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$\rho \to \pi\pi$ IN THE SLAC LATTICE QCD THEORY*

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ABSTRACT

Using the SLAC lattice Hamiltonian QCD theory, we compute the decay width for the decay $\rho^+ \to \pi^+\pi^0$. There is reasonable agreement between the theoretical result and observation. There are two key ingredients involved in our calculation. One is the vacuum insertion technique of Lee, Primack and Treiman for the evaluation of light hadron matrix elements of effective low energy interaction densities. The other is the identification of the SLAC lattice currents with the physical hadron currents to leading order in the SLAC order $1/g^2$ effective Hamiltonian for the fluxless light hadron sector--in the spirit of Gell-Mann's work in current algebra. Our result suggests that the SLAC theory, taken together with these two ingredients, provides a viable technique for calculating large distance light hadron dynamics.

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1. Introduction

One of the most popular candidates for the theory of the usual strong interaction is QCD-quantum chromodynamics.[1] Among the outstanding questions in this (continuum) QCD strong interaction scenario is the detailed mechanism involved in the confinement of quarks. It was with this question in mind that Wilson[2] introduced the lattice approach to the large distance behavior of QCD. For, since the confinement of quarks is presumably a large distance phenomenon, one could hope that the short distance part of the QCD theory, which appears to be consistent with observation, could be cut-off in a gauge invariant way without affecting, significantly, the true large distance properties of the theory. Wilson's lattice QCD theory represents an effort to construct just such a gauge invariant short distance cut-off. And, indeed, Wilson found that his lattice QCD theory confines infinitely heavy colored quarks. The issue of dynamical, light quark confinement on the lattice remains an open question.

Indeed, in Wilson's original formulation, there were a number of unresolved issues, as is always true in any entirely new development. Among these issues was the fact that Wilson's arguments also led to confinement for Abelian lattice gauge theories, while weakly coupled QED does not confine. In addition, in order to avoid spurious fermionic degrees of freedom on his lattice, Wilson had to introduce chiral non-invariant terms into the lattice QCD theory with massless fermions--such terms made the discussion of the chiral aspects of low energy hadron dynamics difficult on the lattice itself, if not impossible.[3] It was with such questions in mind that the SLAC group[4] introduced what we will refer to as the SLAC lattice approach to strong interaction dynamics.

One may naturally ask what are the key differences between the SLAC lattice and the Wilson lattice. A significant calculational difference is that the SLAC theory uses a continuous time variable and latticed spatial coordinates together with a Hamiltonian formalism whereas Wilson's theory is most simply described as a Lagrangian theory in which all four Lorentz coordinates are latticed, with the time coordinate taken to be imaginary (Euclidean quantum field theory). But, from the point of view of chiral symmetry, the primary difference is that Wilson's theory uses the usual difference definition of differentiation on a lattice whereas the SLAC group has introduced into hadron dynamics the lattice derivative

$$\nabla f(ja) \equiv \sum_k ikf(k)e^{ikja} \tag{1}$$

where

$$f(ja) \equiv \sum_k f(k)e^{ikja} \tag{2}$$

for a function f(ja) defined on a one-dimensional lattice. Here, the sum on k is over

$$k = 2m\pi/((2N+1)a) \quad , \qquad m = -N, -N+1, \ldots, N \quad . \tag{3}$$

It has been shown in Ref. 4 that the derivative (1) alleviates spurious fermionic modes on the lattice and, yet, maintains local γ_5-symmetry for massless fermions on the respective lattice. Thus, the chiral symmetry properties of low energy hadron dynamics on a lattice are more readily discussed with (1) than with Wilson's difference derivative.[5-8]

In addition, the SLAC group has been able to argue that, in Abelian lattice gauge theories, there is a phase transition so that such theories only confine (heavy) quarks at strong coupling. At weak coupling, a theory like lattice QED would not confine heavy quarks. This is an important result for the entire lattice framework, since one does not believe that heavy weakly charged particles are confined by QED in the continuum. Thus, a confining result for lattice QED in this case would have indicated that the lattice theory does not accurately represent the large distance properties of the attendant continuum theory. A more detailed proof of the existence of this phase transition in Abelian lattice gauge theory has been given by Guth.[9]

As we have attempted to emphasize, while all of these results about confinement are encouraging, they nonetheless refer to heavy quarks (static quarks). At some point, one must face the issue of dynamical light quarks in the lattice QCD framework. For, one would like to feel that the known low energy phenomena of light hadrons were not inconsistent with the lattice framework. Toward this end, the SLAC group has made substantial progress.

More specifically, working in their QCD lattice Hamiltonian framework, the SLAC group has been able to show[10] that, in order $1/g^2$, where g is the gauge coupling constant, there arises an approximate $SU(6) \times SU(6) \times U(1)$ symmetry of light hadron physics. A number of important results follow from their work: $\mu_P/\mu_N = -3/2$, $g_A/g_V \neq -5/3$, both vector mesons and pseudoscalar mesons are pseudo-Goldstone bosons, etc. Thus, one can say that this SLAC lattice theory for QCD is not obviously inconsistent with the general aspects of the dynamics of light hadrons (hadrons composed of u, d, and s quarks). It is, therefore, tempting to use this theory to address more of the details of light hadron dynamics, that is to say, more of the details of large distance approximately chirally invariant light hadron dynamics. This is our primary objective in the development which follows.

What we shall do is to use a prototypical large distance light hadron process to perform a detailed test of the applicability of the SLAC lattice QCD theory to large distance light hadron dynamics. Our prototypical process will be

$$\rho^+ \to \pi^+\pi^0 \quad . \tag{4}$$

For, experimentally,[11] approximate scaling occurs already for $Q^2 \gtrsim 1_+$ GeV^2, where Q^2 represents the magnitude of the squared four-momentum transfer in lepton-hadron inelastic scattering, or s in e^+e^- annihilation, the squared center of momentum energy, for example. This fact, taken together with the recent theoretical result[12] that the transition from weak coupling asymptotically free behavior[1] to strong coupling confining behavior in QCD is abrupt means that the process (4), with $Q^2 = m_\rho^2 = .602$ GeV^2, should be in the regime of confinement--in the regime of large distance light hadron dynamics. And, indeed, we will find reasonable agreement between the SLAC lattice theory and observation. This will provide further support for the general lattice QCD idea itself as well as the particular SLAC representation of that idea.

Our work will be organized according to the following scheme. In the next section, Section 2, we describe the relevant aspects of the SLAC theory for the computation of $\rho^+ \to \pi^+\pi^0$. In Section 3, we present this computation itself and thereby derive an expression for the width $\Gamma(\rho^+ \to \pi^+\pi^0)$. Section 4 presents the determination of effective value of the lattice constant a, which is needed to evaluate the expression derived in Section 3 for $\Gamma(\rho^+ \to \pi^+\pi^0)$. Section 5 contains the determination of the effective gauge coupling constant g which is needed to evaluate our expression for $\Gamma(\rho^+ \to \pi^+\pi^0)$. Section 6 then presents this evaluation of $\Gamma(\rho^+ \to \pi^+\pi^0)$ and compares our result with observation, with due discussion of the various theoretical procedures involved in our work. Finally, Section 7 contains some concluding remarks.

2. The SLAC Lattice Theory

Our objective is to calculate the process $\rho^+ \rightarrow \pi^+ \pi^0$. We shall do this using the SLAC lattice theory. Here, we wish to delineate those aspects of the SLAC theory which are relevant to our calculation.

More precisely, the SLAC theory consists of the lattice Hamiltonian[10]

$$H(g,a) = \frac{1}{a} \left\{ \sum_{\text{links}} \frac{1}{2} g^2 E^{\alpha^2}_{\vec{j},\hat{\mu}} - \sum_{\text{loops}} \frac{1}{g^2} \,\mathrm{tr} \left[\prod_{\substack{\text{around} \\ \text{loop}}} U_{\vec{j},\hat{\mu}} \right] - \left[i \sum_{\substack{\vec{j},\hat{\mu} \\ n>0}} \delta'(n)\, \psi^{\dagger\alpha f}_{\vec{j}}\, \alpha_\mu\, \psi^{\beta f}_{\vec{j}+n\hat{\mu}} \left[\prod_{m=0}^{n-1} U_{\vec{j}+m\hat{\mu},\hat{\mu}} \right]^{\alpha\beta} + \text{h.c.} \right] \right\} \quad (5)$$

where we have used the notation of Ref. 10. Thus, the spinor field $\psi^{\alpha f}_{\vec{j}}$ at site $\vec{j}$ carries color index α and flavor index f. The operators $E_{\vec{j},\hat{\mu}}$ measure the units of color flux created by the operators $U_{\vec{j},\hat{\mu}}$ on the link joining site $\vec{j}$ to site $\vec{j} + \hat{\mu}$. The α_μ are Dirac's matrices--we will always represent them in the convention of Bjorken and Drell.[13] The parameter a is the lattice spacing and, to repeat, g is the gauge coupling constant. Finally, we note that the quantity $\delta'(n)$ in (5) is defined so that

$$\partial_\mu \psi_{\vec{j}} = \frac{1}{a} \sum_n \delta'(n)\, \psi_{\vec{j}+n\hat{\mu}} \quad (6)$$

is the SLAC derivative on a lattice. Thus, taking 2N + 1 lattice sites along each coordinate axis,

$$\delta'(n) = \frac{1}{2N+1} \sum_{m=-N}^{N} ik(m) \exp[-ik(m)n] \quad (7)$$

where

$$k(m) = 2\pi m/(2N + 1) \quad . \tag{8}$$

In our work, we shall always be interested in the infinite volume limit $N \to \infty$. In this limit, $\delta'(n)$ becomes

$$\delta'(n) \to (-1)^{n+1}/n \quad . \tag{9}$$

This completes the definition of the SLAC lattice QCD theory to the extent required by our analysis.

