

Diffractional excitation in QCD*

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ABSTRACT

Hadronic collision models based on QCD predict remarkably large cross sections for diffractive scattering of hadrons on a nuclear target. The diffraction arises from the overall transparency of a nucleus to the portion of the projectile wavefunction having small transverse separation between its constituents. Correspondingly, the typical transverse momentum within the diffractive system is significantly enhanced. This QCD-based picture leads to large cross sections for diffractive charm production.

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Diffraction dissociation, as explained in the classic papers of Feinberg and Pomeranchuk¹ and Good and Walker², is a distinctive quantum effect which makes up a significant part of the total hadron-hadron cross section at high energy. In a typical dissociation event, the target recoils intact with small momentum transfer, while the projectile is excited to a state with particles having low transverse momentum. The diffraction excitation arises from the variability of the absorption amplitude as a function of the internal coordinates of the projectile wavefunction. We wish to focus attention on the enhanced possibility of studying hadronic wavefunctions by diffractive dissociation on nuclear targets. In a QCD-based picture of hadron-hadron total cross sections^{3,4}, the components of a color singlet projectile wavefunction with small transverse separation pass freely through the nucleus while the large transverse separation components are nearly totally absorbed.⁵ In short, a large target will act as a filter, removing from the beam all but the short-range components of the projectile wavefunction. The cross section for diffractive production of inelastic states is then equal to the elastic scattering cross section of the projectile on the target, multiplied by the probability that sufficiently small transverse separation configurations are present in the wavefunction.

The relevant portions of the wavefunction can be calculated in QCD, because the short distance behavior is in the perturbative domain.^{6,7} The QCD wavefunction is represented in terms of Fock components defined at equal time on the light cone; internal coordinates are the

transverse position $r_{\perp i}$ and the light cone momentum fraction $x_i = \frac{(k_0 + k_3)_i}{p_0 + p_3}$ of each constituent, where p is the projectile momentum in a frame with $p^1 = p^2 = 0$. For the interaction between hadrons we make use of the gluon exchange model of Low and Nussinov³. An immediate consequence⁴ of the model is that the interaction depends strongly on the configuration of the constituents of the hadron. For the Fock components with many constituents, it is very improbable for all the transverse separations to be small. Only the valence Fock configuration has a wavefunction $\xi(x, r_{\perp})$ which is large as $r_{\perp} \rightarrow 0$,^{6,7} and hence a wavefunction component which is totally noninteracting.⁸

At high energy the S-matrix may be written as a function of impact parameter b , and internal projectile coordinates $\xi = (x_i, r_{\perp i})$. The projectile wavefunction ψ is normalized, $\int d\xi |\psi(\xi)|^2 = 1$. After filtering by the target, the surviving hadron wavefunction is

$$S(b, \xi) \psi(\xi) = \psi'(b, \xi) \quad (1)$$

Defining $\bar{S}(b) = \int d\xi |\psi(\xi)|^2 S(b, \xi)$, the expectation value of S for the incoming hadron, we decompose ψ' as

$$\psi'(b, \xi) = \bar{S}(b) \psi(\xi) + [S(b, \xi) - \bar{S}(b)] \psi(\xi) \quad (2)$$

The first term is simply the survival amplitude of the incident state, while the second term is a superposition of excited hadron states orthogonal to the incident state. If for certain values of ξ there is weak absorption, $S(b, \xi) \approx 1$, then the cross section for producing such ξ 's is given by

$$\frac{d\sigma}{d\xi} = \int d^2b (1 - \bar{S}(b))^2 |\psi(\xi)|^2 = \sigma_{el} |\psi(\xi)|^2 \quad (3)$$

where σ_{el} is the elastic scattering cross section. Thus the diffractive cross section ratio $(\sigma_{el})^{-1} d\sigma/d\xi$ directly measures the original wavefunction.

As an example consider a pion projectile. We treat the interaction of the $q\bar{q}$ Fock component, and assume that the higher Fock components are completely absorbed. The asymptotic form of the $q\bar{q}$ Fock state wavefunction can be calculated in perturbative QCD.⁶ With the normalization taken from the $\pi \rightarrow \mu\nu$ decay rate, the wavefunction at $r_{\perp} = 0$ is given by

$$\psi(x, r_{\perp}=0) = \sqrt{4\pi} \sqrt{3} x(1-x) f_{\pi} \quad (4)$$

with $f_{\pi} = 93$ MeV. The associated cross section is then

$$\frac{d\sigma}{dx d^2r_{\perp}} \Big|_{r_{\perp} \rightarrow 0} \approx \sigma_{el}^{\pi} 12\pi f_{\pi}^2 x^2 (1-x)^2 \quad (5)$$

If the typical r_{\perp} which survived the nuclear filtering were small enough the q and \bar{q} would materialize as jets in the final state. In the center of mass of the $q\bar{q}$ jet the x variable would be related to the jet angle θ (relative to the incoming beam direction) by $x = (1 + \cos \theta)/2$, and the internal momentum k_{\perp} of the q (or \bar{q}) would determine the mass of the diffractive system through $M^2 = k_{\perp}^2/x(1-x)$. However, we shall see that the nuclear filter does not generally produce states with large enough k_{\perp} that a jet structure can be expected to appear.

