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HEAVY QUARKS AND PERTURBATIVE QCD CALCULATIONS*

Subhash Gupta and Helen R. Quinn
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We consider a model universe in which the lightest quarks are heavy on the scale of the QCD Λ-parameter (however defined). In this model universe we find that there are nonperturbative effects that are not suppressed by powers of Q^2. We discuss the implications of such effects in the real world — residual effects at large Q^2 could cause deviations from perturbative predictions.

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This paper examines in some detail the physics of e^+e^- annihilation in a model universe. In this universe the usual QCD theory is modified by requiring that the lightest quark mass be very heavy compared to the intrinsic QCD scale Λ, which for the sake of definiteness we will take as the mass of the lightest glueball in this world. According to the standard understanding of QCD this theory would still possess confinement, and furthermore all the usual tools of perturbative QCD can be applied to this model. The purpose of this paper is to point out that contradictory results are obtained when one tries to add the requirement of confinement (i.e., neutralization of triality) to the perturbative QCD jet-production predictions. We will argue that these contradictions indicate that, in this model, there are nonperturbative effects which, unlike higher-twist effects, are not suppressed by powers of Q^2, but in fact give the dominant contributions at large Q^2. We will then argue that this result implies that even in the real world, with light quarks, there can be corrections to QCD perturbative predictions which need not be suppressed by powers of Q^2, but which more realistically depend on m_q/Λ, where m_q is the highest quark in the theory. The fact that the usual perturbative arguments break down when the lightest quark is heavy is perhaps no surprise — we have simply removed the possibility of a soft hadronization mechanism. However not necessarily power suppressed in the real world means that if experiments fail to agree with the perturbative predictions it would not cause us to abandon QCD, but rather could be interpreted as an observation of non-perturbative effects.

The prediction for the total hadronic e^+e^- annihilation is given by

$$R = \frac{\sigma\left(e^+ + e^- \to \text{hadron}\right)}{\sigma\left(e^+ + e^- \to \mu^+\mu^-\right)} = \sum_{\text{quarks}} e_i^2\left(1 + \mathcal{O}(\alpha_s)\right)$$

for $4m_q^2 \lesssim Q^2$. This prediction is based on the operator product expansion for the operator $J_\mu J^\mu$ and its vacuum expectation value.[1] The expansion is rigorously proven in perturbation theory in the space-like regime. To make the prediction for time-like Q^2 one has to rotate the contour and take the imaginary part, this then requires smearing of the data.[2]

In perturbative analysis the only non-zero term comes from the operator 1, all the rest have a zero vacuum expectation value, hence perturbatively there are no other corrections.

$$J_\mu(x)\ J^\mu(0) = \sum_i C_i(x)\ O^i(0)$$

where $C_i(x)$ are coefficient functions and $O^i(0)$ are local operators.

$$\left\langle J_\mu(x)\ J^\mu(0)\right\rangle = C_0(x)\ \langle 1\rangle = C_0(x)\ .$$

At large Q^2, all higher order α_s corrections are also small so we expect to have

$$R = \sum e_i^2\left(1 + \mathcal{O}(\alpha_s)\ , \frac{\Lambda}{Q}\ , \frac{m_q}{Q}\right) \quad .$$

We do not expect the requirement of color singlet formation to alter this conclusion as it is just based on short distance analysis and no large distances are involved. Hence we expect the total cross section to be correct.

Let us now try to look more closely at final states. Consider a two jet cross section as defined by Sterman and Weinberg.[3] Define $\sigma(Q,\theta,\Omega,\varepsilon,\delta)$ in center-of-mass frame as follows: It is the cross section for events which have a fraction of energy less than εQ outside two back to back cones, of opening angle δ, at an angle θ to the beam direction. Then one can define the fraction of two jet events as

$$f(2\text{ jets}) = \frac{\sigma(Q,\theta,\Omega,\varepsilon,\delta)}{\Omega \frac{d\sigma}{d\Omega}(Q,\theta)} \quad \text{where} \quad \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4Q^2}\left(1+\cos^2\theta\right) \quad .$$

Perturbation theory predicts

$$f(2\text{ jets}) = 1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{\Lambda}{Q}, \frac{m_q}{Q}, \delta\right) \quad .$$

This result is based on a detailed analysis of perturbative QCD to all orders. A similar result can be obtained for a somewhat different definition of jets,[4] where the cut-amplitude can be shown to factorize into a hard part and jet cut-vertices which are a generalization of the light cone-expansion.