To continue with our discussion, we further observe that, working to order $1/g^2$ in the large g (large distance) regime in the fluxless sector of the Hilbert space associated with H, the SLAC group has derived the following effective second-order Hamiltonian:

$$H^{(2)}_{eff} = \frac{1}{a} \sum_{\vec{j},n,\mu} \frac{\delta'(n)\delta'(-n)}{(1/2)g^2|n|C_F} \psi^{\dagger\alpha f}_{\vec{j}} \alpha_\mu \psi^{\beta f}_{\vec{j}+n\hat{\mu}} \psi^{\dagger\beta f'}_{\vec{j}+n\hat{\mu}} \alpha_\mu \psi^{\alpha f'}_{\vec{j}} \quad , \tag{10}$$

by the standard degenerate state perturbative methods. Here, N_c is the number of colors, and C_F is the value of the quadratic Casimir operator of $SU(N_c)$ in the fundamental representation:

$$C_F = \frac{N_c^2 - 1}{2N_c} \quad . \tag{11}$$

In the case of primary import, $N_c = 3$. The matrix α_μ in (10) is represented by

$$\alpha_\mu = \gamma^0\gamma^\mu \tag{12}$$

in the familiar notation of Ref. 13.

It is this interaction (10) which we shall employ to compute $\rho^+ \to \pi^+\pi^0$. We turn to this calculation in the next section.

3. The Decay Width $\Gamma(\rho^+ \to \pi^+\pi^0)$

We wish to use the interaction (10) to compute the rate for $\rho^+ \to \pi^+\pi^0$. We begin by discussing the relevant aspects of the particle spectrum associated with (5) and (10).

More specifically, in the particle spectrum attendant to (5) and (10), the SLAC group has found,[10] among other things, that both the ρ and the π are Goldstone particles of an approximate SU(12) symmetry. The corresponding broken charge densities are as follows:

$$\rho^+ : \psi^\dagger_{\vec{j}} \, \vec{\gamma} \otimes \lambda^+ \, \psi_{\vec{j}} \, s_{\vec{\gamma}\lambda^+}(\vec{j}) \tag{13}$$

$$\pi^+ : \psi^\dagger_{\vec{j}} \, \gamma_5 \otimes \lambda^+ \, \psi_{\vec{j}} \tag{14}$$

where

$$\alpha_x^{jx} \, \alpha_y^{jy} \, \alpha_z^{jz} \, \vec{\gamma} \otimes \lambda^+ \, \alpha_z^{jz} \, \alpha_y^{jy} \, \alpha_x^{jx} \equiv \vec{\gamma} \otimes \lambda^+ \, s_{\vec{\gamma}\lambda^+}(\vec{j})$$

and where, here, we suppress color and we take λ^+ to be equal to the usual isospin raising Gell-Mann matrix. The analogous correspondences hold for the ρ^-, ρ^0, π^- and π^0. Of course, the correspondences for the pions are well known. Thus, as a first step toward our computation, we may define, with the idea that the particle states are pseudo-Goldstone manifestations of the same broken symmetry,

$$\left\langle 0 \middle| \psi^\dagger_{\vec{0}} \, \gamma_5 \otimes \lambda^- \, \psi_{\vec{0}} \middle| \pi^+(\bar{p}) \right\rangle = i\sqrt{2} \, f_\pi \, \bar{p}^0 / \sqrt{2\bar{p}^0} \tag{15}$$

$$\vec{\varepsilon}^* \cdot \left\langle 0 \middle| \psi^\dagger_{\vec{0}} \, \gamma_0 \, \vec{\gamma} \otimes \lambda^- \, \psi_{\vec{0}} \middle| \rho^+(p), \vec{\varepsilon} \right\rangle \equiv \sqrt{2} \, f_\rho \, m_\rho / \sqrt{2p^0} \tag{16}$$

where f_π is the familiar pion decay constant, f_ρ is the ρ decay constant, $\vec{\varepsilon}$ is the polarization vector of the ρ^+ in the state $|\rho^+(p), \vec{\varepsilon}\rangle$ of four momentum $p = (m_\rho, \vec{0})$, and $|\pi^+(\bar{p})\rangle$ is the π^+ state of four momentum $\bar{p}$.

We continue to suppress color. It is well known that

$$f_\pi \doteq 98.4 \text{ MeV} \quad . \tag{17}$$

Further, the value of f_ρ is also well known[14] to be (see Appendix I)

$$f_\rho \doteq 140 \text{ MeV} \quad . \tag{18}$$

We shall now show that (17) and (18) allow us to evaluate the amplitude for $\rho^+ \to \pi^+\pi^0$ using (10).

Toward this end, we employ two key techniques well tested in other problems in theoretical particle physics. The first is inspired by the idea of Gell-Mann[15] to abstract the properties of a quasi-realistic model field theory and attribute them to the full hadron interacting currents. Here, we will argue that although the "currents" in (10) are restricted to the lattice, we are only going to use (10) in regimes where the dominant strong interaction effects are already represented if we use the algebraic structure of the interaction in (10). In this regime, the fact that the currents in (10) are on a lattice does not prevent us from remembering that these currents have been derived by restricting the fully interacting QCD theory to the lattice. For the evaluation of our matrix elements, we shall so remember. We discuss this point in more detail in Section 6.

The second key idea to be employed here is borrowed from the work of Lee, Primack and Treiman[16] on $\Delta S \neq 0$, $\Delta Q = 0$ effects in what were once candidate gauge theories of the weak and electromagnetic interaction. Here S is strangeness and Q is electric charge. We have reference to the vacuum insertion technique for evaluating the light hadron matrix elements of four-fermion effective interaction Lagrangians. This technique allows one to understand,[16] quantitatively, phenomena such as $K_L \to \bar{\mu}\mu$ and $m_{K_L} - m_{K_S}$, where m_A is the mass of A, $A = K_L, K_S$. We may now proceed with the computation of $\rho^+ \to \pi^+\pi^0$.

First, we observe that the amplitude for $\rho^+ \to \pi^+\pi^0$ is, to lowest order in $H_{eff}^{(2)}$,

$$\mathscr{A} = \left\langle \pi^+\pi^0 \right| -i \int_{-\infty}^{\infty} dt\ H_{eff}^{(2)} \left| \rho^+ \right\rangle \quad . \tag{19}$$

The relevant kinematics is summarized in Fig. 1, where we work in the ρ^+ rest frame so that $q_1 = (m_\rho, \vec{0})$. The vacuum insertion technique then invites us to evaluate the following four expressions:

$$\left\langle \pi^+\pi^0 \right| \psi_{\vec{j}}^{\dagger\alpha f}\ \alpha_\mu\ \psi_{\vec{j}+n\hat{\mu}}^{\beta f} \left| 0 \right\rangle \left\langle 0 \right| \psi_{\vec{j}+n\hat{\mu}}^{\dagger\beta f'}\ \alpha_\mu\ \psi_{\vec{j}}^{\alpha f'} \left| \rho^+ \right\rangle \quad , \tag{20}$$

$$- \left\langle \pi^+\pi^0 \right| \psi_{\vec{j}\nu_1}^{\dagger\alpha f}\ (\alpha_\mu)_{\nu_1\nu_2}\ \psi_{\vec{j}\sigma_2}^{\alpha f'} \left| 0 \right\rangle \left\langle 0 \right| \psi_{\vec{j}+n\hat{\mu}\sigma_1}^{\dagger\beta f'}\ (\alpha_\mu)_{\sigma_1\sigma_2}\ \psi_{\vec{j}+n\hat{\mu}\nu_2}^{\beta f} \left| \rho^+ \right\rangle \ , \tag{21}$$

$$\left\langle 0 \right| \psi_{\vec{j}}^{\dagger\alpha f}\ \alpha_\mu\ \psi_{\vec{j}+n\hat{\mu}}^{\beta f} \left| \rho^+ \right\rangle \left\langle \pi^+\pi^0 \right| \psi_{\vec{j}+n\hat{\mu}}^{\dagger\beta f'}\ \alpha_\mu\ \psi_{\vec{j}}^{\alpha f'} \left| 0 \right\rangle \quad , \tag{22}$$

$$- \left\langle 0 \right| \psi_{\vec{j}\nu_1}^{\dagger\alpha f}\ (\alpha_\mu)_{\nu_1\nu_2}\ \psi_{\vec{j}\sigma_2}^{\alpha f'} \left| \rho^+ \right\rangle \left\langle \pi^+\pi^0 \right| \psi_{\vec{j}+n\hat{\mu}\sigma_1}^{\dagger\beta f'}\ (\alpha_\mu)_{\sigma_1\sigma_2}\ \psi_{\vec{j}+n\hat{\mu}\nu_2}^{\beta f} \left| 0 \right\rangle \quad . \tag{23}$$

Here, σ_1, σ_2, ν_1, and ν_2 are Dirac spinor indices. We note that the expressions (20) and (22) vanish by the Wigner-Eckart theorem in the flavor space associated with the indices f, f'. Thus, we only have to evaluate (21) and (23).

