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GLUON JETS FROM HEAVY PARAQUARKONIUM*

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ABSTRACT

The GGG and $q\bar{q}G$ jets in the decay of very heavy paraquarkonia are analyzed in perturbative quantum chromodynamics. The thrust distributions are calculated and the result is compared with the thrust distributions in the orthoquarkonium decay and in $e^+e^- \to q\bar{q}G$ jets.

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It is now well established experimentally that final states of e^+-e^- annihilation processes are substantially composed of hadronic jets.[1] In quantum chromodynamics those jets are supposed to come from quarks and gluons produced by short-distance interactions.[2]

A similar effect is also expected in heavy-quarkonium decays where only the gluons are produced primarily in accord with the OZI rule[3] and hadronic jets coming from the gluons should be observed. There have been some indications of such jets from quarkonia.[4] It has, however, been pointed out[5] that present experimental energies (or equivalently quarkonium masses) are not high enough to test real perturbative QCD effects as the nonperturbative hadronization effects still dominate. Hopefully for very heavy quarkonia which may be observed in the future, the nonperturbative effects will become less important so that the perturbative QCD predictions may be directly tested.

In the present paper we consider the heavy paraquarkonium decay into two gluons, three gluons, and quark-antiquark gluons up to order α_s^3. The total width of the decay to this order has already been calculated by Barbieri, Curci, d'Emilio and Remiddi,[6] and Hagiwara, Kim and Yoshino.[7] There, the α_s correction to the Born term is found to be quite large in the $\overline{MS}$ scheme[8] although some arguments have been given on improving the correction term.[7,9] It is also interesting to observe final states of the decay. The average thrust of jets in paraquarkonium decays was calculated by Barbieri, Caffo and Remeddi.[10] In this paper we calculate the thrust (T) distribution of the jets from heavy paraquarkonia and present the detail of the calculation which would be useful for other similar calculations. We check the cancellation of the infrared divergence at T = 1. The resulting thrust distribution looks quite different from that of the orthoquarkonium.

According to the factorization theorem[11] the amplitude of the

OZI-forbidden heavy-quarkonium decay may be separated into the hard and soft parts. The hard part is short-distance dominated and calculable in perturbative QCD while the soft part requires full information on the bound-state nature of quarkonia and is nonperturbative.

For the color-singlet $^1S_0$ $q\bar{q}$ bound state (paraquarkonium) the full decay width Γ is given by the following formula,

$$\Gamma = |\psi(0)|^2 v\sigma(q\bar{q}), \tag{1}$$

where $\psi(0)$ is the paraquarkonium wave function at the origin, v the center-of-mass relative velocity between the quark and antiquark, and $\sigma(q\bar{q})$ the total cross section of the quark-antiquark annihilation in the free color-singlet $^1S_0$ state. The thrust distribution of jets produced in the decay of the heavy paraquarkonium is

$$\frac{1}{\Gamma}\frac{d\Gamma}{dT} = \frac{1}{\sigma}\frac{d\sigma}{dT} , \tag{2}$$

with $d\sigma/dT$ the thrust distribution of the final state jets produced by the color-singlet $^1S_0$ $q\bar{q}$ annihilation.

We calculate σ and $d\sigma/dT$ up to $O(\alpha_s^3)$. Relevant diagrams for calculating σ and $d\sigma/dT$ in perturbative QCD are shown in Fig. 1. To order α_s^3 the set of diagrams shown in Fig. 2 contributes to the cross sections. We omit here all the details of the calculations. The results are given in the Appendix in the form of energy spectra with respect to the final-particle energies. In our calculation the nonrelativistic approximation was employed for the heavy quark-antiquark system. The ultraviolet divergence appearing in the calculation of the contribution of Fig. 2a was regularized dimensionally and renormalized properly by the use of the renormalized QCD coupling constant $g(\alpha_s = g^2/4\pi)$. In the present calculation we adopted the $\overline{MS}$ renormalization scheme.[8]

In the final state, as in Fig. 2d, we take into account only the quarks that are lighter than the heavy initial quarks in conformity with the decoupling argument.[12] The same criterion also applies to the virtual quarks appearing in Fig. 2a. Those light quarks that contribute are assumed to be massless just for calculational convenience. It is straightforward to include quark mass effects.

Most of the diagrams in Fig. 2 give rise to the infrared (soft as well as collinear) divergence which we regularize in dimensional regularization method. According to the KLN theorem[13] such infrared divergences should cancel out in the total cross sections σ and $d\sigma/dT$.

The gluons and light quarks produced in the final state will eventually fragment into hadrons. If the p_T spread of hadrons from the direction of the original gluons and quarks is limited to be small, the hadron jet should be observed experimentally. The thrust of the hadron jets is then approximately equal to that of the original gluons and quarks. Hence the thrust distribution $d\sigma/dT$ of the gluons and quarks is substantially the same as that of hadron jets in the final state of the heavy quarkonium decays.

