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PRODUCTION OF A NEW FLAVOR QUARK IN TWO PHOTON PROCESSES*

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ABSTRACT

We show how the structure function W_3 for two photon scattering can be calculated to $O(\bar{g}^2)$ in perturbative QCD. We have calculated W_3 to O(1) for massive quarks, as well as its smeared value over the energy range of heavy quark production (charm, bottom and top quark, and heavy lepton). We argue that the effect of the top quark may be more readily seen in this measurement of W_3 than in e^+e^- annihilation.

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I. INTRODUCTION

It is a common expectation that the top quark should exist, but so far there has been no experimental evidence for its existence. The effect of the top quark is usually looked for in e^+e^- annihilation, since this process can be calculated in the parton model as well as in QCD.

We suggest here that another process in which the effect of the top quark can be looked for is photon-photon scattering. Since the photon has hadronic components, the structure function W_3 of this process is not completely calculable. It can, however, as we shall show, be calculated to $O(\bar{g}^2)$. This is due to the fact that W_3 is related to a matrix element of the product of electromagnetic currents involving a helicity change of two units. Also this calculation would have been impossible if the three gluon coupling were absent.

A small advantage of looking for the effect of the top quark by measuring W_3 in photon-photon scattering versus e^+e^- annihilation is that W_3 varies with the quark charges as $\sum_i e_i^4$ in the former, but only as $\sum_i e_i^2$ in the latter process. Since $\sum_i e_i^4$ changes from 35/81 to 51/81 as the threshold for the top quark is crossed, whereas $\sum_i e_i^2$ changes from 11/9 to 15/9, the relative change in W_3 is larger in photon-photon scattering than in e^+e^- annihilation. It may thus be seen more easily in such an experiment.

In Sec. II we calculate W_3 to O(1) for massive quarks. In Sec. III we calculate the smeared value of W_3 for future comparison with experiment, and in Sec. IV we show how W_3 can be calculated to $O(\bar{g}^2)$.

II. MASS EFFECT IN W_3

The cross section for two photon scattering[1,2] in the reaction $e^+e^- \to e^+ + e^- + \ldots$ can be written as[3,4] (Fig. 1)

$$\frac{d\sigma}{dE_1' \, dE_2' \, d\cos\theta_1 \, d\phi} = \alpha^2 \frac{EE_2'}{Q^2 \nu^2} N_2 \left(2 - 2y_2 + y_2^2\right) \left\{W_1 + \varepsilon_1 W_L + \varepsilon_1 \varepsilon_2 W_3 \cos\phi\right\} \tag{1}$$

in the beam CM frame. Here θ_1 is the scattering angle of the lepton and is large, ϕ is the angle between the scattering planes of the two leptons, E_1' and E_2' are the final lepton energies, and E is the beam energy. Also

$$\nu = p\cdot q \ , \quad Q^2 = -q^2 \ , \quad y_1 = \frac{\nu}{k_2\cdot q} \ , \quad y_2 = \frac{\nu}{k_1\cdot p} \ , \quad \varepsilon_i = \frac{1 - y_i}{1 - y_i + \frac{1}{2} y_i^2} \tag{2}$$

and

$$N_2 = \frac{\alpha}{\pi} \, \frac{E^2 + E_2'^2}{E} \, E_2' \int \frac{d\cos\theta}{-p^2} \quad . \tag{3}$$

Using the equivalent photon approximation,[5,6] we have

$$N_2 \simeq \frac{\alpha}{\pi} \, \frac{E^2 + E_2'^2}{E^2} \, \ell n \, \frac{E}{m_e} \tag{4}$$

(m_e is the electron mass).

The tensor, whose components define the structure functions, is defined by

$$W_{\mu\nu\rho\tau} = \frac{1}{2\pi} \int d^4x \, e^{iqx} \sum_{a,b=\pm 1} \varepsilon_\rho(a) \left\langle \gamma(a) \left| J_\mu(x) \; J_\nu(0) \right| \gamma(b) \right\rangle \varepsilon_\tau^*(b) \, 2\omega \tag{5}$$

where ω is the energy of the real photon:

$$\omega = E - E_2' \tag{6}$$

and ε_ρ, ε_τ^* are the polarization vectors of the photon. Because the photon is real, a and b take only the values ±1. The helicity amplitudes are defined by

$$\begin{aligned} W_{ab;cd} &= \varepsilon_\mu^*(a)\, \varepsilon_\rho^*(b)\, \varepsilon_\nu(c)\, \varepsilon_\tau(d)\, W^{\mu\nu\rho\tau} \\ &= \frac{1}{2\pi}\int d^4x\, e^{iqx}\, \varepsilon_\mu^*(a)\left\langle\gamma(b)\left|J^\mu(x)\; J^\nu(0)\right|\gamma(d)\right\rangle \varepsilon_\nu(c) 2\omega\ . \end{aligned} \tag{7}$$