To evaluate (21) and (23), we use the standard Dirac matrix algebra to write

$$\psi_{\vec{j}\nu_1}^{\dagger\alpha f}\ \psi_{\vec{j}\sigma_2}^{\alpha f'} = \sum_{\eta=1}^{16} \frac{1}{4}\ \psi_{\vec{j}}^{\dagger\alpha f}\ \mathscr{M}^\eta\ \psi_{\vec{j}}^{\alpha f'}\ \mathscr{M}^\eta_{\sigma_2\nu_1}\ \operatorname{sgn}(\mathscr{M}^\eta) \tag{24}$$

$$\psi_{\vec{j}+n\hat{\mu}\sigma_1}^{\dagger\beta f'}\ \psi_{\vec{j}+n\hat{\mu}\nu_2}^{\beta f} = \sum_{\eta=1}^{16} \frac{1}{4}\ \psi_{\vec{j}+n\hat{\mu}}^{\dagger\beta f'}\ \mathscr{M}^\eta\ \psi_{\vec{j}+n\hat{\mu}}^{\beta f}\ \mathscr{M}^\eta_{\nu_2\sigma_1}\ \operatorname{sgn}(\mathscr{M}^\eta) \quad , \tag{25}$$

where, in accordance with Refs. 10, we take the notation of Ref. 13 for the Dirac matrices $\mathcal{M}^{\eta}$:

$$\mathcal{M}^{\eta} = 1,\ \gamma_{\mu},\ -i\sigma_{\mu\nu},\ \gamma_5,\ \gamma_5\gamma_{\mu} \tag{26}$$

and $\text{sgn}(\mathcal{M}^{\eta})$ is such that

$$(\mathcal{M}^{\eta})^2\ \text{sgn}(\mathcal{M}^{\eta}) = 1 \quad . \tag{27}$$

The results (24) and (25) allow us to write (21) as

$$\begin{aligned} -\frac{1}{16} \sum_{\eta,\eta'} \left\langle \pi^+\pi^0 \middle| \psi^{\dagger\alpha f}_{\vec{j}}\, \mathcal{M}^{\eta}\, \psi^{\alpha f'}_{\vec{j}} \middle| 0 \right\rangle \left\langle 0 \middle| \psi^{\dagger\beta f'}_{\vec{j}+n\hat{\mu}}\, \mathcal{M}^{\eta'}\, \psi^{\beta f}_{\vec{j}+n\hat{\mu}} \middle| \rho^+ \right\rangle \\ \times\ \text{sgn}(\mathcal{M}^{\eta})\ \text{sgn}(\mathcal{M}^{\eta'})\ \text{tr}\left(\mathcal{M}^{\eta}\alpha_{\mu}\mathcal{M}^{\eta'}\alpha_{\mu}\right) \quad ; \end{aligned} \tag{28}$$

an entirely analogous expression may be written for (23). Thus, our problem is now formulated as the evaluation of (28).

For this purpose, we use our variant of Gell-Mann's idea[15] to re-implement the Lorentz group so that, for example, we identify

$$\psi^{\dagger\alpha f}_{\vec{j}} = e^{iP_{op}\cdot x_{\vec{j}}}\ \psi^{\dagger\alpha f}(0)\ e^{-iP_{op}\cdot x_{\vec{j}}} \tag{29}$$

where

$$x_{\vec{j}} \equiv (t,\ \vec{j}a) \quad , \tag{30}$$

P^{μ}_{op} is the 4-momentum operator and, now, $\psi^{\dagger\alpha f}(0)$ is the fully Lorentz covariant Heisenberg field of the QCD theory at the origin of Minkowski space. With this identification, the expression in (28) becomes

$$\begin{aligned} -\frac{1}{16} \sum_{\eta,\eta'} e^{[i(q_2+q_3-q_1)\cdot x_{\vec{j}}]} \left\langle \pi^+\pi^0 \middle| \psi^{\dagger\alpha f}(0)\mathcal{M}^{\eta}\ \psi^{\alpha f'}(0) \middle| 0 \right\rangle \\ \left\langle 0 \middle| \psi^{\dagger\beta f'}(0)\mathcal{M}^{\eta'}\ \psi^{\beta f}(0) \middle| \rho^+ \right\rangle \\ \times\ \text{sgn}(\mathcal{M}^{\eta})\ \text{sgn}(\mathcal{M}^{\eta'})\ \text{tr}\left(\mathcal{M}^{\eta}\alpha_{\mu}\mathcal{M}^{\eta'}\alpha_{\mu}\right) \quad . \end{aligned} \tag{31}$$

For, since $q_1 = (m_\rho, \vec{0})$, the factor $\exp(+i\vec{q}_1 \cdot n\hat{\mu}a)$ produced when $\exp(-iP_{op} \cdot x_{\vec{j}+n\hat{\mu}})$ acts on $|\rho^+\rangle$ is equal to 1. Thus, (31) is independent of n.

Note that the analog of (31) for the expression (23) will also be independent of n ultimately because $\vec{q}_1 = \vec{0}$ so that, after summation over $\vec{j}$, $\vec{q}_2 + \vec{q}_3$ will also be constrained to be $\vec{0}$; this renders $\exp\left\{i(q_2 + q_3) \cdot x_{\vec{j}+n\hat{\mu}}\right\}$ independent of n. And, the latter phase is the only possible source of n-dependence in the analog of (31) for the expression (23). (The neglect of umklapps will be justified presently.)

To proceed further, we need to evaluate the expressions

$$\mathrm{tr}\left(\mathcal{M}^\eta \alpha_\mu \mathcal{M}^{\eta'} \alpha_\mu\right) \tag{32}$$

in (31). One easily verifies the following: if

$$\alpha_\mu \mathcal{M}^{\eta'} \equiv s_{\eta'}(\mu) \mathcal{M}^{\eta'} \alpha_\mu \quad , \tag{33}$$

where $s_{\eta'}(\mu) = \pm 1$, we have

$$\begin{aligned} \mathrm{tr}\left(\mathcal{M}^\eta \alpha_\mu \mathcal{M}^{\eta'} \alpha_\mu\right) &= s_{\eta'}(\mu)\ \mathrm{tr}\left(\mathcal{M}^\eta \mathcal{M}^{\eta'} (\alpha_\mu)^2\right) \\ &= s_{\eta'}(\mu)\ \mathrm{tr}\left(\mathcal{M}^\eta \mathcal{M}^{\eta'}\right) \\ &= 4 s_{\eta'}(\mu)\ \mathrm{sgn}(\mathcal{M}^\eta)\, \delta_{\eta\eta'} \quad . \end{aligned} \tag{34}$$

We list the values of $s_\eta(\mu)$ and $\mathrm{sgn}(\mathcal{M}^\eta)$ in Table I. In the Appendix II, we show that only $\mathcal{M}^\eta = \alpha_i$ and $\mathcal{M}^\eta = \gamma^\ell$ contribute to the amplitude in (28) and (31) and that only the contributions of $\mathcal{M}^\eta = \alpha_i$ are significant. The $\mathcal{M}^\eta = \gamma^\ell$ terms are small because m_q^*/m_ρ is small, where m_q^* is the SU(2) symmetric u and d current quark mass.[17] Using (34), the results in Table I and the results (AII.3) - (AII.18) in Appendix II, we find that (31) is approximately the same as

$$- \frac{1}{4} e^{[i(q_2+q_3-q_1)\cdot x_{\vec{j}}]} \left\{ \langle \pi^+\pi^0 | \psi^{\dagger\alpha f}(0)\ \alpha_i\ \psi^{\alpha f'}(0) | 0 \rangle \right.$$

$$\left. \times \langle 0 | \psi^{\dagger\beta f'}(0)\ \alpha_i\ \psi^{\beta f}(0) | \rho^+ \rangle (-1)^{1-\delta_{i\mu}} \right\}$$

$$= - \frac{1}{4} e^{[i(q_2+q_3-q_1)\cdot x_{\vec{j}}]} (-1)^{1-\delta_{i\mu}} \langle \pi^+\pi^0 | \bar{\psi}^{\alpha f}(0)\ \gamma^i\ \psi^{\alpha f'}(0) | 0 \rangle \tag{35}$$

$$\times \langle 0 | \bar{\psi}^{\beta f'}(0)\ \gamma^i\ \psi^{\beta f}(0) | \rho^+ \rangle$$

$$= - \frac{1}{4} e^{[i(q_2+q_3-q_1)\cdot x_{\vec{j}}]} (-1)^{1-\delta_{i\mu}} \sqrt{2}\, f_\rho \frac{\varepsilon^i\, m_\rho}{\sqrt{2q_1^0}} \sqrt{2}\, F_\pi(m_\rho^2) \frac{(q_3-q_2)^i a^6}{\sqrt{2q_3^0}\sqrt{2q_2^0}} ,$$

where F_π is the pion form factor, and we have restored the factors of a which have been scaled-out of the fields in (10). The expression (23), which only differs from (21) in the interchange $(\langle \pi^+\pi^0|, |0\rangle) \leftrightarrow (\langle 0|, |\rho^+\rangle)$, can easily be seen to give a result which only differs from (35) in that the phase $\exp[i(q_2+q_3-q_1)\cdot x_{\vec{j}}]$ is replaced by $\exp[i((q_2+q_3)\cdot x_{\vec{j}+n\hat{\mu}} - q_1 \cdot x_{\vec{j}})]$. Thus, the sum of (21) and (23) gives

$$- \frac{1}{4} \left\{ e^{[i(q_2+q_3-q_1)\cdot x_{\vec{j}}]} + e^{[i((q_2+q_3)\cdot x_{\vec{j}+n\hat{\mu}} - q_1\cdot x_{\vec{j}})]} \right\} (-1)^{\delta_{i\mu}-1}$$

$$\times \sqrt{2}\, f_\rho\, \varepsilon^i\, m_\rho\, \sqrt{2}\, \frac{F_\pi(m_\rho^2)}{\sqrt{2q_1^0}} \frac{(q_3-q_2)^i a^6}{\sqrt{2q_3^0}\sqrt{2q_2^0}} . \tag{36}$$

The result (36) is sufficient to evaluate (19).