We can define the thrust T of the final state gluons (or quarks) by the largest energy fraction carried by them[14]

$$T = \text{Max}\ (x_1, x_2, x_3) \quad , \tag{3}$$

where the energy fraction x_i for the i-th parton is defined by

$$x_i = 2P \cdot k_i / P^2 \tag{4}$$

with P the total momentum of the initial $q\bar{q}$ state and k_i the momentum of the i-th parton in the final state. Due to the energy-momentum conservation the constraint on x_i,

$$\sum_i x_i = 2 , \tag{5}$$

should be satisfied.

Through the energy spectra given in the Appendix we calculate thrust distributions $d\sigma/dT$ by integrating over irrelevant energy fractions x_i. The contribution of the two-gluon final state shown in Fig. 2a may be obtained from Eq. (A.5) and σ which is given later:

$$\frac{1}{\sigma}\frac{d\sigma^{2G}}{dT} = [1 + G(\varepsilon)a(\varepsilon)]\delta(1 - T) \tag{6}$$

where

$$G(\varepsilon) = \frac{\alpha_s}{\pi}\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} \Gamma(1 + \varepsilon) , \tag{7}$$

$$a(\varepsilon) = C_A\left(-\frac{1}{\varepsilon^2} - \frac{11}{6\varepsilon} - \frac{181}{18} + \frac{23\pi^2}{24}\right) + \frac{2}{3}T_R(N_f - 1)\left(\frac{1}{\varepsilon} + \frac{8}{3}\right) . \tag{8}$$

Here $\varepsilon = (4-D)/2$ with D the number of the space-time dimensions, μ is the regularization mass scale, $s = p^2$, $C_A = N$ and $T_R = 1/2$ for the SU(N) color, and N_f the number of flavors. The total annihilation cross section, σ, of the color singlet ${}^1S_0$ heavy $q\bar{q}$ state is, to order α_s^3, obtained by adding Eqs. (A.5), (A.8), (A.9), (A.10) and (A.11) and integrating over x_i,

$$\sigma = \sigma_B(\varepsilon)\,[1 + G(\varepsilon)\,A]\ , \tag{9}$$

where $\sigma_B(\varepsilon)$ is the Born total cross section calculated in D-dimensions,

$$v\sigma_B(\varepsilon) = \frac{8\pi\alpha_s^2}{s}\,C_F\left(\frac{4\pi\mu^4}{s}\right)^{\varepsilon}\frac{\Gamma(2-\varepsilon)}{\Gamma(1-2\varepsilon)}\quad , \tag{10}$$

with $v = 2\sqrt{1-4m^2/s}$ the relative velocity of the initial quarks (of mass m) in the center-of-mass system, and $C_F = (N^2-1)/2N$ and

$$A = C_F\left(\frac{\pi^2}{v} - 5 + \frac{\pi^2}{4}\right) - \frac{4}{3}T_R\left(\frac{4}{3}(N_f-1) + \ln 2\right) + C_A\left(\frac{199}{18} - \frac{13}{24}\pi^2\right). \tag{11}$$

Note that, in Eq. (8) with Eqs. (7) and (11), the ultraviolet divergence is already renormalized in the $\overline{\text{MS}}$ scheme (see the Appendix).

The three-gluon contribution as shown in Fig. 2b together with the FP-ghost contribution in Fig. 2c is given, through Eqs. (A.8), (A.9) and (A.10), by

$$\frac{1}{\sigma}\frac{d\sigma^{3G}}{dT} = G(\varepsilon)C_A b(\varepsilon,T)\ , \tag{12}$$

where

$$b(\varepsilon,T) = \frac{1}{(1-T)^{1+\varepsilon}}\left[\frac{2}{\varepsilon}\left(\frac{1}{(1-T)^{\varepsilon}} - 1\right)\left(1-\frac{\pi^2\varepsilon^2}{6}\right) - \frac{11}{6} + \left(\frac{\pi^2}{3} - \frac{67}{18}\right)\varepsilon\right] + b(T)\quad , \tag{13}$$

$$b(T) = \frac{2}{1-T}\,\ln(2T-1) + 2\left(-T^2 + T-2 + \frac{1}{T}\right)\ln\frac{2T-1}{1-T}$$

$$+\left[2T^2 - 2T - 4 + \frac{7}{2T} - \frac{3}{T^2} + \frac{7}{2(2-T)} - \frac{6}{(2-T)^2} + \frac{2}{(2-T)^3}\right]\ln\frac{2(1-T)}{T}$$

$$+ 3T + \frac{17}{2} - \frac{79}{12T} + \frac{7}{6T^2} + \frac{1}{T^3} - \frac{9}{4(2-T)} + \frac{1}{(2-T)^2} \quad . \tag{14}$$

