

Radiative  $Z^0$  Production: A Method for  
Neutrino Counting in  $e^+e^-$  Collisions\*

G. Barbiellini<sup>†</sup>, B. Richter, J. L. Siegrist  
Stanford Linear Accelerator Center  
Stanford University, Stanford, CA. 94305

Abstract

Radiative  $Z^0$  production in  $e^+e^-$  collisions is analyzed as a method of measuring the partial width of the  $Z^0$  for decay into neutrinos and thus determining if there exist low mass neutral leptons beyond the three now known ( $\nu_e, \nu_\mu, \nu_\tau$ ). The conditions for observing the  $\gamma\nu\bar{\nu}$  final state are analyzed and background cross sections are determined. We conclude that the experiment is feasible and requires relatively modest apparatus.

(Submitted to Phys. Lett.)

---

<sup>†</sup> Permanent Address: 1211 Geneva 23, CERN, Geneva, Switzerland

\* Work supported by the Department of Energy, contract DE-AC03-76SF00515

In the past decade a new model of the weak interaction has emerged which accounts for all of the observed weak interaction phenomenon seen at energies presently accessible in the laboratory. All variants of the gauge theory predict that the weak interaction is carried by vector mesons of finite mass. A great effort is underway to find these gauge bosons experimentally, for not only would their discovery provide an essential confirmation of a remarkable theoretical development, but also the study of the decay of these bosons could itself tell us more about the structure of matter and of the weak interactions.

In this note we describe a method for doing one of the most important of these decay experiments -- the determination of the partial decay width of the neutral gauge boson ( $Z^0$ ) into neutrinos (or other low mass weakly interacting particles). This measurement would show if there are neutrinos, beyond the three types already known ( $\nu_e, \nu_\mu, \nu_\tau$ ), having charged partners with masses too large to be produced by the accelerators now available. Besides having important implications in particle physics, the number of low mass neutrinos is an important parameter of models of the development of the early universe.

The method involves  $Z^0$  tagging in the study of the reaction

$$e^+e^- \rightarrow \gamma\nu\bar{\nu} . \quad (1)$$

An electron-positron colliding beam facility is to be operated at a center-of-mass energy above the  $Z^0$  mass. A photon is observed with an energy such that the recoil system has an energy about equal to the  $Z^0$  mass. Auxiliary detectors surround the collision point to separate those reactions where no charged leptons, additional photons, or strongly interacting neutral particles emerge ( $Z^0 \rightarrow \nu\bar{\nu}$ ), from those where such additional particles do emerge.

From these measurements the number of different types of neutrinos that contribute to  $Z^0$  decay can be determined.

The suggestion to determine the number of neutrinos via radiative neutrino production in  $e^+e^-$  annihilation is not new.<sup>(1,2)</sup> Here we examine the possibility from the perspective of the experimenter, comparing this method with the most often proposed alternative method of determining the number of neutrinos -- the measurement of the absolute width of the  $Z^0$ ; develop the notion of  $Z^0$  tagging; examine potential background problems; and comment on apparatus requirements. We conclude that the method is simple, clean and that the experiment can be carried out with apparatus much more modest than required for the giant universal detector so often discussed for use at big colliding beam storage rings.

In all of the weak interaction calculations that follow in this note we shall use the standard Weinberg-Salam model. Figure 1 shows the Feynman diagrams for radiative neutrino

pair production. The upper two diagrams involve  $Z^0$  production while the lower three diagrams involve charge boson exchange. The cross section for the reaction is given by<sup>(2)</sup>

$$\frac{d^2\sigma}{dx dy} = \frac{G_F^2 \alpha s(1-x) [(1-x/2)^2 + x^2 y^2/4]}{6\pi^2 x (1-y^2)} \times \left\{ \frac{N_\nu (g_V^2 + g_A^2) + 2(g_V + g_A) [1 - s(1-x)/m_Z^2]}{[1 - s(1-x)/m_Z^2]^2 + \Gamma_Z^2/m_Z^2} + 2 \right\} \quad (3)$$