Indeed, introducing (36) into (19), one finds

$$\mathscr{A} = \frac{i}{2g^2 a C_F} \int_{-\infty}^{\infty} dt\ a^3 \sum_{\vec{j},n,\mu} \frac{\delta'(n)\,\delta'(-n)}{|n|} \Big\{ e^{[i\,(q_2+q_3-q_1)\cdot\, x_{\vec{j}}\,]}$$

$$+ e^{[i((q_2+q_3)\cdot x_{\vec{j}+n\hat{\mu}} - q_1\cdot x_{\vec{j}})]}\Big\} (-1)^{1-\delta_{i\mu}} \frac{2f_\rho\ \varepsilon^i\ m_\rho\ F_\pi(m_\rho^2)(q_3-q_2)^i a^3}{\sqrt{2q_1^0}\sqrt{2q_2^0}\sqrt{2q_3^0}}$$

$$= \frac{(2\pi)^4\delta^4(q_2+q_3-q_1)\ 2i}{2g^2 a C_F} \sum_{n\neq 0} \frac{(-1)^{n+1}(-1)^{-n}}{|n|^3}\,(-1)\,2f_\rho \frac{\varepsilon^i\ m_\rho}{\sqrt{2q_1^0}} F_\pi(m_\rho^2)$$

$$\times \frac{(q_3-q_2)^i a^3}{\sqrt{2q_3^0}\sqrt{2q_2^0}} \tag{37}$$

$$= (2\pi)^4\,\delta^4(q_1-q_2-q_3)\,\frac{(2i)}{g^2 a C_F}\, f_\rho\ F_\pi(m_\rho^2)\,\frac{\vec{\varepsilon}\cdot(\vec{q}_3-\vec{q}_2)\ m_\rho a^3}{\sqrt{2q_3^0}\sqrt{2q_2^0}\sqrt{2q_1^0}} \sum_{n\neq 0}\frac{1}{|n|^3}$$

$$= (2\pi)^4\,\delta^4(q_1-q_2-q_3)\,\frac{(2i)}{g^2 C_F}\, f_\rho\ F_\pi(m_\rho^2)\,\frac{m_\rho a^2\,\vec{\varepsilon}\cdot(\vec{q}_3-\vec{q}_2)}{\sqrt{2q_1^0}\sqrt{2q_2^0}\sqrt{2q_3^0}}\,\big(2\zeta(3)\big) \quad ,$$

where $\zeta(3)$ is the Riemann zeta function evaluated at 3. This expression (37) is our basic result. A few comments about the steps in deriving it are in order.

Namely, we have passed to the limit of the infinite volume. Thus, in (37), we take

$$a^3 \sum_{\vec{j}} e^{-i(\vec{q}_2+\vec{q}_3)\cdot\vec{j}a} \rightarrow (2\pi)^3\,\delta^3(\vec{q}_2+\vec{q}_3) \tag{38}$$

and

$$\sum_{n\neq 0} \frac{1}{|n|^3} = 2\zeta(3) \quad . \tag{39}$$

Here, we anticipate that $2\pi/a \sim 1$ GeV so that, by conservation of energy, no reciprocal lattice vectors appear in (38). With these remarks, we may now proceed with the calculation of $\rho^+ \to \pi^+\pi^0$.

In particular, the standard methods can easily be seen to give

$$\Gamma(\rho^+ \to \pi^+\pi^0) = \left(4/(g^4 C_F^2)\right)\int (2\pi)^4\delta^4(q_1 - q_2 - q_3)\,|f_\rho|^2 |F_\pi(m_\rho^2)|^2\; m_\rho^2\left(2\zeta(3)\right)^2$$
$$\times\; a^4\left(|\vec{q}_3 - \vec{q}_2|^2/3\right) \frac{1}{2q_1^0}\,\frac{1}{2q_2^0}\,\frac{1}{2q_3^0}\,\frac{d^3q_2\; d^3q_3}{(2\pi)^6} \tag{40}$$

or

$$\Gamma(\rho^+ \to \pi^+\pi^0) = \frac{2}{3\pi}\,\frac{\left(2\zeta(3)\right)^2 |F_\pi(m_\rho^2)|^2\, |f_\rho|^2\, a^4\left(m_\rho^2/4 - m_\pi^2\right)^{3/2}}{C_F^2 g^4}\ , \tag{41}$$

where, for simplicity, we take $m_{\pi^0} \doteq m_{\pi^+} \doteq 137.3$ MeV $\equiv m_\pi$. The formula (41) is the advertised result of this paper. The issue of its relationship to the experimental result[18]

$$\Gamma(\rho^+ \to \pi^+\pi^0) \doteq 158 \text{ MeV} \tag{42}$$

will now be discussed in some detail.

More specifically, to evaluate (41), we need theoretical and/or experimental values for the following parameters:

(a) the lattice constant a

(b) the coupling constant g.

For, from (11), we know that, for $N_c = 3$, $N_c^2\, C_F^2 = (4)^2 = 16$. Further, f_ρ is given by (18) and, the experimental results[19] for $F_\pi(m_\rho^2)$ are consistent with

$$F_\pi(m_\rho^2) \doteq 6.0\quad . \tag{43}$$

In addition, the Reimann function of argument 3 in (41) has the value

$$\zeta(3) \doteq 1.202\quad . \tag{44}$$

Thus, values of a and g will allow us to compare (41) with (42). We discuss these two parameters in the next two sections. We consider the constant a first.

4. Determination of the Lattice Constant a

In this section, we wish to determine the value of the lattice constant a to be used in the evaluation of $\Gamma(\rho^+ \to \pi^+\pi^0)$ in (41). We proceed by analyzing an appropriate normalization condition on the lattice.

We observe that a formula such as (15) for f_π allows one, on a lattice, to determine the relationship between f_π, the effective quark mass m_q on the lattice, and the lattice spacing a. For, the momentum transfer on the lattice to the current in (15) is small, m_π^2, and, thus, the current is being probed at large distances. The fact that the pion is a Goldstone particle on this lattice then gives us confidence that the standard PCAC ideas[20] should be applicable for the appropriate m_q. What this means is that we can reduce--in the pion in (15) and use PCAC to write (15) as

$$\frac{\sqrt{2}\, f_\pi\, p^0 i}{\sqrt{2p^0}} = -3 \int_{\text{Lattice}} \frac{d^4k_1}{2\pi}\, \text{tr}\, \frac{\left[i(\not{k}_1 + m_q)\gamma^0\gamma_5\lambda^- \; i(\not{k}_2 + m_q)\gamma^0\gamma_5\lambda^+\right]}{\left[k_1^2 - m_q^2 + i\varepsilon\right]\left[k_2^2 - m_q^2 + i\varepsilon\right]} \times \frac{i m_\pi^2}{\left(-i\sqrt{2}\, f_\pi\, p_0\sqrt{2p_0}\right)} \tag{45}$$

where the kinematics is summarized by Fig. 2 and $k_2 = k_1$. The only unusual things about (45) are the following:

(a) The "integral" over the lattice is

$$\int_{\text{Lattice}} d^4k_1 \equiv \int_{-\infty}^{\infty} dk_1^0 \sum_{\vec{k}_1} \qquad (46)$$

$$\rightarrow \int_{-\infty}^{\infty} dk_1^0 \int_{-\pi/a}^{\pi/a} \frac{dk_1^x}{2\pi} \int_{-\pi/a}^{\pi/a} \frac{dk_1^y}{2\pi} \int_{-\pi/a}^{\pi/a} \frac{dk_1^z}{2\pi}$$

where $\vec{k}_1 \equiv (k_1^x, k_1^y, k_1^z)$. This restriction to latticed Fourier components is forced by the lattice current in (15)--it only contains such Fourier components.

(b) The exact fermion propagators for the quarks in Fig. 2 have been replaced by the effective free propagators

$$\frac{i}{\not{k}_1 - m_q + i\varepsilon} \quad , \quad \frac{i}{\not{k}_2 - m_q + i\varepsilon} \qquad (47)$$

with the effective mass parameter m_q. For, in the theory (5), we are working to leading order in $1/g^2$, in the large distance regime. The interaction (10) already represents the interactions to this order. Thus, we can treat the quarks as "free"[21] (ignore further terms of order $1/g^2$) as long as we use the large distance quark mass: this mass parameter is well known[17] as the constituent mass

$$m_q \doteq 343 \text{ MeV} \quad . \qquad (48)$$

With these remarks, we proceed.[22]

Namely, the result (45) becomes, in the approximation of replacing the integration region $\{-\pi/a \leq k_1^x \leq \pi/a,\ -\pi/a \leq k_1^y \leq \pi/a,\ -\pi/a \leq k_1^z \leq \pi/a\}$ by a sphere of the same volume centered at the origin of $\vec{k}_1$-space,

$$2f_\pi^2 = \frac{3m_q^2}{\pi^2}\left[\ln\left(\frac{\pi(6/\pi)^{1/3}}{am_q} + \left(1 + \pi^2(6/\pi)^{2/3}/(a^2\ m_q^2)\right)^{1/2}\right)\right.$$
$$\left. - \frac{\pi(6/\pi)^{1/3}}{am_q}\left(1/\left(1 + \pi^2(6/\pi)^{2/3}/(a^2 m_q^2)\right)\right)^{1/2}\right] \quad . \tag{49}$$

Solving for a by the Newton method one finds

$$a \doteq 5.74\ \text{GeV}^{-1} \quad . \tag{50}$$

This completes the determination of the effective lattice constant a. Our value for a is consistent with (38). We turn next to the effective value of the coupling g in (41).