$W_{\mu\nu\rho\tau}$ has the following tensor structure:

$$\begin{aligned} W_{\mu\nu\rho\tau} &= \frac{1}{2} W_1\, R_{\mu\nu}\, R_{\rho\tau} \\ &+ \frac{1}{2} W_L\, R_{\rho\tau}\, \frac{1}{\nu}\left(p_\mu - \frac{1}{q^2} p\cdot q q_\mu\right)\left(p_\nu - \frac{1}{q^2} p\cdot q q_\nu\right) \\ &+ \frac{1}{2} W_3\left(R_{\mu\rho}\, R_{\nu\tau} + R_{\mu\tau}\, R_{\nu\rho} - R_{\mu\nu}\, R_{\rho\tau}\right) \\ &+ \frac{1}{2} W_4\left(R_{\mu\rho}\, R_{\nu\tau} - R_{\mu\tau}\, R_{\nu\rho}\right) \end{aligned}$$

where the structure functions W_1, W_L, W_3 and W_4 are related to the helicity amplitudes by

$$\begin{aligned} W_1 &= W_{++,++} + W_{+-,+-} \\ W_L &= 2W_{0+,0+} \\ W_3 &= W_{++,--} \\ W_4 &= W_{++,++} - W_{+-,+-} \ . \end{aligned} \tag{8}$$

Only W_1, W_L and W_3 are related to the unpolarized case. For the definition $R_{\mu\nu}$ see below. It is evident from Eq. (1) that W_3 can be measured, if the dependence of the cross section on the angle ϕ can be measured.

As mentioned in Sec. I, W_3 can be calculated to order $\bar{g}^2$ in perturbation theory. We can learn a lot about gluon physics, if we know W_3. Since the heavy lepton can decay into hadrons, heavy lepton pair production will also contributed to W_3. We proceed then to calculate W_3 in QCD. It can be written[6,7] as

$$W_3 = W_3^{\ell}\left(x,W,m_\ell\right)_{box} + \sum_i W_3^i\left(x,W,m_i\right)_{box} + O\left(\bar{g}^2\right)$$

$$W^2 = \left(p+q\right)^2 \tag{9}$$

where W_3 the contribution due to lepton pair production, $W^i_{3,box}$ that of the i^{th} quark pair production, m_i is the mass of the i^{th} quark. To calculate W_3 and $W_{3,box}$ we use the formula

$$\varepsilon^*_\mu(a)\,\varepsilon_\nu(a) = \frac{1}{2}\left\{R_{\mu\nu} - \frac{i}{\sqrt{X}}\,a\,\varepsilon_{\mu\nu\lambda\eta}\,q_1^\lambda\,q_2^\eta\right\} \tag{10}$$

where

$$R_{\mu\nu} = -g_{\mu\nu} + \frac{1}{X}\left\{p\cdot q\left(p_\mu q_\nu + p_\nu q_\mu\right) - q^2 p_\mu\, p_\nu\right\}$$

and

$$X = \left(p\cdot q\right)^2 \tag{11}$$

If ε_μ is the polarization vertor of the virtual photon, then $q_1^\mu = q^\mu$, $q_2^\mu = p^\mu$, whereas if ε_μ refers to the real photon, then $q_1^\mu = p^\mu$ and $q_2^\mu = q^\mu$. Using Eq. (11) we obtain

$$W_3 = \frac{1}{4}\left(R^{\mu\rho}\,R^{\nu\tau} + R^{\nu\rho}\,R^{\mu\tau} - R^{\mu\nu}\,R^{\rho\tau}\right)W_{\mu\nu\rho\tau} \quad . \tag{12}$$

To calculate $W_{ac,bd}$ we insert a fermion pair between $J_\mu(x)$ and $J_\nu(0)$ for

Eq. (7), and use Eq. (12) to extract W_3 and $W^i_{3,box}$. The results are

$$W_3^\ell = -\frac{e^2}{2\pi^2}\left\{\left(1-\frac{4m_\ell^2}{W^2}\right)^{\frac{1}{2}}\left[x^2+\frac{2m_\ell^2}{W^2}(1-x)^2\right]\right.$$
$$\left.+\frac{4m_\ell^2}{W^2}(1-x)\left[1-(1-x)\left(1-\frac{m_\ell^2}{W^2}\right)\right]\ell n\,\frac{1+\left(1-\frac{4m_\ell^2}{W^2}\right)^{\frac{1}{2}}}{1-\left(\bar{1}-\frac{4m_\ell^2}{W^2}\right)^{\frac{1}{2}}}\right\}\theta\left(W^2-4m_\ell^2\right). \tag{13}$$

$W^i_{3,box}$ is given by Eq. (13) with e^2 replaced by $3e^2e_i^4$ and m_ℓ by m_i and $\theta(W-4m_\ell^2)$ by $\theta(W^2-4m_i^2)$. (Here m_ℓ is the lepton mass and e_i the charge of the i^{th} quark.)

In the massless case our result agrees with that of Ref. 6, except for a factor of 1/4, which is due to normalization.

Due to factor e_i^4 in the result for $W^i_{3,box}$, W_3 varies with the quark charges as $\sum_i e_i^4$ (at high energy). This number jumps from 35/81 to 51/81, as the threshold for the top quark is crossed. The corresponding factor in e^+e^- annihilation, however, is $\sum_i e_i^2$, and this jumps from 11/9 to 15/9, so that the production of the top quark would be more evident in the two photon process than in e^+e^- annihilation.

According to the argument of Ref. 8, because of the occurence of bound states of quarks, lepton and gluons, perturbation theory, is not justified in the physical region. Rather, what should be compared with experiment is the smeared quantity

$$W_{3,box} = \frac{\Delta}{\pi}\int_{W^2_{min}}^{\infty}\left\{W_3\left(x,W',m_\ell\right)+\sum_i W_3^i\left(x,W',m_i\right)\right\}\frac{dW'^2}{\left(W'^2-W^2\right)^2+\Delta^2} \tag{14}$$

where the summation is taken over the quark types. This is what we do here. We also take $\Delta = 3\ \mathrm{GeV}^2$ according to the above reference. This result can then be used to compare with the smeared experimental result for W_3.

