SLAC-PUB-2746
May 1981
(T/E)

HELICITY SELECTION RULES AND TESTS OF GLUON SPIN IN EXCLUSIVE QCD PROCESSES*

Stanley J. Brodsky
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305
and
G. Peter Lepage†
Laboratory of Nuclear Studies
Cornell University, Ithaca, New York 14853

ABSTRACT

We show how the helicity and angular dependence of large-momentum-transfer exclusive processes can be used to test the gluon spin and other basic elements of perturbative QCD. Unlike inclusive reactions, these processes isolate QCD hard-scattering subprocesses in situations where the helicity of all the interacting quarks are controlled. The predictions can be summarized in terms of a general spin selection rule which states that the total hadron helicity is conserved $\left(\sum_{\text{initial}} \lambda_H = \sum_{\text{final}} \lambda_H\right)$ up to corrections falling as an inverse power in the momentum transfer. In particular, the hadrons in $e^+e^- \to \gamma^* \to h_A + h_{\bar{B}}$ are produced at large Q^2 with opposite helicity $\lambda_A + \lambda_B = 0$, and $|\lambda_i| \leq 1/2$. This also implies $d\sigma/d\cos\theta \propto (1+\cos^2\theta)$ for all baryon pairs and $d\sigma/d\cos\theta \propto \sin^2\theta$ for all meson pairs, to leading order in $1/Q$. Applications to many processes are given, including electroweak form factors, two-photon processes, hadron-hadron scattering and heavy quark decays (e.g., $\psi \to p\bar{p}$).

Submitted to Physical Review D

---

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.
† Work supported by the National Science Foundation.

I. INTRODUCTION

Among the most critical tests of any dynamical theory of hadronic phenomena is the correct description of spin effects. In this paper we will focus on the phenomenological predictions of perturbative quantum chromodynamics for large-momentum-transfer exclusive reactions,[1,2] with special emphasis on tests of gluon spin and the helicity structure of the theory. For the most part we will restrict ourselves to results which are valid to all orders in QCD perturbation theory.

The predictions for large-momentum-transfer exclusive reactions are based on a factorization theorem[1] which separates the dynamics of hard scattering quark and gluon amplitudes T_H from process-independent distribution amplitudes $\phi_H(x,Q)$ evolved to a large momentum transfer scale Q. As we shall discuss here, exclusive processes have the potential for isolating the QCD hard-scattering processes in situations where the helicities of all the interacting constituents are controlled.

Let us briefly review the essential points for the calculation of hadronic amplitudes in QCD.[1] Hadronic bound states can be conveniently described in terms of Fock state wave functions $\psi_H^{(n)}(x_i,k_{\perp i},s_i)$, $i=1\ldots n$ defined at equal time $\tau = t+z$ on the light cone. The wave functions are functions of the light-cone longitudinal momentum fractions $x_i = (k^0+k^3)/(p^0+p^3)$, $\sum_{i=1}^{n} x_i=1$, and of the transverse momenta $\sum_{i=1}^{n} k_\perp^{(i)}=0$. Away from possible special points in the x_i integrations (see below), a general hadronic amplitude $\mathcal{M}_{AB\to CD}(Q^2,\theta_{c.m.})$ can be written as a convolution over the x_i of a connected hard-scattering amplitude $T_H(x_i,s_i\,;Q^2,\theta_{c.m.})$ with the valence quark distribution amplitudes:

$$\phi_M(x,\tilde{Q}) = d_F^{-1}(\tilde{Q}) \int^{k_\perp^2 < \tilde{Q}^2} d^2k_\perp \; \psi_{q\bar{q}}(x,k_\perp) \tag{1}$$

and

$$\phi_B(x_i,\tilde{Q}) = d_F^{-3/2}(\tilde{Q}) \int^{k_{\perp i}^2 < Q^2} \left[d^2k_\perp\right] \psi_{qqq}(x_i,k_{\perp i}) \;, \; i=1,2,3 \tag{2}$$

for flavor nonsinglet mesons and baryons, respectively. The pion form factor, for example, is given by[1,2]

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy \; \phi_\pi^*(y,\tilde{Q}_y) \; T_H(x,y;Q^2) \; \phi_\pi(x,\tilde{Q}_x) \tag{3}$$

where $\tilde{Q}_x = \min(x,1-x)Q$.

In T_H each hadron is replaced by massless, collinear valence partons, each carrying some fraction of the hadron's momentum. Thus T_H is the scattering amplitude for the constituents. The distribution amplitude $\phi_\pi(x,Q)$, for example, is the amplitude for finding a quark and antiquark in a pion carrying momentum fractions x and 1-x, respectively, and collinear up to the scale Q. The distribution amplitudes are weakly (logarithmically) Q-dependent due to QCD scaling violation. The detailed dependence[1] can be derived via evolution equations or the operator product expansion at short distances.

The essential behavior of an exclusive amplitude at large Q^2 is determined by T_H. For most x_i, all internal quark and gluon legs are far off-shell $\left[p_j^2 \sim \tilde{Q}^2\right.$, where $\tilde{Q}^2$ is a linear function of Q^2 and the $\left. x_i\right]$ in the lowest-order tree graphs for T_H. This is essential if contributions with $k_\perp^2 \ll Q^2$ are to factorize, and thereby be absorbed into the distribution amplitudes. In higher orders T_H is defined to be "collinear irreducible"; i.e., the transverse momentum integrations are restricted to $k_\perp^2 > \tilde{Q}^2$ since the region $k_\perp^2 < \tilde{Q}^2$ is already included in ϕ. In general there can be endpoint regions of integration ($x_i \to 0$) and/or pinch (Landshoff) singularities[3] at particular values of x_i for which intermediate propagators in the connected quark scattering amplitude approach the mass shell, and factorization is jeopardized. In the case of the meson form factors, and amplitudes such as $\gamma\gamma \to M\bar{M}$,[4] $\gamma^* + \gamma \to M$,[1] and $e^+e^- \to M_1 \ldots M_N$ at fixed angle,[5] these regions of integration lead to power-law suppressed contributions, even at the tree level. We then can obtain rigorous predictions for these large momentum transfer processes; in particular T_H has a consistent perturbative expansion in $\alpha_s(Q^2)$.