5. Determination of the Effective Coupling g

The lone remaining ingredient required for the evaluation of (41) is the value of the parameter g--the strong coupling gauge coupling constant. We determine this in a phenomenological use of the results in Refs. 1.

More specifically, we first observe that g is not to be confused with the value at a in (50) of the function g(a) used by Creutz[12] in his Monte Carlo work. For, Creutz' function is the effective gauge coupling at lattice spacing a in the presence of an appropriate renormalization scheme asymptotic freedom scale. What we want is the value of the gauge coupling that characterizes momentum transfers

$$Q^2 = q_1^2 = m_\rho^2 = (.776)^2\ \text{GeV}^2 = .602\ \text{GeV}^2 \quad . \tag{51}$$

To determine this value of g, we should add that we do not care whether one uses a lattice function g(a) or a continuum space formula[1] such as

$$\frac{g^2}{4\pi} = \frac{12\pi}{23\ \ln(Q^2/\Lambda^2)} \tag{52}$$

with Λ^2 obtained from experiment. We choose to use (52).

More precisely, when one includes non-perturbative effects, the various results[23] from deeply inelastic lepton-hadron scattering and e^+e^- annihilation are consistent with

$$\Lambda \doteq .34 \text{ GeV} \quad . \tag{53}$$

This gives, from (52),

$$g^2 \doteq 12.5 \quad . \tag{54}$$

We now have all of the parameters required for the evaluation (41). This evaluation is effected in the next section, where its ramifications are also discussed.

6. Comparison of Theory and Experiment

Using the formula (41) and the results for a and g, we find the theoretical result

$$\Gamma(\rho^+ \to \pi^+\pi^0) \doteq 162 \text{ MeV} \tag{55}$$

to be compared with the experimental value 158 MeV. As one can see, the theory is in reasonable agreement with observation. This is the advertised state of affairs. One can ask, "What are the major ramifications of this agreement?"

The primary ingredient in (55) is the SLAC lattice interaction (10). Thus, we may consider (55) as a direct support for the deeper significance of (10). The obvious issue of the application of (10) to other large distance phenomena is then extremely pertinent. This issue will be taken up elsewhere.[24]

Somewhat subordinate to the interaction (10) were the various theoretical methods used to apply it to $\rho^+ \to \pi^+\pi^0$. These were as follows:

(a) Our variant of Gell-Mann's idea of abstracting for the physical hadron currents properties derived from quasi-realistic field theory models.

(b) The vacuum insertion technique for hadron matrix elements of four-fermion operators.

(c) The use of the large distance quark mass m_q in an effective free-quark propagator in the evaluation of a.

(d) The identification of g with the asymptotic freedom formula (52) evaluated at $q^2 = m_\rho^2$.

We wish to discuss these key theoretical ingredients in turn.

Concerning (a), it should be realized that the identification of the currents in (10) with the physical hadronic currents is more than an abstraction. For, the interaction Hamiltonian (10) contains, to order $1/g^2$, all of the effects of the strong interaction in the sector of Hilbert space of interest. Thus, using the full hadron currents is entirely justified to leading order in $1/g^2$ in the strong coupling regime.

Concerning (b), we refer the reader to Ref. 16 for a detailed understanding of the nature of the approximation involved in the use of (b) in our work. Phenomenologically, this approximation appears to be accurate to approximately 20% or, perhaps, even better--when properly used.

The procedure (c), which was essential in the evaluation of the lattice constant a, asserts that the dynamics on the lattice is indeed large distance dynamics for the relevant value of a. Please understand that this does not preclude one from considering, in theoretical work such as that in Ref. 12, the limit $a \downarrow 0$, the continuum limit. Rather, our point is that, in our particular application, the theory is probed to large distances.

The short distance regime, which is perturbative,[1] gives, in the simplest view, a small correction to the corresponding large distance effects. It is these large distance effects which we have calculated. Correspondingly, the effective lattice constant a then represents the distance cut-off for large distance phenomena in hadron dynamics. It is seen that a is of the order of the radii of the typical light hadron bags in the M.I.T. bag model, for example.[25] This gives us additional confidence in the procedure (c).

Clearly the crucial point in (c) is the use of the large distance mass m_q. We feel that, given the resulting size of a, the use of $m_q \doteq 343$ MeV is self-consistently justified. Had we found a much smaller value for a, one could then question (50)--but, we didn't.

Finally, we emphasize that the procedure (d) is extremely natural because of the recent results[23] for the QCD scale Λ. For, the result (52) should be reliable whenever α_s/π is small[1] compared to unity, where

$$\alpha_s \equiv \frac{g^2}{4\pi} \quad . \tag{56}$$

Using (52), we see that, in our problem,

$$\alpha_s/\pi \doteq .317 \quad , \tag{57}$$

so that we have reason to believe that the procedure (d) is not a gross approximation.

Our basic conclusion is that the procedures and methods used to compute (55) are all entirely reasonable, although one can hardly claim complete rigor!

7. Discussion

What we have accomplished here is the computation of the width $\Gamma(\rho^+ \to \pi^+\pi^0)$ to leading order in the $1/g^2$ expansion of the SLAC lattice Hamiltonian theory. The agreement between the theoretical result and observation is a reasonable agreement. It gives us additional evidence of the intimate relationship between the confining property of the strong interaction and the detailed structure of light hadron dynamics. More importantly, our result for $\Gamma(\rho^+ \to \pi^+\pi^0)$ supports the specific QCD theory--its short distance behavior as computed in Ref. 1 and its long distance behavior as computed in Refs. 2 and 4. The natural question is how does one obtain further checks of this QCD scenario for the strong interactions of light hadrons?

An obvious answer to this question is to apply the ideas and methods in this paper to further large distance processes, such as $K^* \to K\pi$, $\Phi \to KK$, $\eta \to \pi\pi\pi$, etc. Such processes will be taken up elsewhere.[24]

We do wish to emphasize, however, that the methods in this paper should pertain only to large distance dominated hadron dynamics. Thus, a process such as $\psi/J \to \rho\pi$ would require arguments in addition to those given in this paper. For, $m^2_{\psi/J} \doteq 9.6\ \mathrm{GeV}^2$ is well within the scaling region of the QCD theory, i.e., is well above $1_+\ (\mathrm{GeV})^2$. But, to be sure, one should be able to use the methods presented in the text above for the purely large distance aspects of processes dominated by short distance interactions when the corresponding large distance effects are not trivial. In general, we expect that the respective appropriate synthesis of large and small distance QCD behavior may be quite involved, but tractable.

In summary, we feel the following is a fair assessment of the results of this paper: the decay width $\Gamma(\rho^+ \to \pi^+\pi^0)$ has been used to test the applicability of the SLAC lattice QCD Hamiltonian theory to light hadron dynamics--the theory passed this test.

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APPENDIX I: EVALUATION OF f_ρ FROM $\tau^+ \rightarrow \rho^+ \bar{\nu}_\tau$

In this appendix, we wish, for completeness, to use the decay

$$\tau^+ \rightarrow \rho^+ \bar{\nu}_\tau \tag{AI.1}$$

to determine the value of f_ρ in (16) in the text. For, the result for f_ρ is well known[14,26] and our purpose here is primarily to set our notation and conventions. The relevant Feynman diagram is shown in Fig. 3 for the standard $SU_2 \times U_1$ weak-electromagnetic model[27] in which the mass of $\bar{\nu}_\tau$ is zero. The invariant amplitude is

$$(2\pi)^4 \delta^4(p - k_1 - k_2) \sqrt{\frac{m_{\nu_\tau}}{k_1^0}} \sqrt{\frac{m_\tau}{p^0}} \frac{(-ig_W)}{2\sqrt{2}} \bar{v}_\tau(p) \gamma_\mu (1 - \gamma_5) v_{\bar{\nu}_\tau}(k_1) \left(\frac{-ig^{\mu\nu}}{m_\rho^2 - M_W^2}\right)$$
$$\times \left(\frac{-ig_W}{2\sqrt{2}}\right) \left\langle \rho^+(k_2), \varepsilon \middle| J_\nu^+(0) \middle| 0 \right\rangle \tag{AI.2}$$

where the kinematics is summarized by Fig. 3. Here, ε is the ρ^+ polarization vector, g_W is the SU_2 coupling parameter in the $SU_2 \times U_1$ model, M_W is the mass of the charged intermediate vector boson in the model and the weak vector hadronic current J_ν^+ is the adjoint of the current whose latticed spatial components appear in (16). Thus,

$$\left\langle \rho^+(k_2), \varepsilon \middle| J_\nu^+(0) \middle| 0 \right\rangle = \sqrt{2}\, f_\rho\, \varepsilon_\nu\, m_\rho \frac{1}{\sqrt{2k_2^0}} \quad . \tag{AI.3}$$

The spinors $\bar{v}_\tau$ and $v_{\bar{\nu}_\tau}$ are defined in the well-known convention of Ref. 13.

