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AN IMPEDANCE MEASUREMENT METHOD FOR DOUBLE-GAP KLYSTRON CAVITY*

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ABSTRACT

A new method has been developed for measuring the impedance of a two-gap cavity used in high-power klystrons. The principle is based on network analysis. The cavity under test is considered as a microwave network. The two gaps of the cavity and its output terminal are referred to as three ports. We then can use an impedance matrix to characterize this system, and the six independent impedance parameters can be found by measuring the input impedance seen from the output waveguide when the gaps are in different conditions; viz., either open, shorted or perturbed. The gap impedance then can be deduced therefrom. It is shown that there are three impedances for a double-gap cavity instead of one for a single-gap cavity. Another problem dealt with here is how to evaluate the capacitance introduced by the perturbation. A few typical experimental results are presented.

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INTRODUCTION

It is well known that usually there are two approaches to treating a microwave circuit. One is based on field equations, another is based on network analysis. In principle, field equations are more elegant, but very often the boundary problems are too difficult to get a rigorous solution. Instead, the circuit concept is rather simple, and is preferable for many engineering problems. It has been proven that in many cases the validity is equivalent for both.[1] There are some articles discussing the measurement of the impedance in a periodic structure.[2-5] The measurement of the impedance in a klystron output cavity with single gap and filter output structure has also been discussed by others.[6] Now we are concerned about estimating the impedance of a double-gap cavity used in high-power klystrons, which is not periodic and is strongly coupled to the output waveguide. For a common single-gap cavity the typical process is measuring R/Q by means of perturbation techniques and combining with the Q measurement to calculate the gap impedance. Unfortunately, in the case of high-power klystrons in which the coupling iris of the output cavity is usually very large, the perturbation formula can no longer be used, because it is based on the concept that the whole system is isolated, that electric and magnetic energies are balanced at resonance, and any perturbation destroying the balance will cause the resonance frequency to change. But this condition is no longer satisfied for the cavity mentioned above. It has been found that sometimes a network analysis method is more flexible and more powerful. We will discuss this below.

IMPEDANCE MATRIX

To begin with, we have to clarify our problem. In a single gap cavity, impedance is defined as the ratio of the voltage across the gap and the current by which the voltage is induced.* For a two-gap cavity, although the beam passes through two gaps sequentially, the a.c. components of current are different both in amplitude and phase. Besides, the current will not only induce a certain voltage on the gap it passes but also induces a voltage on the other gap, so there are at least three impedances which we have to obtain. Now we consider a microwave network which represents the system we are concerned with. Two ports represent the two gaps, while the third port represents the output waveguide, as shown in Fig. 1. We then can relate the three ports by an impedance matrix as follows:

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} \tag{1}$$

If we terminate any two terminals, say a,b, with impedances Z_a, Z_b, then they determine a fixed Z_c impedance looking into the third terminal. So that in general one can write:

$$V_1 = Z_1 I_1, \quad V_2 = Z_2 I_2, \quad V_3 = Z_3 I_3 , \tag{2}$$

where one of these impedances would be an input impedance at its terminal, and the other two, output impedances, with appropriate attention to our sign convention.

* Another equivalent definition of the impedance is: $R_{sh} = V^2/2P$.

Using relation (2) we can get unique relations between voltages for various choices of terminating impedance. Substituting (2) into (1), we get the following equation and its expanding form:

$$\begin{vmatrix} Z_{11} - Z_1 & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} - Z_2 & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} - Z_3 \end{vmatrix} = 0 \tag{3}$$

$$Z_3 = Z_{33} + \frac{2Z_{12}Z_{23}Z_{31} - Z_{11}Z_{23}^2 - Z_{22}Z_{13}^2 + Z_1Z_{23}^2 + Z_2Z_{13}^2}{\left(Z_{11} - Z_1\right)\left(Z_{22} - Z_2\right) - Z_{12}^2} \tag{4}$$

Z_3, the input impedance seen from the output waveguide, is easy to measure; and will change when Z_1 or Z_2 is changed. Note that there are six independent parameters. It is necessary to do six different experiments to define them. We can do it this way: let gap 1 or gap 2 be opened, shorted, or connected to some convenient value of impedance, for which the best choice is to perturb the gap by putting a small object into it, which introduces an extra capacitance ΔCp shunting the gap. Then Z_1 or Z_2 will be infinite, zero, or $Z_p = - 1/j\omega\Delta C_p$, respectively. Substituting those values into (4), the input impedance Z_3, which is the only impedance we can measure directly, will be as follows:

Condition in Gaps		Z_{3ab} To Be Measured
$Z_1 = \infty,$	$Z_2 = \infty$	$Z_{3oo} = Z_{33}$
$Z_1 = 0,$	$Z_2 = \infty$	$Z_{3so} = Z_{33} - \dfrac{Z_{13}^2}{Z_{11}}$
$Z_1 = \infty,$	$Z_2 = 0$	$Z_{3os} = Z_{33} - \dfrac{Z_{23}^2}{Z_{22}}$
$Z_1 = Z_{1p},$	$Z_2 = \infty$	$Z_{3po} = Z_{33} + \dfrac{Z_{13}^2}{Z_{1p} - Z_{11}}$
$Z_1 = \infty,$	$Z_2 = Z_{2p}$	$Z_{3op} = Z_{33} + \dfrac{Z_{23}^2}{Z_{2p} - Z_{22}}$
$Z_1 = 0,$	$Z_2 = 0$	$Z_{3ss} = Z_{33} + \dfrac{2Z_{12}Z_{23}Z_{13} - Z_{11}Z_{23}^2 - Z_{22}Z_{13}^2}{Z_{11}Z_{22} - Z_{12}^2}$

(5)

The subscripts o,s,p of Z_3 denote, respectively, the condition that the gap 1 or gap 2 is opened, shorted, or perturbed.

From the set of equations, the parameters Z_{ij} can be solved and expressed by Z_{3ab} (a,b = o,s,p).

GAP IMPEDANCE

What we are concerned with is: "When a bunched beam passes through the gap, how much voltage can it induce?" This problem is just the opposite of that discussed above. Namely, we want to find the input impedance seen from port 1 and 2 when port 3 is terminated. It can also be solved by the basic equation (1). Expanding (1) and using (2) we obtain:

$$V_1 = \zeta_{11} I_1 + \zeta_{12} I_2$$
$$V_2 = \zeta_{21} I_1 + \zeta_{22} I_2 \qquad (6)$$

where

$$\zeta_{11} = Z_{11} + \frac{Z_{13}^2}{Z_3 - Z_{33}}$$

$$\zeta_{22} = Z_{22} + \frac{Z_{23}^2}{Z_3 - Z_{33}} \qquad (7)$$

$$\zeta_{12} = \zeta_{21} = Z_{12} + \frac{Z_{13} Z_{23}}{Z_3 - Z_{33}}$$

Z_3 here is the loaded impedance at port 3. Since the impedance matrix parameters have been found as mentioned before, they are readily calculated.

Up to now, there is no restriction on the loading of the output waveguide, nor on the network under test. Generally, the load is matched; its normalized form is:

$$Z_3 = -1 \qquad (8)$$

The minus sign is due to the definition of the current, which orients toward the network rather than the load as shown in Fig. 1. Note that the reference plane on port 3 is still optional. For simplicity, we choose the reference plane as the position of the standing wave minimum when there is no perturbation. We regard this plane as the origin. Providing the system is lossless, the impedance at the minimum is zero. Thus:

$$Z_{33} = Z_{3oo} = 0 \qquad (9)$$

while the other impedances in (5) can be expressed by:

$$Z_{3ab} = j\tan\theta_{ab} \qquad (a,b = o,s,p) \tag{10}$$

where θ is the phase shift with the origin, when the gaps are opened, shorted, or perturbed. By some algebra, the gap impedances can be expressed as below

$$\zeta_{ij} = \rho_{ij} + jX_{ij}, \qquad i,j = 1,2$$

$$\rho_{11} = \frac{1}{\omega\Delta C_{1p}\left(\cot\theta_{po} - \cot\theta_{so}\right)}, \quad \frac{X_{11}}{\rho_{11}} = -\cot\theta_{so}$$

$$\rho_{22} = \frac{1}{\omega\Delta C_{2p}\left(\cot\theta_{op} - \cot\theta_{os}\right)}, \quad \frac{X_{22}}{\rho_{22}} = -\cot\theta_{os} \tag{11}$$

$$\rho_{12} = \pm\sqrt{\rho_{11}\rho_{22}}, \quad \frac{X_{12}}{\rho_{12}} = \left[\sqrt{\left(\cot\theta_{so} - \cot\theta_{ss}\right)\left(\cot\theta_{os} - \cot\theta_{ss}\right)} - \cot\theta_{ss}\right]$$

where ΔC_{1p} and ΔC_{2p} are the capacitances introduced in the gap 1 or 2 by the perturbations.