To make a quantitative estimate for nuclear targets we shall assume that the absorption is strong for impact parameters less than the nuclear radius, i.e. $\bar{S}(b) \approx 0$ for $b < R \approx 1.2A^{1/3}$. Then the wavefunction of the diffractive excitation is

$$\psi'(x, r_{\perp}) = S(b, r_{\perp})\psi(x, r_{\perp}) \quad b < R \quad (6)$$

We shall now relate $S(b, r_{\perp})$ to the pi-nucleon total cross section. In vector exchange models such as the Low-Nussinov model, the derived cross section depends only on r_{\perp} distribution and not on the x distribution⁴, so we have dropped the x variable. We shall also find for large nuclei that $S(b, r_{\perp})$ is damped rapidly with increasing r_{\perp} and we may neglect the r_{\perp} dependence of ψ in computing the momentum spectrum. The cross section is then given approximately by

$$\frac{d\sigma}{dx d^2 r_{\perp}} \approx \pi R^2 |\psi(x, r_{\perp}=0)|^2 S_A^2(r_{\perp}) \quad (7)$$

where $S_A^2(r_{\perp})$ is the mean S^2 for the nucleus. Transforming ψ to momentum space, the cross section is

$$\frac{d\sigma}{dx d^2 k_{\perp}} \approx \pi R^2 |\psi(x, r_{\perp}=0)|^2 \frac{\tilde{S}_A^2(k_{\perp})}{(2\pi)^2} \quad (8)$$

$$\text{where } \tilde{S}_A(k_{\perp}) = \int e^{ik_{\perp} \cdot r_{\perp}} S_A(r_{\perp}) d^2 r_{\perp}.$$

We now estimate $S_A(r_{\perp})$. In the Low-Nussinov model the S-matrix for scattering on a single nucleon has the following limit as $r_{\perp} \rightarrow 0$,

$$S_{\pi N}(b, r_{\perp}) \approx 1 - \mu(b) r_{\perp}^2 \quad (9)$$

The expression for $\mu(b)$ is complicated and, in any case, its exact normalization can only be determined by reference to an observed cross section. We use the total pi-nucleon cross section to estimate the integrated $\mu(b)$,

$$\sigma_{\pi N}^T = 2 \int d^2 b (1 - \text{Re} S) = 2 \langle r_{\perp}^2 \rangle_{\pi} \int d^2 b \mu(b) \quad (10)$$

We can then treat the scattering from nuclei with an eikonalized pi-nucleon scattering operator,

$$\begin{aligned}
S_A &\approx \exp \left[- \int dz d^2b' u(b-b') \rho(b') r_{\perp}^2 \right] \\
&\approx \exp \left[- \frac{r_{\perp}^2}{2 \langle r_{\perp}^2 \rangle_{\pi}} \int dz \sigma_{\pi N}^T \rho_0 \right] .
\end{aligned} \tag{11}$$

We evaluate this expression using the vector dominance model of the pion radius, $\langle r_{\perp}^2 \rangle = 4/m_{\rho}^2$. With the further estimates of nuclear target parameters $\rho_0 = 0.16/\text{fm}^3$, $\int dz \rho = \rho_0 4/3 R$, and pi-nucleon total cross section $\sigma_{\pi N}^T \approx 25\text{mb}$, we obtain an estimate

$$S_A(b, r_{\perp}) \approx \exp \left[- \frac{r_{\perp}^2 A^{1/3}}{20} \text{GeV}^2 \right] , \tag{12}$$

independent of $b < R$. Combining this with eqs. (4) and (8), this yields a cross section in the momentum representation of

$$\begin{aligned}
\frac{d\sigma_A}{d^2k_{\perp}} &= \pi R^2 \int dx |\psi(x, 0)|^2 \frac{1}{(2\pi)^2} \left(\frac{20\pi}{A^{1/3} \text{GeV}^2} \right)^2 \exp \left[- \frac{10k_{\perp}^2}{A^{1/3} \text{GeV}^2} \right] \\
&\approx 50\text{mb GeV}^{-2} \exp \left[-10k_{\perp}^2/A^{1/3} \text{GeV}^2 \right] .
\end{aligned} \tag{13}$$