On taking the squared modulus of (AI.2), summing over final states, averaging over initial states, and integrating appropriately over $(2\pi)^4\, \delta^4(p - k_1 - k_2)\, d^3k_1\, d^3k_2/(2\pi)^6$, one finds the partial width[14,26]

$$\Gamma(\tau^+ \to \rho^+ \bar{\nu}_\tau) = \frac{G_F^2}{2\pi} \left(\frac{m_\tau^2 - m_\rho^2}{2m_\tau} \right)^2 \left(\frac{m_\tau^2 + 2m_\rho^2}{m_\rho^2} \right) |f_\rho|^2 \, m_\rho^2/m_\tau \qquad \text{(AI.4)}$$

where we have identified the Fermi constant G_F as

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} \qquad \text{(AI.5)}$$

and have neglected m_ρ^2/M_W^2 compared to 1. The experimental result, for the standard model, is[26]

$$\begin{aligned} \Gamma(\tau^+ \to \rho^+ \bar{\nu}_\tau) &= .22 \left(1/(2.7 \times 10^{-13} \text{ sec})\right) \\ &= .00054 \text{ eV} \quad . \end{aligned} \qquad \text{(AI.6)}$$

Using (AI.6) and taking[26] m_τ = 1.784 GeV, one easily finds from (AI.4)

$$f_\rho \doteq 140 \text{ MeV} \quad . \qquad \text{(AI.7)}$$

This is the desired result.[14,26] It coincides with (18) in the text.

APPENDIX II: VACUUM TO PARTICLE STATE MATRIX ELEMENTS FOR $\rho^+ \to \pi^+\pi^0$

For the evaluation of the amplitude $\mathscr{A}$ in (19) for $\rho^+ \to \pi^+\pi^0$, one needs the values of the various matrix elements in expressions of the type (31) in the text; such matrix elements involve all choices of the Dirac matrices $\mathscr{M}^\eta$. (We shall work in the ρ^+ rest frame so that its four momentum is $q_1 = (m_\rho, \vec{0})$. See Fig. 1.) We consider the various choices of $\mathscr{M}^\eta$ in turn and compute the respective matrix element required in (31).

For $\mathscr{M}^\eta = 1$, we observe that (Henceforward, whenever we omit the argument of a field, it is to be understood as evaluated at x = 0.)

$$\langle 0|\psi^{\dagger\beta f'} \mathscr{M}^\eta \psi^{\beta f}|\rho^+\rangle = \langle 0|\bar{\psi}^{\beta f'} \gamma_0 \psi^{\beta f}|\rho^+\rangle$$

$$= \frac{q_1^0}{m_\rho} \langle 0|\bar{\psi}^{\beta f'} \gamma_0 \psi^{\beta f}|\rho^+\rangle$$

$$= \frac{q_1^\mu}{m_\rho} \langle 0|\bar{\psi}^{\beta f'} \gamma_\mu \psi^{\beta f}|\rho^+\rangle$$

$$= \frac{i\partial_\mu}{m_\rho} \langle 0|\bar{\psi}^{\beta f'}(0) \gamma^\mu \psi^{\beta f}(0)\, e^{-iq_1 \cdot x}|\rho^+\rangle \Big|_{x=0}$$

$$= \frac{i}{m_\rho} \partial_\mu \langle 0|e^{iP_{op}\cdot x} \bar{\psi}^{\beta f'}(0)\, e^{-iP_{op}\cdot x} \gamma^\mu$$

$$e^{iP_{op}\cdot x} \psi^{\beta f}(0)\, e^{-iP_{op}\cdot x}|\rho^+\rangle \Big|_{x=0}$$

$$= \frac{i}{m_\rho} \partial_\mu \langle 0|\bar{\psi}^{\beta f'}(x) \gamma^\mu \psi^{\beta f}(x)|\rho^+\rangle \Big|_{x=0}$$

$$= 0 \quad ; \qquad \text{(AII.1)}$$

for, by CVC, the current

$$\bar{\psi}^{\beta f'}(x) \gamma_\mu \psi^{\beta f}(x)$$

is a conserved current. It follows that $\mathcal{M}^\eta = 1$ will not contribute to (31) in our approximations.

For $\mathcal{M}^\eta = \gamma_0$, we have

$$\langle 0|\psi^{\dagger\beta f'} \mathcal{M}^\eta \psi^{\beta f}|\rho^+\rangle = \langle 0|\psi^{\dagger\beta f'} \gamma_0 \psi^{\beta f}|\rho^+\rangle$$

$$= \langle 0|\bar{\psi}^{\beta f'}(0) \psi^{\beta f}(0)|\rho^+\rangle \qquad \text{(AII.2)}$$

$$= 0 \quad ;$$

for, the ρ^+ is a 1^- state whereas the operator

$$\bar{\psi}^{\beta f'}(x) \psi^{\beta f}(x)$$

is a Lorentz scalar. It follows that $\mathcal{M}^\eta = \gamma_0$ does not contribute to (31) in our approximations.

For $\mathcal{M}^\eta = \gamma^\ell$, we have

$$\langle 0|\psi^{\dagger\beta f'} \mathcal{M}^\eta \psi^{\beta f}|\rho^+\rangle = \langle 0|\bar{\psi}^{\beta f'} \gamma_0\gamma^\ell \psi^{\beta f}|\rho^+\rangle \quad . \qquad \text{(AII.3)}$$

The matrix element $\langle 0|\bar{\psi}^{\beta f'} \gamma_0\gamma^\ell \psi^{\beta f}|\rho^+\rangle$ can also be written as

$$\frac{q_1^0}{m_\rho} \langle 0|\bar{\psi}^{\beta f'} \gamma_0\gamma^\ell \psi^{\beta f}|\rho^+\rangle = \frac{1}{m_\rho} \langle 0|\bar{\psi}^{\beta f'} \not{q}_1\gamma^\ell \psi^{\beta f}|\rho^+\rangle$$

$$= \frac{i\partial_\mu}{m_\rho} \langle 0|\bar{\psi}^{\beta f'} \gamma^\mu\gamma^\ell \psi^{\beta f} e^{-iq_1\cdot x}|\rho^+\rangle \Big|_{x=0}$$

$$= \frac{i\partial_\mu}{m_\rho} \langle 0|e^{iP_{op}\cdot x} \bar{\psi}^{\beta f'}(0) e^{-iP_{op}\cdot x} \gamma^\mu\gamma^\ell$$

$$\times e^{iP_{op}\cdot x} \psi^{\beta f}(0) e^{-iP_{op}\cdot x}|\rho^+\rangle \Big|_{x=0}$$

$$= \frac{i\partial_\mu}{m_\rho} \langle 0|\bar{\psi}^{\beta f'}(x) \gamma^\mu\gamma^\ell \psi^{\beta f}(x)|\rho^+\rangle \Big|_{x=0} \quad . \qquad \text{(AII.4)}$$

But, by the standard "current algebra"[15] manipulations,

$$i\partial\!\!\!/\psi^{\beta f}(x) = m_q^* \psi^{\beta f}(x), \; -i\bar{\psi}^{\beta f}(x)\overleftarrow{\partial\!\!\!/} = m_q^* \bar{\psi}^{\beta f}(x) \qquad \text{(AII.5)}$$

where, in the light quark case of interest to us here, we take the flavor SU(2) symmetric limit so that[17]

$$m_q^* \doteq 5.9 \text{ MeV} \quad . \qquad \text{(AII.6)}$$

Thus, we can write

$$\langle 0|\bar{\psi}^{\beta f'} \gamma_0\gamma^\ell \psi^{\beta f}|\rho^+\rangle = \frac{1}{m_\rho}\langle 0|\bar{\psi}^{\beta f'} i\overleftarrow{\partial\!\!\!/}\gamma^\ell \psi^{\beta f}|\rho^+\rangle + \frac{1}{m_\rho}\langle 0|\bar{\psi}^{\beta f'} i\overrightarrow{\partial\!\!\!/}\gamma^\ell \psi^{\beta f}|\rho^+\rangle$$

$$= \frac{-m_q^*}{m_\rho}\langle 0|\bar{\psi}^{\beta f'} \gamma^\ell \psi^{\beta f}|\rho^+\rangle - \frac{1}{m_\rho}\langle 0|\bar{\psi}^{\beta f'} \gamma^\ell i\overrightarrow{\partial\!\!\!/} \psi^{\beta f}|\rho^+\rangle$$

$$+ \frac{1}{m_\rho}\langle 0|\bar{\psi}^{\beta f'}\{\gamma^\ell, i\overrightarrow{\partial\!\!\!/}\} \psi^{\beta f}|\rho^+\rangle$$

$$= \frac{-2m_q^*}{m_\rho}\langle 0|\bar{\psi}^{\beta f'} \gamma^\ell \psi^{\beta f}|\rho^+\rangle + \frac{2}{m_\rho}\langle 0|\bar{\psi}^{\beta f'} i\partial^\ell \psi^{\beta f}|\rho^+\rangle \quad . \qquad \text{(AII.7)}$$

