SLAC-PUB-2772
July 1981
(T)

THE PHASE STRUCTURE OF THE EARLY UNIVERSE IN THE MINIMAL SU(5) GRAND UNIFIED THEORY*

Stephen Parke and So-Young Pi
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

Using the most general, gauge invariant and renormalizable Higgs potential which avoids domain walls, we study the possible phases that can appear at high temperatures in the minimal SU(5) grand unified theory. We emphasize the important role of the fundamental representation of Higgs fields for the phase structure at temperatures of 10^{14} GeV.

Submitted to Physics Letters B

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

Grand unified theories of strong, weak and electromagnetic interactions are based on a simple gauge group G. At very high energies, G is unbroken but at lower energies G undergoes a hierachy of symmetry breakings. A typical energy scale at which G is first broken is of the order 10^{14} GeV [1]. In the standard big-bang model of cosmology, at very early times, the universe was at a temperature exceeding this scale and the gauge symmetry was unbroken. Therefore, as the universe expanded and cooled, it must have undergone several phase transitions before reaching the present state of broken gauge symmetry.

A necessary consequence of the broken gauge symmetry, in grand unified theories, is the existence of superheavy magnetic monopoles [2]. These monopoles would have been produced prolifically in the early universe soon after the big-bang. They first appear when the initial symmetry G is broken down to a subgroup in which U(1) appears explicitly, i.e., $G \to H \times U(1)$. Several authors [3] have studied the initial monopole production and have concluded that the number of monopoles initially produced was at least fourteen orders of magnitude larger than a bound set from present astrophysical data. Therefore, suppression of the initial monopole production in the early universe is a severe constraint on grand unified theories.

Monopole production depends crucially on the history of the phase transitions in the early universe. In a spontaneous broken gauge theory the phase structure is determined by the Higgs sector of the theory. However, the phase sructure at finite temperature need not follow the symmetry breaking pattern at zero temperature [4]. A complicated Higgs sector can make the phase structure at finite temperature very different from the symmetry breaking pattern at zero temperature. In this note, we restrict ourselves to the minimal SU(5) theory [5] which has one adjoint (24-dimensional) and one fundamental (5-dimensional) representation of Higgs fields. First, we present a Higgs potential which avoids the existence of domain walls [6], a desirable feature which is explained below. The exclusion of theories with domain walls is another example of the restrictions on elementary particle models derived from our understanding of cosmology. Then, we study the possible phase structure of the early universe and emphasize the important role of the fundamental Higgs fields for phases at a temperature scale near 10^{14} GeV. We also discuss the implication of these results for magnetic monopole production.

The most general, gauge invariant and renormalizable Higgs potential for the minimal SU(5) theory, with Φ the adjoint Higgs field and H the fundamental Higgs field, has, at zero temperature, the form

$$V(\Phi,H) = -\frac{1}{2}\mu^2 \mathrm{Tr}\Phi^2 + \frac{a}{4}\left(\mathrm{Tr}\Phi^2\right)^2 + \frac{b}{2}\mathrm{Tr}\Phi^4 + \frac{c}{3}\mathrm{Tr}\Phi^3 - \frac{1}{2}\nu^2 H^\dagger H + \frac{\lambda}{4}\left(H^\dagger H\right)^2$$

$$+ \frac{\alpha}{2}\left(H^\dagger H\right)\mathrm{Tr}\Phi^2 + \frac{\beta}{2} H^\dagger \Phi^2 H + \frac{\gamma}{2} H^\dagger \Phi H \quad . \qquad (1)$$

Usually, a discrete symmetry $\Phi \to -\Phi$ is imposed for simplicity which eliminates the $c\mathrm{Tr}\Phi^3$ and $\gamma H^\dagger \Phi H$ terms. However, this discrete symmetry is not part of the gauge symmetry and should not be imposed since it leads to the following cosmologically unacceptable consequence.