From the process discussed above, we note that all the formulas are universally applicable — no matter how complex the system is; no matter if it is resonant or not. Especially, it can be used in a single gap cavity or a filter type output circuit which is often used in wide band klystrons. In this case only ρ_{11} and X_{11} exist.

EVALUATION OF PERTURBATION

All the deductions above are based on the concept of network rather than electromagnetic field. They are valid because in the region of a klystron gap only electric field is important. The shape of a static electric field is a reasonable approximation. So the concept of capacitance will also be adequate in this case. Suppose C_g is the capacitance of the gap, which is contained within the network, while the increment of capacitance ΔC, which is introduced by perturbation and shunted with C_g as shown in Fig. 2, will be referred to as an outer susceptance. We will only be concerned with the electric field in the gap region; although it is quite possible that there is another region where electric field plays an important role as well, it does not affect our problem. The perturbation only affects the amount of C_g, nothing else.

Now consider the electric energy stored in the capacitance. It is

$$W_E = \frac{Q^2}{2C_g} \tag{12}$$

where Q is the charge on the surface of the electrodes, the wall of the gaps. If a metal bead is put inside the gap, it will perturb the field and cause the stored energy to decrease. Let us recall the implication of the perturbation concept. It says that only local field near the perturbing object is influenced; the rest of the field will remain unchanged. (Incidentally, let us point out that this restriction is not necessary in our previous discussion although we use the same terminology-perturbation.) If the bead is small enough, the electric field lines, and so the charge, on the surface will remain unchanged. Therefore, any perturbation causes a decrease of electric energy, and will in the mean time cause an increase of the capacitance. From (12) we obtain

$$\Delta C_b = -\frac{C_g}{W_E}\ \Delta W_E \quad . \tag{13}$$

On the other hand, ΔW_E can be calculated by the well-known perturbation formula:

$$\Delta W_E = -\frac{1}{2}\ \varepsilon_o E^2 k \Delta v \tag{14}$$

where Δv is the volume of the bead, and k is a constant related to the shape of the perturbing object, which is 3 for a sphere. Besides, W_E can be written in another form:

$$W_E = \frac{1}{2}\ C_g V^2 = \frac{1}{2}\ C_g \left[\int_{-\infty}^{\infty} E_m f(z) \cdot dz\right]^2 \quad , \tag{15}$$

f(z) is the field distribution function, z is the longitudinal position along the axis of the gap. Substituting (14) and (15) into (13), we then obtain:

$$\Delta C_b = \varepsilon_o k \Delta v \left[\frac{E_m f(z)}{\int_{-\infty}^{\infty} E_m f(z) \cdot dz}\right]^2 \quad . \tag{16}$$

When the bead is placed at the electric field maximum, where $E = E_m$, $f(z) = 1$, perturbation reaches maximum too. Thus:

$$\Delta C_{bmax} = \frac{\varepsilon_o k \Delta v}{\left[\int_{-\infty}^{\infty} f(z) \cdot dz \right]^2} \tag{17}$$

Since the value of ΔC_b depends on the exact location of the bead, it is not convenient to control its location when frequency response is wanted, so a stable perturbation arrangement is preferable. A metal pin is then selected. Suppose ΔC_p and ΔC_b are capacitances introduced by metal pin and bead, respectively, the corresponding phase shifts of standing wave minimum are θ_{pin} and θ_{be}, then from (11) we can calculate ΔC_p by:

$$\Delta C_p = \Delta C_b \, \frac{\cot\theta_{be} - \cot\theta_{s1}}{\cot\theta_{pin} - \cot\theta_{s1}} \tag{18}$$

Furthermore, the electric field distribution can be tested by moving the bead along the axis and measuring the phase shift in a fixed frequency, and using the following approximate formula:

$$\frac{E^2}{E_m^2} = \frac{\theta_{be}}{\theta_{bem}} \tag{19}$$

where E_m and θ_{bem} are the electric field maximum and corresponding phase shift.