where  $G_F$  is the Fermi coupling constant,  $\alpha$  is the fine structure constant,  $s$  is the square of the  $e^+e^-$  center-of-mass energy,  $x$  is the photon energy in units of the beam energy,  $y$  is the cosine of the photon angle with respect to the incident beam direction,  $N_\nu$  is the number of low mass neutrinos,  $m_Z$  is the  $Z^0$  mass,  $\Gamma_Z$  is the total width of the  $Z^0$ , and in the W-S model  $g_V = -1/2 + 2\sin^2\theta_W$  and  $g_A = -1/2$ . In Eq. (3), the  $(g_V^2 + g_A^2)$  term comes from the square of the  $Z^0$  amplitudes, the "2" term comes from the square of the W exchange amplitudes, and the  $(g_V + g_A)$  term comes from the  $Z^0$ -W interference.

Figure 2 shows the cross section integrated over  $y$  vs. photon energy. To be specific, we have taken the  $e^+e^-$  center-of-mass energy to be 105 GeV and  $|y_{\max}| = 0.94$  ( $160^\circ \leq \theta_\gamma \leq 20^\circ$ ). The figure shows the usual soft photon divergence which occurs in radiative processes, and a peak at a photon energy of 14 GeV corresponding to a  $Z^0$  missing mass recoiling against

the photon. The contribution to the cross section of the W exchange term and the interference term are also shown in Fig. 2. They are not important near the  $Z^0$  peak. The choice of  $e^+e^-$  energy and minimum photon angle must be made to reduce background. These background processes are discussed below.

The  $e^+e^- \rightarrow \gamma\nu\bar{\nu}$  cross section integrated over the region  $E = 14 \pm 2.5$  GeV is  $\approx 0.025$  nb. A 30 day run at an average luminosity of  $5 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$  will produce about 300 events with photon energies within  $\pm 2.5$  GeV of the  $Z^0$  peak, more than enough to determine the number of neutrinos to sufficient accuracy, since the cross section increases by  $\approx 0.008$  nb for each additional neutrino.

In contrast to this large effect on the  $\gamma\nu\bar{\nu}$  cross section, adding a new neutrino has a much smaller effect on the total width of the  $Z^0$ .  $\Gamma_Z$  is given in terms of the number of fundamental fermions by

$$\Gamma_Z = \frac{G_F m_Z^3}{24\sqrt{2}\pi} \left\{ \begin{aligned} &2N_\nu + [1 + (1-4\sin^2\theta_W)^2] N_\ell \\ &+ 3 [1 + (1-\frac{8}{3}\sin^2\theta_W)^2] N_{2/3} \\ &+ 3 [1 + (1-\frac{4}{3}\sin^2\theta_W)^2] N_{-1/3} \end{aligned} \right\} \quad (4)$$

where  $G_F$  is the Fermi coupling constant;  $N_\nu$  is the number of low mass neutrinos;  $N_\ell$  is the number of charged leptons with masses less than  $m_Z/2$ ;  $N_{2/3}$  and  $N_{-1/3}$  are, respectively, the

numbers of  $2/3$  and  $-1/3$  charged quarks with masses less than  $m_Z/2$ ; and  $\theta_W$  is the Weinberg angle. Phase-space effects from finite lepton and quark masses have been ignored.

Taking the accepted values of  $\sin^2\theta = 0.23$ , and  $m_Z = 90$  GeV, and assuming 3 generations of fundamental fermions (including the yet to be discovered t quark), the total width of the  $Z^0$  is 2.6 GeV. The existence of one additional neutrino would increase the width by 0.16 GeV. Thus, to establish the number of neutrinos with 100 to 1 odds against there being one more or one less requires a measurement of the total  $Z^0$  width to about 2%. While there will be no problem in achieving the statistical accuracy in any of the experiments suggested for future  $e^+e^-$  colliding beam machines, it is not at all clear that systematic errors in the  $Z^0$  mass determination, higher order effects on the width, and radiative distortion of the line shape will permit a determination of the width to the required accuracy.