When the hot universe expands and cools through the transition temperature, there are large fluctuations in the Higgs fields. Below this temperature, the Higgs fields have a non-zero vacuum expectation value corresponding to some point in the manifold of degenerate vacua. If this manifold is disconnected, a domain structure in the universe will be created. During the cooling process, at causally disjoint points of the universe, the vacuum expection values will appear in a random fashion within this manifold. The walls between parts of the universe with vacuum expectation values coming from different sectors of this manifold can never terminate and must either form closed surfaces or extend to 'infinity.' For small values (< 1) of the Higgs quartic couplings, these domain walls are so heavy that their existence would lead to a radical change in the cosmological evolution of the universe. The degree of inhomogeneity domain walls was calculated by Zel'dovich, Kobzarev and Okun [6]. They found that it is unacceptably large.

For the adjoint Higgs fields in SU(5), imposing the symmetry $\Phi \to -\Phi$ causes the manifold of degenerate vacua to be disconnected and thus allows the existence of unacceptable domain walls. Therefore, the potential we consider is eq. (1) with c and γ non-zero.

The symmetry breaking patterns given by the Higgs potential of eq. (1) have been analyzed by many authors [7] and their results are summarized in table I. Before we discuss the phases at high temperature, we first constrain the values of the parameters in the potential such that at zero temperature we have the following hierarchy of symmetry breakings

$$SU(5) \to SU(3) \times SU(2) \times U(1) \to SU(3) \times U(1) . \qquad (2)$$

In order to have $SU(5) \to SU(3) \times SU(2) \times U(1)$, b must be chosen to be positive. Also, we must arrange the parameters such that $V(\Phi,H)$ is bounded from below. Sufficient conditions for this, given $b > 0$, are

$$a + \frac{7}{15} b > 0$$
$$\lambda > 0$$

and

$$\left(a + \frac{7}{15} b\right)\lambda > \left[\min\left(\alpha + \frac{4}{5} \beta,\alpha,0\right)\right]^2 . \tag{3}$$

The vacuum expectation value of Φ has the form

$$\langle\Phi\rangle = \frac{v}{\sqrt{30}} \operatorname{diag}(2,2,2,-3-\varepsilon,-3+\varepsilon) \tag{4}$$

where v is given by

$$v = \frac{\sqrt{30}}{4} \left(\frac{c + \left[c^2 + 8\mu^2(15a + 7b)\right]^{1/2}}{15a + 7b}\right) \approx 10^{14} \text{ GeV} . \tag{5}$$

The vacuum expectation value of H is

$$\langle H^\dagger\rangle = \frac{\rho}{\sqrt{2}} (0,0,0,0,1) \tag{6}$$

with

$$\rho^2 = \frac{2}{\lambda} \left[\nu^2 - \alpha v^2 - \frac{3}{10} \beta v^2 + \sqrt{3/10}\, \gamma v + O\left(\varepsilon v^2\right)\right] \approx \left(10^2 \text{ GeV}\right)^2 . \tag{7}$$

Since ε is of $O(\rho^2/v^2)$ and ε vanishes as α, β and γ go to zero, ε will be neglected in our discussion. The parameters in the potential must be chosen so that, a/b and $\mu^2 b/c^2$ are to the right of line I in fig. 1 so that the SU(3) $\times$ SU(2) $\times$ U(1) minimum is lower than any possible SU(4) $\times$ U(1) minimum. Finally, we arbitrarily choose c to be positive.

Equation (7) is the well-known 'hierarchy condition' which requires a 'miraculous cancellation' between the terms of $O(v^2)$, if we choose $\nu^2 \approx \frac{1}{2}\lambda\rho^2$ as in the Weinberg-Salam model. Then, eq. (7) implies

$$\delta \equiv \frac{\gamma}{v} = \sqrt{10/3}\,\left(\alpha + \frac{3}{10}\,\beta\right) . \tag{8}$$

The colored component, H_3, of the fundamental representation of Higgs has a mass given by

$$m^2_{H_3} = \frac{1}{3}\,(5\alpha + \beta)v^2 + O(\rho^2) . \tag{9}$$

H_3 mediates proton decay and therefore should have a mass comparable to the mass of X boson, within a few orders of magnitude. This constraint gives

$$\beta > -5\alpha . \tag{10}$$

There are further constraints on the parameters so that SU(3) $\times$ U(1) with a weakly broken SU(2) $\times$ U(1) is the global minimum of the potential. However, the constraints eqs. (8) and (10) are sufficient for our purpose.