Now, this result cannot be the right result in our model universe if we assume confinement Let us first fix ourselves at $Q^2 \sim 100 m_q^2$. Then, the perturbative analysis implies a quark in each of the jets carrying a large fraction of the total energy, in fact to leading order is $\alpha_s(Q)$, the energy each quark is expected to carry is $\frac{Q}{2}\left\langle 1+\mathcal{O}\left(\frac{\mu}{m_q}\right)\right\rangle$. This final state is one which has triality in each of the jets and hence, by the assumption of confinement, is not a possible final state. To neutralize triality requires one of the following: (i) production of more $q\bar{q}$ pairs; and (ii) turning a quark or antiquark from one jet to another and having one jet as a glueball jet.

The production of noncollinear pairs in our toy world cannot take place as a soft process because $m_q^2 >> \Lambda^2$. Therefore it must be hard, and hence, by the usual rules, perturbatively calculable. We then must ask what contributions can give two color neutral jets. In order to neutralize the triality of the original quark and antiquark we need to produce an additional antiquark almost parallel to the initial quark and similarly a quark almost parallel to the initial antiquark, as in Fig. 1. The phase space for this to occur is restricted. In any reasonable definition of what we mean by almost parallel, we estimate that the phase space for such production is at most of order Λ^2 — and hence cannot compensate for the factors $\alpha_s^2(Q^2)$ and the off-mass-shell quark and gluon propagators in Fig. 1 which provide suppression by powers of Q^2. Hence we estimate this process is suppressed by $\alpha_s^2(Q^2)\ \Lambda^2/Q^2$. Nonparallel production of the additional quark and/or antiquark cannot provide triality neutralization, but simply contributes to the amplitudes for a higher number of jets at the level $\alpha_s^2(Q^2)$. The definition of triality neutralization has not been given explicitly, the form of the suppression factors depends on the definition chosen, but any definition will give more suppression than a factor of $\alpha^2(Q^2)$.[5]

The process (ii), which results in one jet containing both the initial quark and antiquark, is perturbatively seen to be suppressed by powers of Λ/Q and Λ/m_q. In order to turn a quark back to pair up with the antiquark, the hard quark must emit a hard gluon going in the same direction and be itself turned back with essentially the same momentum as the initial antiquark. (Clearly the same applies with the role of quark and antiquark reversed.) These requirements put many propagators highly off-shell, and the phase space is restricted (Fig. 2) in a similar fashion to that discussed for case (i). Thus perturbatively the final state that is a single meson plus many glueball is extremely suppressed.

For Q^2 extremely large, $\alpha\ (Q^2)\ \ln\ (Q^2/m_q^2) \sim O(1)$, large numbers of collinear pairs can be produced without suppressions by factors of $\alpha_s(Q^2)$ (the collinear logarithms may compensate the $1/\ln Q^2$ from the coupling constant). The question then arises whether there is any possibility of color neutralization by this mechanism. A space time analysis of the evolution of these collinear clouds suggests that this does not happen. The smallness of $\alpha(Q^2)$ means that the initial quark and antiquark typically travel a large distance before the first collinear emission happen. The collinear clouds associated with each of the hard particles are thus far separated in space and hence cannot provide a leading order mechanism for color neutralization.[6]

Let us now abandon temporarily the perturbative discussion and consider how the dominant contribution to the cross section could arise nonperturbatively. This problem has been discussed previously by Bjorken[7] and we follow his description here. Although, as perturbative QCD predicts, we expect that the quark and antiquark to start moving back-to-back carrying a large fraction of the energy, the hadronization is so drastic that no trace of this configuration is left in the final state. To understand this let us follow the Bjorken's picture: When the quark and antiquark are separated by a distance of order $(1/\Lambda)$, a string forms between the quark and antiquark. As they separate further, they slow down, pumping the energy into the string, and finally they come to rest. The string then pulls them back together and they emit glueballs which damps the oscillations of the quark and antiquark. The final state then is a onium state essentially at rest, and a lot of glueballs.

Hence, in the heavy quark world, we reach the conclusion that there are nonperturbative effects which, unlike higher-twist effects, are not suppressed by powers of Q^2. These effects can enhance contributions which perturbatively are severely suppressed. Clearly these effects depend on the mass of the lightest quark and on the QCD scale Λ. The question which one must thus address is how big such effects might be in the real world with the lowest quark masses smaller than Λ.