EXPERIMENT RESULTS

An experimental double-gap cavity model was tested. This model is formed essentially by the last two cavities of a standard SLAC XK-5 klystron with a coupling slot between them as shown in Fig. 3. It also shows the shape of the pins and the shorting rods which we used. Inserting the latter can be reasonably regarded as an approximate short. The shorting rods are drilled hollow in order to minimize the influence of a shorting rod to the other gap. The resonance frequencies of the original two cavities are different. They are about 2856MHz and 3100MHz, respectively. The loaded Q of the original output cavity is only about 18, so its R/Q and its field distribution cannot be measured directly by the conventional method without narrowing the coupling iris. Figure 4 shows the measured field shape by means of the present method and making use of (19). The frequency was fixed precisely, and only the standing wave minimum was measured.

In order to estimate the error, two beads with diameters of 1/4 and 1/8 inches were used. The results shown in Fig. 4(a) and (b) coincide pretty well. The field shape shows that the fields of the two gaps overlap each other in the middle region. The individual field of each gap should be that shown by the dashed line in Fig. 4(a). The capacitances introduced by the 1/4 inch bead in gap 1 and gap 2 are 4.78×10^{-15} and 5.48×10^{-15} farad, respectively, while that introduced by the pins are 3.51×10^{-14} and 4.03×10^{-14} farad. The final results of the gap impedance are shown in Fig. 5. The upper part shows the absolute values and real parts of the impedances, and the lower part, their phases.

We have also checked the impedance of a single-gap output cavity of a SLAC XK-5 klystron without knowing R/Q. The obtained value is about 1900 ohms. A modified cavity with a narrower iris, for which both conventional and present methods are valid, has been tested too. The difference between them is about 10 percent.

A CHECK OF THE THEORY

In order to check the validity of the network approximation, we note that from (4) it is possible to predict the input impedance seen from the output waveguide in the case that both gaps are perturbed simultaneously. By some algebra, the following formulas are deduced:

$$\theta_{pp} = \arctan X_{3pp} \tag{20a}$$

where

$$X_{3pp} = \frac{X_{ss}\left(1-\alpha^2\right) - \left(X_{so} + X_{os}\right) + X_{so}X_{os}\left(\frac{1}{X_{po}} + \frac{1}{X_{op}}\right)}{\left(1 - \alpha^2\right) + \left(\frac{X_{so}X_{os}}{X_{po}X_{op}} - 1\right)}$$

$$\alpha = \sqrt{\frac{\left|\left(X_{so} - X_{ss}\right)\left(X_{os} - X_{ss}\right)\right|}{X_{ss}^2}} - \frac{\sqrt{\left|X_{so}X_{os}\right|}}{X_{ss}} \tag{20b}$$

$$X_{ab} = \tan\theta_{ab} \qquad (a,b = o,s,p).$$

The calculated as well as the experimental results are plotted in Fig. 6. The difference between them is about one degree.

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FIGURE CAPTIONS

Fig. 1. An equivalent network of a three-port microwave system.

Fig. 2. The capacitance increment introduced by the perturbation as an outer capacitance connected to the network.

Fig. 3 The double-gap cavity model with (a) the pins, and (b) the shorting rods.

Fig. 4. The field distribution function measured by using a bead with a diameter of (a) 1/4 inches, and (b) 1/8 inches.

Fig. 5. The impedances of a double-gap cavity where

$$\zeta_{ij} = |\zeta_{ij}| e^{j\phi_{ij}} = \rho_{ij} + j\chi_{ij} \quad , \quad i,j = 1,2 .$$

Fig. 6. A check of the theory.