The finite temperature phases may be studied with the help of the temperature dependent effective potential [8]. The high temperature approximation to this potential can be obtained by replacing

$$\mu^2 \to \mu^2(T) = \mu^2 - \frac{1}{2}\sigma T^2$$

$$\nu^2 \to \nu^2(T) = \nu^2 - \frac{1}{2}\tau T^2$$

in eq. (1), where σ and τ are positive functions of the gauge coupling constant and the quartic couplings of the Higgs fields. Using this effective potential, one concludes that the universe was in a SU(5) phase for temperatures higher than the critical temperature T_c, which is approximately $\mu/\sqrt{\sigma}$ and thus of order 10^{14} GeV. As the universe expands and cools below T_c, it passes into a SU(4) $\times$ U(1) phase and in the absence of fundamental Higgs fields finally settles into a SU(3) $\times$ SU(2) $\times$ U(1) phase. Although, this phase structure has been studied by many authors, they have ignored the presence of a fundamental representation of Higgs fields by assuming that they always develop small vacuum expectation values and therefore do not contribute to the phase structure at temperatures much greater than 10^2 GeV. However, we show that for most of the values of the parameters in the potential this is not the case.

To demonstrate this, we concentrate on the stability of SU(4) $\times$ U(1) phase in the presence of the fundamental Higgs fields. The vacuum expectation value of the adjoint Higgs fields in the SU(4) $\times$ U(1) phase has, at temperature T, the form

$$\langle\Phi\rangle = \frac{w(T)}{\sqrt{20}} \text{ diag } (1,1,1,1,-4) \tag{11}$$

where

$$w(T) = \frac{3\sqrt{5}}{2} \left(\frac{c + \left[c^2 + \frac{8}{9}\mu^2(T)(10a + 13b)\right]^{1/2}}{10a + 13b}\right) . \tag{12}$$

In this phase, we write

$$H = \begin{bmatrix} H_4 \\ H_1 \end{bmatrix}$$

where H_4 is the 4-dimensional representation of SU(4) and H_1 is the SU(4) singlet. After expanding the finite temperature effective potential about this vacuum expectation value, one finds that the H_4 and H_1 masses are given by

$$m^2_{H_4} = -\nu^2(T) + \left(\alpha + \frac{1}{20}\beta\right)w^2(T) + \frac{1}{\sqrt{20}}\gamma w(T) \tag{13a}$$

and

$$m^2_{H_1} = -\nu^2(T) + \left(\alpha + \frac{4}{5}\beta\right)w^2(T) - \sqrt{4/5}\,\gamma w(T) \tag{13b}$$

If either of these terms is negative then the SU(4) × U(1) phase is unstable under small perturbations in this direction. In eqs. (13)

$$-\nu^2(T) = -\nu^2 + \frac{1}{2}\tau T^2 \approx \frac{1}{2}\tau T^2 > 0 \tag{14}$$

for $T \gg 10^2$ GeV. However, the hierarchy condition we have imposed for $SU(3) \times SU(2) \times U(1)$ in eq. (7) or (8) does not forbid $m^2_{H_4}$ and/or $m^2_{H_1}$ from becoming negative. If we rewrite eqs. (13) using this hierarchy condition, they come

$$m^2_{H_4} = -\nu^2(T) + \left[\left(\frac{\sqrt{6} + r(T)}{\sqrt{6}}\right)\alpha + \left(\frac{1 + \sqrt{6}\ r(T)}{20}\right)\beta\right]w^2(T) \tag{15a}$$

