a target and to produce quite an amount of N and without too many backgrounds (the cross section for producing N by ν_e and ν_τ could be as big as 10^{-2} of the ν_μ neutral current cross section). To see the properties of N, we list the most important branch ratios in Table I.

TABLE I

Channel	3ν	νe^+e^-	$\nu\pi^o$	$\nu\eta$	$\nu\rho^o$
Br (%)	8.25	16.5	7.47	2.4	4.71
Channel	$e^-\mu^+\nu + e^+\mu^-\nu$	$e^-\tau^+\nu + e^+\tau^-\nu$	$\nu\mu^+\mu^-$	$\nu\omega^o$	$\nu\phi^o$
Br (%)	2 × 3.67	2 × 11.01	1.84	0.55	4.20
Channel	$e^-\pi^+ + e^+\pi^-$	$e^-\rho^+ + e^+\rho^-$	$e^-K^+ + e^+K^-$	$e^-K^{*+} + e^+K^{*-}$	
Br (%)	2 × 1.77	2 × 4.46	2 × 0.119	2 × 0.177	

3. On Neutrino Cosmology

One of the most important physical consequences is cosmological: the mass of neutrinos will be dominant to that of the present universe and the universe will be closed, etc., if neutrinos are massive. Such kinds of problems have been discussed by Szalay and Marx,[13] Schramm and Steigman,[14] Dorshkevich et al.,[15] Sato and Takahara,[16] etc., in China, Ching, Wu, Ho, Chang and Zou reexamined the consequences according to the latest experiment data.[17] They further indicated that

(1) If the neutrino masses are taken to be 30 eV in average and there are three generations in nature as well as the right-handed neutrinos entered the thermal equilibrium from some very early time of the universe until near the same time as the left-handed neutrinos decoupled so that in the present day universe the number of the right-handed neutrinos is nearly equal to that of left-handed, then one could not obtain a solution which can fit both experimental data of the lifetime of the universe ($t_0 \geqslant 6.6 \times 10^9$ year) and the Habble's constant H_0 ($50 \leqslant H_0 \leqslant 100$ km/sec/Mpc) according to the big bang Friedmann model. Therefore we can conclude that not all of the weak interaction models are consistent with the Friedmann model. For example, it is necessary to have m_{W_R}/m_{W_L} bigger than 50-100 in a $SU_L(2) \otimes SU_R(2) \otimes U(1)$ model, however in a GWS model modified by adding a right-handed singlet of a nuetrino for each

generation, it is necessary not to exist any elementary fermion with mass greater than 10^6 GeV.

(2) Considering the uncertainties involved in H_0 and the number of right-handed neutrinos is less than that of left-handed at least one order in the present day universe (this means that the right-handed neutrinos have never entered the thermal equilibrium or they decoupled earlier than hadrons) then the upper bound of the sum of the neutrino masses may be relaxed to $\sum_i m_{\nu_i} \leq 200$ eV instead of $\sum_i m_{\nu_i} \leq 40$ eV.

As the neutrinos contribute to the most part of the universe mass so the possibility of the background neutrinos forming clusters themselves or with other form matter by gravitation would become an interesting problem worth studying. Gao and Ruffini[18] and Ching, Wu, Ho, Chang and Zou,[19] etc., have discussed the problem on neutrino forming clusters themselves. Ching et al., the latter, are not only estimated the possible maximum mass for a stable pure neutrino astronomy object following the Oppenheimer-Volkoff solutions of a neutron star but also discussed the possibility of detecting the background neutrinos.

Fang and Liu,[20] Lu, Lo and Yang[21] (similar to others[22-24]) have discussed the massive neutrinos forming clusters with other form matter such as baryons. Both of them solve this problem by using a two-component fluid model. It seems to me that the Fang et al.'s is more clearer than Lu et al.'s so I will talk about Fang et al.'s work only.

Fang et al. think neutrinos and other matter as two components behaving like fluid. They solved the Vlasov equation of massive neutrino distribution function under the Robertson-Walker matrix so that an effective "pressure" and related Jeans wavelength λ_ν could be defined for the neutrino component and then they established a Jeans instability theory of the two-component fluid. The final result is that the total Jeans wavelength λ can be given by the formula

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_\nu^2} + \frac{1}{\lambda_m^2}, \tag{5}$$

where λ_m is the Jeans wavelength of the other form matter. If each generation neutrino has mass $\simeq$ 30 eV in average and Jeans mass M_J is

$$M_J = \frac{4\pi}{3} \rho_m \lambda^3$$

(counting the part of the the other form matter only), then according to (5) the M_J depends on the temperature can be given (see Fig. 1). It is easy to see λ_ν is dominant to λ before re-combination but λ_m is dominant after recombination. The conditions of forming a cluster which has mass M, $M_{hor} \geqslant M \geqslant M_j$ now become ($T \sim 10^4$ °K):

$$M_{hor} = 1.1 \times 10^{17} M_\odot \geqslant M \geqslant M_J = 10^{12} - 10^{13} M_\odot ,$$

which is just the order of a galaxy cluster. It means that neutrinos played a very important role during the process of forming galaxy clusters and it will partially solve the problem that in the old theory the Jeans mass jumped such a great extent, i.e., 10^6 to 10^{17} $M_\odot$ around before and after the recombination period.[25]

