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VANISHING RENORMALIZATION OF THE D-TERM IN SUPERSYMMETRIC U(1) THEORIES*

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ABSTRACT

The breaking of supersymmetry can be implemented by the Fayet-Iliopoulos D-term in the effective action. In a general left-right asymmetric theory, the renormalization of this term can be quadratically divergent. If this occurs, it would ruin the possibility of naturally large hierarchies as suggested by various authors. According to Witten if the U(1)-factor of the gauge group is unified at high energy in a semi-simple group, such a contribution identically vanishes. In this paper we show that sufficient conditions for the D-counterterm to vanish are much less restrictive than grand unification and only require a vanishing trace of the U(1) charge. In particular, no cancellation between high energy and low energy scales is involved.

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Supersymmetry[1] could provide us with a solution to the hierarchy problem of understanding why the scale of weak interaction breakdown (~ 250 GeV) is so small compared to the Planck scale (10^{19} GeV) or (more modestly) to a possible grand unification scale (10^{15} GeV).[2-5] This hope is based on (i) the unique property of supersymmetric theories to contain naturally light scalars; (ii) surprising results in supersymmetric perturbation theory that some quantities (such as F-terms in the effective potential) cannot be generated in any finite order of perturbation theory.[6]

In a realistic model, the scale of spontaneous supersymmetry breakdown (caused by nonperturbative effects and thus allowing large hierarchies) would then be related to the mass scale of the weak interactions.

The existence of the Fayet-Iliopoulos D-term[7] in supersymmetric theories that include a U(1) gauge group is potentially dangerous for this scenario.[3] It can be generated in perturbation theory, can break supersymmetry and can lead to quadratically divergent mass terms for scalar particles. The only natural mass scale would be the Planck scale. Since the D-term is pseudo-scalar, one could use parity invariance to forbid such a term, but in the $SU(3)_c \times SU(2)_L \times U(1)_Y$ standard model, the properties of U(1)-hypercharge do not allow such a solution.

Recently it has been shown that the $U(1)_Y$-D-term cannot be generated in any order of perturbation theory if $U(1)_Y$ is at some arbitrary scale unified in a semi-simple grand unification group G [e.g., SU(5)].[2] The proof is based on the fact that one cannot construct a G-invariant supersymmetric generalization of the D-term at the grand unified level. This theorem has been interpreted as the necessity for grand unification in a supersymmetric theory.[2] It could, moreover, imply miraculous cancellations of high mass and low mass contributions in perturbation theory, since in principle it can be produced in the low energy theory. We show, however, in this paper that the situation is less exciting than it might appear. Even in the absence of grand unification, the D-term cannot be generated in any order of perturbation theory except at the one loop level. Moreover, the one loop result is proportional to $\mathrm{Tr}\,Q$ [where Q is the $U(1)$ charge] which obviously vanishes in any grand unified model, but does not imply grand unification.

Except for the possibility of generating a D-term at the one-loop level, the situation here is similar to the perturbative results for the F-terms in the effective potential. We do not understand yet the reason for our result.

Let S_a, $a=1\ldots n$, be chiral left-handed superfields with charge Q_a and V a $U(1)_Q$ vector superfield. We use the notation of Ref. 6. Consider

$$\mathscr{L} = \int d^4\theta \left[\left(V D^\alpha \bar{D}^2 D_\alpha V\right) + \sum_a \bar{S}_a \exp\left(g Q_a V\right) S_a\right] + \int d^2\theta \left(\frac{g_{abc}}{3!} S_a S_b S_c + \text{h.c.}\right) \tag{1}$$