and

$$m^2_{H_1} = -\nu^2(T) + \left[\left(\frac{\sqrt{3} - \sqrt{8}\ r(T)}{\sqrt{3}}\right)\alpha + \left(\frac{4 - \sqrt{6}\ r(T)}{5}\right)\beta\right]w^2(T) \tag{15b}$$

where $r(T) \equiv v/w(T)$. Thus, if α and β satisfy either

$$\left(1 + \sqrt{6}\ r(T)\right)\beta < -\frac{20}{\sqrt{6}}\left(\sqrt{6} + r(T)\right)\alpha \tag{16a}$$

or

$$\left(4 - \sqrt{6}\ r(T)\right)\beta < \frac{5}{\sqrt{3}}\left(\sqrt{8}\ r(T) - \sqrt{3}\right)\alpha \tag{16b}$$

then either $m^2_{H_4}$ or $m^2_{H_1}$ can be negative, respectively. For temperatures well below T_c, say $T < 0.3T_c$, $w(T) \sim w(0)$ and we may replace $r(T)$ by $r_0 \equiv r(0)$. The contours for r_0 for various ranges of a/b and $\mu^2 b/c^2$ are shown in fig. 1. For $r = 1.1$, which is a typical value, the regions given by the constraints eqs. (10), (15a) and (15b) are shown in fig. 2.

Region I violates constraint (10) and therefore is forbidden. Region II satisfies constraint (16a) and thus $m^2_{H_4}$ can be negative if the temperature satisfies

$$T \;<\; \left(|f_1|/\tau\right)^{1/2} w(T) \tag{17a}$$

where $f_1 = \left[(\sqrt{6} + r(T))/\sqrt{6}\right]\alpha + \left[(1 + \sqrt{6}\ r(T))/20\right]\beta$ while in the $SU(4) \times U(1)$ phase. This negative mass term would cause the $SU(4) \times U(1)$ phase to unstable and indicates that the universe undergoes a phase transition to a $SU(3) \times U(1)$ phase -- not color and electromagnetism.

In region III the constraint (16b) is satisfied and thus $m^2_{H_1}$ can become negative if the temperature satisfies

$$T \;<\; \left(|f_2|/\tau\right)^{1/2} w(T) \tag{17b}$$

where $f_2 = \left[(\sqrt{3} - \sqrt{8}\ r(T))/\sqrt{3}\right]\alpha + \left[(4 - \sqrt{6}\ r(T))/5\right]\beta$ while in the $SU(4) \times U(1)$ phase. This indicates that the universe has undergone a phase transition and is now a $SU(4)$ phase. Region III has the largest opening angle, approximately 150°, in the allowed part of the (α, β) plane. Also note that if the parameters are chosen such that r_0 is larger (smaller) than 1.1, region III becomes larger (smaller). However r_0 must be larger than 1.0 or the $SU(3)^C \times U(1)^{EM}$ phase cannot be the global minimum.

In region IV none of $m^2_{H_3}$, $m^2_{H_4}$ and $m^2_{H_1}$ are negative at temperatures many orders of magnitude below T_c. So that in this region it is possible to have the Guth and Tye [9] phase transitions

$$SU(5) \to SU(4) \times U(1) \to SU(3) \times SU(2) \times U(1) \to SU(3) \times U(1) \ .$$

In the regions II, III and IV, for a given magnitude of α and β, λ must be chosen such that the $SU(3) \times SU(2) \times U(1)$ minimum is the global minimum for temperatures much smaller than T_c but larger than 10^2 GeV. If λ is chosen to be too small then this is not the case. In regions II and III an order of magnitude estimate for the smallest possible λ is $O[(|\alpha|^2 + |\beta|^2)/(|a| + |b|)]$. Whereas in region IV, numerical calculations indicate that λ in this region can be smaller than in the other regions before the $SU(3) \times SU(2) \times U(1)$ minimum is no longer the global minimum above 10^2 GeV. Of course, λ must be larger than the fourth power of the gauge coupling otherwise gauge loop effects must be included.