III. NUMERICAL RESULTS

In Fig. 4 we plot the smoothed function $-(2\pi^2/e^2)\bar{W}_3(x,W)$ versus W for different x. The solid curves include only up, down, strange, charmed and bottom quarks. The dashed curves include the top quark with mass = 16 GeV and the dashed-dotted curves include the top quark with mass = 40 GeV. We take $m_u = m_d = 20$ MeV, $m_s = 400$ MeV, $m_c = 1.5$ GeV, and $m_b = 4.5$ GeV. The first bump is the result of smoothing the charmed and bottom quark bumps, whereas the second bump is the result of smoothing the top quark bump. The asymptotic values is

$$-\frac{2\pi^2}{e^2}\bar{W}_3(x,W) \underset{W\to\infty}{\longrightarrow} \left(1+3\sum_i e_i^4\right)x^2 = \frac{29}{9}x^2 \quad .$$

IV. NON-LEADING TERM

Since in the two photon scattering the initial state is a photon, box diagrams (with gluon internal lines) contribute to $O(\bar{g}^2)$, unlike the e^+e^- annihilation case, where the initial state is the vacuum. In addition to the box diagram, there are other diagrams that contribute to the same order.

In the unpolarized beam case, the Wilson expansion of the product of two currents have been discussed,[9,10] but here we emphasize how W_3 can be calculated to $O(\bar{g}^2)$, and what is the relationship between this property and gluon physics. The Wilson expansion is

$$\varepsilon^*_\rho(a)\varepsilon_\tau(b)T^{\mu\nu\rho\tau} = i\int d^4x\ 2\omega e^{iqx}\left\langle\gamma(a)\left|T\left\{J^\mu(x)J^\nu(0)\right\}\right|\gamma(b)\right\rangle \tag{15}$$

$$\begin{aligned} T\left\{J^\mu(x)\,J^\nu(0)\right\} = \sum_n i^{n+1}\Big\{&\left(-g_{\mu\nu}\Box + \partial_\mu\partial_\nu\right)x_{\mu_1}\cdots x_{\mu_n}\,C^n_i\left(x^2,g^2\right)\,O_i^{\mu_1\cdots\mu_n} \\ &+ \left(g_{\mu\lambda}g_{\nu\sigma}\Box - g_{\mu\lambda}\partial_\nu\partial_\sigma - g_{\nu\sigma}\partial_\mu\partial_\lambda + g_{\mu\nu}\partial_\lambda\partial_\sigma\right) \\ &\times\ x_{\mu_1}\cdots x_{\mu_n}\,D^n_{a,i}\left(x^2,g^2\right)\,O_{i,a}^{\lambda\sigma\mu_1\cdots\mu_n}\Big\}\quad, \end{aligned} \tag{16}$$

where i = quark, gluon and photon and "a" denotes different types of operators. Obviously, $O_i^{\mu_1\cdots\mu_n}$ is a totally symmetric tensor. From kinematics, the operator $O_{i,a}^{\lambda\sigma\mu_1\cdots\mu_n}$ must be symmetric in $\lambda\sigma$, and $\mu_1\ldots\mu_n$ respectively. There are two such types of operators: $O_{i,1}^{\lambda\sigma\mu_1\ldots\mu_n}$, totally symmetric in λ, σ, and $\mu_1\ldots\mu_n$ and $O_{i,2}^{\lambda\sigma\mu_1\ldots\mu_n}$ symmetric in λ, σ, and $\mu_1\ldots\mu_n$ respectively, and of mixed symmetry in the two groups of indices. The operators $O_i^{\mu_1\cdots\mu_n}$ and $O_{i,1}^{\lambda\sigma\mu_1\cdots\mu_n}$ are quite familiar; they contribute to W_1 and W_L. We discuss here the operator $O_{i,2}^{\lambda\sigma\mu_1\cdots\mu_n}$.

By using the permutation property we obtain four types of such operators:

$$O_{1,2}^{\lambda\sigma\mu_1\cdots\mu_n} = i^{n-2}\,S\left\{F^{\lambda\mu_1}D^{\mu_2}\ldots D^{\mu_{n-1}}F^{\sigma\mu_n} + F^{\sigma\mu_1}D^{\mu_2}\ldots D^{\mu_{n-1}}F^{\lambda\mu_n}\right\} \tag{17}$$