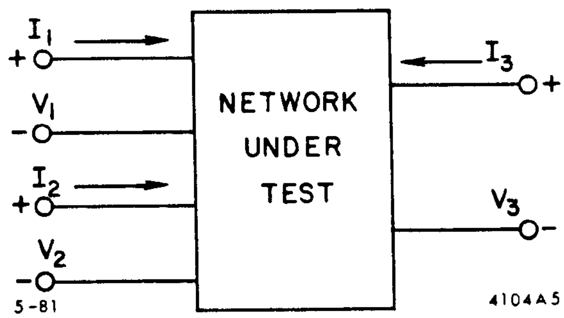


Fig. 1

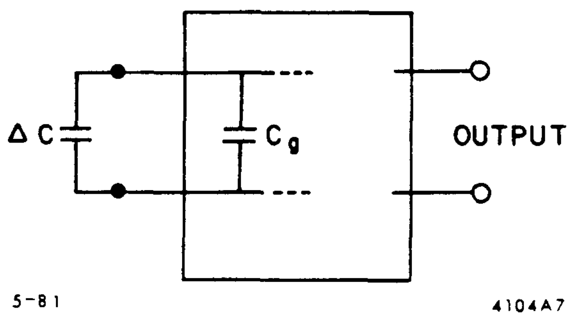
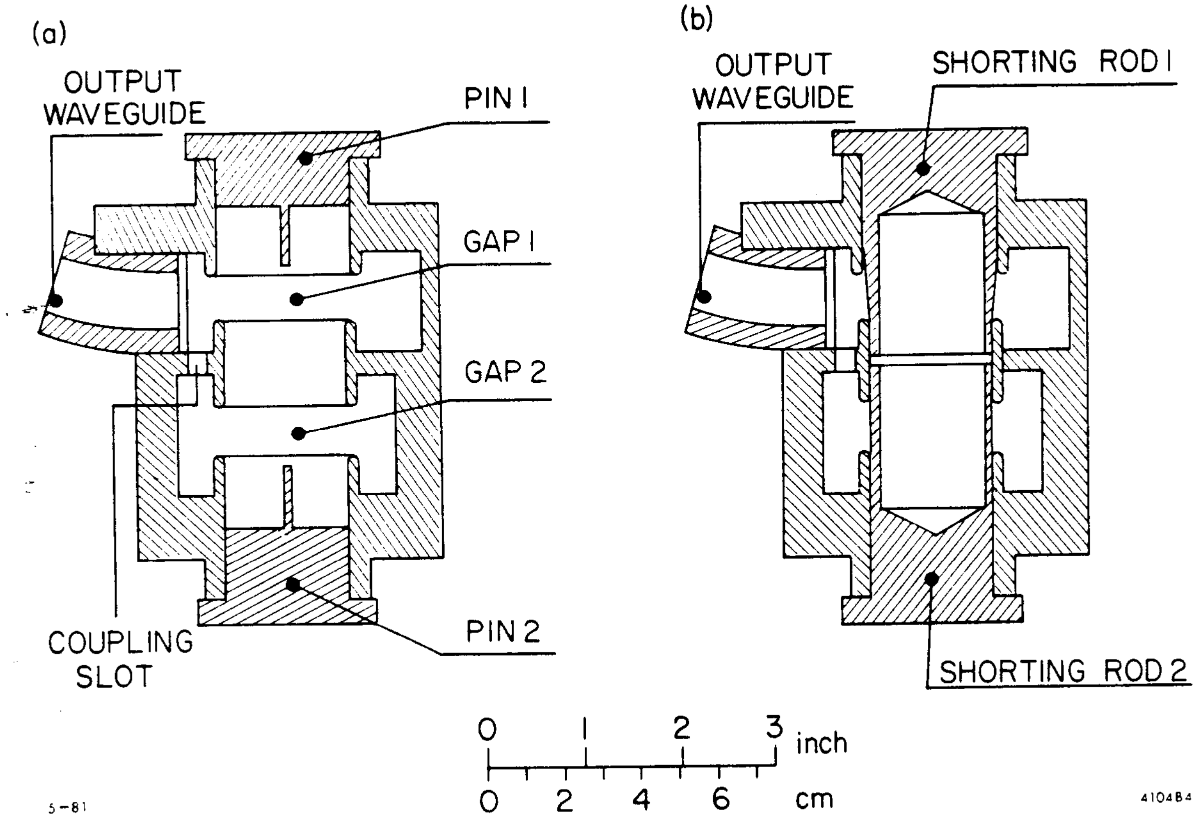