For baryon form factors,[1,6] it is easily seen that any anomalous contribution from the endpoint region $x_1 \sim 1$, $x_2, x_3 \sim O(m/Q)$ is strongly suppressed by the Sudakov form factor which arises from the loop corrections to the near on-shell, high Q^2, $\bar{q}\gamma q$ vertex. The leading contribution to the baryon form factor thus comes from the hard-scattering region.

In the case of hadron-hadron scattering amplitudes, some contributions to T_H have pinch singularities at finite values of the x_i — corresponding to multiple quark-quark scattering at large momentum transfer with nearly on-shell intermediate states. However, these regions of integration are again suppressed by Sudakov form factors at the $q\bar{q}g$ vertices, and the hard-scattering region completely dominates the pinch contributions.[7] In fact, as shown by Mueller,[8] the leading contribution from these diagrams for meson-meson scattering arises from the region $|k_\perp^2| \sim \mathcal{O}(Q^2)^{1-\varepsilon}$ where $\varepsilon = (2c+1)^{-1}$, $c = 8C_F/(11 - 2/3\, n_f)$. [For four flavors, $\varepsilon \cong 0.281$.]

In an Abelian theory where the Sudakov suppression is stronger, $|k_i^2| \sim \mathcal{O}(Q^2)$. Thus for meson-meson scattering at large momentum transfer we have

$$\mathcal{M}_{AB\to CD} = \int [dx_i]\, \phi_C^*(x_c,s_c,\tilde{Q})\, \phi_D(x_d,s_d,\tilde{Q})\, T_H(x_i,s_i,Q^2,\theta_{c.m.}) \times \phi_A(x_a,s_a,\tilde{Q})\, \phi_B(x_b,s_b,\tilde{Q}) \quad . \tag{4}$$

The hard scattering amplitude T_H includes the Sudakov form factors which control and eliminate the pinch region. The effective value of $\tilde{Q}$ varies with the x_i phase-space integration. The leading power computed by Mueller for Eq. (4) is

$$\mathcal{M}_{\pi\pi\to\pi\pi} \sim \left(Q^2\right)^{-3/2 - c\,\ln(2c+1)/2c} \cong \left(Q^2\right)^{-1.922} \tag{5}$$

compared to $(Q^2)^{-2}$ from dimensional counting.

Although detailed results for hadron-hadron scattering have not been completely worked out, we can abstract from QCD some general features of QCD common to all exclusive processes at large momentum transfer:

(1) All of the nonperturbative bound-state physics in the scattering amplitude is isolated in the process-independent distribution amplitudes. This is an essential feature of QCD factorization.

(2) Since the distribution amplitude ϕ is the (orbital angular momentum projection) $L_z = 0$ of the hadron wave function, the sum of the interacting constituents' spin along the hadron's momentum equals the hadron spin:

$$\sum_{i\in H} S_i^z = S_H^z \quad . \tag{6}$$

In contrast, there are any number of noninteracting spectator constituents in inclusive structure functions and reactions and the spin of the active quarks or gluons is only statistically related to the hadron spin (except at the edge of phase space $x \to 1$).

(3) Since all loop integrations in T_H are of order $\tilde{Q}$ the quark and hadron masses can be neglected at large Q^2 up to corrections of order $\sim m/\tilde{Q}$. The vector gluon coupling conserves quark helicity when all masses are neglected — i.e., $\bar{u}_{\downarrow} \gamma^{\mu} u_{\uparrow} = 0$. Thus total quark helicity is conserved in T_H. In addition because of (2), the hadron's helicity is the sum of the helicities of its valence quarks in T_H. We thus have the selection rule

$$\sum_{\text{initial}} \lambda_H - \sum_{\text{final}} \lambda_H = 0 \quad , \tag{7}$$

i.e., total hadronic helicity is conserved up to corrections of order m/Q or higher. Only flavor-singlet mesons in the 0^{-+} nonet can have a two-gluon valence component and thus even for these states the quark helicity equals the hadronic helicity (equals zero). Consequently hadronic helicity conservation applies for all amplitudes involving light meson and baryons.[9] Exclusive reactions which involve hadrons with quarks or gluons in higher orbital angular states are suppressed by powers.

(4) The nominal power-law behavior of an exclusive amplitude at fixed $\theta_{c.m.}$ is $(1/Q)^{n-4}$ where n is the number of external elementary particles (quarks, gluons, leptons, photons,...) in T_H. This dimensional counting rule[10] is modified by the Q^2-dependence of the factors of $\alpha_s(Q^2)$ in T_H, by the Q^2-evolution of the distribution amplitudes, and possibly by a small power correction associated with the Sudakov suppression of pinch singularities in hadron-hadron scattering.

The dimensional counting rules for the power-law fall-off appear to be experimentally well established for a wide variety of processes.[1] In this paper we will emphasize tests of the helicity conservation rule. This rule is one of the most characteristic features of QCD, being a direct consequence of the gluon's spin. A scalar or tensor gluon-quark coupling flips the quark's helicity. Thus, for such theories, helicity may or may not be conserved in any given diagram contributing to T_H, depending upon the number of interactions involved. Only for a vector theory, like QCD, can we have a helicity selection rule valid to all orders in perturbation theory.