For the largest nuclei, $A^{1/3} \approx 6$, and the k_{\perp} of the q and \bar{q} is quite large, $\langle k_{\perp}^2 \rangle \approx 0.6 \text{GeV}^2$. One expects that this large $\langle k_{\perp} \rangle$ will be reflected in a p_{\perp} spectrum for the hadrons which materialize that is considerably harder than the typical spectrum for

diffractive hadron-nucleon collisions, $\langle p^2 \rangle \approx 0.1 \text{ GeV}^2$. The integrated diffractive excitation cross section from eq. (13) is

$$\sigma_A \approx 16 \text{ mb } A^{1/3} \quad (14)$$

which should be compared with the diffractive cross section due to single πN collisions on the periphery of the nucleus. The latter may be estimated in terms of the exponential tail of the nuclear density distribution, $\rho(r) \sim e^{-r/a}$, with $a \sim 0.55 \text{ fm}$. The single-collision cross section, $\sigma_{\pi A}^{(1)}$ is roughly given by

$$\sigma_{\pi A}^{(1)} \approx 2\pi R a \quad (15)$$

and the diffractive component is

$$\sigma_{\pi A}^{\text{diff}} \approx 2\pi R a \frac{\sigma_{\pi N}^{\text{diff}}}{\sigma_{\pi N}^{\text{T}}} \quad (16)$$

Taking $\sigma_{\pi N}^{\text{diff}} \approx 3 \text{ mb}$, we find

$$\sigma_{\pi A}^{\text{diff}} \approx 5 \text{ mb } A^{1/3} \quad (17)$$

considerably smaller than the transmitted component. Thus, if the absorptive interaction is color sensitive, the diffractive cross section on nuclear targets should be dominated by the overall transparency of the nucleus to small r_{\perp} components of the pion wavefunction. A conventional geometrical picture of hadron-hadron scattering, in which the absorption is

proportional to the hadronic density overlap, would not predict this dramatic effect.

The masses typical of the diffractive excitation are determined by the relation

$$M^2 = \frac{k_{\perp}^2}{x(1-x)} \geq 4k_{\perp}^2 \quad (18)$$

With $A^{1/3} \sim 6$, $\langle k_{\perp}^2 \rangle \approx 0.6 \text{ GeV}^2$, and we have

$$M^2 \sim 3 \text{ GeV}^2 \quad (19)$$

The diffractive condition, that the nucleus remains unexcited, requires that the momentum transfer to the nucleus be smaller than the inverse nuclear radius. In terms of the mass of the diffractive excitation this condition is⁹ $M^2 R/p_{\text{lab}} < 1$, which is easily satisfied with present high energy accelerators.

Because substantial masses can be coherently produced by diffractive excitation, it is clear that the r_{\perp} filtering may play an important role in the production of diffractive states containing charm. It has been suggested¹⁰ that the proton contains heavy quarks in some of its intrinsic Fock states. If these states were characterized by small transverse dimension¹⁰, then they would be able to pass through the nucleus without interaction, and eq. (3) could be used as a guide to the magnitude of the expected cross section.

Integrating eq. (3) over all ξ for such states we then obtain

$$\sigma_{\text{charm}}^{\text{diff}} = P_c \sigma_{e1}, \quad (21)$$

where P_c is the probability of an intrinsic charm Fock state in the projectile hadron. It has been estimated¹¹ that for nucleons $P_c \approx 2\%$; with $\sigma_{e1} \approx \pi R^2 \approx 50 A^{2/3} \text{mb}$ we obtain

$$\sigma_{\text{charm}}^{\text{diff}} \approx 1 \text{mb} A^{2/3} \quad (22)$$

for nuclear targets. In collisions with a single nucleon, diffractive charm production may also be significant though we see no reason for it to be dominant as in the case of a nuclear target. Naively employing eq. 21 for charm production on a nucleon target yields $\sigma_{\text{charm}}^{\text{diff}} \sim 0.1 \text{mb}$, i.e. of the same order as the cross section observed at the ISR¹². Note that the $A^{2/3}$ (not $A!$) dependence predicted for the diffractive production helps resolve the discrepancy between FNAL¹³ and ISR measurements.

While the nuclear target filtering creates diffractive production of large M^2 final states, it is clear that the softness of the filter places an effective cutoff on the masses. The cross section for asymptotically large M^2 and k_{\perp}^2 can however be estimated by gluon exchanges between target and projectile. For diffraction, the Low-Nussinov model

requires two-gluon exchange between the projectile and each struck nucleon. The predicted high k_{\perp}^2 behavior (modulo logarithms) of $d\sigma/dk_{\perp}^2$ is $1/k_{\perp}^6$ for pion beams, $1/k_{\perp}^8$ for proton beams, and $1/k_{\perp}^4$ for photon beams.¹⁴ Each is a single power of k_{\perp}^2 more suppressed than the high- k_{\perp}^2 tail of the intrinsic projectile wavefunction because of color cancellations. In the regime where the jet structure is visible, the pion-induced jets will have an angular distribution in the jet c.m. frame of $d\sigma/dM^2 d\cos\theta \sim \sin^2\theta$, reflecting the x dependence of eq. 4.

In conclusion, we emphasize that all our results rely crucially upon the assumption that the absorptive amplitude is color sensitive, as in the Low-Nussinov model, so that Fock states of small intrinsic size interact weakly. The diffractive excitation spectrum then provides an unusual measure of the short-range constituent structure of hadrons.

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