The topological characteristic of the  $\gamma\nu\bar{\nu}$  final state is the presence of a single photon of high energy accompanied by no other interacting particle. All of the potential background processes have additional photons or charged particles in the final state and the apparatus must be designed to detect these extra particles and veto the events.

As with all accelerator experiments there will be a region around the beam direction reserved for the accelerator

beams where no detection equipment may be located. However, there is a relation between the minimum transverse momentum of the detected photon in the apparatus and the minimum angle with respect to the beam direction above which at least one other final state particle must appear. For the photon detection limits used in the example above this angle is about  $2.5^\circ$ . This angle increases, and the  $\gamma\nu\bar{\nu}$  cross section decreases as the minimum  $p_T$  increases.

We now analyze the principle backgrounds to the  $\gamma\nu\bar{\nu}$  process and, for the two main backgrounds, give the background cross sections as a function of the minimum detection angle of the apparatus. It will turn out that it is not necessary to cover the entire angular region down to the kinematic minimum angle, but it is necessary to be able to detect particles at angles smaller than the  $20^\circ$  minimum we used in our example for the cross section measurement.

A potentially serious source of background is Beam-Beam Bremsstrahlung (BBB). The total cross section for the reaction  $e^+e^- \rightarrow e^+e^-\gamma$  is about  $10^9$  larger than that for  $\gamma\nu\bar{\nu}$ . However, the BBB photon angular distribution is more strongly peaked in the forward direction than is that of  $\gamma\nu\bar{\nu}$  -- the fraction of BBB events with a photon at an angle to the beam line

$> \theta_{\min}$  is

$$\epsilon(\theta_{\min}) \approx m_e^2 / \theta_{\min}^2 E^2 \quad (5)$$

where  $m_e$  is the electron rest mass and  $E$  is the beam energy. (4)

For  $|y| \geq 0.94$  and  $E_\gamma = 14^{+2.5}$  GeV as used above for the calculation of the  $\gamma\nu\bar{\nu}$  cross section, the BBB cross section is  $2.2 \times 10^{-2}$  nb -- comparable to the  $\gamma\nu\bar{\nu}$  cross section.

We have done a Monte Carlo study of the BBB cross section (5) to determine the effective cross section for a photon as above and for neither of the final electrons to be in the region  $\pi - \theta_{\min} > \theta > \theta_{\min}$ . The results are shown in Fig. 3a. If the BBB background is to be  $< 10\%$  of  $\gamma\nu\bar{\nu}$ ,  $\theta_{\min}$  must be less than  $6^\circ$ .

The cross section of Beam-Gas Bremsstrahlung (BGB) differs from that for BBB by a factor of  $Z$ , the atomic number of the gas atom (not  $Z^2$ , because of the large momentum transfer in our kinematic region) and a logarithmic factor of order unity. The effect of BGB can be estimated from BBB by comparing the "target density" of the beam and the residual gas in the interaction region.

In a colliding beam machine the area density of the beam can be obtained from the beam-beam linear tune shift

$$n_b/A = \Delta\nu E / r_e m_e \beta_y^* \approx 4 \times 10^{17} \Delta\nu / \beta_y^* \text{ cm}^{-2} \quad (6)$$

where  $r_e$  is the classical electron radius and  $\beta_y^*$  is the vertical betatron function at the collision point. The factor  $\Delta\nu / \beta_y^*$  ranges from  $2 \times 10^{-3}$  in the proposed CERN LEP machine to about one in the Stanford Linear Collider.



For the residual gas

$$Zn_g/A = 4 \times 10^7 Z \ell(\text{cm}) m p(10^{-9}) \text{ cm}^{-2} \quad (7)$$

where  $\ell$  is the effective interaction region length,  $m$  is the number of atoms per gas molecule, and  $p$  is the pressure in units of  $10^{-9}$  torr. Typical numbers might be  $\ell = 100$ ,  $Z = 7$ ,  $m = 2$  and  $p = 10$ . In this case, BGB is negligible compared to BBB.