$$
\begin{aligned}
O_{2,2}^{\lambda\sigma\mu_1\ldots\mu_n} = i^{n-2}\, S\Big\{ & g^{\lambda\sigma} F^{\mu_1\alpha} D^{\mu_2}\ldots D^{\mu_{n-1}} F_{\alpha}^{\mu_n} \\
& + g^{\mu_1\mu_n} F^{\lambda\alpha} D^{\mu_2}\ldots D^{\mu_{n-1}} F_{\alpha}^{\mu_n} \\
& - g^{\lambda\mu_1} F^{\sigma\alpha} D^{\mu_2}\ldots D^{\mu_{n-1}} F_{\alpha}^{\mu_n} \\
& - g^{\lambda\mu_n} F^{\mu_1\alpha} D^{\mu_2}\ldots D^{\mu_{n-1}} F_{\alpha}^{\sigma} \Big\} + (\lambda \leftrightarrow \sigma)
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
O_{3,2}^{\lambda\sigma\mu_1\ldots\mu_n} = i^{n-1}\, S\Big\{ & 2g^{\lambda\sigma} \bar{\psi}\gamma^{\mu_1} D^{\mu_2}\ldots D^{\mu_n}\psi \\
& + g^{\mu_1\mu_n} \bar{\psi}\left(\gamma^{\lambda}D^{\sigma} + \gamma^{\sigma}D^{\lambda}\right) D^{\mu_2}\ldots D^{\mu_{n-1}}\psi \\
& - g^{\lambda\mu_1} \bar{\psi}\left(\gamma^{\sigma}D^{\mu_n} + \gamma^{\mu_n}D^{\sigma}\right) D^{\mu_2}\ldots D^{\mu_{n-1}}\psi \\
& - g^{\sigma\mu_1} \bar{\psi}\left(\gamma^{\lambda}D^{\mu_n} + \gamma^{\mu_n}D^{\lambda}\right) D^{\mu_2}\ldots D^{\mu_{n-1}}\psi \Big\}
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
O_{4,2}^{\lambda\sigma\mu_1\ldots\mu_n} = i^{n-1}\, S\Big\{ & 2g^{\lambda\sigma} \bar{\psi}\lambda_i \gamma^{\mu_1} D^{\mu_2}\ldots D^{\mu_n}\psi \\
& + g^{\mu_1\mu_n} \bar{\psi}\lambda_i\left(\gamma^{\lambda}D^{\sigma} + \gamma^{\sigma}D^{\lambda}\right) D^{\mu_2}\ldots D^{\mu_{n-1}}\psi \\
& - g^{\lambda\mu_1} \bar{\psi}\lambda_i\left(\gamma^{\sigma}D^{\mu_n} + \gamma^{\mu_n}D^{\sigma}\right) D^{\mu_2}\ldots D^{\mu_{n-1}}\psi \\
& - g^{\sigma\mu_1} \bar{\psi}\lambda_i\left(\gamma^{\lambda}D^{\mu_n} + \gamma^{\mu_n}D^{\lambda}\right) D^{\mu_2}\ldots D^{\mu_{n-1}}\psi \Big\}
\end{aligned} \tag{20}
$$

where D_μ is the covariant derivative, λ_i is the flavor matrix and S denotes symmetrization in $\mu_1\ldots\mu_n$. Replacing the gluon field by the photon field in Eqs. (17) and (18) we obtain two more operators of the above type. Calculation of the one loop diagram shows that these operators mix. However, the operator

$$
O_{1,2}^{\lambda\sigma\mu_1\ldots\mu_n} - \frac{1}{2} O_{2,2}^{\lambda\sigma\mu_1\ldots\mu_n} \tag{21}
$$

(for gluon and photon) does not mix with the other operators (there is one more term than Ref. 10), whereas $O_{i,2}^{\lambda\sigma\mu_1\ldots\mu_n}$ $(i=2,3,4)$, and the corresponding ones for photon, mix with each other. Of course, the photon operator $\left(O_{1,2}^{\lambda\sigma\mu_1\ldots\mu_n} - \frac{1}{2}O_{2,2}^{\lambda\sigma\mu_1\ldots\mu_n}\right)_\gamma$ mix with the gluon operator $\left(O_{1,2}^{\lambda\sigma\mu_1\ldots\mu_n} - \frac{1}{2}O_{2,2}^{\lambda\sigma\mu_1\ldots\mu_n}\right)_g$.

The twist of all operators is two. Their ~~photon~~ matrix elements are

$$\left\langle\gamma(p)\left|O_{i,2}^{\lambda\sigma\mu_1\ldots\mu_n}\right|\gamma(p)\right\rangle 2\omega = \varepsilon_\rho^* \varepsilon_\tau M_i^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} \quad . \tag{22}$$

Because of current conservation, the tensor M_i satisfies the following conditions

$$p_\rho M_i^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} = p_\tau M_i^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} = 0 \quad . \tag{23}$$

There are two types of tensors, which satisfy conditions (23) and possess mixed symmetry between $\lambda\sigma$ and $\mu_1\ldots\mu_n$:

$$\begin{aligned} T_1^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} = S\Big\{\Big[&\left(p^\lambda g^{\mu_1\rho} - p^{\mu_1} g^{\lambda\rho}\right)\left(p^\sigma g^{\mu_n\tau} - p^{\mu_n} g^{\sigma\tau}\right) \\ &+ \left(p^\lambda g^{\mu_1\tau} - p^{\mu_1} g^{\lambda\tau}\right)\left(p^\sigma g^{\mu_n\rho} - p^{\mu_n} g^{\sigma\rho}\right)\Big] p^{\mu_2}\ldots p^{\mu_{n-1}}\Big\} \end{aligned} \tag{24}$$

and

$$\begin{aligned} T_2^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} = R^{\rho\tau}\, S\Big\{\Big(&2g^{\lambda\sigma} p^{\mu_1} p^{\mu_n} + 2g^{\mu_1\mu_n} p^\lambda p^\sigma - g^{\mu_1\sigma} p^\lambda p^{\mu_n} \\ &- g^{\mu_n\sigma} p^\lambda p^{\mu_1} - g^{\mu_1\lambda} p^\sigma p^{\mu_n} - g^{\mu_n\lambda} p^\sigma p^{\mu_1}\Big) p^{\mu_2}\ldots p^{\mu_{n-1}}\Big\}. \end{aligned} \tag{25}$$