In Sec. II, we discuss QCD predictions for hadronic form factors as measured using e^+e^- colliding beams. The power-law dependence on s, relative normalizations, and especially the angular distributions can be analyzed. Similar predictions apply to the two-body decays of the $\psi, \psi', \ldots$ etc. when they are produced by e^+e^- annihilations. These are discussed in Sec. III. There already exists evidence supporting hadronic helicity conservation, coming from the decays $\psi \to p\bar{p}$, $n\bar{n}$, $\Sigma\bar{\Sigma}$, ... This is one of the most persuasive demonstrations of the vector nature of the gluon. A detailed leading order analysis of $\psi \to B\bar{B}$ is given in the Appendix. In Sec. IV we review other tests of the helicity rule employing data for the electroweak form factors of baryons, $\gamma\gamma \to \rho\rho$, hadronic scattering amplitudes and so on. Finally in Sec. V, we review the general implications of dimensional counting and of helicity conservation — i.e., what we learn about the strong interaction.

II. $e^+e^- \to h_A\bar{h}_B$

The study of timelike hadronic form factors using e^+e^- colliding beams can provide very sensitive tests of the helicity selection rule. This follows because the virtual photon in $e^+e^- \to \gamma^* \to h_A\bar{h}_B$ always has spin ±1 along the beam axis at high energies.[11] Angular momentum conservation implies that the virtual photon can "decay" with one of only two possible angular distributions in the center of momentum frame: $(1+\cos^2\theta)$ for $|\lambda_A - \lambda_B| = 1$, and $\sin^2\theta$ for $|\lambda_A - \lambda_B| = 0$ where $\lambda_{A,B}$ are the helicities of hadron $h_{A,B}$. Hadronic helicity conservation, Eq. (7), as required by QCD greatly restricts the possibilities. It implies that $\lambda_A + \lambda_B = 0$ (since the photon carries no "quark helicity"), or equivalently that $\lambda_A - \lambda_B = 2\lambda_A = -2\lambda_B$. Consequently, angular momentum conservation requires $|\lambda_A| = |\lambda_B| = 1/2$ for baryons, and $|\lambda_A| = |\lambda_B| = 0$ for mesons; furthermore, the angular distributions are now completely determined:

$$\begin{aligned} &\frac{d\sigma}{d\cos\theta}\left(e^+e^- \to B\bar{B}\right) \propto 1+\cos^2\theta \qquad \text{(Baryons)} \\ &\frac{d\sigma}{d\cos\theta}\left(e^+e^- \to M\bar{M}\right) \propto \sin^2\theta \qquad \text{(mesons)} \end{aligned} \tag{8}$$

We emphasize that these predictions are far from trivial for vector mesons and for all baryons. For example, one expects distributions like $1+\alpha\cos^2\theta$, $-1 < \alpha < 1$, in theories with a scalar or tensor gluon. So simply verifying these angular distributions [Eq. (8)] would give strong evidence in favor of a vector gluon.

The power-law dependence on s of these cross sections is also predicted in QCD, using the dimensional counting rule. Such "all orders" predictions for QCD allowed processes are summarized in Table I.[12] Processes suppressed in QCD are also listed there; these all violate hadronic helicity conservation, and are suppressed by powers of m^2/s in QCD. This would not necessarily be the case in scalar or tensor theories.

Notice that $e^+e^- \to \pi\rho$, $\pi\omega$, KK^*,... are all suppressed in QCD. This occurs because the γ-π-ρ can couple through only a single form factor — $\epsilon^{\mu\nu\tau\sigma}\, \varepsilon_\mu^{(\gamma)}\, \varepsilon_\nu^{(\rho)}\, p_\tau^{(\pi)}\, p_\sigma^{(\rho)}\, F_{\pi\rho}(s)$ — and this requires $|\lambda_\rho| = 1$ in e^+e^- collisions. Hadronic helicity conservation requires $\lambda = 0$ for mesons, and thus these amplitudes are suppressed in QCD (though, again, not in scalar or tensor theories). Notice however that the processes $e^+e^- \to \gamma\pi$, $\gamma\eta$, $\gamma\eta'$ are allowed by the helicity selection rule; helicity conservation applies only to the hadrons. Unfortunately the form factors governing these last processes are not expected to be large $\left[F_{\pi\gamma}(s) \sim 2f_\pi/s\right]$.

These form factors can also tell us about the quark distribution amplitudes ϕ. For example, sum rules require (to all orders in α_s) that $\pi^+\pi^-$, K^+K^-, and $\rho^+\rho^-$ (helicity zero) pairs are produced in the ratio of $f_\pi^4 : f_K^4 : 4f_\rho^4 \sim 1{:}2{:}7$ respectively, if the π, K, and ρ distribution amplitudes are similar shape.[1] These ratios must apply at very large energies, where all distribution amplitudes tend to $\phi \propto x(1-x)$. On the other hand, the kaon's distribution amplitude may be quite asymmetric about $x = 1/2$ at low energies due to the large difference between s and u,d quark masses. This could enhance K^+K^- production. [Distribution amplitudes for π's and ρ's must be symmetric due to isospin.] The process $e^+e^- \to K_L K_S$ is only possible if the kaon distribution amplitude is asymmetric;[13] the presence or absence of $K_L K_S$ pairs relative to K^+K^- pairs is thus a sensitive indicator of asymmetry in the wave function.

III. $\psi \to h_A \bar{h}_B$

The exclusive decays of heavy quark atoms ($\psi, \psi', \ldots$) into light hadrons can also be analyzed in QCD.[14] The decay $\psi \to p\bar{p}$ for example proceeds via diagrams such as those in Fig. 2 (see the Appendix). Since ψ's produced in e^+e^- collisions must also have spin ±1 along the beam direction and since they can only couple to light quarks via gluons, all the properties listed in Table I apply to ψ, ψ', T, T',... decays as well. Already there is considerable experimental data for the ψ and ψ' decays.[15,16]