Therefore, for a wide range of parameters in the potential, eq. (1), the $SU(4) \times U(1)$ phase goes into a $SU(4)$ phase. In order to suppress monopole production one may further arrange parameters such that there is sufficient supercooling in the $SU(4)$ phase. Then the phase structure of the early universe is

$$SU(5) \to SU(4) \times U(1) \to SU(4) \to SU(3) \times SU(2) \times U(1)$$

where the transition from $SU(4)$ to $SU(3) \times SU(2) \times U(1)$ is strongly first order. The major difference between this structure and the one proposed by Guth and Tye is that the supercooling takes place in $SU(4)$ phase instead of the $SU(4) \times U(1)$ phase. The fundamental Higgs field has a large ($\approx 10^{14}$ GeV) vacuum expectation value in this phase. This is allowed because the gauge hierarchy condition imposed for the SU(3) × SU(2) × U(1) phase is not the same as a similar gauge hierarchy condition for the SU(4) × U(1) phase. Therefore, it should be no surprise that the SU(4) × U(1) phase can be broken by the fundamental Higgs fields at a large temperature.

As soon as the universe enters the SU(3) × SU(2) × U(1) phase the fundamental Higgs fields loose their non-zero vacuum expectation value until a temperature of 10^2 GeV. Below this temperature the SU(3) × SU(2) × U(1) phase is weakly broken to $SU(3)^C \times U(1)^{EM}$.

Our scenario for the phase structure of the early universe affects monopole production in a major way. The SU(4) phase unlike the SU(4) × U(1) phase has no monopole-like solitons. Thus, any solitons that existed in the SU(4) × U(1) phase disappeared when the universe passed into the SU(4) phase. Since, there were no solitons available to seed the first order phase transition in the SU(4) phase, as Steinhardt [10] has suggested for the SU(4) × U(1) solitons in the Guth and Tye scenario, there can be sufficient supercooling in the SU(4) phase.

Guth and Weinberg [11] have found that for the phase transition SU(4) × U(1) → SU(3) × SU(2) × U(1), if there is enough supercooling so that the monopole production rate is strongly suppressed, the universe never percolates completely into the SU(3) × SU(2) × U(1) phase. Although a detailed calculation has not been performed for the bubble production rate which converts the SU(4) phase to the SU(3) × SU(2) × U(1) phase, we expect the same results as found by Guth and Weinberg. A similar phase tumbling phenomena in one of the alternate grand unified models could possibly provide a solution to this problem.

While this manuscript was being typed, we received a preprint by Kuzmin, Shaposhnikov and Tkachev [12] who also find an unusual symmetry structure in the SU(5) theory at high temperature. Their phase structure is different from ours because they impose the reflection symmetry $\Phi \to -\Phi$ in the Higgs potential and allow ν to be of order 10^{14} GeV.

ACKNOWLEDGEMENT

We would like to thank the members of the SLAC Theory Group for many stimulating discussions. This work was supported by the Department of Energy under contract DE-AC03-76SF00515.

REFERENCES

[1] We use units such that $\hbar = c = k$(Boltzmann constant) = 1.

[2] G. t'Hooft, Nucl. Phys. B79 (1974) 276; A. M. Polyakov, JETP Lett. 20 (1974) 194; S. Coleman, in New Phenomena in Subnuclear Physics, Part A, ed. A. Zichichi (Plenum Press, N.Y., 1977), p. 297.

[3] Ya. B. Zel'dovich and M. Yu. Khlopov, Phys. Lett. 79B (1978) 239; J. P. Preskill, Phys. Rev. Lett. 43 (1979) 1365; M. B. Einhorn, D. L. Stein and D. Toussaint, Phys. Rev. D21 (1980) 3295.

[4] S. Weinberg, Phys. Rev. D9 (1974) 3357; L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 42 (1979) 1651, and Phys. Rev. D20 (1979) 3390; P. Langacker and S.-Y. Pi, Phys. Rev.Lett. 45 (1980) 1.

[5] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

[6] Ya. B. Zel'dovich, I. Yu. Kobzarev and L. B. Okun, Sov. Phys. JETP 40 (1974) 1.