where $\bar{D}$ and D are covariant derivatives. Note that for the last term to be gauge invariant, we have the constraints that $Q_a + Q_b + Q_c = 0$. We are considering massless chiral fields; extension to the massive case is trivial, treating the mass term as an S^2 vertex. We could add the so-called D-term $\xi\int d^4\theta\, V(x,\theta,\bar{\theta})$ where ξ has dimensions of $(\text{mass})^2$, but we will set $\xi=0$ and see how such a term can be generated in perturbation theory.' The relevant graphs are given in Fig. 1, where the solid (wavy) lines denote chiral (vector) superfields. We use the superfield Feynman rules of Ref. 6, except for working in Minkowski spacetime and keeping factors of D^2 and $\bar{D}^2$ on the chiral propagators. For the one-loop graph in Fig. 1a, we obtain

$$\sum_a gQ_a \int \frac{d^4k}{k^2} \int d^4\theta \; V(p=0,\theta,\bar{\theta}) \qquad . \tag{2}$$

The coefficient is quadratically divergent and proportional to Tr Q. The higher order graphs give

$$\sum_a \sum_{m=1}^{\infty} \frac{i(gQ_a)^{m+1}}{m!} \int \frac{d^4k_1 \ldots d^4k_{m+2}}{(2\pi)^{4m+8}} \int d^4\theta \; d^4\tau \; V(p=0,\theta,\bar{\theta})$$

$$\times \left\{ -\delta^4(\theta-\tau) + \frac{1}{k_1^2} \exp\left[\left(2\tau\sigma_\mu\bar{\theta} - \theta\sigma_\mu\bar{\theta} - \tau\sigma_\mu\bar{\tau}\right)k_1^\mu\right]\right\}$$

$$\times \left\langle S_a\left(k_1,\theta,\bar{\theta}\right) \bar{S}_a\left(k_2,\tau,\bar{\tau}\right) V\left(k_3,\tau,\bar{\tau}\right) \ldots V\left(k_{m+2},\tau,\bar{\tau}\right)\right\rangle \tag{3}$$

where the two terms in curly brackets come from Fig. 1b and 1c, respectively, and

$$-\frac{i}{3!} \sum_{abc} g_{abc} \int \frac{d^4k_1 \; d^4k_2 \; d^4k_3}{(2\pi)^{12}} \; d^4\theta \; d^4\tau \; \delta^2(\bar{\tau}) \; V(p=0,\theta,\bar{\theta})$$

$$\times \frac{1}{k_1^2} \exp\left[\left(2\tau\sigma_\mu\bar{\theta} - \theta\sigma_\mu\bar{\theta} - \tau\sigma_\mu\bar{\tau}\right)k_1^\mu\right]$$

$$\times \left\langle S_a\left(k_1,\theta,\bar{\theta}\right) S_b\left(k_2,\tau,\bar{\tau}\right) S_c\left(k_3,\tau,\bar{\tau}\right)\right\rangle \tag{4}$$

from Fig. 1d.

The chiral conditions, $\bar{D}S = 0$, $D\bar{S} = 0$, and invariance under supersymmetry transformations lead to the general form

$$\left\langle S_a(k_1,\theta_1,\bar{\theta}_1)\, \bar{S}_b(k_2,\theta_2,\bar{\theta}_2)\, V(k_3,\theta_3,\bar{\theta}_3) \cdots V(k_{m+2},\theta_{m+2},\bar{\theta}_{m+2})\right\rangle$$

$$= \delta^4\left(\sum_i k_i\right) \exp\left[\sum_i \left(\theta_i\sigma_\mu\bar{\theta}_2 k_i^\mu - \theta_1\sigma_\mu\bar{\theta}_i k_i^\mu\right)\right]$$

$$\times\, f_{ab}\left(k_1;k_2;k_3,\theta_3-\theta_1,\bar{\theta}_3-\bar{\theta}_2;\, \ldots\, ;k_{m+2},\theta_{m+2}-\theta_1,\bar{\theta}_{m+2}-\bar{\theta}_2\right) \qquad (5)$$

and

$$\left\langle S_a(k_1,\theta_1,\bar{\theta}_1)\, S_b(k_2,\theta_2,\bar{\theta}_2)\, S_c(k_3,\theta_3,\bar{\theta}_3)\right\rangle$$