Perhaps the most significant are the decays $\psi, \psi' \to p\bar{p}$, $n\bar{n}, \ldots$. The predicted angular distribution $1 + \cos^2\theta$ is consistent with published data.[16] This is important evidence favoring a vector gluon since scalar or tensor gluon theories would predict a distribution of $\sin^2\theta + \mathcal{O}(\alpha_s)$. Dimensional counting rules can be checked by comparing the ψ and ψ' rates into $p\bar{p}$, normalized by the total rates into light-quark hadrons so as to remove dependence upon the heavy-quark wave functions. Theory predicts

$$\frac{BR(\psi \to p\bar{p})}{BR(\psi' \to p\bar{p})} \sim \left(\frac{M_{\psi'}}{M_\psi}\right)^8 \tag{9}$$

where

$$BR(\psi \to p\bar{p}) \equiv \frac{\Gamma(\psi \to p\bar{p})}{\Gamma(\psi \to \text{light-quark hadrons})} \quad . \tag{10}$$

Existing data suggests a ratio $(M_{\psi'}/M_\psi)^n$ with $n = 6 \pm 3$, in good agreement with QCD. Finally we can use the data for $\psi \to p\bar{p}$, $\Lambda\bar{\Lambda}$, $\Xi\bar{\Xi}, \ldots$ to estimate the relative magnitudes of the quark distribution amplitudes for baryons. Correcting for phase space, we obtain $\phi_p \sim 1.04(13)\phi_n \sim 0.82(5)\phi_\Xi \sim$

$1.08(8)\phi'_\Sigma \sim 1.14(5)\phi_\Lambda$ by assuming similar functional dependences on the quark momentum fractions x_i for each case.[17]

Another class of interesting decays includes $\psi, \psi' \to \pi\rho$, $KK^*, \ldots$. These are suppressed in QCD because again they violate hadronic helicity conservation. Thus we expect

$$\frac{BR(\psi \to \pi\rho)}{BR(\psi' \to \pi\rho)} \sim \left(\frac{M_{\psi'}}{M_\psi}\right)^n \tag{11}$$

with $n \geq 6$ in QCD, while $n = 4$ is possible in scalar or tensor theories. In fact, existing data shows that $n \gtrsim 10$. While this is consistent with expectations from QCD, the degree of suppression is surprisingly strong. It is also curious that the $\pi\rho$ and KK^* rates are roughly comparable, given that helicity flips are usually associated with factors of the quark mass.[18]

As is well known, the decay $\psi \to \pi^+\pi^-$ must be electromagnetic if G-parity is conserved by the strong interactions. This decay normally would proceed through diagrams such as those in Fig. 3. However these diagrams cancel in pairs (see Fig. 3) if the quark distribution amplitudes are symmetric about $x = 1/2$, which is the case if isospin is a good symmetry. To leading order in α_s then, one expects the decay through a virtual photon (i.e., $\psi \to \gamma^* \to \pi^+\pi^-$), and the rate is determined by the pion's electromagnetic form factor:

$$\frac{\Gamma\left(\psi \to \pi^+\pi^-\right)}{\Gamma\left(\psi \to \mu^+\mu^-\right)} = \frac{1}{4}\left(F_\pi(s)\right)^2\left[1 + \mathcal{O}\left(\alpha_s(s)\right)\right] \tag{12}$$

where $s = (3.1\ \text{GeV})^2$. Taking $F_\pi(s) \simeq (1 - s/m_\rho^2)^{-1}$ gives a rate $\Gamma(\psi \to \pi^+\pi^-) \sim 0.0011\ \Gamma(\psi \to \mu^+\mu^-)$, which compares well with the measured ratio of 0.0015(7). This indicates that there is indeed little asymmetry in the pion's wave function.

The same analysis applied to $\psi \to K^+K^-$ suggests that the kaon's wave function is similarly symmetric about $x = 1/2$.[19] The ratio $\Gamma(\psi \to K^+K^-)/\Gamma(\psi \to \pi^+\pi^-)$ is 2 ± 1, which agrees with the ratio $(f_K/f_\pi)^4 \sim 2$ expected if π and K have similar quark distribution amplitudes. This conclusion is further supported by measurements of $\psi \to K_L K_S$ which vanishes completely if the K distribution amplitudes are symmetric; experimentally the limit is $\Gamma(\psi \to K_L K_S)/\Gamma(\psi \to K^+K^-) \lesssim 1/2$.

IV. OTHER TESTS OF GLUON SPIN

The gluon's spin can be tested in a wide variety of exclusive processes. Included among these are:

(a) $\gamma\gamma \to \rho\rho$, K^*K^*,.... These cross sections can be measured using e^+e^- colliding beams. At large energies ($s \gtrsim 2$-4 GeV^2) and wide angles, the final state helicities must be equal and opposite. These processes can also be used as a sensitive probe of the structure of the quark distribution amplitudes (see Ref. 4).

(b) Electroweak form factors of baryons. Relations, valid to all order in α_s, can be found among the various electromagnetic and weak interaction form factors of the nucleons and of other baryons (see Ref. 6). These relations depend crucially upon quark-helicity conservation and as such test the vector nature of the gluon. Current data for the axial and electromagnetic form factors of the nucleons is in excellent agreement with these QCD predictions, although a definitive test requires higher energies.

(c) $\pi p \to \pi p$, $pp \to pp, \ldots$. QCD predicts that total hadronic helicity is conserved from the initial state to the final state in all high-energy, wide-angle, elastic and quasi-elastic hadronic amplitudes. One immediate consequence of this is the suppression of the backward peak relative to the forward peak in scalar-meson, baryon scattering. This follows because angular momentum cannot be conserved along the beam axis if only the baryons carry helicity, helicity is conserved, and the baryons scatter through 180°. Data[20] for πp and Kp scattering is consistent with this observation. However the hard scattering amplitudes for these processes must be computed before a detailed interpretation of the data is possible.