[7] H. Ruegg, Phys. Rev. D22 (1980) 2040; D. D. Wu, Harvard preprint HUTP-81/A003.

[8] S. Weinberg, and L. Dolan and R. Jackiw, in ref. 4.

[9] A. H. Guth and S.-H. H. Tye, Phys. Rev. Lett. 44 (1980) 631, 963.

[10] P. J. Steinhardt, Harvard preprint HUTP-81/A012.

[11] A. H. Guth and E. J. Weinberg, Phys. Rev. D23 (1981) 876.

[12] V. A. Kuzmin, M. E. Shaposhnikov and I. I. Tkachev, Institute for Nuclear Research of the Academy of Sciences of U.S.S.R. preprint.

TABLE I

Possible symmetry breakings for SU(5) using an adjoint, Φ, and a fundamental, H, of Higgs fields.

	$\langle\Phi\rangle = 0$	$\langle\Phi\rangle \neq 0$
$\langle H\rangle = 0$	SU(5)	SU(4) × U(1) SU(3) × SU(2) × U(1)
$\langle H\rangle \neq 0$	SU(4)	SU(4) SU(3) × U(1) SU(2) × SU(2) × U(1)

FIGURE CAPTIONS

Fig. 1. Minima of the potential for the adjoint Higgs system. The SU(4) × U(1) minimum and SU(3) × SU(2) × U(1) minimum are degenerate along the solid line I. To the right of solid line II there is no SU(4) × U(1) minimum. The dashed lines are contours of r_0.

Fig. 2. Regions I through IV of parameter space for the coupling between the adjoint and fundamental Higgs using $r = v/w = 1.1$.

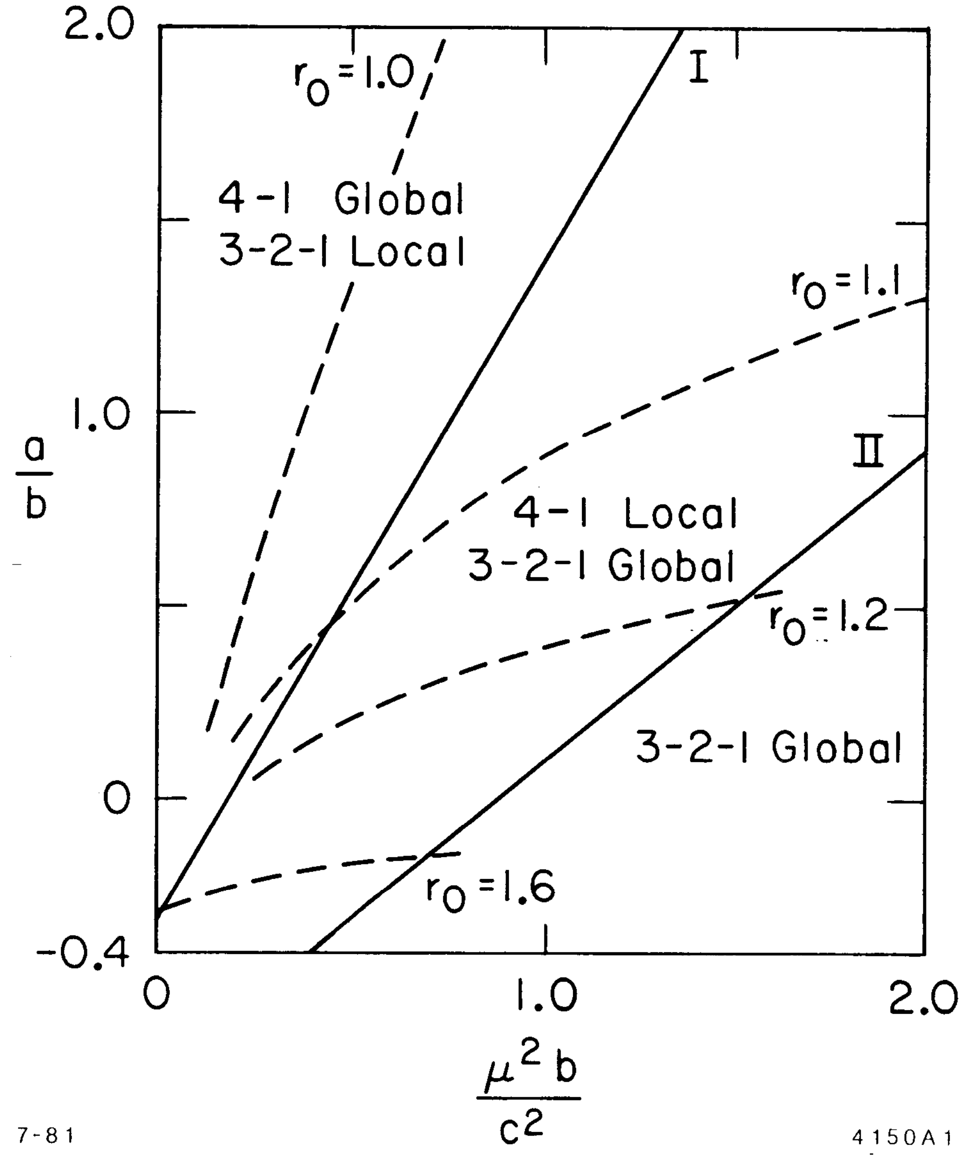
2.0
1.0
0
-0.4
$\frac{a}{b}$
$r_0 = 1.0$
I
4-1 Global
3-2-1 Local
$r_0 = 1.1$
II
4-1 Local
3-2-1 Global
$r_0 = 1.2$
3-2-1 Global
$r_0 = 1.6$
0
1.0
2.0
$\frac{\mu^2 b}{c^2}$
7-81
4150A1