$$= \delta^4\left(\sum_i k_i\right) \Theta_1\Theta_2 \exp\left[-\sum_i \theta_i\sigma_\mu\bar{\theta}_i k_i^\mu\right]$$

$$\times\, h_{abc}\left(k_1,k_2,k_3,\theta_1-\theta_2,\theta_2-\theta_3\right) \qquad (6)$$

where

$$\Theta_\beta = \sum_{i,\alpha} k_{i\mu}\, \theta_\alpha\, \sigma^\mu_{\alpha\beta} \quad . \qquad (7)$$

Using these general forms and performing the $d^4\tau$ integration, the two parts of (3) cancel, while (4) vanishes upon using permutation symmetry of matrix element (6) and charge conservation at the SSS vertex.

We thus have proven that in a theory with $\mathrm{Tr}\, Q = 0$ the D-term is not generated in any order of perturbation theory. If $\mathrm{Tr}\, Q \neq 0$, the D-term is generated at the one-loop level with a quadratically divergent coefficient.

The same result is true in more complicated cases as, e.g., $SU(3) \times SU(2) \times U(1)$. $SU(3)$ and $SU(2)$ gluons may be included with the photon lines in Fig. 1b and 1c without spoiling the cancellations between the two graphs since the group factors are identical in both cases.

In conclusion, we note that the condition for the vanishing of a quadratically divergent D-term is much weaker than grand unification. In particular, no miraculous cancellation between supermassive states and low mass particles is needed.

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FIGURE CAPTION

Fig. 1. General graph with one external line.

(a) The one-loop graph.

(b) External photon attaches to n-point vertex, $n > 3$.

(c) External photon attaches to 3-point vertex, and next vertex along the chiral line (following the arrow) is attachment of one or more photons.

(d) Same as (c), but next vertex along chiral line is SSS.

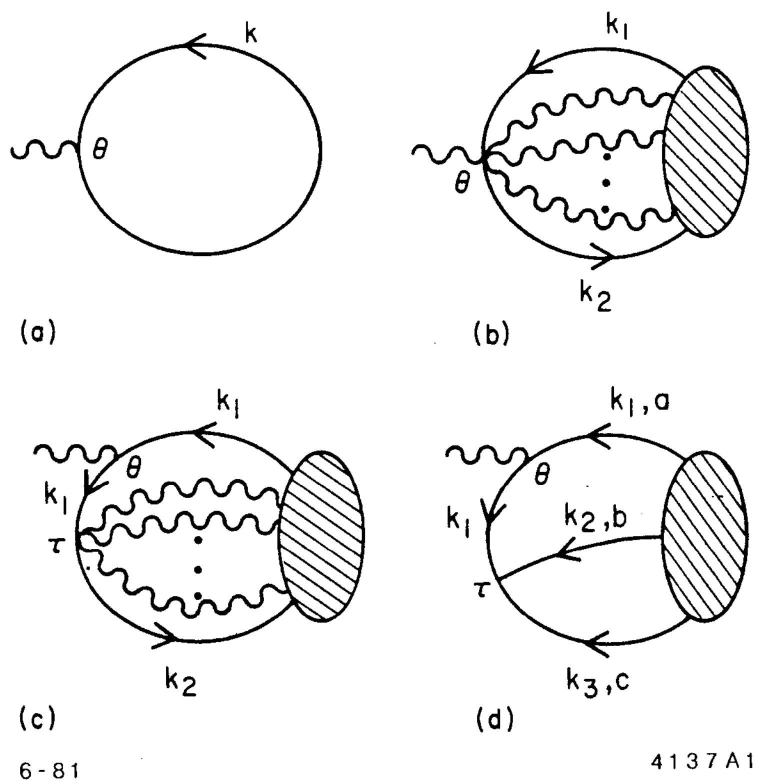

Fig. 1