The second term on the RHS (right-hand side) of (AII.7) can be seen to be small, in a general sense, as follows. Observe that

$$\mathcal{O} \equiv \frac{1}{m_\rho}\bar{\psi}^{\beta f'}(0)\,\partial_\ell\psi^{\beta f}(0) = \bar{\psi}^{\beta f'}(0)\,\psi^{\beta f}(0) + \frac{1}{m_\rho}\bar{\psi}^{\beta f'}(0)\,\partial^\ell\psi^{\beta f}(0)$$

$$- \bar{\psi}^{\beta f'}(0)\,\psi^{\beta f}(0)$$

$$= \bar{\psi}^{\beta f'}(0)\left(\psi^{\beta f}(0) + \frac{a}{am_\rho}\partial_\ell\psi^{\beta f}(0)\right)$$

$$- \bar{\psi}^{\beta f'}(0)\,\psi^{\beta f}(0)$$

$$= \bar{\psi}^{\beta f'}(0)\left(\psi^{\beta f}(0) + .22a\,\partial_\ell\psi^{\beta f}(0)\right) - \bar{\psi}^{\beta f'}(0)\,\psi^{\beta f}(0)$$

$$\doteq \bar{\psi}^{\beta f'}(0)\,\psi^{\beta f}(.22a\hat{\ell}) - \bar{\psi}^{\beta f'}(0)\,\psi^{\beta f}(0) \quad , \qquad \text{(AII.8)}$$

where we have used the result (50) for a. Since $|\rho^+\rangle$ is a 1^- state, the Lorentz scalar $\bar{\psi}^{\beta' f}(0)\ \psi^{\beta f}(0)$ term on the RHS (AII.8) does not contribute to $\langle 0|\mathcal{O}|\rho^+\rangle$. Thus, using the parity operator P we have

$$\begin{aligned}\langle 0|\mathcal{O}|\rho^+\rangle &\doteq \langle 0|\bar{\psi}^{\beta f'}(0)\ \psi^{\beta f}(.22a\hat{\ell})|\rho^+\rangle \\ &= \langle 0|P\ \bar{\psi}^{\beta f'}(0)\ P^{-1}\ P\ \psi^{\beta f}(.22a\hat{\ell})\ P^{-1}\ P|\rho^+\rangle \\ &= -\ \langle 0|\bar{\psi}^{\beta f'}(0)\ \psi^{\beta f}(-.22a\hat{\ell})|\rho^+\rangle \end{aligned} \qquad \text{(AII.9)}$$

so that, from (AII.8) [Here, $O(\delta)$ denotes of the order of δ.],

$$\langle 0|\bar{\psi}^{\beta f'}(0)\ \psi^{\beta f}(.22a\hat{\ell})|\rho^+\rangle = \langle 0|\mathcal{O}|\rho^+\rangle + O\left((.22)^3/3!\right) \quad . \qquad \text{(AII.10)}$$

On our lattice, we have excluded variations in fields described by momenta $>\pi/a$. Thus, variations of the fields over distances smaller than a are suppressed. It follows that, on our lattice,

$$\langle 0|\bar{\psi}^{\beta f'}(0)\ \psi^{\beta f}(.22a\hat{\ell})|\rho^+\rangle \sim \langle 0|\bar{\psi}^{\beta f'}(0)\ \psi^{\beta f}(0)|\rho^+\rangle = 0 \quad , \qquad \text{(AII.11)}$$

in the sense of the size of the respective matrix elements in (AII.7). Thus, we conclude that $(1/m_\rho)\ \langle 0|\bar{\psi}^{\beta f'}(0)\ \partial^\ell\ \psi^{\beta f}(0)|\rho^+\rangle$ is negligible compared with the coefficient of $2m_q^*/m_\rho$ on the RHS of (AII.7), $\langle 0|\bar{\psi}^{\beta f'}(0)\,\gamma^\ell\,\psi^{\beta f}(0)|\rho^+\rangle$, in our calculational scheme. For further reference, note that, since $\langle \pi^+\pi^0|$ also has $J^P = 1^-$ here, we can replace the pair $(\langle 0|, |\rho^+\rangle)$ with the pair $(\langle \pi^+\pi^0|, |0\rangle)$ respectively everywhere in (AII.7) - (AII.11). Thus, $(1/m_\rho)\langle \pi^+\pi^0|\bar{\psi}^{\beta f'}\ \partial^\ell\ \psi^{\beta f}|0\rangle$ will also be negligible compared to $\langle \pi^+\pi^0|\bar{\psi}^{\beta f'}\ \gamma^\ell\ \psi^{\beta f}|0\rangle$ in our scheme.

The contribution of the first term on the RHS of (AII.7) to the amplitude (31) will be evaluated in terms of the respective matrix elements of $\mathcal{M}^\eta = \alpha_i$. Such an evaluation will then be seen to complete the discussion of $\mathcal{M}^\eta = \gamma^\ell$.

Turning now to $\mathcal{M}^\eta = \alpha_i$, we have

$$\langle 0|\psi^{\dagger\beta f'} \mathcal{M}^\eta \psi^{\beta f}|\rho^+\rangle = \langle 0|\psi^{\dagger\beta f'} \alpha_i \psi^{\beta f}|\rho^+\rangle$$

$$= \langle 0|\psi^{\dagger\beta f'} (-i) \sigma^{oi} \psi^{\beta f}|\rho^+\rangle$$

$$= \langle 0|\psi^{\dagger\beta f'} \gamma^o\gamma^i \psi^{\beta f}|\rho^+\rangle$$

$$= \langle 0|\bar{\psi}^{\beta f'} \gamma^i \psi^{\beta f}|\rho^+\rangle \quad . \tag{AII.12}$$

As we indicated in the discussion of (13) - (16) in the text, the $|\rho^+\rangle$ will select, in (31), the combinations of f'f in (AII.12) corresponding to the isospin lowering operator λ^- in Gell-Mann's notation,

$$\lambda^- = I_1 - i\, I_2 \tag{AII.13}$$

if (I_1, I_2, I_3) are the generators of isospin SU(2). Thus, to obtain the contribution of (AII.12) to (31) we will only need to know

$$\langle 0|\bar{\psi}^\beta \gamma^i \lambda^- \psi^\beta|\rho^+\rangle \equiv \sqrt{2}\, f_\rho\, \varepsilon^i\, m_\rho \frac{1}{\sqrt{2q_1^0}} \quad ; \tag{AII.14}$$

where ε_ν is the ρ^+ polarization, and where f_ρ has been computed in the Appendix I but is already well known.[14,26]

Observe that, as we promised, for $i = \ell$ in (AII.7) and (AII.11) the RHS of (AII.12) is equal to $\left(-m_\rho/(2m_q^*)\right)$ times the γ^ℓ-term on the RHS of (AII.7). Thus, we can indeed evaluate the dominant part of the $\langle 0|$ to $|\rho^+\rangle$ matrix element for $\mathcal{M}^\eta = \gamma^\ell$ in terms of the analogous matrix element for $\mathcal{M}^\eta = \alpha_\ell$. For, since $m_q^*/m_\rho << 1$, we may neglect the γ^ℓ-term on the RHS of (AII.7) compared to $\langle 0|\bar{\psi}^{\beta f'} \gamma^\ell \psi^{\beta f}|\rho^+\rangle$. Further, by (AII.11) the ∂^ℓ-term on the RHS of (AII.7) is also negligible compared to $\langle 0|\bar{\psi}^{\beta f'} \gamma^\ell \psi^{\beta f}|\rho^+\rangle$. (The complete neglect of the $\mathcal{M}^\eta = \gamma^\ell$ contribution to (31) relative to the $\mathcal{M}^\eta = \alpha_i$ contributions will be justified when we show that

$\langle \pi^+\pi^0|\bar{\psi}^{\beta f} \gamma_0\gamma^\ell \psi^{\beta f'}|0\rangle$ is negligible compared to $\langle \pi^+\pi^0|\bar{\psi}^{\beta f} \gamma^\ell \psi^{\beta f'}|0\rangle$. We will do this presently.)

To complete the computation of the contribution of $\mathcal{M}^\eta = \alpha_i$ to (31), we will need to know

$$\langle \pi^+\pi^0|\bar{\psi}^{\beta f} \gamma^i \psi^{\beta f'}|0\rangle = \langle \pi^0|\bar{\psi}^{\beta f} \gamma^i \psi^{\beta f'}|\pi^-(-q_2)\rangle \quad , \qquad \text{(AII.15)}$$

where we have used crossing to substitute an anti-particle in the in-state for a particle in the out-state of the appropriate four-momentum. See Fig. 1 for the kinematics. In the case of interest, when the ρ^+ has selected the operator λ^- as in (AII.14), the operator $\lambda^+ = I_1 + i\, I_2$ will have been selected in the contributions of (AII.15) to (31). Thus, we will need

$$\langle \pi^0|\bar{\psi}^{\beta} \gamma^i \lambda^+ \psi^{\beta}|\pi^-(-q_2)\rangle = \sqrt{2}\, F_\pi(m_\rho^2)\ (q_3 - q_2)^i / \left(\sqrt{2q_3^0}\,\sqrt{2q_2^0}\right) , \qquad \text{(AII.16)}$$

where $F_\pi(m_\rho^2)$ is the usual pion form factor. This completes the discussion of the matrix elements required for $\mathcal{M}^\eta = \alpha_i$.

The contribution of $\mathcal{M}^\eta = \gamma^\ell$ will also be complete if we relate

$$\langle \pi^+\pi^0|\bar{\psi}^{\beta f} \gamma_0\gamma^\ell \psi^{\beta f'}|0\rangle$$

to (AII.16). Repeating the steps from (AII.3) to (AII.7), one finds

$$\langle \pi^+\pi^0|\bar{\psi}^{\beta f} \gamma_0\gamma^\ell \psi^{\beta f'}|0\rangle = \frac{2m_q^*}{m_\rho} \langle \pi^+\pi^0|\bar{\psi}^{\beta f} \gamma^\ell \psi^{\beta f'}|0\rangle - \frac{2}{m_\rho}\langle \pi^+\pi^0|\bar{\psi}^{\beta f'} i\partial^\ell \psi^{\beta f}|0\rangle . \qquad \text{(AII.17)}$$

Thus, using the analog of (AII.11) we may conclude that, for the f,f', of interest,

$$|\langle \pi^+\pi^0|\bar{\psi}^{\beta f} \gamma_0\gamma^\ell \psi^{\beta f'}|0\rangle| \ll |\langle \pi^+\pi^0|\bar{\psi}^{\beta f} \gamma^\ell \psi^{\beta f'}|0\rangle| \quad . \qquad \text{(AII.18)}$$

The matrix elements necessary to compute the contribution of $\mathcal{M}^\eta = \gamma^\ell$ to (31) are all negligible compared with the respective matrix elements involved in the contribution of $\mathcal{M}^\eta = \alpha_i$ to (31). It follows that the contribution of $\mathcal{M}^\eta = \gamma^\ell$ to (31) is negligible.