For the $q\bar{q}G$ final state of Fig. 2d we obtain from Eq. (A.11)

$$\frac{1}{\sigma}\frac{d\sigma^{q\bar{q}G}}{dT} = 2G(\varepsilon)T_R(N_f - 1)c(\varepsilon,T) \quad , \tag{15}$$

where

$$c(\varepsilon,T) = \frac{3+5\varepsilon}{9(1-T)^{1+\varepsilon}} + c(T) \quad , \tag{16}$$

$$c(T) = -\,3T + 2 + \frac{1}{3T} - \frac{2}{3T^2} + (2T^2 - 2T + 1)\ln\frac{2T-1}{1-T}$$

$$+ 2T(1-T)\ln\frac{2(1-T)}{T} \quad . \tag{17}$$

In the above results, Eqs. (6), (12) and (15), the infrared (soft as well as collinear) divergence explicitly shows up. This divergence, however, should cancel out by the KLN theorem[13] if we combine them into the total cross section. In fact, by the use of the relation

$$\frac{1}{(1-T)^{1+\varepsilon}} = -\frac{1}{\varepsilon}\delta(1-T) + \frac{1}{(1-T)_+} - \varepsilon\left(\frac{\ln(1-T)}{1-T}\right)_+ \tag{18}$$

where $1/(1-T)_+$ and $(\ln(1-T)/(1-T))_+$ are distributions defined, e.g., in

Ref. 15, we may rewrite Eqs. (12) and (15) in the following way,

$$\frac{1}{\sigma}\frac{d\sigma^{3G}}{dT} = G(\varepsilon)C_A\left[\left(\frac{1}{\varepsilon^2} + \frac{11}{6\varepsilon} + \frac{67}{18} - \frac{\pi^2}{2}\right)\delta(1-T) - \frac{11}{6(1-T)_+} - 2\left(\frac{\ln(1-T)}{1-T}\right)_+ + b(T)\right] , \tag{19}$$

$$\frac{1}{\sigma}\frac{d\sigma^{q\bar{q}G}}{dT} = 2G(\varepsilon)T_R(N_f-1)\left[-\frac{1}{3}\left(\frac{1}{\varepsilon} + \frac{5}{3}\right)\delta(1-T) + \frac{1}{3(1-T)_+} + c(T)\right] . \tag{20}$$

On summing Eqs. (6), (19) and (20), one sees immediately that the infrared divergences cancel out. We find

$$\begin{aligned}\frac{1}{\Gamma}\frac{d\Gamma}{dT} &= \frac{1}{\sigma}\left(\frac{d\sigma^{2G}}{dT} + \frac{d\sigma^{3G}}{dT} + \frac{d\sigma^{q\bar{q}G}}{dT}\right) \\ &= \delta(1-T)\left[1 + \frac{\alpha_s}{\pi}\left(C_A\left(-\frac{19}{3} + \frac{11\pi^2}{24}\right) + \frac{2}{3}T_R(N_f-1)\right)\right] \\ &+ \frac{\alpha_s}{\pi}\left[C_A\left(-\frac{11}{6(1-T)_+} - 2\left(\frac{\ln(1-T)}{1-T}\right)_+ + b(T)\right)\right. \\ &\left.+ 2T_R(N_f-1)\left(\frac{1}{3(1-T)_+} + c(T)\right)\right] .\end{aligned} \tag{21}$$

The average thrust can be easily calculated by using Eq. (21) or Eqs. (19) and (20). The result reads

$$\langle 1-T\rangle = \langle 1-T\rangle_{3G} + \langle 1-T\rangle_{q\bar{q}G} , \tag{22}$$

where $\langle 1-T\rangle_{3G}$ and $\langle 1-T\rangle_{q\bar{q}G}$ are the contributions of Eqs. (19) and (20) to the average thrust, respectively, and are given by

$$\langle 1-T\rangle_{3G} = \frac{\alpha_s}{\pi} C_A \frac{1}{16}\left[\frac{148}{3} + \frac{424}{3}\ln 2 - \frac{297}{2}\ln 3 + 16\ln^2 2 + \frac{46}{3}\pi^2 + 288L\left(-\frac{1}{2}\right)\right] , \quad (23)$$

$$\langle 1-T\rangle_{q\bar{q}G} = \frac{\alpha_s}{\pi} T_R\left(N_f-1\right) \frac{1}{4}\left(-\frac{8}{3} - \frac{20}{3}\ln 2 + \frac{29}{4}\ln 3\right) , \quad (24)$$

with L(x) the Spence function (or dilogarithm $Li_2(x)$ in Ref. 16). The above results (23) and (24) agree with the previous calculation by Barbieri, Caffo and Remiddi.[10] For SU(3) color we have

$$\langle T\rangle_{3G} = 1 - 0.837\,\alpha_s$$

$$\langle T\rangle_{q\bar{q}G} = 1 - 0.0269\left(N_f-1\right)\alpha_s . \quad (25)$$

Equation (25) shows that the thrust distribution due to the $q\bar{q}G$ jet is much more sharply peaked toward $T=1$ than that due to the 3G jet.