The three photon annihilation cross section ( $e^+e^- \rightarrow \gamma\gamma\gamma$ ) has been calculated by Dicus<sup>(6)</sup> who found three  $\gamma$  production to be an important background to  $\gamma\nu\bar{\nu}$ . His relatively large background arises from the acceptance of too broad a range of photon energies (which contributes little to the signal but much to the noise), and from not taking full advantage of the potential for vetoing events with more than one photon in the detector.

We have also done a Monte Carlo study of this cross section<sup>(5)</sup> requiring one photon of  $14 \pm 2.5$  GeV with  $|\gamma| \leq 0.94$ , and no other photon with  $E > 1$  GeV in the angular range  $\pi - \theta_{\min} > \theta > \theta_{\min}$ . The results are shown in Fig. 3b. The  $3\gamma$  background falls below 1% of the  $\gamma\nu\bar{\nu}$  signal for  $\theta_{\min} < 9^\circ$ .

The so-called two photon process,  $e^+e^- \rightarrow e^+e^-x$  where  $x$  is any leptonic or hadronic final state, has a large total cross section, but a small cross section for the production of particles with large transverse momentum with respect to

the beam direction. Brodsky<sup>(7)</sup> has given the cross section for the production of hadronic jets or leptons where the jet or lepton transverse momentum is greater than some minimum  $p_T$  as

$$\sigma_{2\gamma}(p_T > p_{T_{\min}}) = \frac{8\alpha^4 R_{\gamma\gamma}}{3\pi p_{T_{\min}}^2} \ln^2\left(\frac{s}{m_e^2}\right) \left(\ln\frac{s}{p_{T_{\min}}} - \frac{19}{6}\right) \quad (8)$$

where  $R_{\gamma\gamma}$  is the sum of the fourth powers of the quark and lepton charges. For  $p_{T_{\min}} = 4$  GeV which is the minimum  $p_T$  of the  $\gamma\nu\bar{\nu}$  photons detected in our example,  $\sigma_{2\gamma}$  is  $2.7 \times 10^{-2}$  nb -- comparable to the  $\gamma\nu\bar{\nu}$  cross section. Since the  $2\gamma$  process has many charged particles and photon, or leptons, in the final state, they are easy to veto and should give no background problems.

We have investigated the effect of the choice of other center-of-mass energies and minimum detection angles. Qualitatively, as the minimum  $p_T$  of the detected photon decreases, both the signal and the backgrounds increase. The BBB process remains the dominant background, and the minimum detection angle for the BBB final state electrons must decrease to maintain a good signal-to-noise ratio. For example, at  $\sqrt{s} = 97$  GeV,  $E_\gamma = 7 \pm 2.5$  GeV, and  $|y_{\min}| = 0.94$ , the cross section for  $\gamma\nu\bar{\nu}$  is  $5.9 \times 10^{-2}$  nb and the minimum angle,  $\theta_{\min}$ , must be reduced to about  $2^\circ$  to keep the background small.

The apparatus required for this limited experiment is, not surprisingly, modest when compared to the large universal detectors being designed for the next generation of large  $e^+e^-$  machines.<sup>(8)</sup> The requirements for the  $\gamma\nu\bar{\nu}$  experiment are a photon detector and charged particle tracking system which covers the sphere to within a few degrees of the incident beam direction. The photon detectors need sufficient segmentation to measure the photon angular distribution, and an energy resolution of about 5% at 10 GeV to allow the observation of the  $Z^0$  peak in the spectrum. Large volume magnetic fields or calorimeters are not required. Backgrounds are not large and are easy to measure.

We conclude the  $Z^0$  tagging and the determination of the  $\gamma\nu\bar{\nu}$  cross section in the  $e^+e^-$  annihilation is an eminently practical method of determining the number of low mass neutrinos.