In the case of $pp \to pp$ scattering, there are in general five independent parity-conserving and time-reversal-invariant amplitudes $\mathcal{M}(++ \to ++)$, $\mathcal{M}(+- \to +-)$, $\mathcal{M}(-+ \to +-)$, $\mathcal{M}(++ \to +-)$, and $\mathcal{M}(-- \to ++)$. Total hadron helicity conservation implies that $\mathcal{M}(++ \to +-)$ and $\mathcal{M}(-- \to ++)$ are power-law suppressed. The vanishing of the double-flip amplitude implies $A_{NN} = A_{SS}$, and

$$2A_{NN} - A_{LL} = 1 \qquad \left(\theta_{c.m.} = 90^\circ\right) \quad . \tag{13}$$

Here A_{NN} is the spin asymmetry for incident nucleons polarized normal ($\hat{x}$) to the scattering plane. A_{LL} refers to initial spins polarized along the laboratory beam direction ($\hat{z}$) and A_{SS} refers to initial spin polarized (sideways) along y. Preliminary data at $p_{lab} = 11.75$ GeV/c from Argonne[21] appears to be consistent with the prediction (13).

(d)′ Zeros of meson form factors. Asymptotically, the electromagnetic form factors of charged π's, K's, and $\rho(\lambda = 0)$'s are positive in QCD. In a theory of scalar gluons, these form factors become negative for Q^2 large, and thus must vanish at some finite Q^2 since $F(Q^2 = 0) = 1$ by definition. Consequently the absence of zeros in $F_\pi(Q^2)$ is further evidence for a vector gluon.[1]

V. CONCLUSIONS

The experimental verification of the quantitative predictions of perturbative QCD is generally complicated by the presence of large $\mathcal{O}(\alpha_s)$ (and higher) corrections, strong renormalization-scheme dependence, and/or numerous higher twist contributions. For this reason it is worthwhile to examine more general features of the strong interaction, as predicted by QCD — especially those features valid in each order of perturbation theory and not easily obscured by higher twist effects. In this paper we have emphasized the use of large momentum transfer exclusive processes as a means to this end. The two most prominent characteristics of these processes (in QCD) are the approximate power-law dependence of the amplitudes on energy at fixed angles, and the conservation of hadronic helicity. The experimental verification of these 'all orders' features — i.e., the success of dimensional counting, and present evidence for helicity conservation from $\psi \to p\bar{p}$, $\pi\rho$ — already tells us much about the nature of the strong interaction:

(1) The unrenormalized interactions of the short-distance theory are consistent with scale invariance; i.e., the coupling constant is dimensionless. This is necessary for dimensional counting arguments.

(2) The powers of Q^2 in an exclusive cross section at large momentum transfer count the number of constituents. Experiment verifies that the meson has two constituents in its valence (minimal Fock) state while a baryon has three. This of course also implies that the constituents of a baryon have half-integer spin.

(3) If the strong interaction theory is a gauge theory (like QCD) hadrons must be singlet states. Otherwise infrared gluon radiation would result in amplitudes which fall faster than any power. While singlet states are possible in QCD where a meson is $q\bar{q}$ and a baryon is qqq, it is impossible in an SU(4) gauge theory, for example, to make a singlet baryon of three quarks in any simple way.

(4) The variation of the running coupling $\alpha_s(Q^2)$ must be small at current energies, as otherwise explicit powers of α_s in T_H would have resulted in substantial deviations from the dimensional counting predictions. Thus we require the β-function $Q^2\, d/dQ^2\, \alpha_s(Q^2)$ to be small. [Setting $\alpha_s(Q^2) \simeq 4\pi/\beta\, \ell n\, (Q^2/\Lambda^2)$, $\Lambda^2 \lesssim .1\ \mathrm{GeV}^2$ seems necessary.]

(5) The quarks interact via exchange of a vector gluon.

These features are all entirely consistent with perturbative QCD.

Large momentum transfer exclusive processes are particularly well suited to the study of the spin structure of the theory underlying strong interactions. This is true because a hadron's helicity equals the sum of the helicities of its valence partons in all dominant amplitudes. Consequently the helicities of the external hadrons can strongly affect the microscopic subprocesses which determine the gross features of an amplitude. Finally, e^+e^- colliding beams are particularly useful for studying the helicity structure of amplitudes, because the intermediate virtual photon, or heavy quark resonance is always polarized along the beam axis. Consequently, hadronic helicity conservation can be verified simply by measuring the angular distributions of final states.

ACKNOWLEDGEMENTS

We wish to thank Dr. T. Huang for helpful conversations. This work was supported in part by the Department of Energy, contract DE-AC03-76SF00515 and by the National Science Foundation.

APPENDIX

$\psi \to p\bar{p}$

In this Appendix we examine in detail the decay $\psi \to p\bar{p}$, where the ψ, being produced by e^+e^- annihilation, is transversely polarized along the beam direction. Since the angular distribution is $1+\cos^2\theta$ in QCD (see Table I), we need only evaluate the amplitude for $\theta = 0$. The general structure of this amplitude is

$$T(s,\theta=0) = \int_0^1 [dx][dy]\ \phi^*(y_i,s)\ T_H(x_i,y_i,s)\ \phi(x_i,s) \ . \qquad \text{(A-1)}$$

The amplitudes contributing to T_H have the general structure shown in Fig. 4(a). The external quarks are collinear and massless; all collinear mass singularities are absorbed into the protons' distribution amplitudes in the usual fashion. Consequently all loop momenta are hard, i.e., of order $s = (M_\psi)^2 \simeq 9.6\ \mathrm{GeV}^2$, and perturbation theory is viable. To lowest and first order, the decay of the heavy quark state into the intermediate gluons can also be analyzed perturbatively. The diagrams contributing in leading order are shown in Fig. 4(b). The resulting hard scattering amplitude is (for $\psi_\uparrow \to p_\uparrow \bar{p}_\downarrow$ at $\theta = 0$)

$$T_H = -\frac{\left(4\pi\alpha_s(s)\right)^3 32C}{s^{5/2}}\ \varphi_{NR}(0)\ \frac{1}{y_1y_2y_3}$$

$$\times\ \frac{x_1y_3 + x_3y_1}{[x_1(1-y_1)+y_1(1-x_1)][x_3(1-y_3)+y_3(1-x_3)]}\ \frac{1}{x_1x_2x_3} \qquad \text{(A-2)}$$

where $\varphi_{NR}'(0)$ is the heavy-quark nonrelativistic wave function evaluated at the origin, and C is a color factor:

$$C = \frac{(n_c+1)(n_c+2)}{8n_c^2\sqrt{n_c}} = \frac{5}{18\sqrt{3}} \quad . \tag{A-3}$$