The thrust distribution of Eq. (21) is completely free from the infrared divergence and is expected to be a testable prediction of perturbative QCD for sufficiently heavy quarkonia for which the hadronization effect is relatively unimportant. As an example Eq. (21) is plotted in Fig. 3 for $\alpha_s = 0.19$ and 3 colors with $N_f = 4$ together with $(d\Gamma^{3G}/dT)/\Gamma$ and $(d\Gamma^{q\bar{q}G}/dT)/\Gamma$. We also compare in Fig. 4 our result Eq. (21) for paraquarkonium decays with the thrust distributions for orthoquarkonium decays[17] and for $e^+e^-\to q\bar{q}G$.[18] The thrust distribution for orthoquarkonium decays in the lowest order[17] is independent of α_s,

$$\frac{1}{\Gamma}\frac{d\Gamma}{dT} = \frac{6}{\pi^2-9}\left[\frac{2(1-T)(5T^2-12T+8)}{T^2(2-T)^3}\ln\frac{2(1-T)}{T} + \frac{(3T-2)(2-T^2)}{T^3(2-T)^2}\right] . \qquad (26)$$

From Fig. 4 we notice that the shape of the thrust distribution of the paraquarkonium decay is somewhat similar to that of $e^+e^-\to q\bar{q}G$ while the thrust distribution for 3G jets in the orthoquarkonium decay is much flatter than those of the others. It would seem to be quite important to check whether this tendency would persist in the orthoquarkonium decay when the next order effects are taken into account.

Finally, we make a few remarks. In this paper we have calculated the thrust distribution in heavy paraquarkonium decays to the next order. However, when looking at the region $T \simeq 1$, the large effects of soft gluon emissions may spoil the naive perturbative expansion. This fact is manifested in the term $(1/(1-T)\cdot\ln(1-T))_+$ of Eq. (21). Then in order to obtain the more reliable prediction in this region, the so-called resummation procedure will be needed as stressed by many authors recently.[19] In addition there is always nonperturbative nuisance which is considered to be more important in the $T \simeq 1$ region. Therefore, our results should be compared with "experimental data" in the $T < 1$ region where the perturbative effects can be expected to dominate.

The above problems which concern the $T \simeq 1$ region are left for future studies.

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Note added

We have recently received a preprint (K. Koller, K. H. Streng, T. F. Walsh and P. M. Zerwas, DESY 80/132) which partially overlaps our paper.

APPENDIX

A somewhat detailed description of our calculation is presented here. Throughout our calculation we use the dimensional regularization method for the regularization of both infrared and ultraviolet divergences and adopt the Feynman gauge so that the Faddeev-Popov ghost has to be taken into account in the final state. All the final quark masses are taken to be zero. We present our result in the form of the distribution in the energy fractions x_i, defined in Eq. (4).

I. TWO-GLUON FINAL STATE

The calculation of the contribution of Fig. 2a is essentially the same as that of Refs. 6 and 7. We, however, repeated the calculation so that our result serves as the third independent check of the previous calculations.[6,7] We find

$$\frac{d\sigma^{2G}}{dx_1 dx_2} = \sigma_{B0}(\varepsilon)\left[1 + G_0(\varepsilon)F(\varepsilon,\varepsilon')\right]\delta(1-x_1)\delta(1-x_2) \quad , \tag{A.1}$$

where $G_0(\varepsilon)$ and $\sigma_{B0}(\varepsilon)$ are the same as those of Eqs. (7) and (10) with α_s replaced by the bare coupling constant α_{s0} and

$$F(\varepsilon,\varepsilon') = C_F\left(\frac{\pi^2}{2v} - 5 + \frac{\pi^2}{4}\right) + \frac{2}{3}T_R\left(-\frac{N_f}{\varepsilon'} + \frac{N_f-1}{\varepsilon} - 2\ln 2\right)$$
$$+ C_A\left(-\frac{1}{\varepsilon^2} - \frac{11}{6\varepsilon} + \frac{11}{6\varepsilon'} + 1 + \frac{5\pi^2}{12}\right) \quad . \tag{A.2}$$

Here we distinguish the ultraviolet regulator $\varepsilon' = (4-D)/2$ from the infrared one ε in order to make sure which divergence is going to be renormalized. The result (A.1), in fact, agrees with the one in Refs. 6 and 7. Performing the charge renormalization

$$\alpha_{s0} = Z\alpha_s \quad , \tag{A.3}$$

where

$$Z = 1 + \frac{1}{12\varepsilon'} G_0(\varepsilon)\left(11C_A - 4T_R N_f\right) \quad , \tag{A.4}$$

we obtain

$$\frac{d\sigma^{2G}}{dx_1 dx_2} = \sigma_B(\varepsilon)[1 + G(\varepsilon)F(\varepsilon)]\,\delta(1-x_1)\,\delta(1-x_2) \quad , \tag{A.5}$$

where

$$F(\varepsilon) = C_F\left(\frac{\pi^2}{2v} - 5 + \frac{\pi^2}{4}\right) + \frac{2}{3}T_R\left(\frac{N_f-1}{\varepsilon} - 2\ell n2\right)$$

$$+ C_A\left(-\frac{1}{\varepsilon^2} - \frac{11}{6\varepsilon} + 1 + \frac{5\pi^2}{12}\right) \quad . \tag{A.6}$$

Note that, by choosing the expression of Z in the form of Eq. (A.4), we have adopted the $\overline{\text{MS}}$ renormalization scheme. In fact, the factor

$$G_0(\varepsilon) = \frac{\alpha_{s0}}{\pi}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left[1+\varepsilon(\ell n4\pi-\gamma)\right] \quad , \tag{A.7}$$

takes care of $\ell n4\pi-\gamma$ with γ the Euler constant.