In terms of the nonperturbative picture discussed above the probability of two jet formation is related to the probability of the string breaking. If it does not break then we finish up with a final state of one onium and many glueballs. The likelihood of string breaking, which requires producing a quark-antiquark pair, depends on the mass of the quarks and the energy density in the string, and hence is clearly a function of m_q/Λ. As quark masses get small the probability that the string does not break certainly reduces. The question which we cannot answer is whether it becomes zero for finite m_q/Λ or not. Any non-zero probability would provide a contribution which does not vanish as a power of Q^2. At sufficiently large Q^2, even in the heavy quark world the string may break, simply because it takes a long time to slow down the leading quark and antiquark. However even this possibility would not restore the perturbative jet distributions.

We now turn to a discussion of various processes. We will examine the question of where we expect the nonperturbative effects may provide significant corrections.

(i) R

The prediction is based only on the short distance behavior, so we expect it to be least influenced by the nonperturbative effects. But, at some level nonperturbative effects must come in. The perturbative analysis, in a universe with all quark masses 0, tells us that the total cross section is just due to operator 1 with no power corrections of the form mass/Q. Therefore, the complete perturbative answer for any Q^2 is of the form $\sum_n a_n \, \alpha(Q)^n$ where a_n's have no dependence on Q^2, and are constants. The true answer with all the bound states is expected to have knowledge about the glueball mass and other resonances. Individual channels are expected to have power-law corrections; an example of this is the exclusive process $e^+ + e^- \to \pi + \pi$ which is power suppressed as a power of (f_π/Q), the scale f_π being determined nonperturbatively. Therefore, part of the higher order terms may resum to produce these powers and hence the perturbative prediction may not be correct at some order in α. It is possible that when one sums over all exclusive channels the result will in fact give only logarithms which are just $\mathcal{O}(\alpha_s)$ terms. However, this seems quite unlikely as it requires an intricate relation between various f_π, f_p and so on, which are only determined nonperturbatively. Unitarity does not impose such a restriction. Hence in this case we expect nonperturbative effects may modify the prediction to higher order in α_s, but are very unlikely to affect the leading term.

(ii) Deep Inelastic and Light Cone Processes

In the heavy quark world these processes would also suffer from nonperturbative effects. Here it is easy to see that the twist two operators and their anomalous dimensions might have some nonperturbative contributions. Let us examine deep inelastic scattering for a heavy onium in the toy model. The cut-diagrams that give rise to anomalous dimensions are as shown in Fig. 3. Let us try to draw an equivalent space time picture, as shown in Fig. 3(b), for the frame where the onium is initially at rest. Clearly, the effect of confinement is directly proportional to the distance of quark and antiquark which does not go to zero as the Q^2 becomes larger. In fact, for any light cone process this distance would increase as one goes to higher Q^2.

Once again the question of what residual corrections remain in the light quark world cannot at present be answered. The creation of pairs from the vacuum can give rise to local color singlets and allow the quark along light cone to become independent of the stationary quark. Thus again in the question of how big the nonperturbative effects may be is related to the probability that the string does not break.

(iii) Exclusive Processes

The problems found in our heavy quark world came from attempting to reconcile the requirement of confinement with the perturbative analysis. Since for exclusive processes the color-singlet formation is already required in the standard analysis[8] one might naively think that in such problems our analysis would support the standard result. However in order to resolve the conflict between the total cross section and color-singlet jet production rate we have had to appeal to a nonperturbative picture which suggests that a process which is strongly suppressed in perturbation theory can in fact be enhanced by nonperturbative processes to give the dominant contribution. We suggest that this can also happen for exclusive processes. For example, consider an onium form factor. In perturbation theory the dominant contributions come from the diagrams of the type shown in Fig. 4 and in particular from the regime where all the propagators in τ are off-shell by an amount of order Q^2. We can consider this process in the Breit frame. In this frame perturbatively dominant contribution can roughly be described as follows:[9] the meson, Lorentz contracted to a disc, undergoes a quantum fluctuation which causes it to shrink to a small ball. The photon then interacts with this ball and reverses its direction. The ball now again undergoes quantum fluctuations which again expand it to a disc. Although this particular contribution may not suffer large nonperturbative corrections, there are other regimes, which perturbatively are 1/Q suppressed, which are not so local. For example, the double-flow regime,[8] which in the Breit frame corresponds to a quantum fluctuation which slows one of the quarks in the incoming pion and then the other quark, which now carries all the momentum, interacts with the photon, is back scattered, and subsequently picks up the stopped quark. In this process the physical separation between the two quarks becomes large both before and after the scattering and hence it is plausible that the rate at which it proceeds is substantially changed by nonperturbative effects.