We have set the charmed quark mass equal to $M_\psi/2$. This introduces a small error which is largely cancelled when we compute the branching ratio [Eq. (10)]. The total rate into $p\bar{p}$ is therefore

$$\Gamma(\psi \to p\bar{p}) = \frac{|\vec{p}_{CM}|}{6\pi\sqrt{s}} |T(\theta=0)|^2 \quad . \tag{A-4}$$

This can be compared with the rate into all hadrons which to the same order is

$$\Gamma(\psi \to \text{hadrons}) = \frac{160}{81}\,\alpha_s^3(s)\left(\pi^2-9\right)\frac{|\varphi_{NR}(0)|^2}{s} \quad . \tag{A-5}$$

The branching ratio is then:

$$\frac{\Gamma(\psi \to p\bar{p})}{\Gamma(\psi \to \text{hadrons})} = \left(3.2\times 10^6\right)\alpha_s^3(s)\,\frac{|\vec{p}_{CM}|}{\sqrt{s}}\,\frac{\langle T\rangle^2}{s^4} \tag{A-6}$$

where $|\vec{p}_{CM}|/\sqrt{s} \simeq .4$, $s = 9.6\ \text{GeV}^2$, and

$$\langle T\rangle \equiv \int_0^1 [dx][dy]\,\frac{\phi^*(y_i,s)}{y_1y_2y_3}\,\frac{x_1y_3+x_3y_1}{[x_1(1-y_1)+y_1(1-x_1)][x_3(1-y_3)+y_3(1-x_3)]} \times \frac{\phi(x_i,s)}{x_1x_2x_3} \quad . \tag{A-7}$$

Notice that there can be no endpoint singularities in the x_i and y_i integrations; the integrations are finite so long as $\phi(x_i,s) \lesssim Kx_i^{\varepsilon}$ as

$x_i \to 0$ for some $\varepsilon > 0$. For this reason the present analysis is perhaps more reliable than that of the electromagnetic form factor. However, calculations of $\Gamma(\psi \to p\bar{p})$ cannot be carried beyond the first order corrections without a deeper understanding of the heavy-quark wave function. Still, the power-law behavior and hadronic helicity conservation are features valid to all orders and, given the uncertainties involved in analyzing heavy quark wave functions, they remain the most interesting aspects of this and similar decays.

REFERENCES

1. G. P. Lepage and S. J. Brodsky, Phys. Rev. D22, 2157 (1980), and Phys. Lett. 87B, 359 (1979); S. J. Brodsky and G. P. Lepage, SLAC-PUB-2294, published in "Quantum Chromodynamics," Proceedings of the 1978 La Jolla Institute Summer Workshop, Wm. Frazer and F. Henyey (eds.), AIP (1979), Phys. Rev. Lett. 43, 545 (1979), Erratum, ibid. 43, 1625 (1979), and SLAC-133, Proceedings of the SLAC Summer Institute, 1979; S. J. Brodsky, Y. Frishman, G. P. Lepage and C. Sachrajda, Phys. Lett. 91B, 239 (1980). A report on some of the results of this paper has been given in S. J. Brodsky and G. P. Lepage, SLAC-PUB-2656, presented at the 1980 International Symposium on High Energy Physics with Polarized Beams and Polarized Targets, Lausanne, Switzerland, 1980.
2. Meson form factors are also discussed in G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. 43, 246 (1979); V. L. Chernyak and A. R. Vhitnishii, JETP Lett. 25, 11 (1977); A. V. Efremov and A. V. Radyushkin, Dubna preprints JINR-E2-11535, 11983 and 12384, Riv. Nuovo Cimento 3, 1 (1980), and Phys. Lett. 94B, 245 (1980); A. Duncan and A. H. Mueller, Phys. Rev. D21, 1636 (1980), and Phys. Lett. 90B, 159 (1980); G. Parisi, Phys. Lett. 43, 246 (1979); M. K. Chase, Nucl. Phys. B167, 125 (1980).
3. P. V. Landshoff, Phys. Rev. D10, 1024 (1974); P. Cvitanovic, Phys. Rev. D10, 338 (1974); S. J. Brodsky and G. Farrar, Phys. Rev. D11, 1309 (1975).

4. S. J. Brodsky and G. P. Lepage, SLAC-PUB-2587, presented at the XXth International Conference on High Energy Physics, Madison, Wisconsin, 1980, and SLAC-PUB-2733 (1981).

5. S. Gupta, SLAC-PUB-2692 (1981).

6. S. J. Brodsky, G. P. Lepage and S. A. A. Zaidi, Phys. Rev. D23, 1152 (1981).

7. P. V. Landshoff and D. J. Pritchard, Z. Phys. C6, C9 (1980); G. P. Lepage and S. J. Brodsky, Ref. 1.

8. A. H. Mueller, Columbia preprint CU-TP-192 (1981).

9. Arguments for the conservation of baryon chirality in large momentum transfer processes have been given by B. L. Ioffe, Phys. Lett. 63B, 425 (1976). For some processes this rule leads to different predictions than the QCD results given here.

10. S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. 31, 1153 (1973), and Phys. Rev. D11, 1309 (1975); V. A. Matveev, R. M. Muradyan and A. V. Tavkheldize, Lett. Nuovo Cimento 7, 719 (1973).