Fig. 2



(a)
OUTPUT WAVEGUIDE
PIN I
GAP I
GAP 2
COUPLING SLOT
PIN 2
(b)
OUTPUT WAVEGUIDE
SHORTING ROD I
SHORTING ROD 2
0 1 2 3 inch
0 2 4 6 cm
5-81
4104B4

Fig. 3

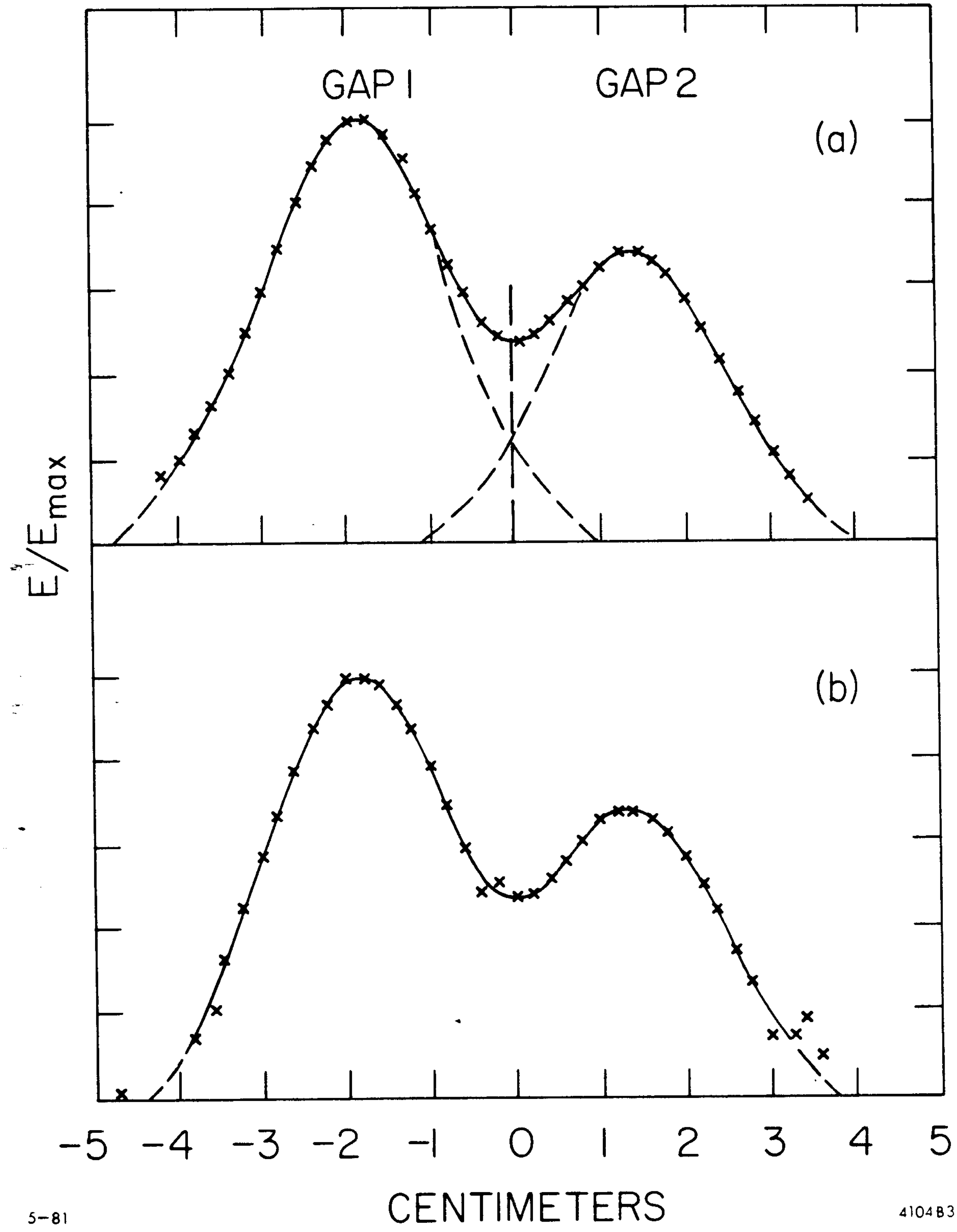


Fig. 4

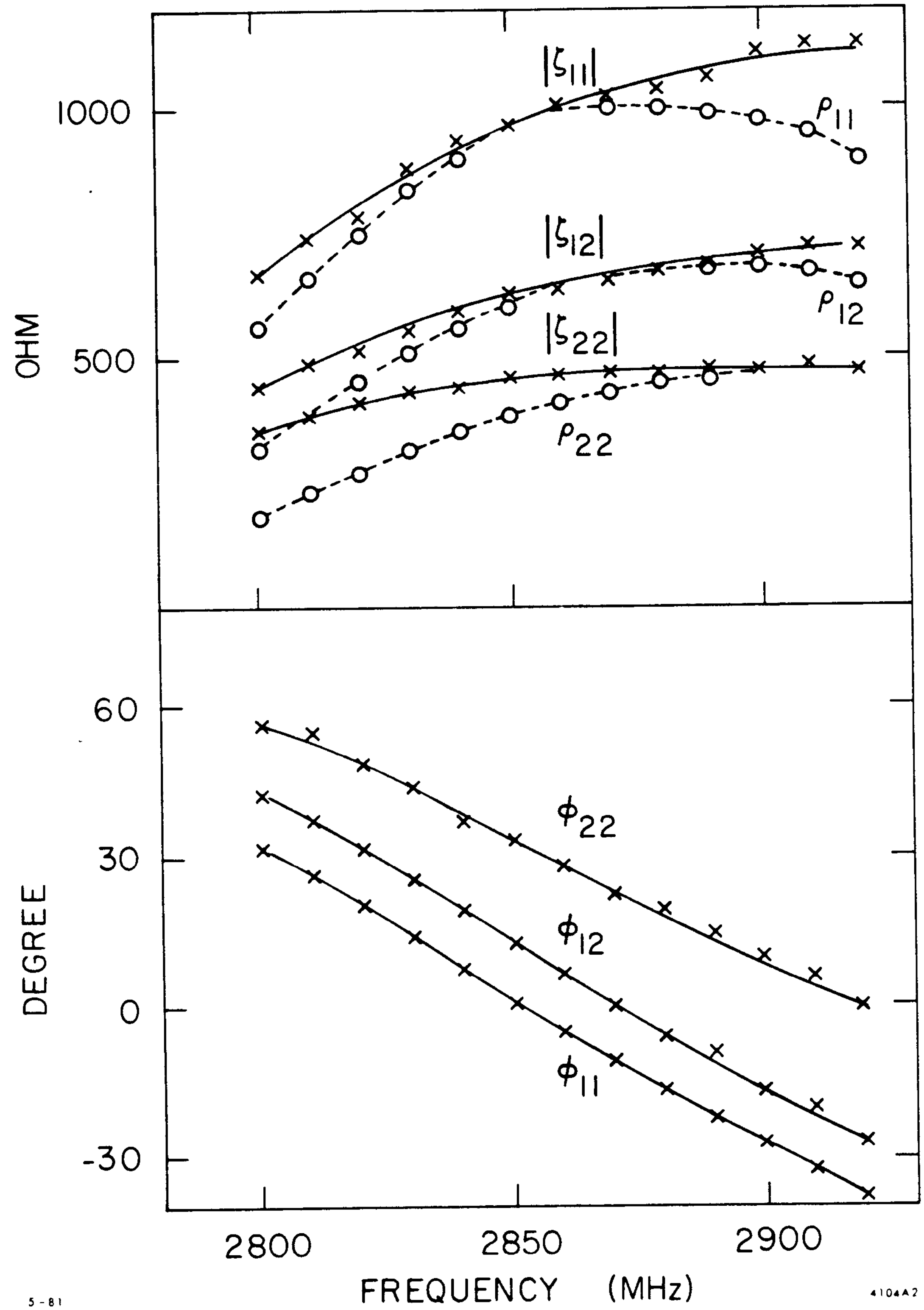
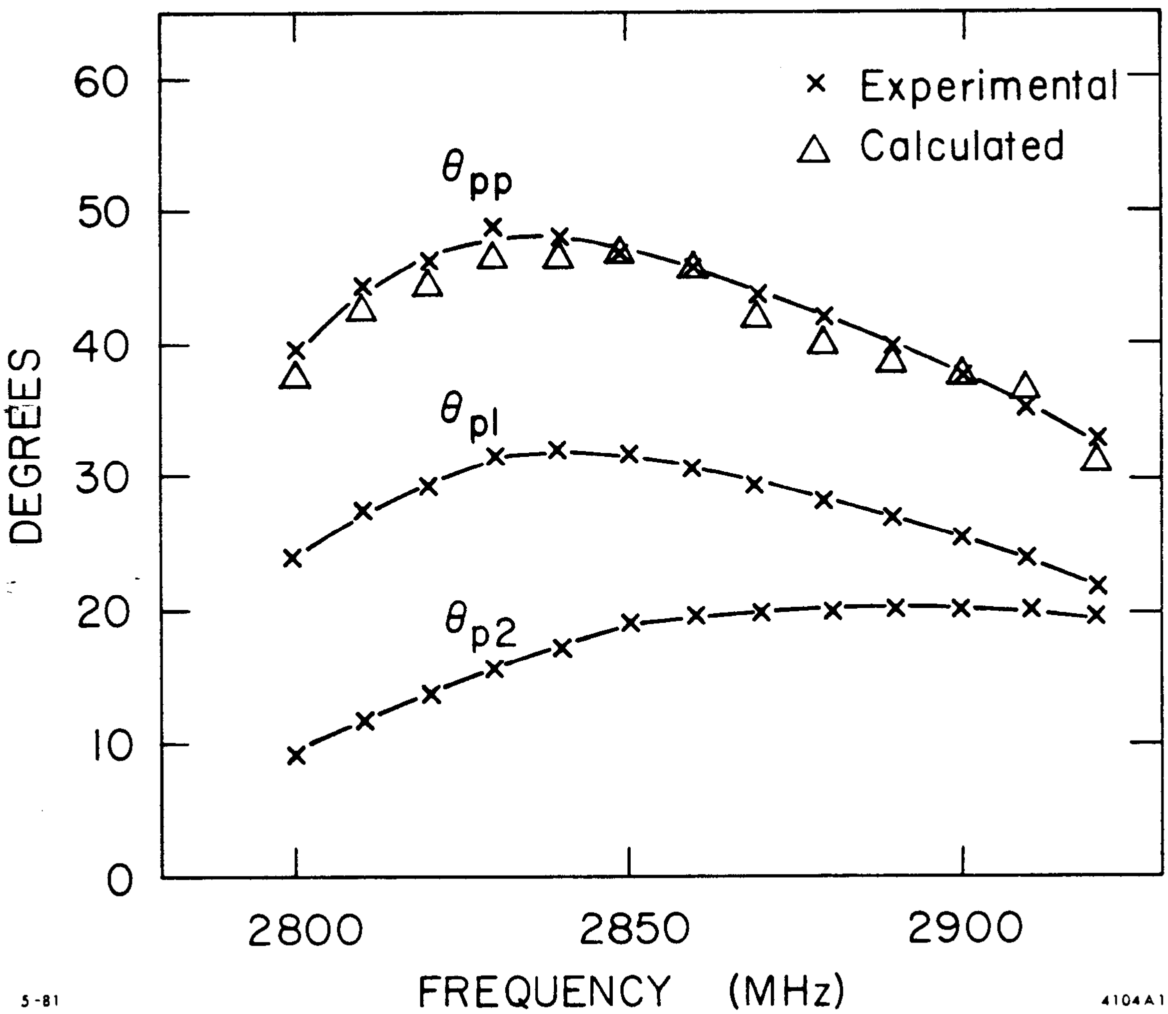

|ζ11|
ρ11
|ζ12|
ρ12
|ζ22|
ρ22
1000
500
OHM
φ22
φ12
φ11
60
30
0
-30
DEGREE
2800
2850
2900
FREQUENCY (MHz)
5-81
4104A2

Fig. 5



60
50
40
30
20
10
0
DEGREES
× Experimental
△ Calculated
θ_{pp}
θ_{p1}
θ_{p2}
2800
2850
2900
FREQUENCY (MHz)
5-81
4104A1

Fig. 6