Taking into account the dimension of the operators, we have:

$$\left\langle\gamma(p)\left| i^{n-2} F^{\mu_1\alpha} D^{\mu_2} \ldots D^{\mu_{n-1}} F_\alpha^{\mu_n}\right|\gamma(p)\right\rangle 2\omega = a_n p^{\mu_1} \ldots p^{\mu_n} R^{\rho\tau} \varepsilon_\rho^* \varepsilon_\tau \tag{26}$$

$$\left\langle\gamma(p)\left| i^{n-1} \bar{\psi}\gamma^{\mu_1} D^{\mu_2} \ldots D^{\mu_n}\psi\right|\gamma(p)\right\rangle 2\omega = a_n' p^{\mu_1} \ldots p^{\mu_n} R^{\rho\tau} \varepsilon_\rho^*(p) \varepsilon_\tau(p)$$

where n is even. By using the definition of $O_{i,2}$ $(i = 2,3,4)$ and Eq. (26) we obtain

$$\left\langle\gamma(p)\left| O_{i,2}^{\lambda\sigma\mu_1\ldots\mu_n}\right|\gamma(p)\right\rangle 2\omega = a_i^n T_j^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} \varepsilon_\rho^*(p) \varepsilon_\tau(p) \tag{27}$$

$$(i = 1,\ j = 1 \quad ; \quad i = 2,3,4,\ j = 2 \text{ and } n = \text{even}) \quad .$$

For the photon operator we obtain

$$a_i^n = 2 \tag{28}$$

and in general

$$\left\langle\gamma(p)\left| O_{1,2}^{\lambda\sigma\mu_1\ldots\mu_n} - \frac{1}{2} O_{2,2}^{\lambda\sigma\mu_1\ldots\mu_n}\right|\gamma(p)\right\rangle 2\omega$$

$$= \varepsilon_\rho^*(p) \varepsilon_\tau(p) \left(b_1^n T_1^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} + b_2^n T_2^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n}\right) \quad . \tag{29}$$

Since the contraction of λ and σ on the left-hand side of Eq. (29) is zero, using Eqs. (24) and (25) we obtain

$$b_2^n = -\frac{1}{2} b_1^n \quad . \tag{30}$$

Substituting Eqs. (27), (29) and (30) into (26) and (25), we see that there are two types of tensors

$$q_{\mu_1} \ldots q_{\mu_n} T_1^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} = (p \cdot q)^n \left(R^{\lambda\rho} R^{\sigma\tau} + R^{\lambda\tau} R^{\sigma\rho}\right) \quad , \tag{31}$$

and

$$q_{\mu_1}\cdots q_{\mu_n} T_2^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} = 2(p\cdot q)^n R^{\rho\tau} R^{\lambda\sigma} \quad . \tag{32}$$

Hence

$$q_{\mu_1}\cdots q_{\mu_n}\left(T_1^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n} - \frac{1}{2} T_2^{\rho\tau\lambda\sigma\mu_1\ldots\mu_n}\right)$$

$$= (p\cdot q)^n\left(R^{\lambda\rho} R^{\sigma\tau} + R^{\lambda\tau} R^{\sigma\rho} - R^{\rho\tau} R^{\lambda\sigma}\right) \quad . \tag{33}$$

Using the relations $g^{\mu\nu}R_{\mu\nu} = -2$, $R^{\mu\nu}R_{\mu\nu} = 2$, and $R^{\mu\nu}R^{\nu'\alpha}g_{\nu\nu'} = -R^{\mu\alpha}$, it can be seen that the tensors given in Eqs. (32) and (33) are orthogonal to each other. Hence, the operators $O_{i,2}^{\lambda\sigma\mu_1\ldots\mu_n}$ $(i=2,3,4)$ contribute to W_1 and W_L and only the operator $O_{1,2}^{\lambda\sigma\mu_1\ldots\mu_n} - \frac{1}{2} O_{2,2}^{\lambda\sigma\mu_1\ldots\mu_n}$ (photon and gluon) contributes to W_3, i.e., quark operators do not contribute to W_3.

The physical reason for this is the following. W_3 is proportional to $\left\langle\gamma(+)\left|O_{i,2}^{\lambda\sigma\mu_1\ldots\mu_n}\right|\gamma(-)\right\rangle$. Since the quark spin is 1/2, the twist two quark operators can only change the photon helicity from -1 to -2, -1 or 0. Thus the photon matrix elements of the quark operators (twist 2) are zero. In contrast, since photons and gluons have spin one, twist two photon and gluon operators can change the photon helicity from -1 to -3, -2, 0, or 1. Thus, the photon matrix elements of the photon and gluon operators are not zero. Thus, the reason only photon and gluon operators contribute to W_3 is basically the photon and gluon spin.