11. This follows from helicity conservation as well, which is a well-known property of QED at high energies. The electron and positron must have opposite helicities; i.e., $\lambda_e + \lambda_{\bar{e}} = 0$, since it is the total helicity carried by fermions (alone) which is conserved, and there are no fermions in the intermediate state. In the lab frame ($\vec{p}_e = -\vec{p}_{\bar{e}}$), their spins must be parallel, resulting in a virtual photon with spin ±1 along the beam.

12. See also A. I. Vainshtain and V. I. Zakharov, Phys. Lett. 72B, 368 (1978); G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. 35, 1416 (1975); B. L. Ioffe, Ref. 9.

13. For example, this amplitude vanishes under the (stronger) assumption of exact flavor — SU(3) symmetry. This is easily seen by defining G_U-parity, in analogy to G-parity: $G_U = C\ \exp(i\pi U_2)$ where the U_i are the isospin-like generators of $SU(3)_f$ which connect the K_0 and $\bar{K}_0$. The final state in $e^+e^- \to K_L K_S$ has positive G_U-parity, while the intermediate photon has negative G_U-parity. G_U-parity is conserved if $SU(3)_f$ is exact, and $e^+e^- \to K_L K_S$ then vanishes.

14. For other applications, see A. Duncan and A. H. Mueller, Phys. Lett. 93B, 119 (1980); S. Gupta, SLAC-PUB-2653 (1980).

15. Unless otherwise noted, the data used here is from the compilation of the Particle Data Group, Rev. Mod. Phys. 52, No. 2, Part II (1980).

16. I. Peruzzi et al., Phys. Rev. D17, 2901 (1978).

17. An analysis of the constraints imposed by the $\psi \to p\bar{p}$ decay rate together with the nucleon form factors is being carried out in collaboration with T. Huang. See also S. J. Brodsky, T. Huang and G. P. Lepage, SLAC-PUB-2540, contributed to the 20th International Conference on High Energy Physics, Madison, Wisconsin, 1980; T. Huang, SLAC-PUB-2580, to be published in the Proc. of the 20th International Conference on High Energy Physics, Madison, Wisconsin, 1980.

18. One possibility is that these form factors are dominated by endpoint contributions (see Ref. 1) for which quark masses may be less relevant. Such terms are expected to be strongly suppressed by quickly falling Sudakov form factors. This could also explain the rapid fall-off of the ψ-π-ρ form factor with increasing M_ψ^2.

19. This amplitude vanishes in the limit of exact $SU(3)_f$, since it violates conservation of $G_V = C\exp(i\pi U_2)$ (cf., Ref. 13).

20. D. Sivers, S. J. Brodsky and R. Blankenbecler, Phys. Reports 23C, 1 (1976).

21. A. Yokosawa, contributed to the 1980 International Symposium on High Energy Physics with Polarized Targets, Lausanne, Switzerland, 1980. For a review of the model calculations for the pp spin asymmetries, see S. J. Brodsky and G. P. Lepage, SLAC-PUB-2656 (1980), contributed to the 1980 International Symposium on High Energy Physics with Polarized Targets, Lausanne, Switzerland, 1980; J. Szwed, Jagellonian University preprint TPJU-13/80 (1980), and references therein. A review of the nucleon-nucleon scattering experiments is given by A. Yokosawa, Phys. Reports 64, 47 (1980).

TABLE I

Exclusive channels in e^+e^- annihilation. The $h_A\bar{h}_B\gamma^*$ couplings in allowed processes are $-ie(p_A - p_B)^\mu F(s)$ for mesons, $-ie\bar{v}(p_B)\gamma^\mu G(s)u(p_A)$ for baryons, and $-ie^2\epsilon_{\mu\nu\rho\sigma}p_M^\nu\epsilon^\rho p_\gamma^\sigma F_{M\gamma}(s)$ for meson-photon final states. Similar predictions apply to decays of heavy-quark vector states, like the $\psi,\psi',\ldots$, produced in e^+e^- collisions.

	$e^+e^- \to h_A(\lambda_A)\ \bar{h}_B(\lambda_B)$	Angular Distribution	$\dfrac{\sigma(e^+e^- \to h_A\bar{h}_B)}{\sigma(e^+e^- \to \mu^+\mu^-)}$
Allowed in QCD	$e^+e^- \to \pi^+\pi^-, K^+K^-$	$\sin^2\theta$	$\frac{1}{4}\lvert F(s)\rvert^2 \sim c/s^2$
	$\rho^+(0)\rho^-(0), K^{*+}K^{*-}$	$\sin^2\theta$	$\frac{1}{4}\lvert F(s)\rvert^2 \sim c/s^2$
	$\pi^0\gamma(\pm1), \eta\gamma, \eta'\gamma$	$1 + \cos^2\theta$	$(\pi\alpha/2)s\lvert F_{M\gamma}(s)\rvert^2 \sim c/s$
	$e^+e^- \to p(\pm\frac{1}{2})\bar{p}(\mp\frac{1}{2}), n\bar{n}, \ldots$	$1 + \cos^2\theta$	$\lvert G(s)\rvert^2 \sim c/s^4$
	$p(\pm\frac{1}{2})\bar{\Delta}(\mp\frac{1}{2}), \bar{n}\Delta, \ldots$	$1 + \cos^2\theta$	$\lvert G(s)\rvert^2 \sim c/s^4$
	$\Delta(\pm\frac{1}{2})\bar{\Delta}(\mp\frac{1}{2}), y^*\bar{y}^*, \ldots$	$1 + \cos^2\theta$	$\lvert G(s)\rvert^2 \sim c/s^4$
Suppressed in QCD	$e^+e^- \to \rho^+(0)\rho^-(\pm1), \pi^+\rho^-, K^+K^{*-}, \ldots$	$1 + \cos^2\theta$	$< c/s^3$
	$\rho^+(\pm1)\rho^-(\pm1), \ldots$	$\sin^2\theta$	$< c/s^3$
	$e^+e^- \to p(\pm\frac{1}{2})\bar{p}(\pm\frac{1}{2}), p\bar{\Delta}, \Delta\bar{\Delta}, \ldots$	$\sin^2\theta$	$< c/s^5$
	$p(\pm\frac{1}{2})\bar{\Delta}(\pm\frac{3}{2}), \Delta\bar{\Delta}, \ldots$	$1 + \cos^2\theta$	$< c/s^5$
	$\Delta(\pm\frac{3}{2})\bar{\Delta}(\pm\frac{3}{2}), \ldots$	$\sin^2\theta$	$< c/s^5$

FIGURE CAPTIONS

Fig. 1. Leading contributions to the pion form factor in QCD.