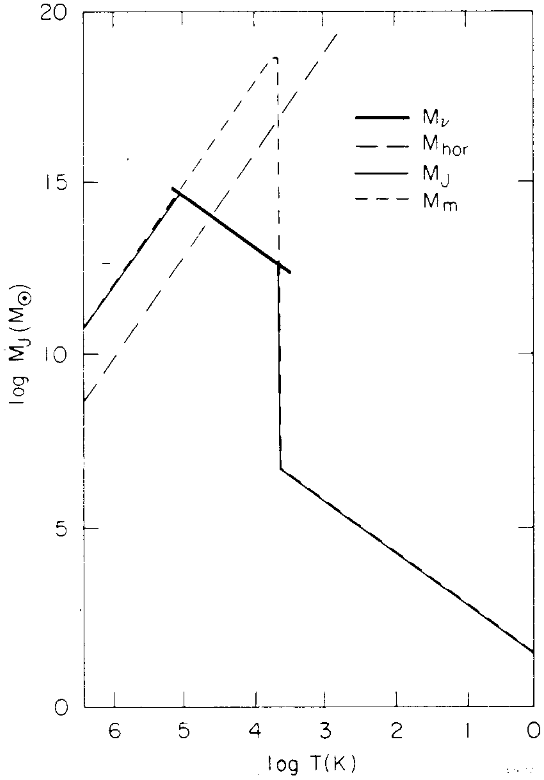

Fig. 1. The curves of Jeans masses related to the temperature of the universe.

Fang et al. also pointed out that if the neutrino masses are too small $m_\nu < 3.3$ eV then $\lambda_\nu > \lambda_m$ in any time so that neutrinos will give no evident effect to the formation of clusters. But the fluctuation theory for the process of forming clusters, various aspects of the linear theory of density perturbations, etc., which have been discussed by others[22-24] were not considered in Fang et al.'s paper.

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REFERENCES

1. C. R. Ching and T. H. Ho, preprint AS-ITP-81-008.
2. V. A. Lyubimov, E. G. Novikov, V. Z. Nozik, E. F. Tretyakov and V. S. Kowsik, preprint ITEP-62 (1980).
3. C. R. Ching, T. H. Ho and C. H. Chang, preprint AS-ITP-81-010.
4. See, for instance, M. Morita, Beta-Decay and Muon Capture (1973).
5. S. L. Glashow, Nucl. Phys. 22, 579 (1961); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in Elementary Particle Theory: Relativity Group and Analyticity, Ed. by N. Svartholm, Wiley, N.Y. (1968).
6. H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
7. Y. S. Wu and N. W. Cao, preprint AS-ITP-81-005.
8. R. E. Marshak and P. N. Mohapatra, Phys. Lett. B91, 222 (1980); R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. 44, 1316 (1980).
9. H. Georgi, Particles and Fields (1974), Ed. by C. E. Carlson, AIP, N.Y. (1975); H. Fritzsch and P. Minkowski, Ann. Phys. 93, 193 (1975).
10. R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
11. C. H. Chang, into preprint.
12. D. S. Baranov et al., Phys. Lett. 70B, 269 (1977).
13. A. S. Sazalay and G. Marx, Astron. Astrophys. 49, 437 (1976).
14. D. N. Schramm and G. Steigman, Gravity Research Foundation Gravity Prize Essay.
15. A. G. Doroshkevich, M. Yu. Khlopov, R. A. Sunyaev, A. S. Szalay and Ya. B. Zeldovich, Proceedings of the Xth Texas Symposium on Relativistic Astrophysics, Baltimore, 15-19 December 1980.
16. H. Sato and F. Tahahara, Prog. Theor. Phys. 64, 2029 (1980).
17. C. R. Ching, Y. S. Wu, T. H. Ho, C. H. Chang and Z. L. Zou, ACTA ASTROPHYSICIA 1, 9 (1981).
18. J. G. Gao and R. Ruffini, ACTA ASTROPHYSICIA. 1, 19 (1981), Phys. Lett. 97B, 388 (1980), and Phys. Lett. 100B, 47 (1981).
19. C. R. Ching, Y. S. Wu, T. H. Ho, C. H. Chang and Z. O. Zou, preprint AS-ITP-002.
20. L. Z. Fang and Y. Z. Liu, to be published in ACTA ASTROPHYSICIA SINICA.
21. T. Lu, L. F. Lo and K. S. Yang, to be published in ACTA ASTROPHYSICIA SINICA.
22. I. Wasserman, preprint CRSR-760.
23. A. S. Szalay, Proceedings of the Neutrino-81 Conference, Hawaii, 1-7 July 1981; M. Crawford, Proceedings of the Neutrino-81 Conference, Hawaii, 1-7 July 1981.
24. J. R. Bond, G. Efstathiov and J. Silk, Phys. Rev. Lett. 45, 1980 (1980); A. L. Melott, preprint of University of Texas, Austin (1980).
25. F. Reines, H. W. Sobel and E. Pasierb, Phys. Rev. Lett. 45, 1307 (1980).
26. P. Nemethy et al., Phys. Rev. D23, 262 (1981).