Another point to note here is that the operators $O_{i,2}^{\lambda\sigma\mu_1\ldots\mu_n}$ $(i=2,3,4)$, which contribute to W_1 and W_L, as the operators $O_i^{\mu_1\ldots\mu_n}$ do, are not really different operators than the $O_{i,1}^{\lambda\sigma\mu_1\ldots\mu_n}$. To see this replace the expression $x_{\mu_1}\ldots x_{\mu_n} D^n_{a,i}(x^2,g^2)$ in the Wilson expansion given in Eq. (16) by $\partial_{\mu_1}\cdots\partial_{\mu_n} D'^{\,n}_{a,i}(x^2,g^2)$. Because of the mixed symmetry of $O^{\lambda\sigma\mu_1\cdots\mu_n}_{i,2}$ between $\lambda\sigma$ and $\mu_1\cdots\mu_n$, we have then

$$\partial_\lambda \partial_{\mu_1}\cdots\partial_{\mu_n} D'^{\,n}_{2,i}\left(x^2,g^2\right) O^{\lambda\sigma\mu_1\cdots\mu_n}_{i,2} = \partial_\sigma \partial_{\mu_1}\cdots\partial_{\mu_n} D'^{\,n}_{2,i}\left(x^2,g^2\right) O^{\lambda\sigma\mu_1\cdots\mu_n}_{i,2}$$

$$= 0 \tag{34}$$

Using Eq. (34), we obtain:

$$\sum_{n(\text{even})} i^{n+1}\left(g_{\mu\lambda}g_{\nu\sigma}\Box - g_{\mu\lambda}\partial_\nu\partial_\sigma - g_{\nu\sigma}\partial_\mu\partial_\lambda + g_{\mu\nu}\partial_\lambda\partial_\sigma\right) \times \partial_{\mu_1}\cdots\partial_{\mu_n} D'^{\,n}_{2,i}\left(x^2,g^2\right) O^{\lambda\sigma\mu_1\cdots\mu_n}_{i,2}$$

$$= \sum_{n(\text{even})} i^{n+1}\,\Box\,\partial_{\mu_1}\cdots\partial_{\mu_n} D'^{\,n}_{2,j}\left(x^2,g^2\right) O^{\mu_1\cdots\mu_n}_{j,2\,\mu\nu} \tag{35}$$

By using the forms of the $O^{\mu\nu\mu_1\cdots\mu_n}_{j,2}$ $(j=2,3,4)$ [Eqs. (28)-(34)], the right hand side of Eq. (35) becomes

$$\sum_{n(\text{even})} i^{n+1}\left(\partial_\lambda\partial_\sigma g_{\mu\nu} + \Box g_{\mu\lambda}g_{\nu\sigma} - \partial_\mu\partial_\sigma g_{\lambda\nu} - \partial_\lambda\partial_\nu g_{\mu\sigma}\right) \times \partial_{\mu_2}\cdots\partial_{\mu_{n-1}} D'^{\,n}_{2,i}\left(x^2,g^2\right) O^{\lambda\sigma\mu_1\cdots\mu_n}_{i,1} .$$

We thus see that the operators $O_{i,2}$ are the same as $O_{i,1}$.

By calculating the diagrams of Fig. 2, we obtain the anomalous dimension (this formulae is different from Ref. 10) of the operator $O_{1,2} - \frac{1}{2}O_{2,2}$; it is given by

$$\frac{g^2}{16\pi^2}\gamma^n_{gg} = \frac{g^2}{8\pi^2}\left\{\left(\frac{1}{3} + 4\sum_{j=2}^{n}\frac{1}{j}\right)C_2(G) + \frac{4}{3}T(R)\right\} . \tag{36}$$

The moments of W_3 are given by

$$\int_0^1 dx\, x^{n-1}\, W_3\left(x,Q^2\right) = \frac{1}{2}\sum_i b_n^i W_{ij}\, D_j^{(n)}\left(1,\bar{g}^2\right) \tag{37}$$

where

$$D_j^{(n)}\left(\frac{q^2}{\mu^2}, g^2\right) = \left(q^2\right)^{n+1}\left(\frac{\partial}{\partial q^2}\right)^n \int d^4x\, e^{iqx}\, D_j^{(n)}\left(x^2, g^2\right) \tag{38}$$

$$W = T\left\{\exp\int_{\bar{g}}^{g} \frac{\gamma(g')}{\beta(g')}\, dg'\right\} \tag{39}$$

$$\gamma(g) = \frac{g^2}{16\pi^2}\begin{pmatrix} \gamma_{gg}^n & \dfrac{e^2}{16\pi^2}\gamma_{g\gamma} \\ \dfrac{e^2}{16\pi^2}\gamma_{\gamma g} & \dfrac{e^2}{g^2}\gamma_{\gamma\gamma} \end{pmatrix} . \tag{40}$$

In Eq. (37) we only keep the term of $O(\alpha_\gamma)$. From Eqs. (39) and (40), we obtain

$$W_{\gamma\gamma} = 1 + O\left(\alpha_\gamma\right)$$

$$W_{\gamma g} = \frac{\alpha_\gamma}{4\pi}\,\frac{\gamma_{\gamma g}}{\gamma_{gg}}\left\{\left(\frac{\bar{g}^2}{g^2}\right)^{d_n} - 1\right\}$$

$$W_{gg} = \left(\frac{\bar{g}^2}{g^2}\right)^{d_n} \tag{41}$$

where $d_n = \gamma_{gg}^n/2\beta_0$, $\beta_0 = \frac{1}{3}\left\{11C_2(G) - 4T(R)\right\}$. Substituting Eq. (41) into (37) we obtain