Fig. 1

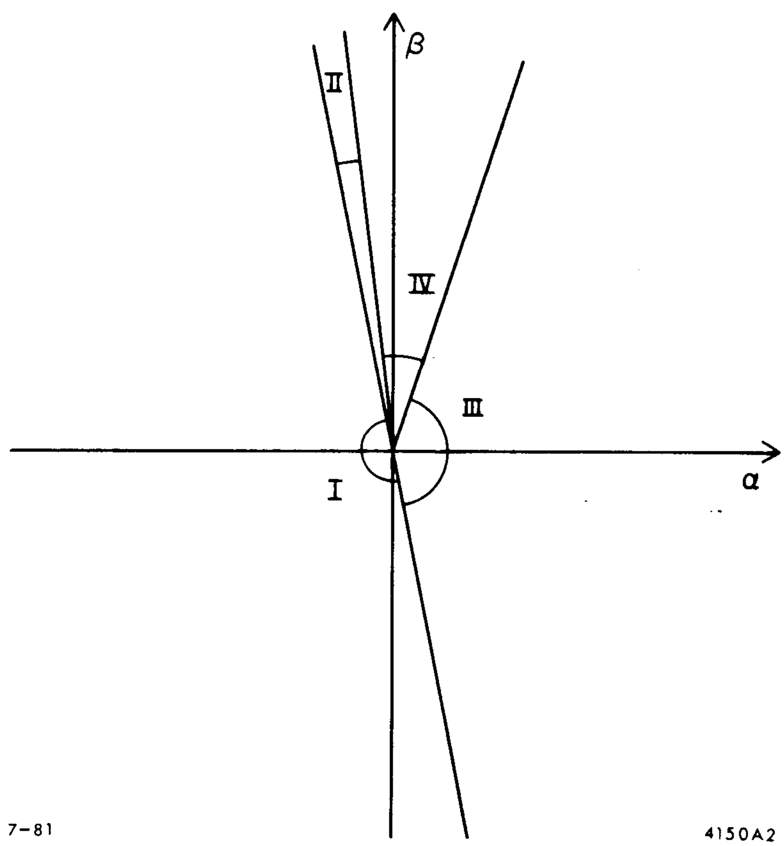

Fig. 2