Fig. 2. Quark-gluon subprocesses for $\psi \to B\bar{B}$.

Fig. 3. Cancelling quark-gluon subprocesses for $\psi \to \pi^+\pi^-$.

Fig. 4. (a) QCD analysis of $\psi \to B\bar{B}$.

(b) Helicity-labeled quark-gluon subprocesses for $\psi \to p\bar{p}$.

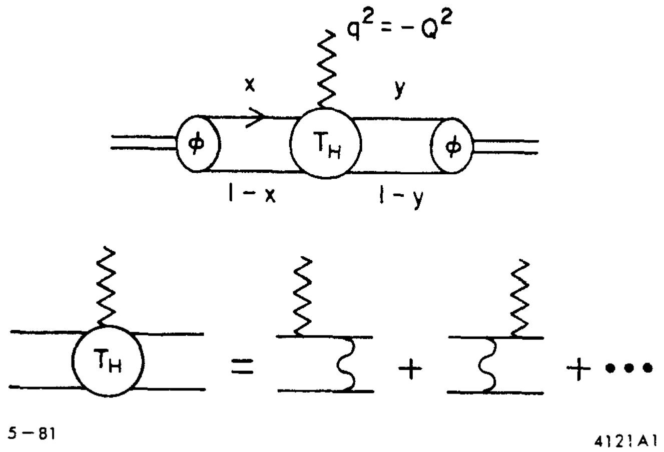

Fig. 1

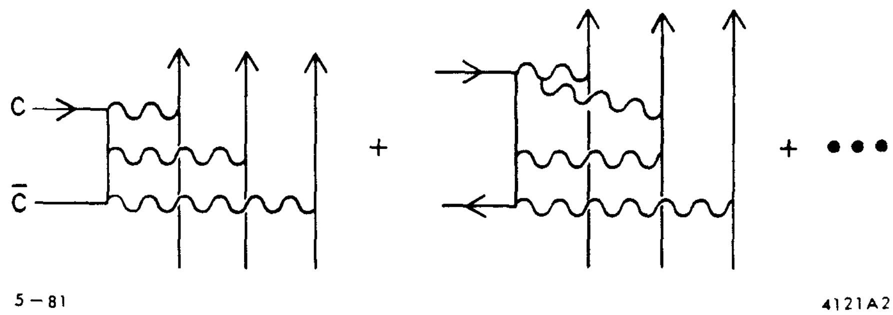

Fig. 2

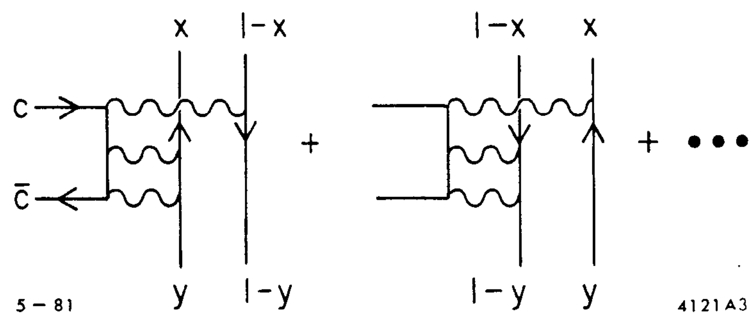

Fig. 3

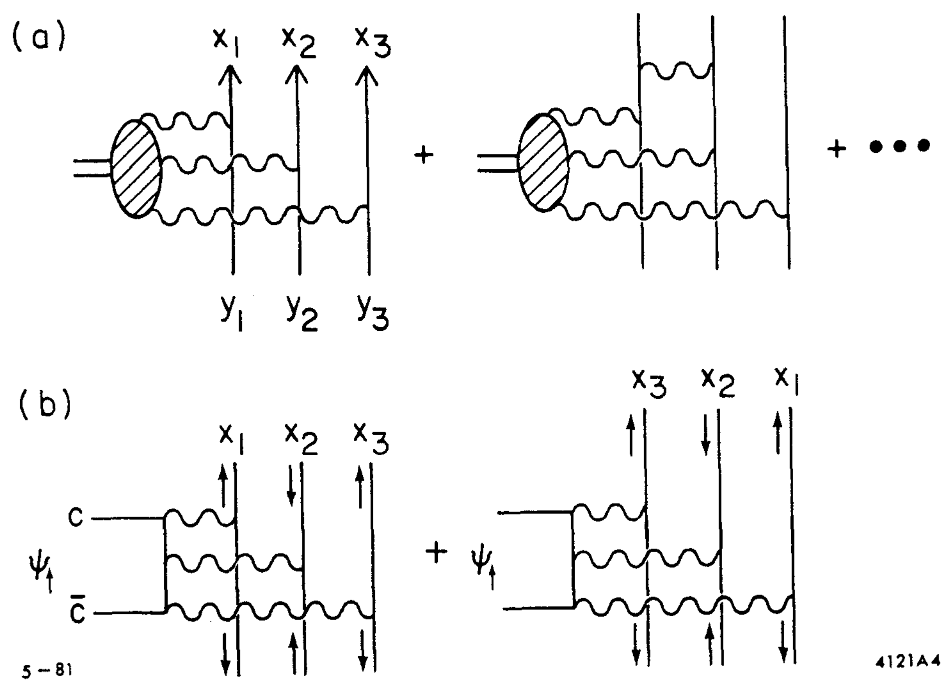

Fig. 4