In short, we would like to stress that perturbative QCD and the assumption of confinement give contradictory results for this model universe. This suggests that nonperturbative effects can in principle contribute in the leading order in $1/Q^2$, even in a world with light quarks. This would imply that the rigorous proofs of perturbative analysis do not imply complete control of a process even at large Q^2. If experiment and perturbative caclulations differ one can interpret the difference as due to nonperturbative effects rather, than a failure of QCD, unless the non-perturbative effects can be estimated theoretically, or at least bounded, in world with light quarks. The experiments are still interesting in that by discovering how big these nonperturbative effects can be in various situations we may learn to understand the theory better. One interesting possibility is that the theory undergoes a phase transition at some finite (m_q/Λ) [10] and that the effects which we find in our heavy quark world vanish identically in the real world.

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5. The power counting works essentially as in exclusive processes. See S. J. Brodsky and G. Farrar, Phys. Rev. D11, 1309 (1975).
6. Of course, at very large energies, the collinear and soft gluon emission regime has Sudakov double logarithms. These regimes do enhance radiation for local color non-singlets. This could possibly help to produce color singlets at extremely large Q^2, but such effects would not be relevant at any experimental energies. All it implies is that the nonperturbative effects may finally go to zero as some function of logarithm of (Q^2/m_q^2).
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10. Such a phase transition is plausible. For light quarks the mass of pseudo-scalar meson goes as $m_\pi^2 \to m_q\langle\bar{q}q\rangle/f_\pi^2$, whereas for the heavy quarks the pseudo-scalar (mass) appears to be of the order of $m^2 \propto m_q^2$. The existence of non-zero vacuum expectation value for $\langle\bar{q}q\rangle$ could be related to a vanishing of the nonperturbative effects from string formation.

FIGURE CAPTIONS

Fig. 1. A diagram for color neutralization by quark-antiquark pair production. The diagram is also to indicate space-time evolution in the center-of-mass frame. The dashed line indicates light cone.

Fig. 2. Diagram for color neutralization by turning around the quark by gluon emission. The diagram also indicates space-time evolution.

Fig. 3. (a) Factorization for deep inelastic scattering.
(b) A space-time diagram of the above in the rest-frame of the target onium.

Fig. 4. Dominant contributions for onium form factor.

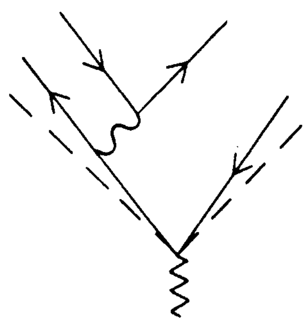

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Fig. 1

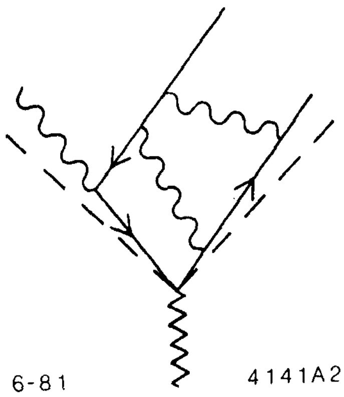
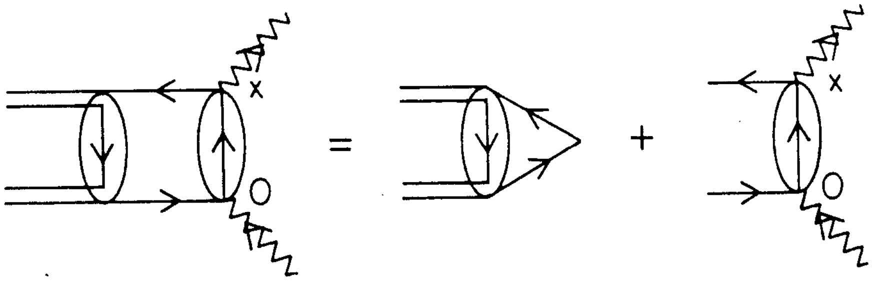


Fig. 2



(a)

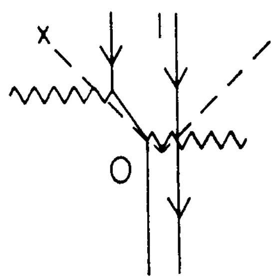


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Fig. 3

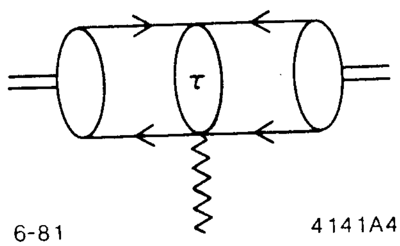

Fig. 4