Considering next $\mathcal{M}^\eta = -i\sigma_i \equiv (-i/2)\ \varepsilon_{ijk}\ \sigma^{jk}$ where ε_{ijk} is the totally antisymmetric symbol on three labels and $\varepsilon_{123} = 1$, we need to evaluate

$$-i\langle 0|\psi^{\dagger\beta f'}\ \sigma_i\ \psi^{\beta f}|\rho^+\rangle = \langle 0|\bar{\psi}^{\beta f'}\ \gamma_0\gamma^j\gamma^k\ \psi^{\beta f}|\rho^+\rangle \tag{AII.19}$$

where i,j,k are cyclic in (1,2,3). The operator

$$\bar{\psi}^{\beta f'}\ \gamma_0\gamma^j\gamma^k\ \psi^{\beta f}$$

is even under parity whereas ρ^+ is of odd parity. Thus, by parity conservation, the RHS of (AII.19) is zero. It follows that $\mathcal{M}^\eta = -i\sigma_i$ does not contribute to (31) in our approximations.

For $\mathcal{M}^\eta = \gamma_5\gamma^0$, we have to consider

$$\langle 0|\psi^{\dagger\beta f'}\ \mathcal{M}^\eta\ \psi^{\beta f}|\rho^+\rangle = \langle 0|\psi^{\dagger\beta f'}\gamma_5\gamma^0\ \psi^{\beta f}|\rho^+\rangle$$

$$= -\langle 0|\bar{\psi}^{\beta f'}\ \gamma_5\ \psi^{\beta f}|\rho^+\rangle\ . \tag{AII.20}$$

But, the pseudo-scalar operator $\bar{\psi}^{\beta f'}\ \gamma_5\ \psi^{\beta f}$ cannot annihilate the spin 1 ρ^+ and conserve angular momentum. Thus, the RHS of (AII.20) vanishes. It follows that $\mathcal{M}^\eta = \gamma_5\gamma^0$ does not contribute to (31) in our computation scheme.

Considering $\mathcal{M}^\eta = \gamma_5\gamma^i$, we need

$$\langle 0|\psi^{\dagger\beta f'}\ \mathcal{M}^\eta\ \psi^{\beta f}|\rho^+\rangle = \langle 0|\psi^{\dagger\beta f'}\ \gamma_5\gamma^i\ \psi^{\beta f}|\rho^+\rangle$$

$$= \langle 0|\bar{\psi}^{\beta f'}\ \gamma_0\gamma_5\gamma^i\ \psi^{\beta f}|\rho^+\rangle\ . \tag{AII.21}$$

The matrix element

$$\langle 0|\bar{\psi}^{\beta f'}\ \gamma_0\gamma_5\gamma^i\ \psi^{\beta f}|\rho^+\rangle \tag{AII.22}$$

is the same as

$$\left\langle 0 \middle| P^{-1} P(\bar{\psi}^{\beta f'} \gamma_0\gamma_5\gamma^i \psi^{\beta f}) P^{-1} P \middle| \rho^+ \right\rangle = \left\langle 0 \middle| \bar{\psi}^{\beta f'} \gamma_0\gamma_0\gamma_5\gamma^i\gamma_0 \psi^{\beta f} P \middle| \rho^+ \right\rangle$$

$$= -\left\langle 0 \middle| \bar{\psi}^{\beta f'} \gamma_0\gamma_5\gamma^i \psi^{\beta f} \middle| \rho^+ \right\rangle \qquad \text{(AII.23)}$$

--it vanishes due to the parity symmetry difference between ρ^+ and $\bar{\psi}^{\beta f'} \gamma_0\gamma_5\gamma^i \psi^{\beta f}$, the former has P = -1, the latter has P = +1. Thus, $\mathcal{M}^\eta = \gamma_5\gamma^i$ will not contribute to (31) here.

Finally, we consider $\mathcal{M}^\eta = \gamma_5$. We need

$$\left\langle 0 \middle| \psi^{\dagger\beta f'} \mathcal{M}^\eta \psi^{\beta f} \middle| \rho^+ \right\rangle = \left\langle 0 \middle| \psi^{\dagger\beta f'} \gamma_5 \psi^{\beta f} \middle| \rho^+ \right\rangle$$

$$= \left\langle 0 \middle| \bar{\psi}^{\beta f'} \gamma_0\gamma_5 \psi^{\beta f} \middle| \rho^+ \right\rangle$$

$$= \frac{q_1^0}{m_\rho} \left\langle 0 \middle| \bar{\psi}^{\beta f'} \gamma_0\gamma_5 \psi^{\beta f} \middle| \rho^+ \right\rangle$$

$$= \frac{i\partial_\mu}{m_\rho} \left\langle 0 \middle| \bar{\psi}^{\beta f'} \gamma^\mu\gamma_5 \psi^{\beta f} \middle| \rho^+ \right\rangle$$

$$= \frac{i}{m_\rho} \left\langle 0 \middle| \partial_\mu (\bar{\psi}^{\beta f'} \gamma^\mu\gamma_5 \psi^{\beta f}) \middle| \rho^+ \right\rangle \quad . \qquad \text{(AII.24)}$$

The RHS of (AII.24) is easily seen to vanish by conservation of angular momentum and G-parity. It follows that $\mathcal{M}^\eta = \gamma_5$ does not contribute to (31) in our approximations.

This completes the discussion of the various matrix elements involved in evaluating (31) in our calculational scheme. We have found that, in this scheme, only $\mathcal{M}^\eta = \gamma^\ell$ and $\mathcal{M}^\eta = \alpha_i$ will make contributions to (31) within the framework of our approximations. For all other choices of $\mathcal{M}^\eta$, the matrix element $\left\langle 0 \middle| \psi^{\dagger\beta f'} \mathcal{M}^\eta \psi^{\beta f} \middle| \rho^+ \right\rangle$ in the respective contribution to the expression in (31) is 0. Further, the contribution of $\mathcal{M}^{\eta} = \gamma^{\ell}$ to (31) has been argued to be negligible.

Our findings here are consistent with what is stated in the text.

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22. To arrive at (45), one must remember that, since m_π^2 is small, only the large distance behavior of the time-ordered products represented by the loop in Fig. 3 is relevant: $\langle 0|T(\psi_{\vec{0}}\bar{\psi}(x))|0\rangle$, $\langle 0|T(\psi(x)\bar{\psi}_{\vec{0}})|0\rangle$.

In this regime the Fourier components of $\psi_{\vec{0}}$ may be identified with the respective components of $\psi(\vec{x} = \vec{0})$. Here, $\psi(x)$ is the continuum spinor quark field and $\psi_{\vec{j}}$ is its lattice analog, where we suppress color and flavor. In this way, one can readily verify (45).

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TABLE I

The α_μ-commutation signs $s_\eta(\mu)$ and squares' signs $\mathrm{sgn}(\mathcal{M}^\eta)$ of the 16 Dirac matrices $\mathcal{M}^\eta$ as defined in (27) and (33) in the text. Here, ε_{ijk} is the totally antisymmetric tensor in three dimensions, with $\varepsilon_{123} = 1$; and δ_{ij} is the Kronecker delta function.

Dirac Matrix $\mathcal{M}^\eta$	$s_\eta(\mu)$	$\mathrm{sgn}(\mathcal{M}^\eta)$
1	1	1
γ^o	-1	1
γ^ℓ	$(-1)^{\delta_{\ell\mu}}$	-1
$\alpha_j = -i\sigma^{oj}$	$(-1)^{1-\delta_{j\mu}}$	1
$\frac{-i}{2}\varepsilon_{\ell jk}\sigma^{jk}$	$(-1)^{1-\delta_{\ell\mu}}$	-1
$\gamma_5\gamma^o$	-1	-1
$\gamma_5\gamma^\ell$	$(-1)^{\delta_{\ell\mu}}$	1
γ_5	1	1

Figure Captions

1. The decay $\rho^+ \to \pi^+ \pi^0$.

2. The PCAC soft pion equation for f_π on a lattice. The factor of 3 in Eq. (45) represents the sum over color for the quark loop.

3. The decay $\tau^+ \to \rho^+ \bar{\nu}_\tau$ in the tree approximation in the standard $SU_2 \times U_1$ model.

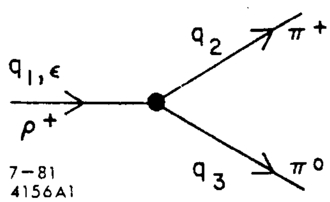

Fig. 1

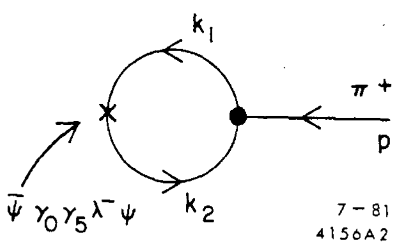

Fig. 2

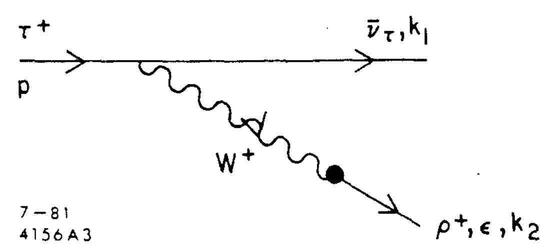

Fig. 3