$$\int_0^1 dx\, x^{n-1}\, W_3(x,Q) = \frac{1}{2}\left\{b_n^{\gamma} D_{\gamma}'^{(n)}\left(1,\bar{g}^2\right) - \frac{\alpha_{\gamma}}{4\pi}\, \frac{\gamma_{\gamma g}}{\gamma_{gg}}\right.$$

$$\left. \times\, b_n^{\gamma}\left[1 - \left(\frac{\bar{g}^2}{g^2}\right)^{d_n}\right] D_g'^{(n)}\left(1,\bar{g}^2\right) + b_n^g \left(\frac{\bar{g}^2}{g^2}\right)^{d_n} D_g'^{(n)}\left(1,\bar{g}^2\right)\right\} \tag{42}$$

where $D_g'^{(n)}(1,\bar{g}^2) \sim O(\bar{g}^2)$. From Eqs. (36) and (41), we obtain

$$\left(d_n\right)_{min} = \frac{7C_2(G) + 4T(R)}{11C_2(G) - 4T(R)} \quad . \tag{43}$$

Since the number of quark flavors is greater than three, we obtain from Eqs. (43) that

$$\left(d_n\right)_{min} > 1 \quad . \tag{44}$$

Keeping only terms of order $O(1)$ and $O(\bar{g}^2)$ in Eq. (42) it becomes

$$\int_0^1 dx\, x^{n-1}\, W_3\left(x,Q^2\right) = \frac{1}{2}\, b_n^{\gamma}\left\{D_{\gamma}'^{(n)}\left(1,0\right)^{(0)}\right.$$

$$\left. + \frac{\bar{g}^2}{16\pi^2}\, D_{\gamma}'^{(n)}\left(1,\bar{g}^2\right)^{(1)} - \frac{\alpha_{\gamma}}{4\pi}\, \frac{\gamma_{\gamma g}}{\gamma_{gg}}\, \frac{\bar{g}^2}{16\pi^2}\, A_n\right\} \tag{45}$$

where

$$b_n^{\gamma} = 2$$

$$\frac{\bar{g}^2}{16\pi^2}\, A_n = D_g'^{(n)}\left(1,\bar{g}^2\right) \quad . \tag{46}$$

By using Eq. (13) (suitably modified for $W^i_{3,box}$), we can obtain $D_{\gamma}^{(n)}(1,0)^{(0)}$, and, by replacing the factor $3e_i^4$ by $(e_i^2/f)T(R)\bar{g}^2$ (f = number of quark flavors), we can obtain $D_g^{(n)}(1,\bar{g}^2)$. $D_\gamma^{(n)}(1,\bar{g}^2)^{(1)}$ can be obtained by calculating two loop diagrams, which are box diagrams with an internal gluon line. Finally $\gamma_{\gamma g}$ can be obtained from two loop diagrams, such as that shown in Fig. 3. In contrast to the e^+e^- annihilation case, apart from box diagrams (one and two loops), we have, in the case of W_3, to take into account the contribution of gluon operators.

We then see from the above discussion, that W_3 can be calculated to order $\bar{g}^2$ in perturbation theory. The matrix element b_n^g cannot be calculated, but since $d_n > 1$, this term is $O(\bar{g}^4)$. This possibility, of calculating the O(1) and $O(\bar{g}^2)$ contributions to W_3, is based on two facts: the gluon spin is one and the existence of the three-gluon coupling. The latter determines d_n. The three-gluon coupling is an important property of QCD, which distinguishes it from Abelian theories. Therefore, by measuring W_3 and comparing with the QCD result, we may hope to establish whether the non-Abelian character of QCD is reflected in the experimental data.

To summarize, we have shown that: (a) due to the spin of the gluon and the three-gluon coupling, the structure function W_3 in two photon reaction can be calculated in QCD to order $O(\bar{g}^2)$, and have calculated it and its smeared value in the physical region to order O(1); (b) a measurement of W_3 and comparison with the theoretical result would yield more information about gluon physics, it would also tell us about the existence of the top quark; (c) the threshold behavior of W_3 differs from that in e^+e^- annihilation; and (d) a study of W_3 would provide a clean test of QCD.

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FIGURE CAPTIONS

Fig. 1. The two photon reaction considered. Photon at lower vertex is considered in the mass shell ($p^2 \simeq 0$).

Fig. 2. The diagrams contributing to the anomalous dimension of the operator $O_{1,2} - \frac{1}{2}O_{2,2}$ in order $O(\bar{g}^2)$.

Fig. 3. Example of higher order diagrams that would have to be calculated to obtain the anomalous dimension matrix element.

Fig. 4. $-(2\pi^2/e^2)\cdot\bar{W}_3(x,W)$ versus W. Solid curves only up, down, strange, charm, bottom quark and lepton contributions at different x. Dashed curves include top quark with mass = 16 GeV. Dash-dot curves include top quark with mass = 40 GeV.