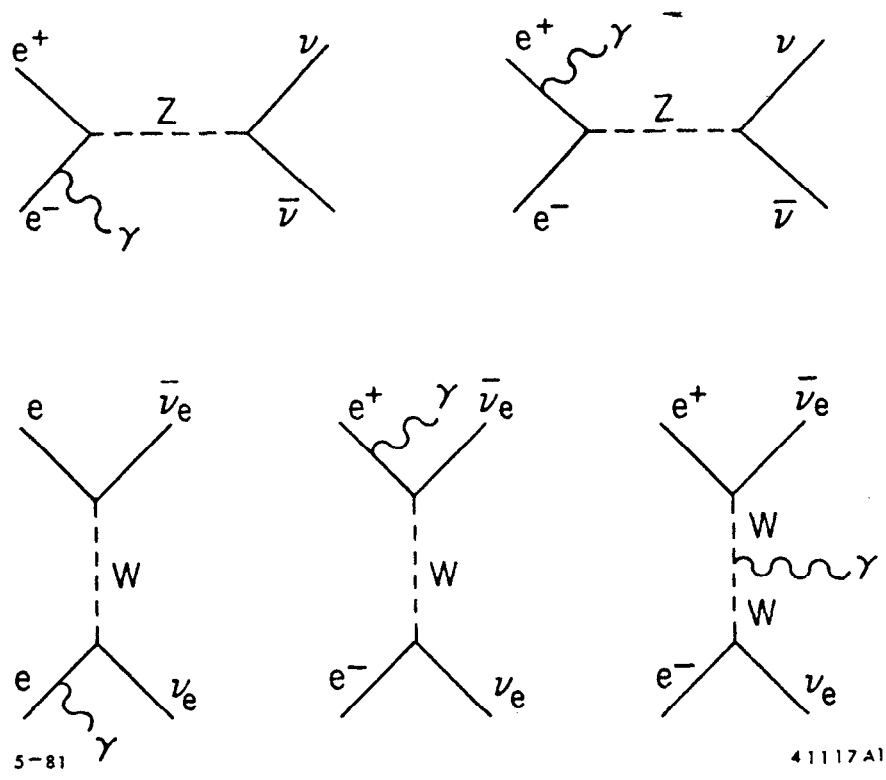
References

1. E. Ma and J. Okada, Phys. Rev. Lett. 41, 287 (1978).
2. K. J. F. Gaemers, R. Gastmans, R. M. Renard, Phys. Rev. D19, 1605 (1979).
3. The  $\gamma\nu\bar{\nu}$  cross section integrated over the  $Z^0$  peak in the photon spectrum measures a quantity proportional to  $N_\nu m_Z^2/\Gamma_Z$ . The shape of the photon spectrum determines both  $m_Z$  and  $\Gamma_Z$  to sufficient accuracy to unambiguously determine  $N_\nu$ . However,  $m_Z$  and  $\Gamma_Z$  will undoubtedly be determined with better precision (and sooner) by the conventional experiment of scanning over the peak in  $e^+e^- \rightarrow Z^0 \rightarrow x$ .
4. S. Brodsky has estimated the angular distribution of higher order contributions to  $e^+e^- \rightarrow \gamma e^+e^-$  and finds them to be of the same form as the lowest order term. S. Brodsky, private communication.
5. Programs for the simulation of events from the Beam-Beam Bremsstrahlung and  $3\gamma$  process were provided by R. Kleiss. See F. A. Berends and R. Kleiss, Nucl. Phys. B177, 237 (1981).
6. D. A. Dicus, Phys. Rev. D21, 1767 (1980).

7. S. Brodsky, invited talk presented at 1979 International Conference on Two Photon Interactions, Lake Tahoe, California.
8. See, for example, C. Fabjan in CERN 79-01, page 689.