II. THREE-GLUON FINAL STATE

The contribution of Fig. 2b together with Fig. 2c is split into three parts as shown in Fig. 5; (a), (b), (c) + (d). Noting the fact that the final gluon state in Fig. 5 is antisymmetric in color indices, we find for the contribution of (a) (The computer manipulation, SCHOONSCHIP and REDUCE 2 were used here and after.)

$$\frac{d\sigma^{(a)}}{dx_1 dx_2 dx_3} = \sigma_B(\varepsilon) G(\varepsilon) C_A \frac{1}{12} \left[2\frac{1-x_1}{x_2^2 x_3^2} + \frac{3-4x_1}{x_2 x_3} + \text{cyclic perm.} \right]$$

$$\times \left[(1-x_1)(1-x_2)(1-x_3)\right]^{-\varepsilon} \delta(x_1 + x_2 + x_3 - 2) \quad . \qquad \text{(A.8)}$$

The contribution of (b) is given by

$$\frac{d\sigma^{(b)}}{dx_1 dx_2 dx_3} = \sigma_B(\varepsilon) G(\varepsilon) C_A \frac{\sin\pi\varepsilon}{6\pi\varepsilon} \left[\frac{2(1-x_1)^2}{x_1 x_3^2} + \frac{2(1-x_2)^2}{x_2 x_3^2} + \frac{x_3^2 + x_3 + 2}{x_1 x_2 x_3} \right.$$

$$\left. - \left(1 + \frac{1}{x_3}\right)\left(\frac{1}{x_1} + \frac{1}{x_2}\right) - 2\frac{3+\varepsilon}{1-\varepsilon} \frac{(1-x_1)(1-x_2)}{x_1 x_2 x_3} + \text{cycl. perm.} \right]$$

$$\times \left[(1-x_1)(1-x_2)(1-x_3)\right]^{-\varepsilon} \delta(x_1 + x_2 + x_3 - 2) \quad . \qquad \text{(A.9)}$$

The contribution of (c) + (d) is as follows,

$$\frac{d\sigma^{(c)+(d)}}{dx_1 dx_2 dx_3} = \sigma_B(\varepsilon) G(\varepsilon) C_A \frac{\sin\pi\varepsilon}{6\pi\varepsilon} \left[2\frac{(1-x_1)(1-x_2)}{x_3^2\,(1-x_3)} + \frac{2x_1-x_2}{(1-x_1)(1-x_3)} \right.$$

$$\left. - \frac{2(2-x_2)}{x_1 x_3} + 8\,\frac{1}{1-\varepsilon}\,\frac{1-x_2}{x_1 x_3} + \text{cycl. perm.} \right]$$

$$\times \left[(1-x_1)(1-x_2)(1-x_3)\right]^{-\varepsilon}\, \delta(x_1 + x_2 + x_3 - 2) \quad . \tag{A.10}$$

It should be noted that the factor $\left[(1-x_1)(1-x_2)(1-x_3)\right]^{-\varepsilon}$ appears according to the three massless-particle phase space in D-dimensions.

III. $q\bar{q}G$ FINAL STATE

The contribution of Fig. 2d may be calculated in the same way as in the three-gluon case. The result is

$$\frac{d\sigma^{q\bar{q}G}}{dx_1 dx_2 dx_3} = \sigma_B(\varepsilon) G(\varepsilon) T_R (N_f-1) \left[\frac{1}{1-x_3} - 2(1+\varepsilon)\, \frac{(1-x_1)(1-x_2)}{x_3^2\,(1-x_3)} \right]$$

$$\times \left[1-x_1)(1-x_2)(1-x_3)\right]^{-\varepsilon}\, \delta(x_1 + x_2 + x_3 - 2) \quad , \tag{A.11}$$

where x_3 refers to the gluon and $x_1(x_2)$ refers to the quark (antiquark).

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FIGURE CAPTIONS

Fig. 1. Relevant QCD diagrams for heavy quarkonium decays. Wavy, broken and solid lines represent gluon, FP-ghost and quark, respectively. The FP-ghost has to be taken into account in the final state in order to maintain the gauge invariance unless we use the physical gauge.

Fig. 2. The QCD diagrams for heavy quarkonium decays up to order α_s^3.