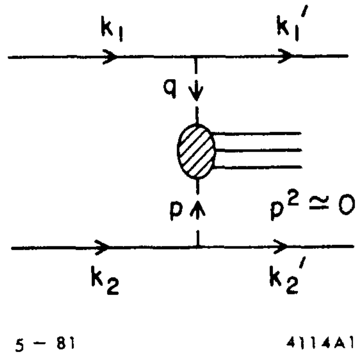

Fig. 1

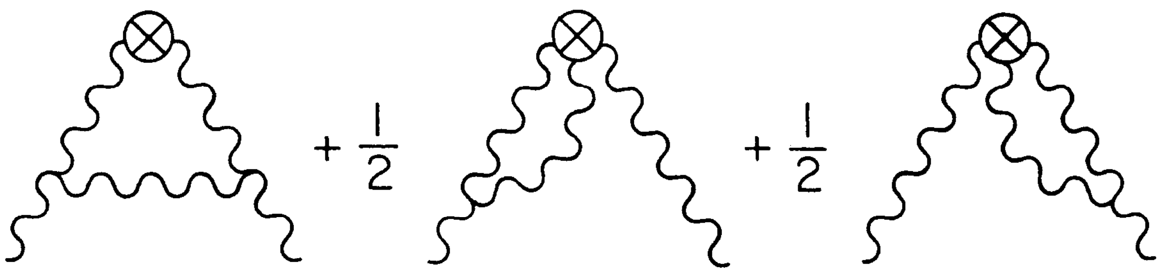

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Fig. 2

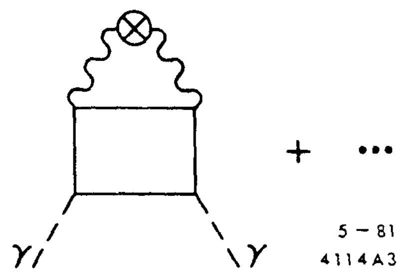

Fig. 3

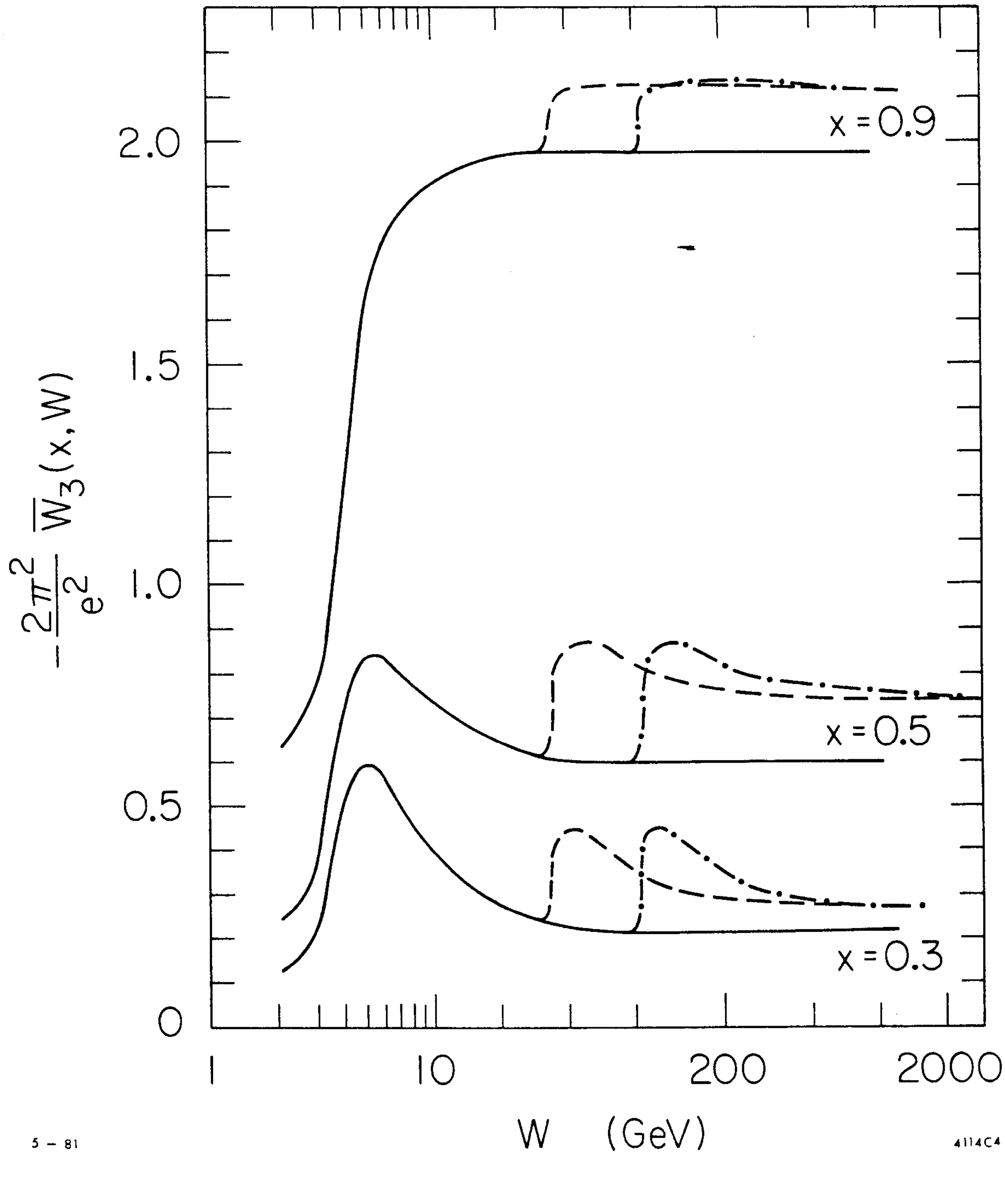

Fig. 4