Figure Captions

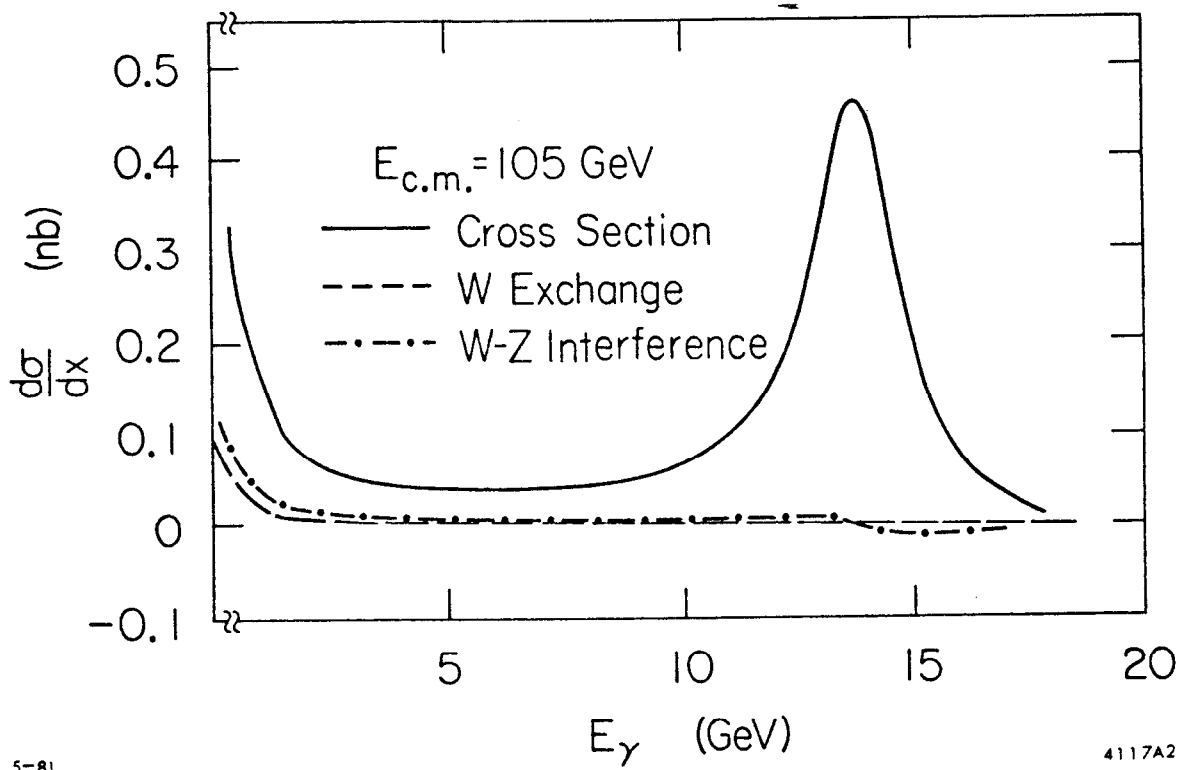
1. Lowest order Feynman diagrams contributing to the process  $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ .
2. Differential cross section  $d\sigma/dx$  vs. the photon energy  $E_\gamma$ . The three curves are for the total cross section, the contribution from W exchange, and the W-Z interference term.
3. a) Total cross section for Beam-Beam Bremsstrahlung requiring that the photon have  $|y| \leq 0.94$ ,  $E_\gamma = 14 \pm 2.5$  GeV, and that neither electron have an angle  $> \theta_{\min}$  vs.  $\theta_{\min}$ . The error bars are the statistical errors from the Monte Carlo calculation.  
b) Total cross section for  $e^+e^- \rightarrow 3\gamma$  requiring that one photon have  $|y| \leq 0.94$ ,  $E_\gamma = 14 \pm 2.5$  GeV, and that any other photons of energy  $> 1$  GeV have angles  $< \theta_{\min}$  vs.  $\theta_{\min}$ . The error bars are the statistical errors from the Monte Carlo calculation.