Fig. 3. The thrust distribution of jets from paraquarkonia with $\alpha_s = 0.19$, $N_f = 4$ and 3 colors. Solid line: total [Eq. (21)]; Dotted line (broken line): contribution from $3G(q\bar{q}G)$ final state.

Fig. 4. The thrust distribution of jets from paraquarkonia (solid line) compared with that of jets from orthoquarkonia (dotted line) and in $e^+e^- \to q\bar{q}G$ (broken line).

Fig. 5. The three-gluon jets together with the FP-ghost contribution.

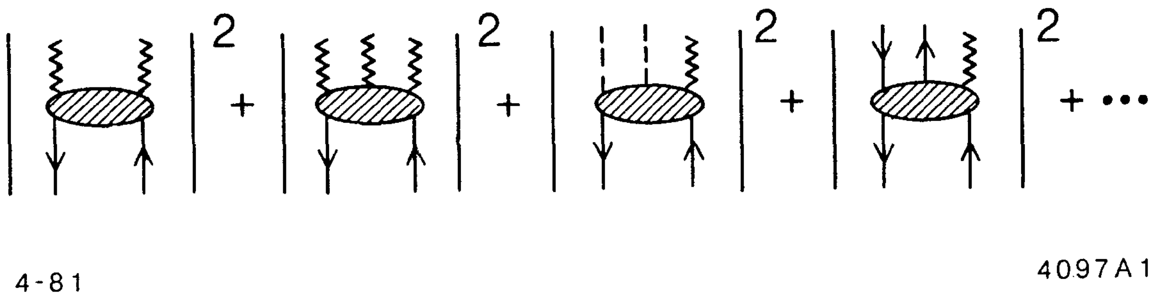

Fig. 1

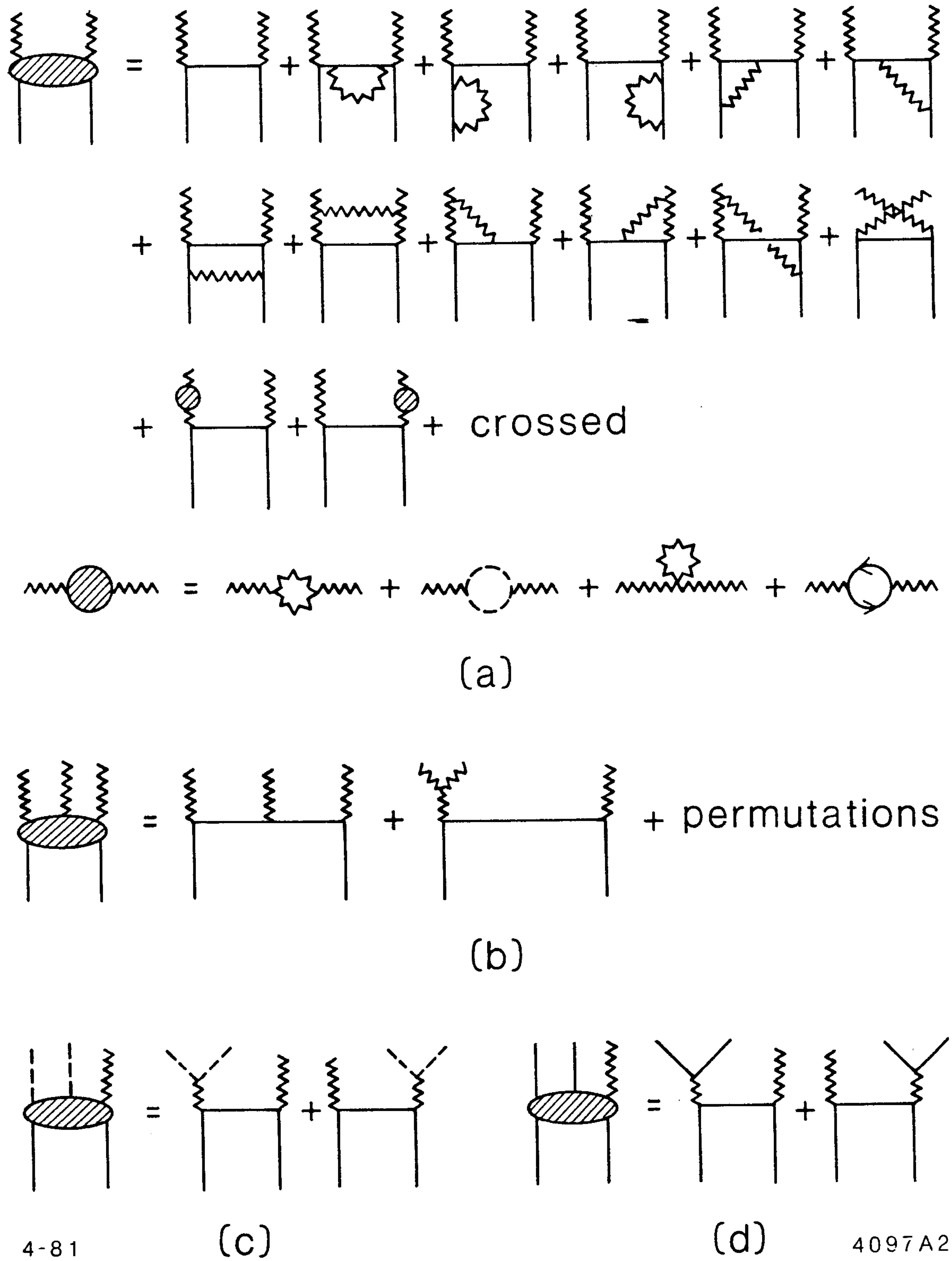

Fig. 2

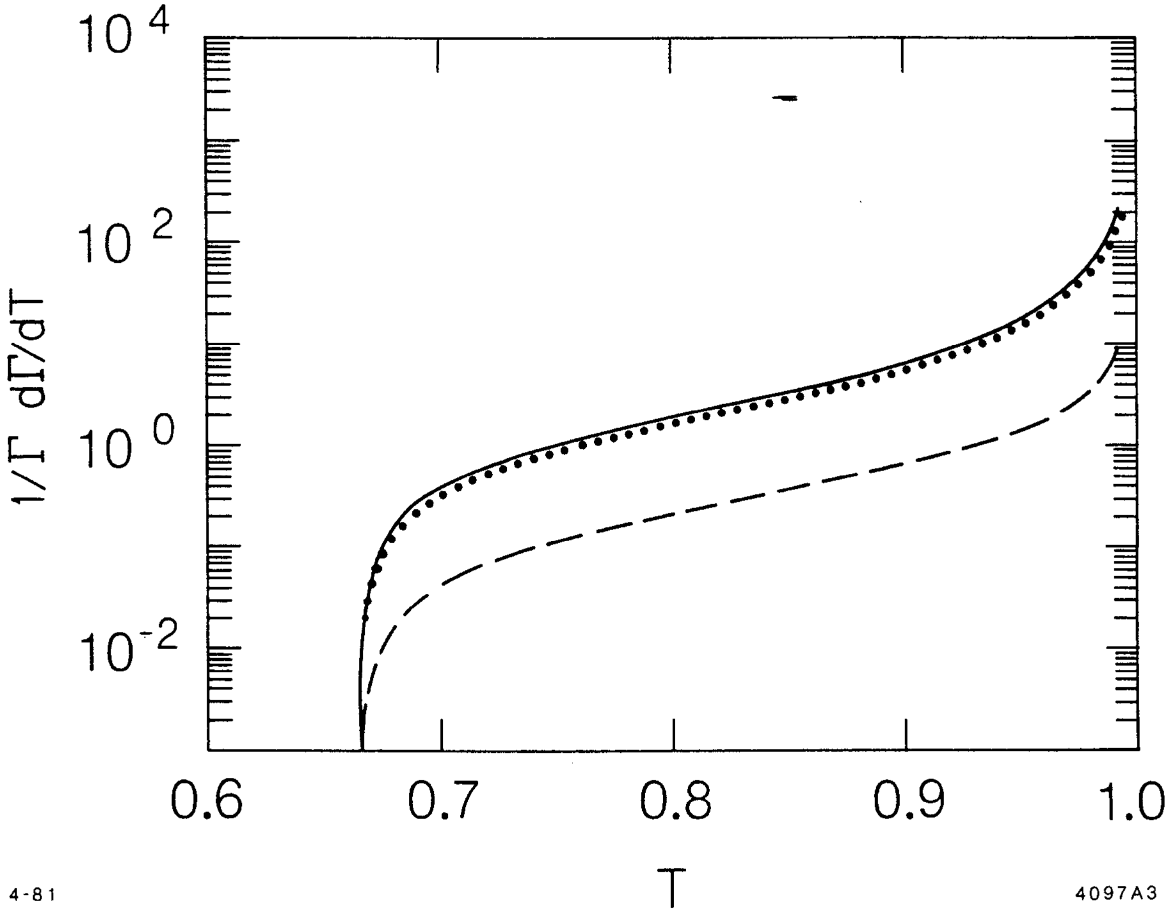

Fig. 3

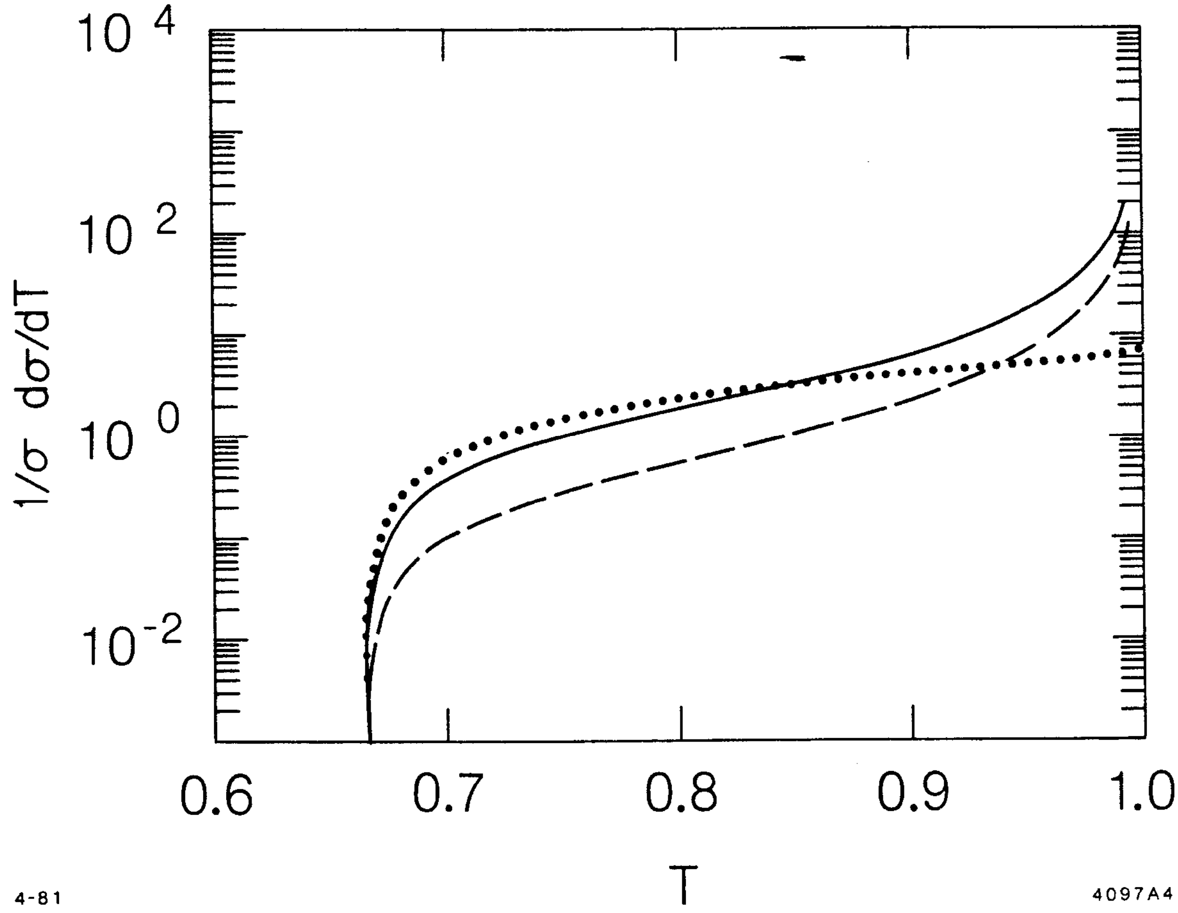

Fig. 4

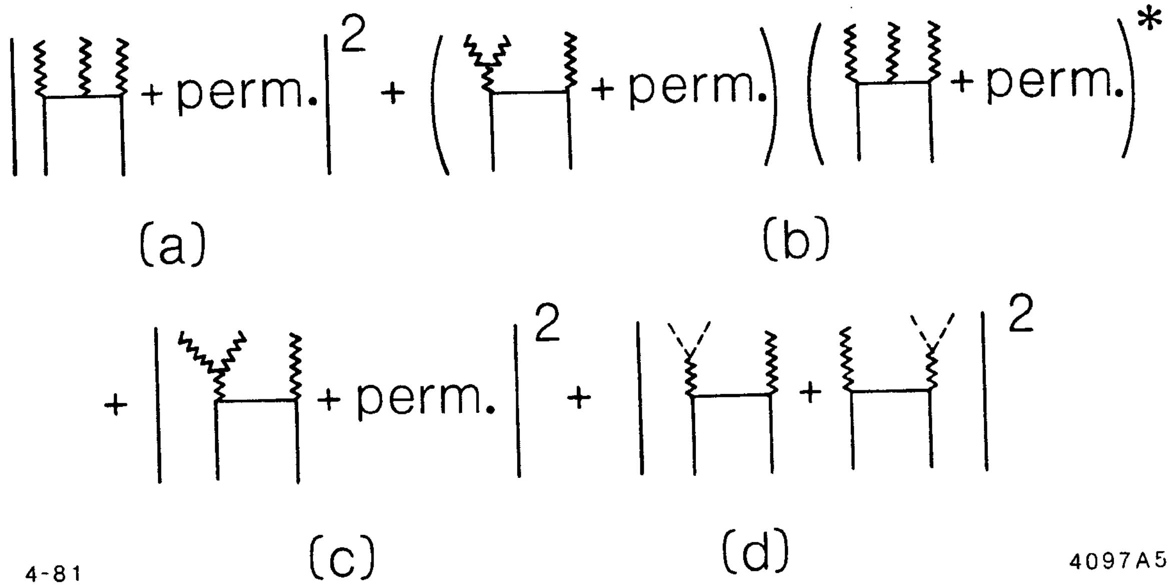

Fig. 5