5-81

41117A1

Fig. 1



5-81

4117A2

Fig. 2

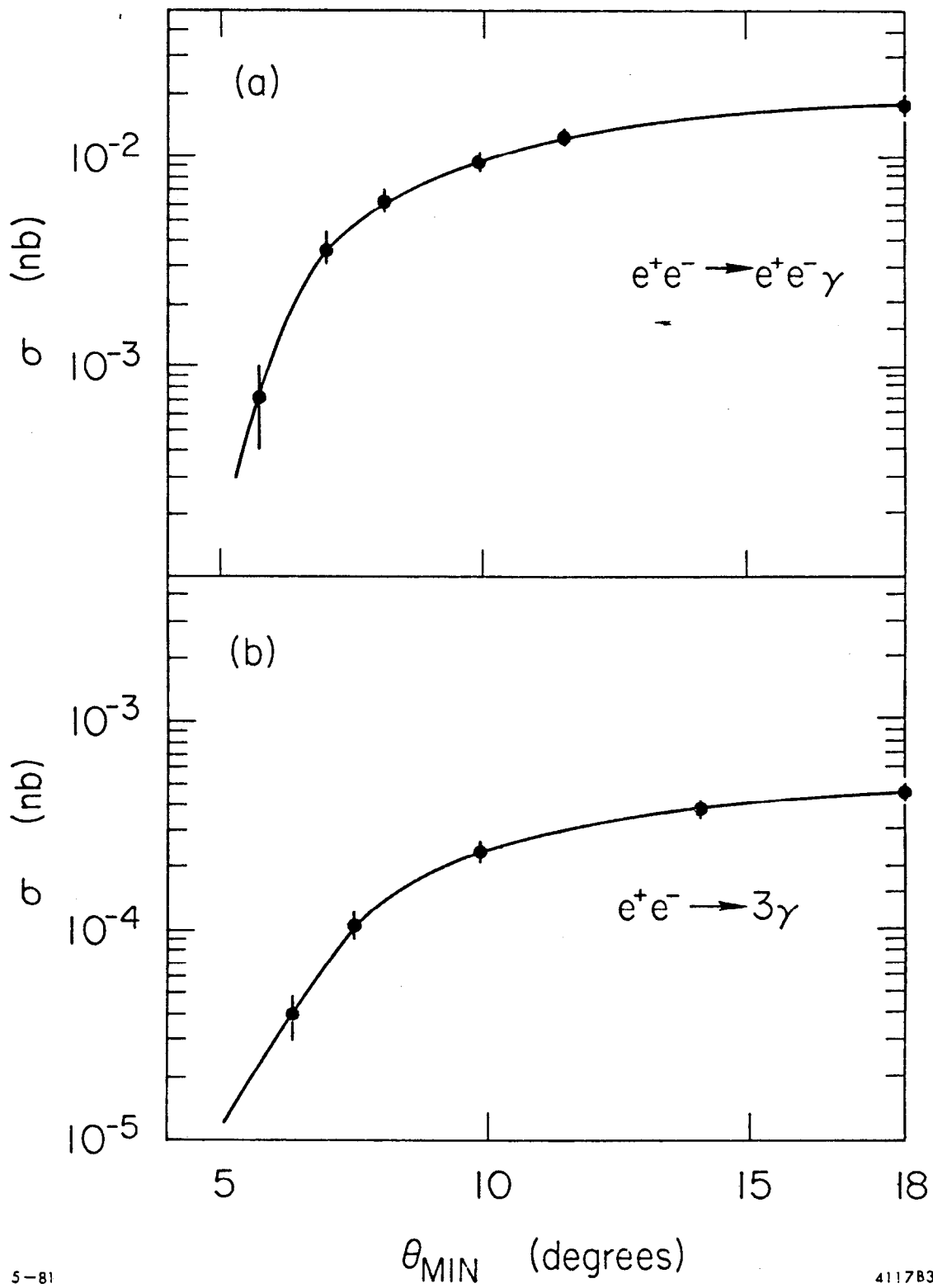


Fig. 3